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BIOMETRIKA

A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS

FOUNDED BY

W. F. R. WELDON, FRANCIS GALTON AND KARL PEARSON

EDITED BY

KARL PEARSON

ASSISTED BY

EGON S. PEARSON

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DEPARTMENT OF APPLIED STATISTICS

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A JOURNAL FOR THE SCIENTIFIC STUDY OF RACIAL PROBLEMS

Edited by KARL PEARSON, assisted by ETHEL M. ELDERTON

I have no Faith in anything short of actual Measurement and the Rule of Three.—CHARLES DARWIN

ISSUED BY THE GALTON LABORATORY FOR NATIONAL EUGENICS, UNIVERSITY OF LONDON, AND PRINTED AT THE UNIVERSITY PRESS, CAMBRIDGE

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VOLUME XXI

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With this volume—the twenty-first—*Biometrika* reaches its majority, but it is the twenty-ninth year of its life. A history of its inception and early struggles by the Editor of this Journal will be found in the forthcoming third volume of *The Life, Letters, and Labours of Francis Galton.*

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1749—1827.

"This delicate calculus applies to the most important questions in life, which are for the most part only problems in probability." *Théorie analytique des Probabilités.*

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VOLUME XXI DECEMBER, 1929

BIOMETRIKA

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In a recent paper* Harold Hotelling has obtained the frequency curve for the distribution of η^2, the square of the correlation ratio, subject to the conditions:

(*a*) Independence of the variates in the sampled population.

(*b*) An "indefinitely large" sampled population.

(*c*) Normal distribution of the two variates.

This frequency curve is:

$$z = \frac{\Gamma(\frac{1}{2}(N-1))}{\Gamma(\frac{1}{2}(n-1))\,\Gamma(\frac{1}{2}(N-n))}(\eta^2)^{\frac{n-3}{2}}(1-\eta^2)^{\frac{N-n-2}{2}} \quad \text{...........(i)},$$

where N is the size of the sample, and n the number of arrays on which η^2 is based. Conditions (*a*)—(*c*) of course very much limit the field of application of the above formula, the more so as η^2 is generally used to investigate dependence, when the distribution is not certainly normal. Still there appears to be a number of cases, in which the primary variable is given in broad categories, but the secondary variable in quantitative measure, where the above result may be of considerable service, and we know so little about the distribution of η^2, that any contribution to our knowledge is of value and likely to be suggestive. We have accordingly written down some of the values which flow from the above result and are likely to be useful to the practical statistician.

Modal value of η^2, or $\check{\eta}^2$, $\quad = \dfrac{n-3}{N-5}$.

Mean value of η^2, or $\bar{\eta}^2$, $\quad = \dfrac{n-1}{N-1}$ †.

$$_{\eta^2}\mu_2 = \sigma_{\eta^2}^2 = \frac{2}{N+1}\,\frac{n-1}{N-1}\left(1-\frac{n-1}{N-1}\right) = \frac{2\bar{\eta}^2(1-\bar{\eta}^2)}{N+1}$$

* *Proceedings of the National Academy of Sciences*, Vol. II. pp. 657—662, Washington, 1925. The same form was reached by R. A. Fisher three years earlier (*Journal of the Royal Statistical Society*, Vol. LXXXV. p. 605, 1922). Fisher's proof may be more general than Hotelling's, although we must confess to finding it harder to follow. Hotelling assumes his variates have a normal distribution; Fisher that the arrays for which η^2 is computed are normally distributed. Both deal only with the case in which the variates are in the sampled population uncorrelated.

† The value in customary use is $\bar{\eta}^2 = \dfrac{n-1}{N}$, which is, of course, practically identical with the above for reasonably large samples.

hence $$\sigma_{\eta^2} = \frac{1}{\sqrt{N+1}} \sqrt{2\bar{\eta}^2 (1-\bar{\eta}^2)}.$$

Again: $${}_{\eta^2}\mu_3 = 8\frac{n-1}{N-1}\left(1-\frac{n-1}{N-1}\right)\left(1-2\frac{n-1}{N-1}\right)\frac{1}{(N+1)(N+3)}$$
$$= \frac{8\bar{\eta}^2(1-\bar{\eta}^2)(1-2\bar{\eta}^2)}{(N+1)(N+3)}.$$

Further: $${}_{\eta^2}\mu_4 = 12\frac{n-1}{N-1}\left(1-\frac{n-1}{N-1}\right)$$
$$\times\left\{\frac{n-1}{N-1}\left(1-\frac{n-1}{N-1}\right)(N+3)+4\left(1-2\frac{n-1}{N-1}\right)^2\right\}\frac{1}{(N+1)(N+3)(N+4)}$$
$$= \frac{12\bar{\eta}^2(1-\bar{\eta}^2)}{(N+1)(N+3)(N+5)}\{\bar{\eta}^2(1-\bar{\eta}^2)(N+3)+4(1-2\bar{\eta}^2)^2\}.$$

Thus all the moment coefficients can be expressed in terms of $\bar{\eta}^2$. From these results flow:

$$\beta_1 = 8\frac{N+1}{N+3}\frac{1}{N+3}\frac{(1-2\bar{\eta}^2)^2}{\bar{\eta}^2(1-\bar{\eta}^2)},$$
$$\beta_2 = 3\frac{N+1}{N+5}\left(1+\frac{4(1-2\bar{\eta}^2)^2}{(N+3)\bar{\eta}^2(1-\bar{\eta}^2)}\right),$$

or, $$\beta_2 = 3\frac{N+1}{N+5}+\frac{3}{2}\frac{N+3}{N+5}\beta_1.$$

The latter equation shows that for N very large

$$2\beta_2 = 6 + 3\beta_1,$$

or, the line for β_1, β_2 always lies above, but approaches, as N increases, the line for curves of Type III. Clearly, whatever be the value of n, the β's lie on a straight line nearly passing through the Normal Point and of slope less than 1·5. The order of β_1 is best seen from the equation:

$$\beta_1 = 8\frac{N+1}{N+3}\cdot\frac{N-1}{N+3}\frac{(1-2\bar{\eta}^2)^2}{1-\bar{\eta}^2}\frac{1}{n-1},$$

or, as N grows large while n remains finite, this approaches the limit, $\beta_1 = \frac{8}{n-1}$. For the usual range of values of n, β_1 will be of the order 2·0 to 0·5, i.e. fairly considerable. Accordingly β_2 will differ somewhat widely from 3. Thus, for all samples, the curve of distribution will be a leptokurtic curve differing very sensibly from symmetry, owing to the (in practice) limited number of arrays.

(2) The above values for η^2 must be clearly distinguished from those for η. The frequency curve for η is:

$$z = \frac{2\,\Gamma(\frac{1}{2}(N-1))}{\Gamma(\frac{1}{2}(n-1))\,\Gamma(\frac{1}{2}(N-n))}\eta^{n-2}(1-\eta^2)^{\frac{1}{2}(N-n-2)} \quad \text{...........(ii).}$$

Modal value of η, or $\tilde{\eta}$,

$$\sqrt{\frac{n-2}{N-4}}.$$

Mean value of η, or $\bar{\eta}$,

$$=\frac{\Gamma(\frac{1}{2}(N-1))}{\Gamma(\frac{1}{2}N)}\frac{\Gamma(\frac{1}{2}n)}{\Gamma(\frac{1}{2}(n-1))}.$$

$${}_\eta\mu_2=\sigma_\eta^2=\frac{n-1}{N-1}-(\bar{\eta})^2,$$

$${}_\eta\mu_3=\bar{\eta}\left(\frac{N-n}{N(N-1)}-2\sigma_\eta^2\right)=2\bar{\eta}\left\{\frac{N-n}{N-1}\left(1+\frac{1}{2N}\right)-(1-(\bar{\eta})^2)\right\},$$

$${}_\eta\mu_4=\frac{n-1}{N-1}\frac{n+1}{N+1}+2\left(\left(1+\frac{2}{N}\right)\frac{n-1}{N-1}-\frac{2}{N}\right)(\bar{\eta})^2-3(\bar{\eta})^4.$$

From these equations β_1 and β_2 can be found and hence the distribution of η appreciated in the usual way. The process, however, appears to have no advantage over the discussion of the distribution of η^2. It is of interest, however, to note what error results from supposing $\bar{\eta}=\sqrt{\bar{\eta^2}}$ and $\bar{\eta^2}=(\bar{\eta})^2$. It is worth noting that $\sigma_\eta=\sqrt{\bar{\eta^2}-(\bar{\eta})^2}$.

Table of $\bar{\eta}$ and $\bar{\eta^2}$ as compared with $\sqrt{\bar{\eta^2}}$ and $(\bar{\eta})^2$.

Size of Population	Number of Arrays					
	$n=5$		$n=10$		$n=20$	
50	$\bar{\eta}=\cdot2699 \pm\cdot0936$ $\sqrt{\bar{\eta^2}}=\cdot2857$	$(\bar{\eta})^2=\cdot0729$ $\bar{\eta^2}=\cdot0816$	$\bar{\eta}=\cdot4190 \pm\cdot0901$ $\sqrt{\bar{\eta^2}}=\cdot4286$	$(\bar{\eta})^2=\cdot1756$ $\bar{\eta^2}=\cdot1837$	$\bar{\eta}=\cdot6177 \pm\cdot0787$ $\sqrt{\bar{\eta^2}}=\cdot6227$	$(\bar{\eta})^2=\cdot3816$ $\bar{\eta^2}=\cdot3878$
100	$\bar{\eta}=\cdot1894 \pm\cdot0673$ $\sqrt{\bar{\eta^2}}=\cdot2010$	$(\bar{\eta})^2=\cdot0359$ $\bar{\eta^2}=\cdot0404$	$\bar{\eta}=\cdot2940 \pm\cdot0668$ $\sqrt{\bar{\eta^2}}=\cdot3015^+$	$(\bar{\eta})^2=\cdot0864$ $\bar{\eta^2}=\cdot0909$	$\bar{\eta}=\cdot4335 \pm\cdot0635$ $\sqrt{\bar{\eta^2}}=\cdot4381$	$(\bar{\eta})^2=\cdot1879$ $\bar{\eta^2}=\cdot1919$
500	$\bar{\eta}=\cdot0842 \pm\cdot0304$ $\sqrt{\bar{\eta^2}}=\cdot0895^+$	$(\bar{\eta})^2=\cdot0071$ $\bar{\eta^2}=\cdot0080$	$\bar{\eta}=\cdot1307 \pm\cdot0309$ $\sqrt{\bar{\eta^2}}=\cdot1343$	$(\bar{\eta})^2=\cdot0171$ $\bar{\eta^2}=\cdot0180$	$\bar{\eta}=\cdot1927 \pm\cdot0308$ $\sqrt{\bar{\eta^2}}=\cdot1951$	$(\bar{\eta})^2=\cdot0371$ $\bar{\eta^2}=\cdot0381$
1000	$\bar{\eta}=\cdot0595^- \pm\cdot0216$ $\sqrt{\bar{\eta^2}}=\cdot0633$	$(\bar{\eta})^2=\cdot0035^+$ $\bar{\eta^2}=\cdot0040$	$\bar{\eta}=\cdot0923 \pm\cdot0219$ $\sqrt{\bar{\eta^2}}=\cdot0949$	$(\bar{\eta})^2=\cdot0085^+$ $\bar{\eta^2}=\cdot0090$	$\bar{\eta}=\cdot1361 \pm\cdot0221$ $\sqrt{\bar{\eta^2}}=\cdot1379$	$(\bar{\eta})^2=\cdot0185^+$ $\bar{\eta^2}=\cdot0190$

Thus for $N=50$, the difference ranges from ·005 to ·016 for $\bar{\eta}$ and $\sqrt{\bar{\eta^2}}$, and from ·006 to ·009 for $(\bar{\eta})^2$ and $\bar{\eta^2}$. As there is less range in the variation of $(\bar{\eta})^2$ and $\bar{\eta^2}$, and as it is η^2 which arises first in our calculations, the tables which accompany this introduction are adapted to η^2.

(3) The conception in the tables is very simple. Starting from Equation (i) we measure the ratio of the area of the curve beyond $\eta^2 = \bar{\eta}^2 + \lambda\sigma_{\bar{\eta}}{}^2$ to the area of the whole curve; this is the "probability integral" of the frequency distribution of η^2. Actually λ is given two values which will give P approximately the values ·01 and ·02. To make P *exactly* ·01 and ·02 would have involved five or six times the amount of calculation. Actually the values selected for λ bring P sufficiently near ·01 and ·02 to allow us to determine whether η^2 may be considered to differ significantly from $\bar{\eta}^2$. If P lies above ·02, i.e. 1 in 50, then we cannot emphasise the difference of η^2 and $\bar{\eta}^2$ as probably having significance; if P is below ·01, then we usually claim significance for η^2. Between these values differences may be of a doubtful character, and really indicate that our sample is hardly large enough to admit of a definite judgment of significance.

It will be noted that

$$\lambda = \frac{\text{Observed } \eta^2 - \text{Mean } \bar{\eta}^2}{\text{Standard Deviation of } \eta^2},$$

a function familiar enough in the case of the probability integral of the normal curve. In that case $\lambda = 2{\cdot}33$ for $P = {\cdot}01$, and $= 2{\cdot}05$ for $P = {\cdot}02$. A comparison of the values for λ in the accompanying tables indicates how far the distribution of η^2 on the side towards unity differs from a normal distribution; the divergence when n and N are not small is not very great.

We have no need to consider deviations of η^2 from $\bar{\eta}^2$ when λ is negative. For in such cases η^2 is less than the value $\bar{\eta}^2$ which is the mean value of η^2 for no correlation, and we cannot therefore predict any significance for it on the basis of our size of sample.

The tables give in the first column $\bar{\eta}^2$, and in the second σ_{η^2}, both on the assumption of zero association and of sampling from a normal population. The third and fourth columns give two values of λ corresponding to two values of P approaching respectively ·01 and ·02. It was not possible to deduce the whole table from the Tables of the Incomplete Beta Function, because the latter are confined to the range of $B(p, q)$, in which p and q are both 50 or less. The Incomplete Beta Function Tables were accordingly only used for checking some of the lower values. Weddle's quadrature formula was also used for checking. The values of P for given values of λ were obtained for odd values of n by means of the formula:

$$\frac{\int_z^1 y\,dx}{\int_0^1 y\,dx}$$

$$= \frac{(1-z)^{\frac{1}{2}(N-2s-1)}\left\{1 - \frac{s-1}{1!}\frac{N-2s-1}{N-2s+1}(1-z) + \frac{N(s-1)(s-2)}{2!}\frac{N-2s-1}{N-2s+3}(1-z)^2 - \frac{(s-1)(s-2)(s-3)}{3!}\frac{N-2s-1}{N-2s+5}(1-z)^3 + \dots\right\}}{1 - \frac{s-1}{1!}\frac{N-2s-1}{N-2s+1} + \frac{(s-1)(s-2)}{2!}\frac{N-2s-1}{N-2s+3} - \frac{(s-1)(s-2)(s-3)}{3!}\frac{N-2s-1}{N-2s+5} + \dots}$$

where $n = 2s+1$. For n odd both series are finite and as n was not taken greater than 21, s did not exceed ten, nor the number of terms in numerator and denominator exceed ten each. The coefficients depend only on N and s, thus the powers $(1-z)^t$ could be relatively easily modified until a suitable value of P was found. No attempt was made to reach exact values for P. The values given to N were fairly close together until $N=100$, when an argument change of 50 was found adequate for graphical interpolation. This interpolation was carried out by Miss Ida McLearn.

An appropriate system of λ's and P's having thus been determined for all values of N from 50 to 1000, and for all odd values of n from 3 to 21, it was needful to determine the corresponding values of λ for the even values of n. This task of rather troublesome graphical interpolation was kindly undertaken by Dr E. S. Pearson. The values of P and λ are only tabled to three and two decimals respectively. This is as much as the processes adopted justify, but such approximate values of P and λ are adequate for the end in view, i.e. to determine whether η^2 differs significantly from $\bar{\eta}^2$, the mean value of η^2 when there is no association.

In the following tables $\bar{\eta}^2$ denotes the mean value of η^2 when there is no correlation, i.e. it is not $(\bar{\eta})^2$, but printed briefly for $(\overline{\eta^2})$, and in order to indicate that σ_{η^2} is not the general standard deviation of η^2, whether or no there be correlation, but only when there is no association between the variables, we have printed σ_{η^2} for no association in the population sampled as $\sigma_{\bar{\eta}^2}$, thus linking it up with $\bar{\eta}^2$, which is now in pretty wide use—not for the mean of any η^2— but for the mean when we suppose no association.

(4) *Illustrations.* Throughout we shall use the observed value of η^2 uncorrected for number of arrays.

(i) *Influence of Crowding on General Astigmatism.* We require the correlation ratio η of general astigmatism on crowding, i.e. number of persons per room, $\eta_{A.p}$. Our observations are on 716 schoolboys*, and we have for eight arrays:

$$\eta^2_{A.p} = \cdot 022{,}821, \quad \bar{\eta}^2_{A.p} = \cdot 009{,}790, \quad \sigma_{\bar{\eta}^2} = \cdot 005{,}200,$$
$$\lambda_d = (\eta^2_{A.p} - \bar{\eta}^2_{A.p})/\sigma_{\bar{\eta}^2} = 2\cdot 51.$$

This value of λ_d for the data is a trifle lower than the value of $\lambda_2 = 2\cdot 54$, and accordingly by rough extrapolation it would be about once in 45 or 46 trials that such a value of $\eta^2_{A.p}$ would occur, if there were no association between general astigmatism and crowding. It is possible that there may be some association but no great stress could be laid on the result.

(ii) *Influence of Familial Income on Corneal Astigmatism.* We wish to consider the influence of poverty or its reverse on corneal astigmatism. Our data †

* *Annals of Eugenics*, Vol. III. p. 29, *et seq.*
† *Ibid.* p. 46.

consist of 228 boys arranged in nine arrays, if we include the "comfortable group" as one array. We have

$$\eta^2_{CA.I} = \cdot 139{,}397, \quad \bar{\eta}^2_{CA.I} = \cdot 035{,}242, \quad \sigma_{\bar{\eta}}^2 = \cdot 017{,}232,$$
$$\lambda_d = (\eta^2_{CA.I} - \bar{\eta}^2_{CA.I})/\sigma_{\bar{\eta}^2} = 6\cdot 04.$$

This value for the data is much above λ_1 and the odds against η^2 being a result of random sampling are very much greater than 99 to 1.

(iii) *Hours of Homework and Age.* Here we have data for 322 boys*, and find for eight arrays:

$$\eta^2_{H.A} = \cdot 027{,}433, \quad \bar{\eta}^2_{H.A} = \cdot 021{,}807, \quad \sigma_{\bar{\eta}}^2 = \cdot 011{,}493, \quad \lambda_d = 0\cdot 49.$$

λ_d is accordingly *much* below $\lambda_2 (= 2\cdot 54)$, and it does not appear that older boys work longer hours.

(iv) *Distance of Nearpoint and Colour of Iris.* The data are for 770 boys in seven arrays of eye-colour determined by Martin's scale†. We have

$$\eta^2_{NP.EC} = \cdot 021{,}727, \quad \bar{\eta}^2_{NP.EC} = \cdot 007{,}802, \quad \sigma_{\bar{\eta}}^2 = \cdot 004{,}481,$$

and accordingly

$$\lambda_d = \frac{\cdot 021{,}727 - \cdot 007{,}802}{\cdot 004{,}481} = 3\cdot 11,$$

and therefore by rough extrapolation P = about ·009, or the odds are about 111 to 1 against η^2 for these data having arisen from a population in which distance of nearpoint and eye-colour are unassociated.

(v) *Mental Capacity and Place in Class.* 249 boys arranged in four classes, Excellent, Good, Moderate, Dull. The data gave†:

$$\eta^2_{P.I} = \cdot 525{,}045, \quad \bar{\eta}^2_{P.I} = \cdot 012{,}097. \quad \sigma_{\bar{\eta}}^2 = \cdot 009{,}778,$$
$$\lambda_d = (\cdot 525{,}045 - \cdot 012{,}097)/\cdot 009{,}778 = 5\cdot 25.$$

This value of λ_d is so far in excess of λ_1 that we have no hesitation in asserting significance. The reader will see that while we can on the basis of our tables predict significance, it would need a far larger series of values for P and λ for each value of n and N to obtain even a rough measure of the actual degree of significance. The labour of calculating such tables would be excessive, and when calculated the cost of printing would render publication impossible. It is for these reasons that we have limited our values of P to two in the neighbourhood of those where most practical statisticians would consider significance to be assured.

In this particular case, although our data are sparse and the arrays few, we may certainly assert that the uncorrected correlation ratio of ·7246 is very definitely significant.

* *Annals of Eugenics*, Vol. III. p. 75.

† *Biometrika*, Vol. VIII. p. 544.

(vi) *Intelligence Test Marks and Teacher's Estimate of Intelligence**. The correlation ratio for a table containing 63 girls, divided into four arrays, gave:

$$\eta^2_{T.M:E.I} = \cdot 335{,}825, \quad \bar{\eta}^2_{T.M:E.I} = \cdot 048{,}387, \quad \sigma_{\bar{\eta}^2} = \cdot 037{,}933.$$

Hence $\lambda_d = 7\cdot 58$, which is far above $\lambda_1 = 3\cdot 20$. The chance that the association is merely due to random sampling from unassociated material is extremely slight.

Dr Isserlis gives another table (Table I) for the correlation between School Examination Marks and Intelligence Test Marks. This is based on only 50 girls, and falls outside the present tables, yet the constancy of λ and P for a given number of arrays is so great that it is easy to test the significance. We have for five arrays:

$$\eta^2_{T.M:S.M} = \cdot 164{,}761, \quad \bar{\eta}^2_{T.M:S.M} = \cdot 081{,}633, \quad \sigma_{\bar{\eta}^2} = \cdot 054{,}221.$$

Hence $\lambda_d = 1\cdot 53$. This is considerably below $\lambda_2 = 2\cdot 68$, or the odds against such a value of η arising from random sampling are very small. In other words, the data are too sparse for us to predict whether there is any relation between School Marks and Intelligence Test Marks.

(5) Fisher has given† for the case of no correlation and normal distribution of the variates the following frequency curve for the distribution of the square of the multiple correlation coefficient (R^2).

Probability that R^2 lies between R^2 and $R^2 + dR^2$

$$= \frac{\Gamma\left(\frac{1}{2}(N-1)\right)}{\Gamma\left(\frac{1}{2}(N-n-1)\right)\Gamma\left(\frac{1}{2}n\right)}(R^2)^{\frac{1}{2}(n-2)}(1-R^2)^{\frac{1}{2}(N-n-3)}\,dR^2 \quad \ldots\ldots\text{(iii)}.$$

Accordingly, if we write in the Equation (i) to the frequency curve for η^2 above, $n+1$ for n, we obtain a curve identical with that for R^2, or the tables for the determination of the significance of η^2 can be used for the significance of R^2 provided we enter these tables with n taken equal to the number of the variates $x_1, x_2 \ldots x_n$ on which R is based plus unity, i.e. if x_0 be the variate, which we are multiply correlating, we must enter the tables with the total number, $n+1$, of variates concerned, $x_0, x_1, x_2 \ldots x_n$. We thus obtain the mean value of R^2 by the corresponding $\bar{\eta}^2$, and the standard deviation, σ_{R^2}, of R^2 by the corresponding value of $\sigma_{\bar{\eta}^2}$.

Illustration (vii). *Rainfall in relation to Longitude, Latitude and Altitude.* The data referred to 57 recording stations in Hertfordshire, and Fisher‡ found $R^2 = \cdot 4431$. Here $N = 57$, $n = 4$ (longitude, latitude, altitude, *and rainfall*) and thus our tables give:

$$\bar{R}^2 = \cdot 0536, \quad \sigma_{\bar{R}^2} = \cdot 0418.$$

It follows that: $\lambda_d = (\cdot 4431 - \cdot 0536)/\cdot 0418 = 9\cdot 32$. This is nearly three times as

* "The Relation between Home Conditions and the Intelligence of School Children." By L. Isserlis, D.Sc., Medical Research Council, Special Report, Series 74.

† *Phil. Trans.* Vol. 218, B. p. 91, 1924.

‡ *Statistical methods for Research Workers*, pp. 185 and 228.

great as λ_1 for which the chance is about 1 in 100. Hence the multiple correlation found is certainly significant.

Readers of this paper and users of the present tables are once again warned of their limitations. The theory on which they are based depends upon our sampling being made from an indefinitely large *normal* population; the argument is based, in the case of both η^2 and R^2, on the improbability of the observed result, *supposing the variates are uncorrelated.* The tables tell us the probability of association, but if we conclude that the variates are correlated, they really throw no light on the closeness with which our sample value of the association probably approaches the actual association in the sampled population. The distribution of η^2, even for a normal surface, when correlation does exist would be of undoubted service; but we cannot overlook the point that the chief value of the correlation ratio arises in cases where we have at least grave doubts as to the nature of the regression being linear.

The illustrations provided should suffice to indicate the method of using the tables and their value in saving labour.

TABLES FOR ASCERTAINING THE SIGNIFICANCE OR NON-SIGNIFICANCE OF ASSOCIATION MEASURED BY THE CORRELATION RATIO

n = number of arrays

N = size of sample	$n=3$: $\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1, $\lambda_1=3\cdot50$	P_2, $\lambda_2=2\cdot94$	$n=4$: $\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1, $\lambda_1=3\cdot20$	P_2, $\lambda_2=2\cdot80$	$n=5$: $\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1, $\lambda_1=3\cdot14$	P_2, $\lambda_2=2\cdot68$
51	·040000	·038431	·010	·019	·060000	·046575 −	·011	·018	·080000	·053205 −	·010	·019
52	·039216	·037707	·010	·019	·058824	·045709	·011	·018	·078431	·052226	·010	·019
53	·038462	·037010	·010	·019	·057692	·044873	·011	·018	·076923	·051282	·010	·019
54	·037736	·036338	·010	·019	·056604	·044067	·011	·018	·075472	·050372	·010	·019
55	·037037	·035690	·010	·019	·055556	·043288	·011	·018	·074074	·049493	·010	·019
56	·036364	·035065 −	·010	·019	·054545 +	·042538	·011	·018$_5$	·072727	·048644	·010	·019
57	·035714	·034461	·010	·019	·053571	·041813	·011	·019	·071429	·047824	·010	·019
58	·035088	·033877	·010	·019	·052632	·041112	·011	·019	·070175 +	·047031	·010	·019
59	·034483	·033314	·010	·019	·051724	·040435 −	·011	·019	·068966	·046263	·010	·019
60	·033898	·032768	·010	·019	·050847	·039779	·011	·019	·067797	·045521	·010	·019
61	·033333	·032240	·010	·019	·050000	·039144	·011	·019	·066667	·044801	·010	·019
62	·032787	·031729	·010	·019	·049180	·038529	·011	·019	·065574	·044104	·010	·019
63	·032258	·031234	·010	·019	·048387	·037933	·011	·019	·064516	·043429	·010	·019
64	·031746	·030754	·010	·019	·047619	·037355 +	·011	·019	·063492	·042773	·010	·019
65	·031250	·030288	·010	·019	·046875	·036794	·011	·019	·062500	·042137	·010	·019
66	·030769	·029837	·010	·019	·046154	·036251	·011	·019	·061538	·041520	·010	·019
67	·030303	·029398	·010	·019	·045455 −	·035723	·011	·019	·060606	·040921	·010	·019
68	·029851	·028973	·010	·019	·044776	·035210	·011	·019	·059701	·040338	·010	·019
69	·029412	·028559	·010	·019	·044118	·034712	·011$_5$	·019	·058824	·039772	·010	·019
70	·028986	·028157	·010	·019	·043478	·034227	·012	·019	·057971	·039221	·010	·019
71	·028571	·027766	·010	·019	·042857	·033756	·012	·019	·057143	·038686	·010	·019
72	·028169	·027386	·010	·019	·042254	·033297	·012	·019	·056338	·038165 −	·010	·019
73	·027778	·027017	·010	·019	·041667	·032851	·012	·019	·055556	·037657	·010	·019
74	·027397	·026657	·010	·019	·041096	·032417	·012	·019	·054795 −	·037163	·010	·019
75	·027027	·026306	·010	·019	·040541	·031994	·012	·019	·054054	·036682	·010	·019
76	·026667	·025965 −	·010	·019	·040000	·031582	·012	·019	·053333	·036213	·010$_5$	·019
77	·026316	·025632	·010	·019	·039474	·031180	·012	·019	·052632	·035756	·011	·019
78	·025974	·025308	·010	·019	·038961	·030788	·012	·019	·051948	·035310	·011	·019
79	·025641	·024992	·010	·019	·038462	·030407	·012	·019	·051282	·034876	·011	·019$_5$
80	·025316	·024683	·010	·019	·037975 −	·030034	·012	·019	·050633	·034451	·011	·020
81	·025000	·024383	·010	·019	·037500	·029670	·012	·019	·050000	·034037	·011	·020
82	·024691	·024089	·010	·019	·037037	·029316	·012	·019	·049383	·033633	·011	·020
83	·024390	·023802	·010	·019	·036585 +	·028969	·012	·019	·048780	·033238	·011	·020
84	·024096	·023523	·010	·019	·036145 −	·028631	·012	·019	·048193	·032853	·011	·020
85	·023810	·023249	·010	·019	·035714	·028300	·012	·019	·047619	·032476	·011	·020
86	·023529	·022982	·010	·019	·035294	·027977	·012	·019	·047059	·032108	·011	·020
87	·023256	·022721	·010$_5$	·019	·034884	·027661	·012	·019	·046512	·031748	·011	·020
88	·022989	·022466	·011	·019	·034483	·027353	·012	·019	·045977	·031396	·011	·020
89	·022727	·022216	·011	·019	·034091	·027051	·012	·019	·045455 −	·031051	·011	·020
90	·022472	·021972	·011	·019	·033708	·026755 +	·012	·019	·044944	·030715 −	·011	·020
91	·022222	·021734	·011	·019	·033333	·026467	·012	·019	·044444	·030385 −	·011	·020
92	·021978	·021500 +	·011	·019	·032967	·026184	·012	·019	·043956	·030062	·011	·020
93	·021739	·021272	·011	·019	·032609	·025907	·012	·019	·043478	·029746	·011	·020
94	·021505 +	·021048	·011	·019	·032258	·025636	·012	·019	·043011	·029437	·011	·020
95	·021277	·020829	·011	·019	·031915 −	·025371	·012	·019	·042553	·029134	·011	·020
96	·021053	·020614	·011	·019	·031579	·025111	·012	·019	·042105 +	·028837	·011	·020
97	·020833	·020404	·011	·019	·031250	·024856	·012	·019	·041667	·028547	·011	·020
98	·020619	·020198	·011	·019	·030928	·024607	·012	·019	·041237	·028262	·011	·020
99	·020408	·019996	·011	·019	·030612	·024362	·012	·019	·040816	·027982	·011	·020
100	·020202	·019798	·011	·019	·030303	·024122	·012	·019	·040404	·027708	·011	·020

N = size of sample	$n=6$: $\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1, $\lambda_1=3\cdot06$	P_2, $\lambda_2=2\cdot60$	$n=7$: $\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1, $\lambda_1=2\cdot98$	P_2, $\lambda_2=2\cdot52$	$n=8$: $\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1, $\lambda_1=2\cdot92$	P_2, $\lambda_2=2\cdot48$
51	·100000	·058835 −	·010	·019	·120000	·063730	·009	·019	·140000	·068050 −	·009	·019
52	·098039	·057766	·010	·019	·117647	·062588	·009	·019$_5$	·137255 −	·066847	·009	·019
53	·096154	·056735	·010	·019	·115385 −	·061485 +	·009	·020	·134615 +	·065686	·009	·019
54	·094340	·055740	·010	·019	·113208	·060420	·009	·020	·132075 +	·064563	·009	·019
55	·092593	·054779	·010	·019	·111111	·059391	·009	·020	·129630	·063478	·009	·019
56	·090909	·053850 −	·010	·019	·109091	·058397	·009$_5$	·020	·127273	·062429	·009	·020
57	·089286	·052952	·010	·019	·107143	·057435 −	·010	·020	·125000	·061413	·009$_5$	·020
58	·087719	·052084	·010	·019	·105263	·056503	·010	·020	·122807	·060429	·010	·020
59	·086207	·051243	·010	·019	·103448	·055602	·010	·020	·120690	·059477	·010	·020
60	·084746	·050429	·010	·019	·101695 −	·054728	·010	·020	·118644	·058553	·010	·020
61	·083333	·049640	·010	·019$_5$	·100000	·053882	·010	·020	·116667	·057657	·010	·020
62	·081967	·048876	·010	·020	·098361	·053061	·010	·020	·114754	·056789	·010	·020
63	·080645 +	·048134	·010	·020	·096774	·052264	·010	·020	·112903	·055945 −	·010	·020
64	·079365 +	·047415 +	·010	·020	·095238	·051491	·010	·020	·111111	·055127	·010	·020
65	·078125	·046717	·010	·020	·093750	·050740	·010	·020	·109375	·054331	·010	·020
66	·076923	·046039	·010	·020	·092308	·050011	·010	·020	·107692	·053558	·010	·020
67	·075758	·045380	·010	·020	·090909	·049302	·010	·020	·106061	·052807	·010	·020
68	·074627	·044740	·010	·020	·089552	·048613	·010	·020	·104478	·052076	·010	·020
69	·073529	·044118	·010	·020	·088235 +	·047943	·010	·020	·102941	·051365 +	·010	·020
70	·072464	·043512	·010	·020	·086957	·047291	·010	·020	·101449	·050674	·010	·020
71	·071429	·042923	·010	·020	·085714	·046657	·010	·020	·100000	·050000	·010	·020
72	·070423	·042350 −	·010	·020	·084507	·046039	·010	·020	·098592	·049344	·010	·020
73	·069444	·041792	·010	·020	·083333	·045437	·010	·020$_5$	·097222	·048705 −	·010	·020
74	·068493	·041248	·010	·020	·082192	·044851	·010	·021	·095890	·048082	·010	·020$_5$
75	·067568	·040718	·010	·020	·081081	·044280	·010	·021	·094595 −	·047475 −	·010	·021
76	·066667	·040202	·010	·020	·080000	·043723	·010	·021	·093333	·046883	·010	·021
77	·065789	·039698	·010	·020	·078947	·043180	·010	·021	·092105 +	·046305 −	·010	·021
78	·064935 +	·039207	·010	·020	·077922	·042650 −	·010	·021	·090909	·045741	·010	·021
79	·064103	·038728	·010	·020	·076923	·042133	·010	·021	·089744	·045191	·010	·021
80	·063291	·038260	·010	·020	·075949	·041628	·010	·021	·088608	·044654	·010	·021
81	·062500	·037804	·010	·020	·075000	·041135 −	·010	·021	·087500	·044129	·010	·021
82	·061728	·037358	·010$_5$	·020	·074074	·040653	·010	·021	·086420	·043617	·010	·021
83	·060976	·036923	·011	·020	·073171	·040183	·010	·021	·085366	·043116	·010	·021
84	·060241	·036497	·011	·020	·072289	·039724	·010	·021	·084337	·042627	·010	·021
85	·059524	·036082	·011	·020	·071429	·039274	·010	·021	·083333	·042148	·010	·021
86	·058824	·035675 +	·011	·020	·070588	·038835 +	·010$_5$	·021	·082353	·041680	·010$_5$	·021
87	·058140	·035278	·011	·020	·069767	·038406	·011	·021	·081395 +	·041223	·011	·021
88	·057471	·034889	·011	·020	·068966	·037986	·011	·021	·080460	·040775 +	·011	·021
89	·056818	·034509	·011	·020	·068182	·037575 −	·011	·021	·079545 +	·040337	·011	·021
90	·056180	·034137	·011	·020	·067416	·037172	·011	·021	·078652	·039908	·011	·021
91	·055556	·033773	·011	·020	·066667	·036778	·011	·021	·077778	·039488	·011	·021
92	·054945 +	·033417	·011	·020	·065934	·036393	·011	·021	·076923	·039077	·011	·021
93	·054348	·033068	·011	·020	·065217	·036015 +	·011	·021	·076087	·038674	·011	·021
94	·053763	·032726	·011	·020	·064516	·035646	·011	·021	·075269	·038280	·011	·021
95	·053191	·032392	·011	·020	·063830	·035283	·011	·021	·074468	·037893	·011	·021
96	·052632	·032064	·011	·020	·063158	·034928	·011	·021	·073684	·037514	·011	·021
97	·052083	·031742	·011	·020	·062500	·034580	·011	·021	·072917	·037143	·011	·021
98	·051546	·031427	·011	·020	·061856	·034239	·011	·021	·072165 −	·036779	·011	·021
99	·051020	·031118	·011	·020	·061224	·033905 −	·011	·021	·071429	·036422	·011	·021
100	·050505 +	·030815 +	·011	·020	·060606	·033577	·011	·021	·070707	·036071	·011	·021

N = size of sample	9: $\bar{\eta}^2$	9: $\sigma_{\bar{\eta}^2}$	9: P_1 $\lambda_1=2{\cdot}86$	9: P_2 $\lambda_2=2{\cdot}44$	10: $\bar{\eta}^2$	10: $\sigma_{\bar{\eta}^2}$	10: P_1 $\lambda_1=2{\cdot}81$	10: P_2 $\lambda_2=2{\cdot}41$
51	·160000	·071897	·009	·019	·180000	·075345 +	·009	·019
52	·156863	·070646	·009	·019	·176471	·074055 −	·009	·019
53	·153846	·069436	·009	·019	·173077	·072807	·009	·019
54	·150943	·068267	·009	·019	·169811	·071599	·009	·019
55	·148148	·067135 +	·009	·019	·166667	·070430	·009	·019
56	·145455 −	·066040	·009	·020	·163636	·069297	·009	·019$_5$
57	·142857	·064980	·009	·020	·160714	·068200	·009	·020
58	·140351	·063952	·009$_5$	·020	·157895 −	·067136	·009$_5$	·020
59	·137931	·062957	·010	·020	·155172	·066104	·010	·020
60	·135593	·061991	·010	·020	·152542	·065103	·010	·020
61	·133333	·061054	·010	·020	·150000	·064132	·010	·020
62	·131148	·060145 −	·010	·020	·147541	·063188	·010	·020
63	·129032	·059262	·010	·020	·145161	·062272	·010	·020
64	·126984	·058404	·010	·020	·142857	·061381	·010	·020
65	·125000	·057571	·010	·020	·140625	·060515 +	·010	·020
66	·123077	·056761	·010	·020	·138462	·059673	·010	·020
67	·121212	·055973	·010	·020	·136364	·058854	·010	·020
68	·119403	·055206	·010	·020	·134328	·058057	·010	·020
69	·117647	·054460	·010	·020	·132353	·057280	·010	·020
70	·115942	·053734	·010	·020	·130435 −	·056524	·010	·020$_5$
71	·114286	·053026	·010	·020	·128571	·055787	·010	·021
72	·112676	·052337	·010	·020	·126761	·055070	·010	·021
73	·111111	·051666	·010	·020$_5$	·125000	·054370	·010	·021
74	·109589	·051011	·010	·021	·123288	·053687	·010	·021
75	·108108	·050372	·010	·021	·121622	·053022	·010	·021
76	·106667	·049750 −	·010	·021	·120000	·052372	·010	·021
77	·105263	·049142	·010	·021	·118421	·051738	·010	·021
78	·103896	·048549	·010	·021	·116883	·051119	·010$_5$	·021
79	·102564	·047970	·010	·021	·115385 −	·050515 +	·011	·021
80	·101266	·047405 −	·010	·021	·113924	·049925 −	·011	·021
81	·100000	·046852	·010$_5$	·021	·112500	·049348	·011	·021
82	·098765 +	·046312	·011	·021	·111111	·048784	·011	·021
83	·097561	·045785 −	·011	·021	·109756	·048233	·011	·021
84	·096386	·045269	·011	·021	·108434	·047694	·011	·021
85	·095238	·044765 −	·011	·021	·107143	·047167	·011	·021
86	·094118	·044272	·011	·021	·105882	·046651	·011	·021
87	·093023	·043789	·011	·021	·104651	·046147	·011	·021
88	·091954	·043317	·011	·021	·103448	·045653	·011	·021
89	·090909	·042855 −	·011	·021	·102273	·045170	·011	·021
90	·089888	·042403	·011	·021	·101124	·044696	·011	·021
91	·088889	·041960	·011	·021	·100000	·044233	·011	·021
92	·087912	·041526	·011	·021	·098901	·043778	·011	·021
93	·086957	·041101	·011	·021	·097826	·043333	·011	·021
94	·086022	·040684	·011	·021	·096774	·042897	·011	·021
95	·085106	·040276	·011	·021	·095745 −	·042470	·011	·021
96	·084211	·039876	·011	·021	·094737	·042051	·011	·021
97	·083333	·039484	·011	·021	·093750	·041640	·011	·021
98	·082474	·039099	·011	·021	·092784	·041237	·011	·021
99	·081633	·038722	·011	·021	·091837	·040842	·011	·021
100	·080808	·038352	·011	·021	·090909	·040454	·011	·021

N = size of sample	11: $\bar{\eta}^2$	11: $\sigma_{\bar{\eta}^2}$	11: P_1 $\lambda_1=2{\cdot}76$	11: P_2 $\lambda_2=2{\cdot}38$	12: $\bar{\eta}^2$	12: $\sigma_{\bar{\eta}^2}$	12: P_1 $\lambda_1=2{\cdot}71$	12: P_2 $\lambda_2=2{\cdot}35$
51	·200000	·078446	·009	·019	·220000	·081240	·009	·019
52	·196078	·077126	·009	·019	·215686	·079898	·009	·019
53	·192308	·075847	·009	·019	·211538	·078596	·009	·019
54	·188679	·074609	·009	·019	·207547	·077335 +	·009	·019
55	·185185 +	·073410	·009	·019	·203704	·076113	·009	·019
56	·181818	·072247	·009	·019	·200000	·074927	·009	·019
57	·178571	·071120	·009	·019	·196429	·073776	·010	·019
58	·175439	·070027	·009$_5$	·019	·192982	·072659	·010	·019
59	·172414	·068966	·010	·020	·189655 +	·071574	·010	·019
60	·169492	·067935 +	·010	·020	·186441	·070520	·010	·019$_5$
61	·166667	·066935 −	·010	·020	·183333	·069496	·010	·020
62	·163934	·065963	·010	·020	·180328	·068501	·010	·020
63	·161290	·065018	·010	·020	·177419	·067533	·010	·020
64	·158730	·064100	·010	·020	·174603	·066591	·010	·020
65	·156250	·063206	·010	·020	·171875	·065675 −	·010	·020
66	·153846	·062337	·010	·020	·169231	·064782	·010	·020
67	·151515 +	·061491	·010	·020	·166667	·063914	·010	·020
68	·149254	·060667	·010	·020	·164179	·063068	·010	·020
69	·147059	·059865 −	·010	·020	·161765 −	·062243	·010	·020
70	·144928	·059083	·010	·020	·159420	·061439	·010$_5$	·020
71	·142857	·058321	·010	·020	·157143	·060656	·011	·020
72	·140845 −	·057578	·010	·020	·154930	·059892	·011	·020
73	·138889	·056854	·010	·020$_5$	·152778	·059146	·011	·021
74	·136986	·056148	·010$_5$	·021	·150685 −	·058419	·011	·021
75	·135135 +	·055458	·011	·021	·148649	·057709	·011	·021
76	·133333	·054785 +	·011	·021	·146667	·057016	·011	·021
77	·131579	·054129	·011	·021	·144737	·056339	·011	·021
78	·129870	·053487	·011	·021	·142857	·055677	·011	·021
79	·128205 +	·052860	·011	·021	·141026	·055031	·011	·021
80	·126582	·052248	·011	·021	·139241	·054400	·011	·021
81	·125000	·051650 −	·011	·021	·137500	·053782	·011	·021
82	·123457	·051065 −	·011	·021	·135802	·053179	·011	·021
83	·121951	·050493	·011	·021	·134146	·052588	·011	·021
84	·120482	·049933	·011	·021	·132530	·052010	·011	·021
85	·119048	·049386	·011	·021	·130952	·051445 +	·011	·021
86	·117647	·048850 +	·011	·021	·129412	·050892	·011	·021
87	·116279	·048326	·011	·021	·127907	·050350 +	·011	·021
88	·114943	·047813	·011	·021	·126437	·049820	·011	·021
89	·113636	·047311	·011	·021	·125000	·049301	·011	·021
90	·112360	·046819	·011	·021	·123596	·048792	·011	·021
91	·111111	·046337	·011	·021	·122222	·048293	·011$_5$	·021
92	·109890	·045864	·011	·021	·120879	·047805 +	·012	·021$_5$
93	·108696	·045401	·011	·021	·119565 +	·047326	·012	·022
94	·107527	·044948	·011	·021	·118280	·046857	·012	·022
95	·106383	·044503	·011	·021	·117021	·046397	·012	·022
96	·105263	·044067	·011	·021	·115789	·045945 +	·012	·022
97	·104167	·043640	·011	·021	·114583	·045503	·012	·022
98	·103093	·043220	·011	·021	·113402	·045068	·012	·022
99	·102041	·042809	·011	·021	·112245 −	·044642	·012	·022
100	·101010	·042405 −	·011	·021	·111111	·044224	·012	·022

N = size of sample	13: $\bar{\eta}^2$	13: $\sigma_{\bar{\eta}^2}$	13: P_1 $\lambda_1=2{\cdot}66$	13: P_2 $\lambda_2=2{\cdot}32$	14: $\bar{\eta}^2$	14: $\sigma_{\bar{\eta}^2}$	14: P_1 $\lambda_1=2{\cdot}63$	14: P_2 $\lambda_2=2{\cdot}30$
51	·240000	·083758	·009	·019	·260000	·086023	·009	·018
52	·235294	·082400	·009	·019	·254902	·084658	·009	·018
53	·230769	·081084	·009	·019	·250000	·083333	·009	·019
54	·226415 +	·079807	·009	·019	·245283	·082046	·009	·019
55	·222222	·078567	·009$_5$	·019	·240741	·080796	·009$_5$	·019
56	·218182	·077364	·010	·019	·236364	·079581	·010	·019
57	·214286	·076196	·010	·019	·232143	·078401	·010	·019
58	·210526	·075060	·010	·019	·228070	·077252	·010	·019
59	·206897	·073957	·010	·020	·224138	·076136	·010	·019
60	·203390	·072885 −	·010	·020	·220339	·075050 −	·010	·019$_5$
61	·200000	·071842	·010	·020	·216667	·073993	·010	·020
62	·196721	·070828	·010	·020	·213115 −	·072964	·010	·020
63	·193548	·069841	·010	·020	·209677	·071962	·010	·020
64	·190476	·068880	·010	·020	·206349	·070986	·010	·020
65	·187500	·067945 −	·010	·020	·203125	·070036	·010	·020
66	·184615 +	·067034	·010	·020	·200000	·069109	·010$_5$	·020
67	·181818	·066146	·010	·020	·196970	·068207	·011	·020
68	·179104	·065281	·010$_5$	·020	·194030	·067326	·011	·020
69	·176471	·064438	·011	·020	·191176	·066468	·011	·020
70	·173913	·063616	·011	·020$_5$	·188406	·065630	·011	·020
71	·171429	·062814	·011	·021	·185714	·064813	·011	·021
72	·169014	·062031	·011	·021	·183099	·064015 −	·011	·021
73	·166667	·061268	·011	·021	·180556	·063236	·011	·021
74	·164384	·060523	·011	·021	·178082	·062475 +	·011	·021
75	·162162	·059795 −	·011	·021	·175676	·061732	·011	·021
76	·160000	·059084	·011	·021	·173333	·061006	·011	·021
77	·157895 −	·058390	·011	·021	·171053	·060297	·011	·021
78	·155844	·057711	·011	·021	·168831	·059604	·011	·021
79	·153846	·057048	·011	·021	·166667	·058926	·011	·021
80	·151899	·056399	·011	·021	·164557	·058262	·011$_5$	·021
81	·150000	·055765 +	·011	·021	·162500	·057614	·012	·021
82	·148148	·055145 −	·011	·021	·160494	·056979	·012	·021
83	·146341	·054538	·011	·021	·158537	·056358	·012	·021
84	·144578	·053945 −	·011	·021	·156627	·055750 +	·012	·021
85	·142857	·053363	·011	·021	·154762	·055155 +	·012	·021$_5$
86	·141176	·052794	·011	·021$_5$	·152941	·054573	·012	·022
87	·139535 −	·052237	·012	·022	·151163	·054002	·012	·022
88	·137931	·051692	·012	·022	·149425 +	·053443	·012	·022
89	·136364	·051157	·012	·022	·147727	·052895 −	·012	·022
90	·134831	·050634	·012	·022	·146067	·052358	·012	·022
91	·133333	·050121	·012	·022	·144444	·051832	·012	·022
92	·131868	·049618	·012	·022	·142857	·051316	·012	·022
93	·130435 −	·049125 −	·012	·022	·141304	·050810	·012	·022
94	·129032	·048641	·012	·022	·139785 −	·050314	·012	·022
95	·127660	·048167	·012	·022	·138298	·049827	·012	·022
96	·126316	·047702	·012	·022	·136842	·049350 −	·012	·022
97	·125000	·047246	·012	·022	·135417	·048881	·012	·022
98	·123711	·046798	·012	·022	·134021	·048421	·012	·022
99	·122449	·046358	·012	·022	·132653	·047970	·012	·022
100	·121212	·045927	·012	·022	·131313	·047527	·012	·022

n = number of arrays

N = size of sample	15: $\bar{\eta}^2$	15: $\sigma_{\bar{\eta}^2}$	15: P_1 λ_1 = 2·60	15: P_2 λ_2 = 2·28	16: $\bar{\eta}^2$	16: $\sigma_{\bar{\eta}^2}$	16: P_1 λ_1 = 2·57	16: P_2 λ_2 = 2·26	17: $\bar{\eta}^2$	17: $\sigma_{\bar{\eta}^2}$	17: P_1 λ_1 = 2·54	17: P_2 λ_2 = 2·24
51	·280000	·088056	·009	·018	·300000	·089872	·009	·018	·320000	·091483	·009	·018
52	·274510	·086691	·009	·018	·294118	·088512	·009	·018	·313725 −	·090137	·009	·018
53	·269231	·085363	·009	·018	·288462	·087189	·009	·018$_5$	·307692	·088823	·009	·018
54	·264151	·084073	·009	·019	·283019	·085900	·009	·019	·301887	·087542	·009	·019
55	·259259	·082817	·010	·019	·277778	·084646	·010	·019	·296296	·086294	·010	·019
56	·254545 +	·081596	·010	·019	·272727	·083424	·010	·019	·290909	·085076	·010	·019
57	·250000	·080408	·010	·019	·267857	·082234	·010	·019	·285714	·083889	·010	·019
58	·245614	·079252	·010	·019	·263158	·081075 −	·010	·019	·280702	·082731	·010	·019
59	·241379	·078127	·010	·019	·258621	·079945 −	·010	·019	·275862	·081601	·010	·019$_5$
60	·237288	·077031	·010	·019	·254237	·078844	·010	·019$_5$	·271186	·080499	·010	·020
61	·233333	·075965 −	·010	·020	·250000	·077771	·010	·020	·266667	·079424	·010	·020
62	·229508	·074925 +	·010	·020	·245902	·076725 +	·010	·020	·262295 +	·078376	·010	·020
63	·225806	·073913	·010	·020	·241935 +	·075706	·010	·020	·258065 −	·077352	·010$_5$	·020
64	·222222	·072926	·010	·020	·238095 +	·074711	·011	·020	·253968	·076353	·011	·020
65	·218750	·071963	·010$_5$	·020	·234375	·073741	·011	·020	·250000	·075378	·011	·020
66	·215385 −	·071025 +	·011	·020	·230769	·072794	·011	·020	·246154	·074426	·011	·020
67	·212121	·070110	·011	·020	·227273	·071870	·011	·020	·242424	·073496	·011	·020$_5$
68	·208955 +	·069218	·011	·020	·223881	·070968	·011	·020	·238806	·072587	·011	·021
69	·205882	·068347	·011	·020	·220588	·070088	·011	·020$_5$	·235294	·071700	·011	·021
70	·202899	·067497	·011	·020$_5$	·217391	·069228	·011	·021	·231884	·070833	·011	·021
71	·200000	·066667	·011	·021	·214286	·068388	·011	·021	·228571	·069985 +	·011	·021
72	·197183	·065856	·011	·021	·211268	·067567	·011	·021	·225352	·069157	·011	·021
73	·194444	·065065 −	·011	·021	·208333	·066765 +	·011	·021	·222222	·068347	·011	·021
74	·191781	·064291	·011	·021	·205479	·065981	·011	·021	·219178	·067555 +	·011	·021
75	·189189	·063535 +	·011	·021	·202703	·065215 +	·011	·021	·216216	·066781	·011$_5$	·021
76	·186667	·062797	·011	·021	·200000	·064466	·011$_5$	·021	·213333	·066023	·012	·021
77	·184211	·062075 −	·011	·021	·197368	·063733	·012	·021	·210526	·065281	·012	·021
78	·181818	·061368	·011	·021	·194805 +	·063016	·012	·021	·207792	·064556	·012	·021
79	·179487	·060678	·011	·021	·192308	·062315 −	·012	·021	·205128	·063846	·012	·021$_5$
80	·177215 +	·060002	·012	·021	·189873	·061628	·012	·021	·202532	·063150 +	·012	·022
81	·175000	·059341	·012	·021	·187500	·060957	·012	·021	·200000	·062470	·012	·022
82	·172840	·058694	·012	·021	·185185 +	·060299	·012	·022	·197531	·061803	·012	·022
83	·170732	·058060	·012	·021	·182927	·059655 −	·012	·022	·195122	·061150 −	·012	·022
84	·168675 −	·057440	·012	·021$_5$	·180723	·059024	·012	·022	·192771	·060510	·012	·022
85	·166667	·056833	·012	·022	·178571	·058406	·012	·022	·190476	·059883	·012	·022
86	·164706	·056238	·012	·022	·176471	·057800	·012	·022	·188235 +	·059268	·012	·022
87	·162791	·055655 +	·012	·022	·174419	·057207	·012	·022	·186047	·058666	·012	·022
88	·160920	·055084	·012	·022	·172414	·056626	·012	·022	·183908	·058075 +	·012	·022
89	·159091	·054524	·012	·022	·170455 −	·056055 +	·012	·022	·181818	·057496	·012	·022
90	·157303	·053976	·012	·022	·168539	·055497	·012	·022	·179775 +	·056928	·012$_5$	·022
91	·155556	·053438	·012	·022	·166667	·054948	·012	·022	·177778	·056371	·013	·022
92	·153846	·052910	·012	·022	·164835 +	·054411	·012	·022	·175824	·055824	·013	·022
93	·152174	·052393	·012	·022	·163043	·053883	·012	·022	·173913	·055288	·013	·022
94	·150538	·051886	·012	·022	·161290	·053366	·013	·022	·172043	·054762	·013	·022$_5$
95	·148936	·051388	·012	·022	·159574	·052858	·013	·022	·170213	·054245 −	·013	·023
96	·147368	·050899	·012	·022	·157895 −	·052360	·013	·022	·168421	·053738	·013	·023
97	·145833	·050420	·012	·022	·156250	·051870	·013	·022	·166667	·053240	·013	·023
98	·144330	·049949	·012	·022	·154639	·051390	·013	·022	·164948	·052751	·013	·023
99	·142857	·049487	·012	·022	·153061	·050918	·013	·022$_5$	·163265 +	·052270	·013	·023
100	·141414	·049033	·012	·022	·151515 +	·050455 +	·013	·023	·161616	·051799	·013	·023

N = size of sample	18: $\bar{\eta}^2$	18: $\sigma_{\bar{\eta}^2}$	18: P_1 λ_1 = 2·53	18: P_2 λ_2 = 2·23	19: $\bar{\eta}^2$	19: $\sigma_{\bar{\eta}^2}$	19: P_1 λ_1 = 2·52	19: P_2 λ_2 = 2·22	20: $\bar{\eta}^2$	20: $\sigma_{\bar{\eta}^2}$	20: P_1 λ_1 = 2·51	20: P_2 λ_2 = 2·21
51	·340000	·092902	·008$_5$	·017$_5$	·360000	·094136	·008	·017	·380000	·095192	·008	·017
52	·333333	·091574	·009	·018	·352941	·092833	·008	·017	·372549	·093920	·008	·017
53	·326923	·090276	·009	·018	·346154	·091557	·009	·017$_5$	·365385 −	·092672	·008	·018
54	·320755 −	·089009	·009	·018	·339623	·090308	·009	·018	·358491	·091448	·008	·018
55	·314815 −	·087771	·009	·018	·333333	·089087	·009	·018	·351852	·090248	·008$_5$	·018
56	·309091	·086563	·009	·019	·327273	·087893	·009	·018	·345455 −	·089072	·009	·018$_5$
57	·303571	·085383	·009	·019	·321429	·086724	·009	·018	·339286	·087921	·009	·019
58	·298246	·084230	·010	·019	·315789	·085582	·009	·019	·333333	·086793	·009	·019
59	·293103	·083105 +	·010	·019	·310345 −	·084465 +	·009	·019	·327586	·085688	·009	·019
60	·288136	·082006	·010	·019	·305085 −	·083373	·010	·019	·322034	·084607	·009	·019
61	·283333	·080933	·010	·019	·300000	·082305 +	·010	·019	·316667	·083548	·009	·019
62	·278689	·079885 +	·010	·019$_5$	·295082	·081262	·010	·019	·311475 +	·082512	·010	·019
63	·274194	·078861	·010	·020	·290323	·080241	·010	·019	·306452	·081498	·010	·019
64	·269841	·077861	·010	·020	·285714	·079243	·010	·019$_5$	·301587	·080505 −	·010	·019
65	·265625	·076884	·010	·020	·281250	·078267	·010	·020	·296875	·079533	·010	·020
66	·261538	·075929	·011	·020	·276923	·077312	·010	·020	·292308	·078581	·010	·020
67	·257576	·074996	·011	·020	·272727	·076379	·010	·020	·287879	·077650 +	·010	·020
68	·253731	·074084	·011	·020	·268657	·075466	·010$_5$	·020	·283582	·076738	·010	·020
69	·250000	·073193	·011	·020	·264706	·074572	·011	·020	·279412	·075846	·010	·020
70	·246377	·072321	·011	·020	·260870	·073698	·011	·020	·275362	·074972	·010$_5$	·020
71	·242857	·071468	·011	·021	·257143	·072843	·011	·020	·271429	·074116	·011	·020
72	·239437	·070634	·011	·021	·253521	·072006	·011	·020$_5$	·267606	·073278	·011	·020
73	·236111	·069819	·011	·021	·250000	·071187	·011	·021	·263889	·072457	·011	·020
74	·232877	·069021	·011	·021	·246575 +	·070385 −	·011	·021	·260274	·071653	·011	·021
75	·229730	·068240	·011	·021	·243243	·069600	·011	·021	·256757	·070865 +	·011	·021
76	·226667	·067476	·011	·021	·240000	·068831	·011	·021	·253333	·070094	·011	·021
77	·223684	·066727	·011$_5$	·021	·236842	·068078	·011	·021	·250000	·069338	·011	·021
78	·220779	·065995 −	·012	·021	·233766	·067340	·011	·021	·246753	·068596	·011	·021
79	·217949	·065278	·012	·021	·230769	·066617	·011	·021	·243590	·067870	·011	·021
80	·215190	·064575 +	·012	·021	·227848	·065909	·011$_5$	·021	·240506	·067158	·011	·021
81	·212500	·063887	·012	·021	·225000	·065215 +	·012	·021	·237500	·066460	·011	·021
82	·209877	·063213	·012	·022	·222222	·064535 +	·012	·021	·234568	·065775 +	·011$_5$	·021
83	·207317	·062552	·012	·022	·219512	·063869	·012	·021$_5$	·231707	·065104	·012	·021
84	·204819	·061905 −	·012	·022	·216867	·063215 +	·012	·022	·228916	·064446	·012	·021
85	·202381	·061270	·012	·022	·214286	·062574	·012	·022	·226190	·063800	·012	·022
86	·200000	·060648	·012	·022	·211765 −	·061946	·012	·022	·223529	·063166	·012	·022
87	·197674	·060038	·012	·022	·209302	·061329	·012	·022	·220930	·062545 −	·012	·022
88	·195402	·059439	·012	·022	·206897	·060724	·012	·022	·218391	·061934	·012	·022
89	·193182	·058852	·012	·022	·204545 +	·060131	·012	·022	·215909	·061336	·012	·022
90	·191011	·058277	·012	·022	·202247	·059548	·012	·022	·213483	·060748	·012	·022
91	·188889	·057712	·012	·022	·200000	·058977	·012	·022	·211111	·060171	·012	·022
92	·186813	·057157	·012$_5$	·022	·197802	·058416	·012	·022	·208791	·059604	·012	·022
93	·184783	·056613	·013	·022	·195652	·057865 −	·012	·022	·206522	·059047	·012	·022
94	·182796	·056079	·013	·022	·193548	·057324	·012	·022	·204301	·058501	·012	·022
95	·180851	·055555 −	·013	·022	·191489	·056793	·012	·022	·202128	·057964	·012	·022
96	·178947	·055040	·013	·022	·189474	·056271	·012	·022	·200000	·057437	·012	·022
97	·177083	·054534	·013	·022$_5$	·187500	·055759	·012$_5$	·022	·197917	·056918	·012	·022
98	·175258	·054037	·013	·023	·185567	·055255 +	·013	·022	·195876	·056409	·012	·022
99	·173469	·053550 −	·013	·023	·183673	·054761	·013	·022$_5$	·193878	·055909	·012$_5$	·022$_5$
100	·171717	·053070	·013	·023	·181818	·054275 −	·013	·023	·191919	·055417	·013	·023

N = size of sample	3: $\overline{\eta}^2$	3: $\sigma_{\overline{\eta}^2}$	3: P_1 $\lambda_1=3·50$	3: P_2 $\lambda_2=2·94$	4: $\overline{\eta}^2$	4: $\sigma_{\overline{\eta}^2}$	4: P_1 $\lambda_1=3·20$	4: P_2 $\lambda_2=2·80$
101	·020000	·019604	·011	·019	·030000	·023887	·012	·019
102	·019802	·019414	·011	·019	·029703	·023656	·012	·019
103	·019608	·019227	·011	·019	·029412	·023430	·012	·019
104	·019417	·019044	·011	·019	·029126	·023208	·012	·019
105	·019231	·018864	·011	·019	·028846	·022991	·012	·019
106	·019048	·018688	·011	·019	·028571	·022777	·012	·019
107	·018868	·018515 +	·011	·019	·028302	·022567	·012	·019
108	·018692	·018345 +	·011	·019	·028037	·022361	·012	·019
109	·018519	·018179	·011	·019	·027778	·022159	·012	·019
110	·018349	·018015 −	·011	·019	·027523	·021960	·012	·019
111	·018182	·017854	·011	·019	·027273	·021765 +	·012	·019
112	·018018	·017696	·011	·019	·027027	·021574	·012	·019
113	·017857	·017541	·011	·019	·026786	·021385 +	·012	·019
114	·017699	·017389	·011	·019	·026549	·021200	·012	·019
115	·017544	·017239	·011	·019	·026316	·021019	·012	·019
116	·017391	·017091	·011	·019	·026087	·020840	·012	·019
117	·017241	·016947	·011	·019	·025862	·020664	·012	·019
118	·017094	·016804	·011	·019	·025641	·020491	·012	·019
119	·016949	·016664	·011	·019	·025424	·020321	·012	·019
120	·016807	·016527	·011	·019	·025210	·020154	·012	·019
121	·016667	·016391	·011	·019	·025000	·019990	·012	·019
122	·016529	·016258	·011	·019	·024793	·019828	·012	·019
123	·016393	·016127	·011	·019	·024590	·019669	·012	·019
124	·016260	·015998	·011	·019	·024390	·019512	·012	·019
125	·016129	·015871	·011	·019	·024194	·019358	·012	·019
126	·016000	·015746	·011	·019	·024000	·019206	·012	·019
127	·015873	·015623	·011	·019	·023810	·019057	·012	·019
128	·015748	·015502	·011	·019	·023622	·018910	·012	·019
129	·015625	·015383	·011	·019	·023438	·018765 +	·012	·019
130	·015504	·015265 +	·011	·019	·023256	·018622	·012	·019
131	·015385 −	·015150 −	·011	·019	·023077	·018482	·012	·019
132	·015267	·015036	·011	·019	·022901	·018344	·012	·019
133	·015152	·014924	·011	·019	·022727	·018207	·012	·019
134	·015038	·014813	·011	·019	·022556	·018073	·012	·019
135	·014925 +	·014704	·011	·019	·022388	·017941	·012	·019
136	·014815 −	·014597	·011	·019	·022222	·017810	·012	·019
137	·014706	·014491	·011	·019	·022059	·017682	·012	·019
138	·014599	·014387	·011	·019	·021898	·017555 −	·012	·019
139	·014493	·014284	·011	·019	·021739	·017430	·012	·019
140	·014388	·014183	·011	·019	·021583	·017307	·012	·019
141	·014286	·014083	·011	·019	·021429	·017186	·012	·019
142	·014184	·013985 −	·011	·019	·021277	·017066	·012	·019
143	·014085 −	·013888	·011	·019	·021127	·016948	·012	·019
144	·013986	·013792	·011	·019	·020979	·016831	·012	·019
145	·013889	·013697	·011	·019	·020833	·016717	·012	·019
146	·013793	·013604	·011	·019	·020690	·016603	·012	·019
147	·013699	·013512	·011	·019	·020548	·016491	·012	·019
148	·013605 +	·013422	·011	·019	·020408	·016381	·012	·019
149	·013514	·013332	·011	·019	·020270	·016272	·012	·019
150	·013423	·013244	·011	·019	·020134	·016165 +	·012	·019

N = size of sample	5: $\overline{\eta}^2$	5: $\sigma_{\overline{\eta}^2}$	5: P_1 $\lambda_1=3·14$	5: P_2 $\lambda_2=2·68$	6: $\overline{\eta}^2$	6: $\sigma_{\overline{\eta}^2}$	6: P_1 $\lambda_1=3·11$	6: P_2 $\lambda_2=2·63$
101	·040000	·027440	·011	·020	·050000	·030518	·010	·020
102	·039604	·027176	·011	·020	·049505 −	·030227	·010	·020
103	·039216	·026918	·011	·020	·049020	·029941	·010	·020
104	·038835 −	·026664	·011	·020	·048544	·029661	·010	·020
105	·038462	·026415 +	·011	·020	·048077	·029385 +	·010	·020
106	·038095 +	·026171	·011	·020	·047619	·029115 +	·010	·020
107	·037736	·025931	·011	·020	·047170	·028850 −	·010	·020
108	·037383	·025696	·011	·020	·046729	·028589	·010	·020
109	·037037	·025465 −	·011	·020	·046296	·028333	·010	·020
110	·036697	·025238	·011	·020	·045872	·028082	·010	·020
111	·036364	·025015 −	·011	·020	·045455 −	·027835 +	·010	·020
112	·036036	·024796	·011	·020	·045045 +	·027592	·010	·020
113	·035714	·024580	·011	·020	·044643	·027354	·010	·020
114	·035398	·024369	·011	·020	·044248	·027120	·010	·020
115	·035088	·024161	·011	·020	·043860	·026889	·010	·020
116	·034783	·023956	·011	·020	·043478	·026663	·010	·020
117	·034483	·023755	·011	·020	·043103	·026440	·010	·020
118	·034188	·023557	·011	·020	·042735 +	·026221	·010	·020
119	·033898	·023363	·011	·020	·042373	·026006	·010	·020
120	·033613	·023172	·011	·020	·042017	·025794	·010	·020
121	·033333	·022983	·011	·020	·041667	·025585 +	·010	·020
122	·033058	·022798	·011	·020	·041322	·025380	·010	·020
123	·032787	·022616	·011	·020	·040984	·025178	·010	·020
124	·032520	·022437	·011	·020	·040650 +	·024979	·010	·020
125	·032258	·022260	·011	·020	·040323	·024784	·010	·020
126	·032000	·022086	·011	·020	·040000	·024591	·010	·020
127	·031746	·021915 +	·011	·020	·039683	·024402	·010	·020
128	·031496	·021747	·011	·020	·039370	·024215 −	·010	·020
129	·031250	·021581	·011	·020	·039063	·024031	·010	·020
130	·031008	·021418	·011	·020	·038760	·023850 −	·010	·020
131	·030769	·021257	·011	·020	·038462	·023671	·010	·020
132	·030534	·021098	·011	·020	·038168	·023496	·010	·020
133	·030303	·020942	·011	·020	·037879	·023323	·010	·020
134	·030075 +	·020788	·011	·020	·037594	·023152	·010	·020
135	·029851	·020637	·011	·020	·037313	·022984	·010	·020
136	·029630	·020487	·011	·020	·037037	·022818	·010	·020
137	·029412	·020340	·011	·020	·036765 −	·022655 −	·010	·020
138	·029197	·020195 −	·011	·020	·036496	·022494	·010	·020
139	·028986	·020052	·011	·020	·036232	·022335 −	·010	·020
140	·028777	·019911	·011	·020	·035971	·022178	·010	·020
141	·028571	·019772	·011	·020	·035714	·022024	·010	·020
142	·028369	·019634	·011	·020	·035461	·021872	·010	·020
143	·028169	·019499	·011	·020	·035211	·021722	·010	·020
144	·027972	·019366	·011	·020	·034965 +	·021573	·010	·020
145	·027778	·019234	·011	·020	·034722	·021427	·010	·020
146	·027586	·019104	·011	·020	·034483	·021283	·010	·020
147	·027397	·018976	·011	·020	·034247	·021141	·010	·020
148	·027211	·018850 −	·011	·020	·034014	·021001	·010	·020
149	·027027	·018725 −	·011	·020	·033784	·020862	·010	·020
150	·026846	·018602	·011	·020	·033557	·020726	·010	·020

N = size of sample	7: $\overline{\eta}^2$	7: $\sigma_{\overline{\eta}^2}$	7: P_1 $\lambda_1=3·08$	7: P_2 $\lambda_2=2·58$	8: $\overline{\eta}^2$	8: $\sigma_{\overline{\eta}^2}$	8: P_1 $\lambda_1=3·02$	8: P_2 $\lambda_2=2·54$
101	·060000	·033255 −	·009	·019	·070000	·035728	·009	·019
102	·059406	·032939	·009	·019	·069307	·035391	·009	·019
103	·058824	·032629	·009	·019	·068627	·035060	·009	·019
104	·058252	·032325 +	·009	·019	·067961	·034735 +	·009	·019
105	·057692	·032027	·009	·019	·067308	·034416	·009	·019
106	·057143	·031734	·009	·019	·066667	·034103	·009	·019
107	·056604	·031447	·009	·019	·066038	·033796	·009	·019
108	·056075 −	·031164	·009	·019	·065421	·033494	·009	·019
109	·055556	·030887	·009	·019	·064815 −	·033197	·009	·019
110	·055046	·030614	·009	·019	·064220	·032906	·009	·020
111	·054545 +	·030346	·009	·019	·063636	·032620	·009	·020
112	·054054	·030083	·009	·019	·063063	·032338	·009	·020
113	·053571	·029824	·009	·019	·062500	·032062	·009	·020
114	·053097	·029570	·009	·019	·061947	·031790	·009	·020
115	·052632	·029320	·009	·020	·061404	·031523	·009	·020
116	·052174	·029075 −	·009	·020	·060870	·031260	·009	·020
117	·051724	·028833	·009	·020	·060345 −	·031001	·009	·020
118	·051282	·028595 +	·009	·020	·059829	·030747	·009	·020
119	·050847	·028361	·009	·020	·059322	·030497	·009	·020
120	·050420	·028131	·009	·020	·058824	·030251	·009	·020
121	·050000	·027905 −	·009	·020	·058333	·030008	·009	·020
122	·049587	·027682	·009	·020	·057851	·029770	·010	·020
123	·049180	·027463	·009	·020	·057377	·029535 +	·010	·020
124	·048780	·027247	·010	·020	·056911	·029304	·010	·020
125	·048387	·027035 −	·010	·020	·056452	·029077	·010	·020
126	·048000	·026826	·010	·020	·056000	·028853	·010	·020
127	·047619	·026620	·010	·020	·055556	·028633	·010	·020
128	·047244	·026417	·010	·020	·055118	·028416	·010	·020
129	·046875	·026217	·010	·020	·054688	·028202	·010	·020
130	·046512	·026021	·010	·020	·054264	·027991	·010	·020
131	·046154	·025827	·010	·020	·053846	·027783	·010	·020
132	·045802	·025636	·010	·020	·053435 +	·027579	·010	·020
133	·045455 −	·025448	·010	·020	·053030	·027377	·010	·020
134	·045113	·025262	·010	·020	·052632	·027179	·010	·020
135	·044776	·025080	·010	·020	·052239	·026983	·010	·020
136	·044444	·024900	·010	·020	·051852	·026790	·010	·020
137	·044118	·024722	·010	·020	·051471	·026600	·010	·020
138	·043796	·024547	·010	·020	·051095 −	·026412	·010	·020
139	·043478	·024374	·010	·020	·050725 −	·026227	·010	·020
140	·043165 +	·024204	·010	·020	·050360	·026045 +	·010	·020
141	·042587	·024036	·010	·020	·050000	·025865 +	·010	·020
142	·042553	·023871	·010	·020	·049645 +	·025688	·010	·020
143	·042254	·023708	·010	·020	·049296	·025513	·010	·020
144	·041958	·023547	·010	·020	·048951	·025340	·010	·020
145	·041667	·023388	·010	·020	·048611	·025170	·010	·020
146	·041379	·023231	·010	·020	·048276	·025002	·010	·020
147	·041096	·023077	·010	·020	·047945 +	·024836	·010	·020
148	·040816	·022924	·010	·020	·047619	·024673	·010	·020
149	·040541	·022773	·010	·020	·047297	·024511	·010	·020
150	·040268	·022625 −	·010	·020	·046980	·024352	·010	·020

n = number of arrays

N = size of sample	$n=9$: $\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1, $\lambda_1 = 2·96$	P_2, $\lambda_2 = 2·50$	$n=10$: $\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1, $\lambda_1 = 2·92$	P_2, $\lambda_2 = 2·47$
101	·080000	·037989	·009	·019	·090000	·040073	·009	·019
102	·079208	·037632	·009	·019	·089109	·039700	·009	·019
103	·078431	·037283	·009	·019	·088235 +	·039333	·009	·019
104	·077670	·036939	·009	·019	·087379	·038973	·009	·019
105	·076923	·036602	·009	·019	·086538	·038620	·009	·019
106	·076190	·036271	·009	·019	·085714	·038273	·009	·019
107	·075472	·035946	·009	·019$_5$	·084906	·037932	·009	·019$_5$
108	·074766	·035627	·009	·020	·084112	·037597	·009	·020
109	·074074	·035313	·009	·020	·083333	·037268	·009	·020
110	·073394	·035005 +	·009	·020	·082569	·036944	·009	·020
111	·072727	·034702	·009	·020	·081818	·036626	·009	·020
112	·072072	·034405 −	·009	·020	·081081	·036314	·009	·020
113	·071429	·034112	·009	·020	·080357	·036007	·009	·020
114	·070796	·033824	·009$_5$	·020	·079646	·035705 −	·009	·020
115	·070175 +	·033541	·010	·020	·078947	·035408	·009$_5$	·020
116	·069565 +	·033263	·010	·020	·078261	·035115 +	·010	·020
117	·068966	·032989	·010	·020	·077586	·034828	·010	·020
118	·068376	·032720	·010	·020	·076923	·034545 +	·010	·020
119	·067797	·032455 +	·010	·020	·076271	·034267	·010	·020
120	·067227	·032195 −	·010	·020	·075630	·033993	·010	·020
121	·066667	·031938	·010	·020	·075000	·033724	·010	·020
122	·066116	·031686	·010	·020	·074380	·033459	·010	·020
123	·065574	·031437	·010	·020	·073770	·033197	·010	·020
124	·065041	·031192	·010	·020	·073171	·032940	·010	·020
125	·064516	·030952	·010	·020	·072581	·032687	·010	·020
126	·064000	·030714	·010	·020	·072000	·032438	·010	·020
127	·063492	·030481	·010	·020	·071429	·032192	·010	·020
128	·062992	·030251	·010	·020	·070866	·031951	·010	·020
129	·062500	·030024	·010	·020	·070313	·031712	·010	·020
130	·062016	·029801	·010	·020	·069767	·031478	·010	·020
131	·061538	·029581	·010	·020	·069231	·031246	·010	·020
132	·061069	·029364	·010	·020	·068702	·031018	·010	·020
133	·060606	·029150 −	·010	·020	·068182	·030794	·010	·020
134	·060150 +	·028940	·010	·020	·067669	·030572	·010	·020
135	·059701	·028732	·010	·020	·067164	·030354	·010	·020
136	·059259	·028528	·010	·020	·066667	·030139	·010	·020
137	·058824	·028326	·010	·020	·066176	·029927	·010	·020
138	·058394	·028127	·010	·020	·065693	·029718	·010	·020
139	·057971	·027931	·010	·020	·065217	·029511	·010	·020
140	·057554	·027738	·010	·020	·064748	·029308	·010	·020
141	·057143	·027547	·010	·020	·064286	·029107	·010	·020
142	·056738	·027359	·010	·020	·063830	·028909	·010	·020
143	·056338	·027173	·010	·020	·063380	·028714	·010	·020
144	·055944	·026990	·010	·020	·062937	·028521	·010	·020
145	·055556	·026810	·010	·020	·062500	·028331	·010	·020
146	·055172	·026631	·010	·020	·062069	·028144	·010	·020
147	·054795 −	·026455 +	·010	·020	·061644	·027958	·010	·020
148	·054422	·026282	·010	·020	·061224	·027776	·010	·020
149	·054054	·026111	·010	·020	·060811	·027595 +	·010	·020
150	·053691	·025941	·010	·020	·060403	·027417	·010	·020

N = size of sample	$n=11$: $\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1, $\lambda_1 = 2·88$	P_2, $\lambda_2 = 2·44$	$n=12$: $\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1, $\lambda_1 = 2·84$	P_2, $\lambda_2 = 2·42$
101	·100000	·042008	·009	·019	·110000	·043813	·009	·019
102	·099010	·041619	·009	·019	·108911	·043410	·009	·019
103	·098039	·041237	·009	·019	·107843	·043015 −	·009	·019
104	·097087	·040862	·009	·019$_5$	·106796	·042626	·009	·019
105	·096154	·040494	·009	·020	·105769	·042244	·009	·019$_5$
106	·095238	·040132	·009	·020	·104762	·041869	·009	·020
107	·094340	·039777	·009	·020	·103774	·041501	·009	·020
108	·093458	·039428	·009	·020	·102804	·041139	·009	·020
109	·092593	·039085 −	·009	·020	·101852	·040783	·009	·020
110	·091743	·038748	·009	·020	·100917	·040433	·009	·020
111	·090909	·038416	·009	·020	·100000	·040089	·009	·020
112	·090090	·038090	·009	·020	·099099	·039751	·009	·020
113	·089286	·037770	·009	·020	·098214	·039419	·009	·020
114	·088496	·037455 −	·009	·020	·097345 +	·039092	·009	·020
115	·087719	·037145 −	·009	·020	·096491	·038770	·009$_5$	·020
116	·086957	·036840	·009	·020	·095652	·038454	·010	·020
117	·086207	·036540	·009	·020	·094828	·038142	·010	·020
118	·085470	·036245 −	·009$_5$	·020	·094017	·037836	·010	·020
119	·084746	·035955 −	·010	·020	·093220	·037534	·010	·020
120	·084034	·035669	·010	·020	·092437	·037238	·010	·020
121	·083333	·035388	·010	·020	·091667	·036946	·010	·020
122	·082645 −	·035111	·010	·020	·090909	·036658	·010	·020
123	·081967	·034838	·010	·020	·090164	·036375 −	·010	·020
124	·081301	·034570	·010	·020	·089431	·036096	·010	·020
125	·080645 −	·034305 +	·010	·020	·088710	·035821	·010	·020
126	·080000	·034045 −	·010	·020	·088000	·035551	·010	·020
127	·079365 −	·033788	·010	·020	·087302	·035285 −	·010	·020
128	·078740	·033536	·010	·020	·086614	·035022	·010	·020
129	·078125	·033287	·010	·020	·085938	·034763	·010	·020
130	·077519	·033042	·010	·020	·085271	·034509	·010	·020
131	·076923	·032800	·010	·020	·084615 +	·034257	·010	·020
132	·076336	·032562	·010	·020	·083969	·034010	·010	·020
133	·075758	·032327	·010	·020	·083333	·033766	·010	·020
134	·075188	·032096	·010	·020	·082707	·033525 −	·010	·020
135	·074627	·031868	·010	·020	·082090	·033288	·010	·020
136	·074074	·031643	·010	·020	·081481	·033054	·010	·020
137	·073529	·031421	·010	·020	·080882	·032824	·010	·020
138	·072993	·031202	·010	·020	·080292	·032596	·010	·020
139	·072464	·030987	·010	·020	·079710	·032372	·010	·020
140	·071942	·030774	·010	·020	·079137	·032151	·010	·020
141	·071429	·030564	·010	·020	·078571	·031933	·010	·020
142	·070922	·030357	·010	·020	·078014	·031717	·010	·020
143	·070423	·030153	·010	·020	·077465 −	·031505 −	·010	·020
144	·069930	·029952	·010	·020	·076923	·031295 +	·010	·020
145	·069444	·029753	·010	·020	·076389	·031088	·010	·020
146	·068966	·029557	·010	·020	·075862	·030884	·010	·020
147	·068493	·029363	·010	·020	·075342	·030683	·010	·020
148	·068027	·029172	·010	·020	·074830	·030484	·010	·020
149	·067568	·028983	·010	·020	·074324	·030288	·010	·020
150	·067114	·028797	·010	·020	·073826	·030094	·010	·020

N = size of sample	$n=13$: $\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1, $\lambda_1 = 2·80$	P_2, $\lambda_2 = 2·40$	$n=14$: $\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1, $\lambda_1 = 2·79$	P_2, $\lambda_2 = 2·39$
101	·120000	·045504	·009	·019	·130000	·047092	·009	·019
102	·118812	·045088	·009$_5$	·019	·128713	·046665 −	·009	·019
103	·117647	·044680	·010	·019	·127451	·046245 +	·009	·019
104	·116505 −	·044279	·010	·019	·126214	·045833	·009	·019
105	·115385 −	·043885 −	·010	·019	·125000	·045428	·009	·019
106	·114286	·043498	·010	·019	·123810	·045030	·009	·019
107	·113208	·043117	·010	·019	·122642	·044639	·009	·019
108	·112150 −	·042744	·010	·019	·121495 +	·044254	·009	·019
109	·111111	·042376	·010	·019	·120370	·043876	·009	·019
110	·110092	·042015 −	·010	·019$_5$	·119266	·043505 −	·009	·019
111	·109091	·041660	·010	·020	·118182	·043139	·009	·019
112	·108108	·041311	·010	·020	·117117	·042780	·009	·019
113	·107143	·040967	·010	·020	·116071	·042426	·009	·019
114	·106195 −	·040629	·010	·020	·115044	·042078	·009	·019
115	·105263	·040297	·010	·020	·114035 −	·041736	·009$_5$	·019
116	·104348	·039970	·010	·020	·113043	·041400	·010	·019
117	·103448	·039648	·010	·020	·112069	·041068	·010	·019
118	·102564	·039332	·010	·020	·111111	·040742	·010	·019
119	·101695 −	·039020	·010	·020	·110169	·040421	·010	·019$_5$
120	·100840	·038713	·010	·020	·109244	·040105 +	·010	·020
121	·100000	·038411	·010	·020	·108333	·039794	·010	·020
122	·099174	·038114	·010	·020	·107438	·039488	·010	·020
123	·098361	·037821	·010	·020	·106557	·039186	·010	·020
124	·097561	·037532	·010	·020	·105691	·038889	·010	·020
125	·096774	·037248	·010	·020	·104839	·038596	·010	·020
126	·096000	·036969	·010	·020	·104000	·038308	·010	·020
127	·095238	·036693	·010	·020	·103175 −	·038023	·010	·020
128	·094488	·036421	·010	·020	·102362	·037743	·010	·020
129	·093750	·036154	·010	·020	·101563	·037467	·010	·020
130	·093023	·035890	·010	·020	·100775 +	·037195 +	·010	·020
131	·092308	·035630	·010	·020	·100000	·036927	·010	·020
132	·091603	·035374	·010	·020	·099237	·036663	·010	·020
133	·090909	·035121	·010	·020	·098485	·036403	·010	·020
134	·090226	·034872	·010	·020	·097744	·036146	·010	·020
135	·089552	·034627	·010	·020	·097015 −	·035893	·010	·020
136	·088889	·034385 −	·010	·020	·096296	·035643	·010	·020
137	·088235 +	·034146	·010	·020	·095588	·035397	·010	·020
138	·087591	·033910	·010	·020	·094891	·035154	·010	·020
139	·086957	·033678	·010	·020	·094203	·034914	·010	·020
140	·086331	·033449	·010	·020	·093525 +	·034677	·010	·020
141	·085714	·033223	·010	·020	·092857	·034444	·010	·020
142	·085106	·033000	·010	·020	·092199	·034214	·010	·020
143	·084507	·032780	·010	·020	·091549	·033987	·010	·020
144	·083916	·032563	·010	·020	·090909	·033763	·010	·020
145	·083333	·032348	·010	·020	·090278	·033542	·010	·020
146	·082759	·032137	·010	·020	·089655 +	·033323	·010	·020
147	·082192	·031928	·010	·020	·089041	·033108	·010	·020
148	·081633	·031722	·010	·020	·088435 +	·032895 −	·010	·020
149	·081081	·031519	·010	·020	·087838	·032685 −	·010	·020
150	·080537	·031318	·010	·020	·087248	·032477	·010	·020

N = size of sample	15				16			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·78	P_2 $\lambda_2=$ 2·38	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·77	P_2 $\lambda_2=$ 2·37
101	·140000	·048588	·009	·019	·150000	·050000	·009	·018$_5$
102	·138614	·048150 +	·009	·019	·148515 −	·049553	·009	·019
103	·137255 −	·047720	·009	·019	·147059	·049114	·009	·019
104	·135922	·047298	·009	·019	·145631	·048682	·009	·019
105	·134615 +	·046883	·009	·019	·144231	·048258	·009	·019
106	·133333	·046475 −	·009	·019	·142857	·047841	·009	·019
107	·132075 +	·046074	·009	·019	·141509	·047431	·009	·019
108	·130841	·045680	·009	·019	·140187	·047028	·009	·019
109	·129630	·045292	·009	·019	·138889	·046632	·009	·019
110	·128440	·044911	·009	·019	·137615 −	·046242	·009	·019
111	·127273	·044536	·009	·019	·136364	·045859	·009	·019
112	·126126	·044167	·009	·019	·135135 +	·045481	·009	·019
113	·125000	·043805 −	·009	·019	·133929	·045110	·009	·019
114	·123894	·043448	·009	·019	·132743	·044745 +	·009	·019
115	·122807	·043097	·009	·019	·131579	·044386	·009	·019
116	·121739	·042751	·009	·019	·130435 −	·044032	·009	·019
117	·120690	·042411	·009	·019	·129310	·043684	·009	·019
118	·119658	·042076	·009	·019	·128205 −	·043341	·009	·019
119	·118644	·041747	·009	·019	·127119	·043004	·009	·019
120	·117647	·041422	·009	·019	·126050 +	·042671	·009	·019
121	·116667	·041103	·009	·019	·125000	·042344	·009	·019
122	·115702	·040788	·009	·019	·123967	·042022	·009	·019
123	·114754	·040478	·009	·019	·122951	·041704	·009	·019
124	·113821	·040173	·009	·019	·121951	·041392	·009	·019
125	·112903	·039872	·009	·019	·120968	·041083	·009	·019
126	·112000	·039576	·009	·019	·120000	·040780	·009	·019
127	·111111	·039284	·009	·019	·119048	·040481	·009	·019
128	·110236	·038996	·009	·019	·118110	·040186	·009	·019
129	·109375	·038712	·009$_5$	·019$_5$	·117188	·039895 −	·009	·019
130	·108527	·038433	·010	·020	·116279	·039608	·009	·019
131	·107692	·038157	·010	·020	·115385 −	·039326	·009	·019
132	·106870	·037886	·010	·020	·114504	·039047	·009	·019
133	·106061	·037618	·010	·020	·113636	·038773	·009	·019
134	·105263	·037354	·010	·020	·112782	·038502	·009	·019
135	·104478	·037093	·010	·020	·111940	·038235 −	·009	·019
136	·103704	·036836	·010	·020	·111111	·037971	·009	·019
137	·102941	·036583	·010	·020	·110294	·037712	·009	·019
138	·102190	·036333	·010	·020	·109489	·037455 +	·009	·019
139	·101449	·036087	·010	·020	·108696	·037202	·010	·019
140	·100719	·035843	·010	·020	·107914	·036953	·010	·019
141	·100000	·035603	·010	·020	·107143	·036707	·010	·019
142	·099291	·035367	·010	·020	·106383	·036464	·010	·019
143	·098592	·035133	·010	·020	·105634	·036224	·010	·019
144	·097902	·034902	·010	·020	·104895 +	·035987	·010	·019
145	·097222	·034675 −	·010	·020	·104167	·035753	·010	·019$_5$
146	·096552	·034450 −	·010	·020	·103448	·035523	·010	·020
147	·095890	·034228	·010	·020	·102740	·035295 −	·010	·020
148	·095238	·034009	·010	·020	·102041	·035070	·010	·020
149	·094595 −	·033793	·010	·020	·101351	·034848	·010	·020
150	·093960	·033579	·010	·020	·100671	·034629	·010	·020

N = size of sample	17				18			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·76	P_2 $\lambda_2=$ 2·36	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·75	P_2 $\lambda_2=$ 2·35
101	·160000	·051335 +	·008	·018	·170000	·052599	·008	·018
102	·158416	·050880	·008	·018	·168317	·052136	·008	·018
103	·156863	·050432	·009	·018	·166667	·051681	·008	·018
104	·155340	·049992	·009	·018	·165049	·051234	·008	·018
105	·153846	·049560	·009	·018$_5$	·163462	·050794	·008	·018
106	·152381	·049135 −	·009	·019	·161905 −	·050362	·008	·018
107	·150943	·048717	·009	·019	·160377	·049936	·008$_5$	·018
108	·149533	·048306	·009	·019	·158879	·049518	·009	·018$_5$
109	·148148	·047901	·009	·019	·157407	·049107	·009	·019
110	·146789	·047504	·009	·019	·155963	·048702	·009	·019
111	·145455 −	·047113	·009	·019	·154545 +	·048304	·009	·019
112	·144144	·046728	·009	·019	·153153	·047912	·009	·019
113	·142857	·046349	·009	·019	·151786	·047526	·009	·019
114	·141593	·045976	·009	·019	·150442	·047146	·009	·019
115	·140351	·045609	·009	·019	·149123	·046773	·009	·019
116	·139130	·045248	·009	·019	·147826	·046405 −	·009	·019
117	·137931	·044893	·009	·019	·146552	·046042	·009	·019
118	·136752	·044543	·009	·019	·145299	·045686	·009	·019
119	·135593	·044198	·009	·019	·144068	·045334	·009	·019
120	·134454	·043859	·009	·019	·142857	·044988	·009	·019
121	·133333	·043524	·009	·019	·141667	·044647	·009	·019
122	·132231	·043195 −	·009	·019	·140496	·044312	·009	·019
123	·131148	·042870	·009	·019	·139344	·043981	·009	·019
124	·130081	·042551	·009	·019	·138211	·043655 −	·009	·019
125	·129032	·042236	·009	·019	·137097	·043334	·009	·019
126	·128000	·041925 +	·009	·019	·136000	·043017	·009	·019
127	·126984	·041619	·009	·019	·134921	·042705 −	·009	·019
128	·125984	·041318	·009	·019	·133858	·042397	·009	·019
129	·125000	·041021	·009	·019	·132813	·042094	·009	·019
130	·124031	·040728	·009	·019	·131783	·041795 −	·009	·019
131	·123077	·040439	·009	·019	·130769	·041500 +	·009	·019
132	·122137	·040154	·009	·019	·129771	·041209	·009	·019
133	·121212	·039873	·009	·019	·128788	·040922	·009	·019
134	·120301	·039596	·009	·019	·127820	·040640	·009	·019
135	·119403	·039323	·009	·019	·126866	·040361	·009	·019
136	·118519	·039053	·009	·019	·125926	·040085 +	·009	·019
137	·117647	·038787	·009	·019	·125000	·039814	·009	·019
138	·116788	·038525 −	·009	·019	·124088	·039546	·009	·019
139	·115942	·038266	·009	·019	·123188	·039282	·009	·019
140	·115108	·038010	·009	·019	·122302	·039021	·009	·019
141	·114286	·037758	·009	·019	·121429	·038763	·009	·019
142	·113475 +	·037510	·009	·019	·120567	·038509	·009	·019
143	·112676	·037264	·009	·019	·119718	·038258	·009	·019
144	·111888	·037022	·009	·019	·118881	·038011	·009	·019
145	·111111	·036782	·009	·019	·118056	·037766	·009	·019
146	·110345 −	·036546	·009	·019	·117241	·037525 −	·009	·019
147	·109589	·036313	·009	·020	·116438	·037286	·009	·019
148	·108844	·036083	·009	·020	·115646	·037051	·009	·019$_5$
149	·108108	·035855 +	·009	·020	·114865 −	·036819	·009	·020
150	·107383	·035631	·009	·020	·114094	·036589	·009	·020

N = size of sample	19				20			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·74	P_2 $\lambda_2=$ 2·34	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·73	P_2 $\lambda_2=$ 2·33
101	·180000	·053797	·008	·018	·190000	·054933	·008	·018
102	·178218	·053327	·008	·018	·188119	·054458	·008	·018
103	·176471	·052866	·008	·018	·186275 −	·053990	·008	·018
104	·174757	·052412	·008	·018	·184466	·053530	·008	·018
105	·173077	·051965 +	·008	·018	·182692	·053078	·008	·018
106	·171429	·051526	·008	·018	·180952	·052633	·008	·018
107	·169811	·051095 −	·008	·018	·179245 +	·052196	·008	·018
108	·168224	·050670	·008	·018	·177570	·051765 −	·008	·018
109	·166667	·050252	·008	·018	·175926	·051341	·008	·018
110	·165138	·049841	·008	·018$_5$	·174312	·050924	·008	·018
111	·163636	·049436	·008	·019	·172727	·050514	·008	·018$_5$
112	·162162	·049038	·008	·019	·171171	·050110	·008	·019
113	·160714	·048646	·008	·019	·169643	·049712	·008	·019
114	·159292	·048260	·008$_5$	·019	·168142	·049321	·008	·019
115	·157895 −	·047880	·009	·019	·166667	·048935 +	·008	·019
116	·156522	·047506	·009	·019	·165217	·048555 +	·008	·019
117	·155172	·047137	·009	·019	·163793	·048181	·008	·019
118	·153846	·046775 −	·009	·019	·162393	·047813	·008	·019
119	·152542	·046417	·009	·019	·161017	·047450 +	·008$_5$	·019
120	·151261	·046065 +	·009	·019	·159664	·047093	·009	·019
121	·150000	·045718	·009	·019	·158333	·046740	·009	·019
122	·148760	·045377	·009	·019	·157025 −	·046393	·009	·019
123	·147541	·045040	·009	·019	·155738	·046051	·009	·019
124	·146341	·044708	·009	·019	·154472	·045714	·009	·019
125	·145161	·044381	·009	·019	·153226	·045382	·009	·019
126	·144000	·044059	·009	·019	·152000	·045054	·009	·019
127	·142857	·043741	·009	·019	·150794	·044731	·009	·019
128	·141732	·043428	·009	·019	·149606	·044412	·009	·019
129	·140625	·043119	·009	·019	·148438	·044098	·009	·019
130	·139535 −	·042814	·009	·019	·147287	·043789	·009	·019
131	·138462	·042514	·009	·019	·146154	·043483	·009	·019
132	·137405 −	·042218	·009	·019	·145038	·043182	·009	·019
133	·136364	·041925 +	·009	·019	·143939	·042885 −	·009	·019
134	·135338	·041637	·009	·019	·142857	·042592	·009	·019
135	·134328	·041353	·009	·019	·141791	·042303	·009	·019
136	·133333	·041072	·009	·019	·140741	·042017	·009	·019
137	·132353	·040796	·009	·019	·139706	·041736	·009	·019
138	·131387	·040523	·009	·019	·138686	·041458	·009	·019
139	·130435 −	·040253	·009	·019	·137681	·041183	·009	·019
140	·129496	·039987	·009	·019	·136691	·040913	·009	·019
141	·128571	·039725 −	·009	·019	·135714	·040645 +	·009	·019
142	·127660	·039465 +	·009	·019	·134752	·040382	·009	·019
143	·126761	·039210	·009	·019	·133803	·040121	·009	·019
144	·125874	·038957	·009	·019	·132867	·039864	·009	·019
145	·125000	·038708	·009	·019	·131944	·039610	·009	·019
146	·124138	·038462	·009	·019	·131034	·039360	·009	·019
147	·123288	·038218	·009	·019	·130137	·039112	·009	·019
148	·122449	·037978	·009	·019	·129252	·038867	·009	·019
149	·121622	·037741	·009	·019	·128378	·038626	·009	·019
150	·120805 +	·037507	·009	·019$_5$	·127517	·038387	·009	·019$_5$

n = number of arrays

N = size of sample	3: $\bar{\eta}^2$	3: $\sigma_{\bar{\eta}^2}$	3: P_1 $\lambda_1 = 3·50$	3: P_2 $\lambda_2 = 2·94$	4: $\bar{\eta}^2$	4: $\sigma_{\bar{\eta}^2}$	4: P_1 $\lambda_1 = 3·20$	4: P_2 $\lambda_2 = 2·80$	5: $\bar{\eta}^2$	5: $\sigma_{\bar{\eta}^2}$	5: P_1 $\lambda_1 = 3·14$	5: P_2 $\lambda_2 = 2·68$
151	·013333	·013157	·011	·019	·020000	·016059	·012	·019	·026667	·018480	·011	·020
152	·013245 +	·013071	·011	·019	·019868	·015955 −	·012	·019	·026490	·018360	·011	·020
153	·013158	·012986	·011	·019	·019737	·015851	·012	·019	·026316	·018242	·011	·020
154	·013072	·012902	·011	·019	·019608	·015749	·012	·019	·026144	·018125 +	·011	·020
155	·012987	·012819	·011	·019	·019481	·015649	·012	·019	·025974	·018010	·011	·020
156	·012903	·012738	·011	·019	·019355 −	·015549	·012	·019	·025806	·017896	·011	·020
157	·012821	·012657	·011	·019	·019231	·015451	·012	·019	·025641	·017783	·011	·020
158	·012739	·012578	·011	·019	·019108	·015355 −	·012	·019	·025478	·017672	·011	·020
159	·012658	·012499	·011	·019	·018987	·015259	·012	·019	·025316	·017563	·011	·020
160	·012579	·012421	·011	·019	·018868	·015164	·012	·019	·025157	·017454	·011	·020
161	·012500	·012345 −	·011	·019	·018750	·015071	·012	·019	·025000	·017347	·011	·020
162	·012422	·012269	·011	·019	·018634	·014979	·012	·019	·024845 −	·017242	·011	·020
163	·012346	·012194	·011	·019	·018519	·014888	·012	·019	·024691	·017137	·011	·020
164	·012270	·012120	·011	·019	·018405 −	·014798	·012	·019	·024540	·017034	·011	·020
165	·012195 +	·012047	·011	·019	·018293	·014709	·012	·019	·024390	·016932	·011	·020
166	·012121	·011975 +	·011	·019	·018182	·014621	·012	·019	·024242	·016831	·011	·020
167	·012048	·011904	·011	·019	·018072	·014535 −	·012	·019	·024096	·016732	·011	·020
168	·011976	·011833	·011	·019	·017964	·014449	·012	·019	·023952	·016633	·011	·020
169	·011905 −	·011764	·011	·019	·017857	·014364	·012	·019	·023810	·016536	·011	·020
170	·011834	·011695 +	·011	·019	·017751	·014281	·012	·019	·023669	·016440	·011	·020
171	·011765 −	·011627	·011	·019	·017647	·014198	·012	·019	·023529	·016345 +	·011	·020
172	·011696	·011560	·011	·019	·017544	·014116	·012	·019	·023392	·016251	·011	·020
173	·011628	·011493	·011	·019	·017442	·014035 +	·012	·019	·023256	·016158	·011	·020
174	·011561	·011428	·011	·019	·017341	·013955 +	·012	·019	·023121	·016067	·011	·020
175	·011494	·011363	·011	·019	·017241	·013876	·012	·019	·022989	·015976	·011	·020
176	·011429	·011299	·011	·019	·017143	·013798	·012	·019	·022857	·015886	·011	·020
177	·011364	·011235 +	·011	·019	·017045 +	·013721	·012	·019	·022727	·015797	·011	·020
178	·011299	·011172	·011	·019	·016949	·013644	·012	·019	·022599	·015710	·011	·020
179	·011236	·011110	·011	·019	·016854	·013569	·012	·019	·022472	·015623	·011	·020
180	·011173	·011049	·011	·019	·016760	·013494	·012	·019	·022346	·015537	·011	·020
181	·011111	·010988	·011	·019	·016667	·013420	·012	·019	·022222	·015452	·011	·020
182	·011050 −	·010928	·011	·019	·016575 −	·013347	·012	·019	·022099	·015368	·011	·020
183	·010989	·010869	·011	·019	·016484	·013275 −	·012	·019	·021978	·015285 +	·011	·020
184	·010929	·010810	·011	·019	·016393	·013203	·012	·019	·021858	·015203	·011	·020
185	·010870	·010752	·011	·019	·016304	·013132	·012	·019	·021739	·015122	·011	·020
186	·010811	·010695 −	·011	·019	·016216	·013062	·012	·019	·021622	·015042	·011	·020
187	·010753	·010638	·011	·019	·016129	·012993	·012	·019	·021505 +	·014962	·011	·020
188	·010695 +	·010581	·011	·019	·016043	·012924	·012	·019	·021390	·014883	·011	·020
189	·010638	·010526	·011	·019	·015957	·012857	·012	·019	·021277	·014805 +	·011	·020
190	·010582	·010471	·011	·019	·015873	·012789	·012	·019	·021164	·014728	·011	·020
191	·010526	·010416	·011	·019	·015789	·012723	·012	·019	·021053	·014652	·011	·020
192	·010471	·010362	·011	·019	·015707	·012657	·012	·019	·020942	·014577	·011	·020
193	·010417	·010309	·011	·019	·015625	·012592	·012	·019	·020833	·014502	·011	·020
194	·010363	·010256	·011	·019	·015544	·012528	·012	·019	·020725 +	·014428	·011	·020
195	·010309	·010204	·011	·019	·015464	·012464	·012	·019	·020619	·014355 −	·011	·020
196	·010256	·010152	·011	·019	·015385 −	·012401	·012	·019	·020513	·014282	·011	·020
197	·010204	·010100	·011	·019	·015306	·012339	·012	·019	·020408	·014210	·011	·020
198	·010152	·010050 −	·011	·019	·015228	·012277	·012	·019	·020305 −	·014139	·011	·020
199	·010101	·009999	·011	·019	·015152	·012216	·012	·019	·020202	·014069	·011	·020
200	·010050 +	·009950 −	·011	·019	·015075 +	·012155 −	·012	·019	·020101	·013999	·011	·020

N = size of sample	6: $\bar{\eta}^2$	6: $\sigma_{\bar{\eta}^2}$	6: P_1 $\lambda_1 = 3·11$	6: P_2 $\lambda_2 = 2·63$	7: $\bar{\eta}^2$	7: $\sigma_{\bar{\eta}^2}$	7: P_1 $\lambda_1 = 3·08$	7: P_2 $\lambda_2 = 2·58$	8: $\bar{\eta}^2$	8: $\sigma_{\bar{\eta}^2}$	8: P_1 $\lambda_1 = 3·02$	8: P_2 $\lambda_2 = 2·54$
151	·033333	·020591	·010	·020	·040000	·022478	·010	·020	·046667	·024195 −	·010	·020
152	·033113	·020458	·010	·020	·039735 +	·022333	·010	·020	·046358	·024039	·010	·020
153	·032895 −	·020326	·010	·020	·039474	·022190	·010	·020	·046053	·023886	·010	·020
154	·032680	·020196	·010	·020	·039216	·022049	·010	·020	·045752	·023735 −	·010	·020
155	·032468	·020068	·010	·020	·038961	·021910	·010	·020	·045455 −	·023585 +	·010	·020
156	·032258	·019942	·010$_5$	·020	·038710	·021772	·010	·020	·045161	·023438	·010	·020
157	·032051	·019817	·011	·020	·038462	·021636	·010	·020	·044872	·023292	·010	·020
158	·031847	·019694	·011	·020	·038217	·021502	·010	·020	·044586	·023148	·010	·020
159	·031646	·019572	·011	·020	·037975 −	·021370	·010	·020	·044304	·023006	·010	·020
160	·031447	·019351	·011	·020	·037736	·021239	·010	·020	·044025 +	·022865 +	·010	·020
161	·031250	·019333	·011	·020	·037500	·021109	·010	·020	·043750	·022726	·010	·020
162	·031056	·019215 +	·011	·020	·037267	·020982	·010	·020	·043478	·022589	·010	·020
163	·030864	·019099	·011	·020	·037037	·020855 +	·010	·020	·043210	·022454	·010	·020
164	·030675 −	·018984	·011	·020	·036810	·020731	·010	·020	·042945 −	·022320	·010	·020
165	·030488	·018871	·011	·020	·036585 +	·020607	·010	·020	·042683	·022188	·010	·020
166	·030303	·018759	·011	·020	·036364	·020486	·010	·020	·042424	·022057	·010	·020
167	·030120	·018649	·011	·020	·036145 −	·020365 +	·010	·020	·042169	·021928	·010	·020
168	·029940	·018539	·011	·020	·035928	·020246	·010	·020	·041916	·021800	·010	·020
169	·029762	·018431	·011	·020	·035714	·020129	·010	·020	·041667	·021674	·010	·020
170	·029586	·018325 −	·011	·020	·035503	·020012	·010	·020	·041420	·021549	·010	·020
171	·029412	·018219	·011	·020	·035294	·019898	·010	·020	·041176	·021426	·010	·020
172	·029240	·018115 −	·011	·020	·035088	·019784	·010	·020	·040936	·021304	·010	·020
173	·029070	·018012	·011	·020	·034884	·019672	·010	·020	·040698	·021184	·010	·020
174	·028902	·017910	·011	·020	·034682	·019561	·010	·020	·040462	·021065 −	·010	·020
175	·028736	·017809	·011	·020	·034483	·019451	·010	·020	·040230	·020947	·010	·020
176	·028571	·017709	·011	·020	·034286	·019342	·010	·020	·040000	·020830	·010	·020
177	·028409	·017611	·011	·020	·034091	·019235 +	·010	·020	·039773	·020715 −	·010	·020
178	·028249	·017513	·011	·020	·033898	·019129	·010	·020	·039548	·020601	·010	·020
179	·028090	·017417	·011	·020	·033708	·019024	·010	·020	·039326	·020488	·010	·020
180	·027933	·017321	·011	·020	·033520	·018920	·010	·020	·039106	·020377	·010	·020
181	·027778	·017227	·011	·020	·033333	·018817	·010	·020	·038889	·020267	·010	·020
182	·027624	·017134	·011	·020	·033149	·018716	·010	·020	·038674	·020157	·010	·020
183	·027473	·017041	·011	·020	·032967	·018615 +	·010	·020	·038462	·020049	·010	·020
184	·027322	·016950 +	·011	·020	·032787	·018516	·010	·020	·038251	·019943	·010	·020
185	·027174	·016860	·011	·020	·032609	·018417	·010	·020	·038043	·019837	·010	·020
186	·027027	·016770	·011	·020	·032432	·018320	·010	·020	·037838	·019732	·010	·020
187	·026882	·016682	·011	·020	·032258	·018224	·010	·020	·037634	·019629	·010	·020
188	·026738	·016594	·011	·020	·032086	·018128	·010	·020	·037433	·019527	·010	·020
189	·026596	·016508	·011	·020	·031915 −	·018034	·010	·020	·037234	·019425 +	·010	·020
190	·026455 +	·016422	·011	·020	·031746	·017941	·010	·020	·037037	·019325 +	·010	·020
191	·026316	·016337	·011	·020	·031579	·017848	·010	·020	·036842	·019226	·010	·020
192	·026178	·016253	·011	·020	·031414	·017757	·010	·020	·036649	·019128	·010	·020
193	·026042	·016170	·011	·020	·031250	·017666	·010	·020	·036458	·019030	·010	·020
194	·025907	·016088	·011	·020	·031088	·017577	·010	·020	·036269	·018934	·010	·020
195	·025773	·016007	·011	·020	·030928	·017488	·010	·020	·036082	·018839	·010	·020
196	·025641	·015926	·011	·020	·030769	·017400	·010	·020	·035897	·018745 −	·010	·020
197	·025510	·015846	·011	·020	·030612	·017313	·010	·020	·035714	·018651	·010	·020
198	·025381	·015767	·011	·020	·030457	·017227	·010	·020	·035533	·018559	·010	·020
199	·025253	·015689	·011	·020	·030303	·017142	·010	·020	·035354	·018467	·010	·020
200	·025126	·015612	·011	·020	·030151	·017058	·010	·020	·035176	·018377	·010	·020

N = size of sample	9 $\bar{\eta}^2$	9 $\sigma_{\bar{\eta}^2}$	9 P_1 $\lambda_1=2{\cdot}96$	9 P_2 $\lambda_2=2{\cdot}50$	10 $\bar{\eta}^2$	10 $\sigma_{\bar{\eta}^2}$	10 P_1 $\lambda_1=2{\cdot}92$	10 P_2 $\lambda_2=2{\cdot}47$
151	·053333	·025775 −	·010	·020	·060000	·027242	·010	·020
152	·052980	·025610	·010	·020	·059603	·027068	·010	·020
153	·052632	·025447	·010	·020	·059211	·026897	·010	·020
154	·052288	·025286	·010	·020	·058824	·026728	·010	·020
155	·051948	·025128	·010	·020	·058442	·026561	·010	·020
156	·051613	·024971	·010	·020	·058065 −	·026396	·010	·020
157	·051282	·024816	·010	·020	·057692	·026233	·010	·020
158	·050955 +	·024664	·010	·020	·057325 −	·026072	·010	·020
159	·050633	·024513	·010	·020	·056962	·025913	·010	·020
160	·050314	·024363	·010	·020	·056604	·025756	·010	·020
161	·050000	·024216	·010	·020	·056250	·025600	·010	·020
162	·049689	·024071	·010	·020	·055901	·025447	·010	·020
163	·049383	·023927	·010	·020	·055556	·025296	·010	·020
164	·049080	·023785 −	·010	·020	·055215 −	·025146	·010	·020
165	·048780	·023644	·010	·020	·054878	·024998	·010	·020
166	·048485 −	·023505 +	·010	·020	·054545 +	·024852	·010	·020
167	·048193	·023368	·010	·020	·054217	·024707	·010	·020
168	·047904	·023233	·010	·020	·053892	·024564	·010	·020
169	·047619	·023099	·010	·020	·053571	·024423	·010	·020
170	·047337	·022966	·010	·020	·053254	·024284	·010	·020
171	·047059	·022835 +	·010	·020	·052941	·024145 +	·010	·020
172	·046784	·022706	·010	·020	·052632	·024009	·010	·020
173	·046512	·022578	·010	·020	·052326	·023874	·010	·020
174	·046243	·022451	·010	·020	·052023	·023741	·010	·020
175	·045977	·022326	·010	·020	·051724	·023609	·010	·020
176	·045714	·022202	·010	·020	·051429	·023478	·010	·020
177	·045455 −	·022080	·010	·020	·051136	·023349	·010	·020
178	·045198	·021959	·010	·020	·050847	·023222	·010	·020
179	·044944	·021839	·010	·020	·050562	·023095 +	·010	·020
180	·044693	·021720	·010	·020	·050279	·022970	·010	·020
181	·044444	·021603	·010	·020	·050000	·022847	·010	·020
182	·044199	·021487	·010	·020	·049724	·022725 −	·010	·020
183	·043956	·021372	·010	·020	·049451	·022604	·010	·020
184	·043716	·021259	·010	·020	·049180	·022484	·010	·020
185	·043478	·021147	·010	·020	·048913	·022366	·010	·020
186	·043243	·021036	·010	·020	·048649	·022248	·010	·020
187	·043011	·020926	·010	·020	·048387	·022133	·010	·020
188	·042781	·020817	·010	·020	·048128	·022018	·010	·020
189	·042553	·020709	·010	·020	·047872	·021904	·010	·020
190	·042328	·020603	·010	·020	·047619	·021792	·010	·020
191	·042105 +	·020497	·010	·020	·047368	·021681	·010	·020
192	·041885 −	·020393	·010	·020	·047120	·021570	·010	·020
193	·041667	·020289	·010	·020	·046875	·021461	·010	·020
194	·041451	·020187	·010	·020	·046632	·021354	·010	·020
195	·041237	·020086	·010	·020	·046392	·021247	·010	·020
196	·041026	·019985 +	·010	·020	·046154	·021141	·010	·020
197	·040816	·019886	·010	·020	·045918	·021036	·010	·020
198	·040609	·019788	·010	·020	·045685 +	·020933	·010	·020
199	·040404	·019690	·010	·020	·045455 −	·020830	·010	·020
200	·040201	·019594	·010	·020	·045226	·020728	·010	·020

N = size of sample	11 $\bar{\eta}^2$	11 $\sigma_{\bar{\eta}^2}$	11 P_1 $\lambda_1=2{\cdot}88$	11 P_2 $\lambda_2=2{\cdot}44$	12 $\bar{\eta}^2$	12 $\sigma_{\bar{\eta}^2}$	12 P_1 $\lambda_1=2{\cdot}84$	12 P_2 $\lambda_2=2{\cdot}42$
151	·066667	·028613	·010	·020	·073333	·029902	·010	·020
152	·066225 +	·028432	·010	·020	·072848	·029713	·010	·020
153	·065789	·028252	·010	·020	·072368	·029527	·010	·020
154	·065359	·028075 +	·010	·020	·071895 +	·029343	·010	·020
155	·064935 +	·027901	·010	·020	·071429	·029161	·010	·020
156	·064516	·027728	·010	·020	·070968	·028981	·010	·020
157	·064103	·027557	·010	·020	·070513	·028803	·010	·020
158	·063694	·027389	·010	·020	·070064	·028628	·010	·020
159	·063291	·027223	·010	·020	·069620	·028455 −	·010	·020
160	·062893	·027058	·010	·020	·069182	·028283	·010	·020
161	·062500	·026896	·010	·020	·068750	·028114	·010	·020
162	·062112	·026735 +	·010	·020	·068323	·027947	·010	·020
163	·061728	·026577	·010	·020	·067901	·027782	·010	·020
164	·061350 −	·026420	·010	·020	·067485 −	·027619	·010	·020
165	·060976	·026265 +	·010	·020	·067073	·027457	·010	·020
166	·060606	·026112	·010	·020	·066667	·027298	·010	·020
167	·060241	·025961	·010	·020	·066265 +	·027140	·010	·020
168	·059880	·025811	·010	·020	·065868	·026984	·010	·020
169	·059524	·025663	·010	·020	·065476	·026830	·010	·020
170	·059172	·025517	·010	·020	·065089	·026678	·010	·020
171	·058824	·025372	·010	·020	·064706	·026528	·010	·020
172	·058480	·025230	·010	·020	·064327	·026379	·010	·020
173	·058140	·025088	·010	·020	·063953	·026231	·010	·020
174	·057803	·024948	·010	·020	·063584	·026086	·010	·020
175	·057471	·024810	·010	·020	·063218	·025942	·010	·020
176	·057143	·024674	·010	·020	·062857	·025799	·010	·020
177	·056818	·024538	·010	·020	·062500	·025658	·010	·020
178	·056497	·024405 −	·010	·020	·062147	·025519	·010	·020
179	·056180	·024272	·010	·020	·061798	·025381	·010	·020
180	·055866	·024142	·010	·020	·061453	·025245 −	·010	·021
181	·055556	·024012	·010	·020	·061111	·025110	·010	·021
182	·055249	·023884	·010	·020	·060773	·024977	·010	·021
183	·054945 +	·023757	·010	·021	·060440	·024844	·010	·021
184	·054645 −	·023632	·010	·021	·060109	·024714	·010	·021
185	·054348	·023508	·010	·021	·059783	·024584	·010	·021
186	·054054	·023385 +	·010	·021	·059459	·024456	·010	·021
187	·053763	·023264	·010	·021	·059140	·024330	·010	·021
188	·053476	·023144	·010	·021	·058824	·024204	·010	·021
189	·053191	·023025 −	·010	·021	·058511	·024080	·010	·021
190	·052910	·022907	·010	·021	·058201	·023958	·010	·021
191	·052632	·022790	·010	·021	·057895 −	·023836	·010	·021
192	·052356	·022675 −	·010	·021	·057592	·023716	·010	·021
193	·052083	·022560	·010	·021	·057292	·023597	·010	·021
194	·051813	·022447	·010	·021	·056995 −	·023479	·010	·021
195	·051546	·022335 +	·010	·021	·056701	·023362	·010	·021
196	·051282	·022225 −	·010	·021	·056410	·023246	·010	·021
197	·051020	·022115 −	·010	·021	·056122	·023132	·010	·021
198	·050761	·022006	·010	·021	·055838	·023018	·010	·021
199	·050505 +	·021898	·010	·021	·055556	·022906	·010	·021
200	·050251	·021792	·010	·021	·055276	·022795 −	·010	·021

N = size of sample	13 $\bar{\eta}^2$	13 $\sigma_{\bar{\eta}^2}$	13 P_1 $\lambda_1=2{\cdot}80$	13 P_2 $\lambda_2=2{\cdot}40$	14 $\bar{\eta}^2$	14 $\sigma_{\bar{\eta}^2}$	14 P_1 $\lambda_1=2{\cdot}79$	14 P_2 $\lambda_2=2{\cdot}39$
151	·080000	·031119	·010	·020	·086667	·032273	·010	·020
152	·079470	·030924	·010	·020	·086093	·032070	·010	·020
153	·078947	·030730	·010	·020	·085526	·031871	·010	·020
154	·078431	·030539	·010	·020	·084967	·031673	·010	·020
155	·077922	·030351	·010	·020	·084416	·031478	·010	·020
156	·077419	·030164	·010	·020	·083871	·031286	·010	·020
157	·076923	·029980	·010	·020	·083333	·031096	·010	·020
158	·076433	·029798	·010	·020	·082803	·030908	·010	·020
159	·075949	·029619	·010	·020	·082278	·030722	·010	·020
160	·075472	·029441	·010	·020	·081761	·030539	·010	·020
161	·075000	·029266	·010	·020	·081250	·030358	·010	·020
162	·074534	·029092	·010	·020	·080745 +	·030179	·010	·020
163	·074074	·028921	·010	·020	·080247	·030001	·010	·020
164	·073620	·028752	·010	·020	·079755 −	·029827	·010	·020
165	·073171	·028584	·010	·020	·079268	·029654	·010	·020
166	·072727	·028419	·010	·020	·078788	·029483	·010	·020
167	·072289	·028256	·010	·020	·078313	·029314	·010	·020
168	·071856	·028094	·010	·020	·077844	·029147	·010	·020
169	·071429	·027934	·010	·020	·077381	·028981	·010	·020
170	·071006	·027776	·010	·020	·076923	·028818	·010	·020
171	·070588	·027620	·010	·020	·076471	·028657	·010	·020
172	·070175 +	·027465 +	·010	·020	·076023	·028497	·010	·020
173	·069767	·027313	·010	·020	·075581	·028339	·010	·020
174	·069364	·027161	·010	·020	·075145 −	·028183	·010	·020
175	·068966	·027012	·011	·020	·074713	·028028	·010	·020
176	·068571	·026864	·011	·020	·074286	·027875 +	·010	·020
177	·068182	·026718	·011	·020	·073864	·027724	·010	·020
178	·067797	·026573	·011	·020	·073446	·027575 −	·010	·020
179	·067416	·026430	·011	·020	·073034	·027427	·010	·020
180	·067039	·026289	·011	·020	·072626	·027280	·010	·020
181	·066667	·026149	·011	·020	·072222	·027135 +	·010	·020
182	·066298	·026010	·011	·020	·071823	·026992	·010	·020
183	·065934	·025873	·011	·021	·071429	·026850 +	·010	·020
184	·065574	·025738	·011	·021	·071038	·026710	·010	·020
185	·065217	·025603	·011	·021	·070652	·026571	·010	·020
186	·064865 −	·025470	·011	·021	·070270	·026434	·010	·020
187	·064516	·025339	·011	·021	·069892	·026298	·010	·020
188	·064171	·025209	·011	·021	·069519	·026163	·010	·020
189	·063830	·025080	·011	·021	·069149	·026030	·010	·020
190	·063492	·024952	·011	·021	·068783	·025898	·010	·020
191	·063158	·024826	·011	·021	·068421	·025767	·010	·020
192	·062827	·024701	·011	·021	·068063	·025638	·010	·020
193	·062500	·024578	·011	·021	·067708	·025510	·010	·020
194	·062176	·024455 +	·011	·021	·067358	·025383	·010	·020
195	·061856	·024334	·011	·021	·067010	·025258	·010	·020
196	·061538	·024214	·011	·021	·066667	·025134	·010	·020
197	·061224	·024095 −	·011	·021	·066327	·025011	·010	·020
198	·060914	·023977	·011	·021	·065990	·024889	·010	·020
199	·060606	·023861	·011	·021	·065657	·024768	·010	·020
200	·060302	·023745 +	·011	·021	·065327	·024649	·010	·020

n = number of arrays

N = size of sample	15: $\bar{\eta}^2$	15: $\sigma_{\bar{\eta}^2}$	15: P_1 λ_1 = 2·78	15: P_2 λ_2 = 2·38	16: $\bar{\eta}^2$	16: $\sigma_{\bar{\eta}^2}$	16: P_1 λ_1 = 2·77	16: P_2 λ_2 = 2·37	17: $\bar{\eta}^2$	17: $\sigma_{\bar{\eta}^2}$	17: P_1 λ_1 = 2·76	17: P_2 λ_2 = 2·36
151	·093333	·033368	·010	·020	·100000	·034412	·010	·020	·106667	·035409	·009	·020
152	·092715 +	·033160	·010	·020	·099338	·034199	·010	·020	·105960	·035190	·009	·020
153	·092105 +	·032954	·010	·020	·098684	·033987	·010	·020	·105263	·034974	·009	·020
154	·091503	·032751	·010	·020	·098039	·033779	·010	·020	·104575 −	·034760	·009	·020
155	·090909	·032551	·010	·020	·097403	·033573	·010	·020	·103896	·034549	·010	·020
156	·090323	·032352	·010	·020	·096774	·033369	·010	·020	·103226	·034340	·010	·020
157	·089744	·032157	·010	·020	·096154	·033168	·010	·020	·102564	·034134	·010	·020
158	·089172	·031963	·010	·020	·095541	·032969	·010	·020	·101911	·033930	·010	·020
159	·088608	·031772	·010	·020	·094937	·032773	·010	·020	·101266	·033729	·010	·020
160	·088050 +	·031583	·010	·020	·094340	·032579	·010	·020	·100629	·033530	·010	·020
161	·087500	·031396	·010	·020	·093750	·032387	·010	·020	·100000	·033333	·010	·020
162	·086957	·031212	·010	·020	·093168	·032197	·010	·020	·099379	·033139	·010	·020
163	·086420	·031029	·010	·020	·092593	·032010	·010	·020	·098765 −	·032947	·010	·020
164	·085890	·030849	·010	·020	·092025 −	·031825 −	·010	·020	·098160	·032757	·010	·020
165	·085366	·030671	·010	·020	·091463	·031641	·010	·020	·097561	·032569	·010	·020
166	·084848	·030495 −	·010	·020	·090909	·031460	·010	·020	·096970	·032384	·010	·020
167	·084337	·030321	·010	·020	·090361	·031281	·010	·020	·096386	·032200	·010	·020
168	·083832	·030148	·010	·020	·089820	·031104	·010	·020	·095808	·032019	·010	·020
169	·083333	·029978	·010	·020	·089286	·030929	·010	·020	·095238	·031839	·010	·020
170	·082840	·029810	·010	·020	·088757	·030756	·010	·020	·094675 −	·031662	·010	·020
171	·082353	·029643	·010	·020	·088235 +	·030585 +	·010	·020	·094118	·031486	·010	·020
172	·081871	·029479	·010	·020	·087719	·030416	·010	·020	·093567	·031313	·010	·020
173	·081395 +	·029316	·010	·020	·087209	·030249	·010	·020	·093023	·031141	·010	·020
174	·080925 −	·029155 −	·010	·020	·086705 +	·030083	·010	·020	·092486	·030971	·010	·020
175	·080460	·028996	·010	·020	·086207	·029919	·010	·020	·091954	·030803	·010	·020
176	·080000	·028838	·010	·020	·085714	·029757	·010	·020	·091429	·030637	·010	·020
177	·079545 +	·028682	·010	·020	·085227	·029597	·010	·020	·090909	·030473	·010	·020
178	·079096	·028528	·010	·020	·084746	·029439	·010	·020	·090395 +	·030310	·010	·020
179	·078652	·028376	·010	·020	·084270	·029282	·010	·020	·089888	·030149	·010	·020
180	·078212	·028225 −	·010	·020	·083799	·029127	·010	·020	·089385 +	·029990	·010	·020
181	·077778	·028075 +	·010	·020	·083333	·028973	·010	·020	·088889	·029832	·010	·020
182	·077348	·027928	·010	·020	·082873	·028821	·010	·020	·088398	·029677	·010	·020
183	·076923	·027781	·010	·020	·082418	·028671	·010	·020	·087912	·029522	·010	·020
184	·076503	·027637	·010	·020	·081967	·028522	·010	·020	·087432	·029369	·010	·020
185	·076087	·027493	·010	·020	·081522	·028375 −	·010	·020	·086957	·029218	·010	·020
186	·075676	·027352	·010	·020	·081081	·028229	·010	·020	·086486	·029069	·010	·020
187	·075269	·027211	·010	·020	·080645 +	·028085 −	·010	·020	·086022	·028921	·010	·020
188	·074866	·027073	·010	·020	·080214	·027942	·010	·020	·085561	·028774	·010	·020
189	·074468	·026935 +	·010	·020	·079787	·027800	·010	·020	·085106	·028629	·010	·020
190	·074074	·026799	·010	·020	·079365 +	·027660	·010	·020	·084656	·028485 +	·010	·020
191	·073684	·026664	·010	·020	·078947	·027522	·010	·020	·084211	·028343	·010	·020
192	·073298	·026531	·010	·020	·078534	·027385 −	·010	·020	·083770	·028202	·010	·020
193	·072917	·026399	·010	·020	·078125	·027249	·010	·020	·083333	·028063	·010	·020
194	·072539	·026268	·010	·020	·077720	·027114	·010	·020	·082902	·027925 −	·010	·020
195	·072165 −	·026139	·010	·020	·077320	·026981	·010	·020	·082474	·027788	·010	·020
196	·071795 −	·026011	·010	·020	·076923	·026849	·010	·020	·082051	·027652	·010	·020
197	·071429	·025884	·010	·020	·076531	·026718	·010	·020	·081633	·027518	·010	·020
198	·071066	·025758	·010	·020	·076142	·026589	·010	·020	·081218	·027386	·010	·020
199	·070707	·025633	·010	·020	·075758	·026461	·010	·020	·080808	·027254	·010	·020
200	·070352	·025510	·010	·020	·075377	·026334	·010	·020	·080402	·027124	·010	·020

N = size of sample	18: $\bar{\eta}^2$	18: $\sigma_{\bar{\eta}^2}$	18: P_1 λ_1 = 2·75	18: P_2 λ_2 = 2·35	19: $\bar{\eta}^2$	19: $\sigma_{\bar{\eta}^2}$	19: P_1 λ_1 = 2·74	19: P_2 λ_2 = 2·34	20: $\bar{\eta}^2$	20: $\sigma_{\bar{\eta}^2}$	20: P_1 λ_1 = 2·73	20: P_2 λ_2 = 2·33
151	·113333	·036362	·009	·020	·120000	·037276	·009	·020	·126667	·038152	·009	·020
152	·112583	·036138	·009	·020	·119205 −	·037047	·009	·020	·125828	·037919	·009	·020
153	·111842	·035917	·009	·020	·118421	·036821	·009	·020	·125000	·037689	·009	·020
154	·111111	·035699	·009	·020	·117647	·036598	·009	·020	·124183	·037462	·009	·020
155	·110390	·035483	·009	·020	·116883	·036378	·009	·020	·123377	·037237	·009	·020
156	·109677	·035269	·009	·020	·116129	·036160	·009	·020	·122581	·037015 +	·009	·020
157	·108974	·035059	·009	·020	·115385 −	·035945 +	·009	·020	·121795 −	·036796	·009	·020
158	·108280	·034850 +	·009	·020	·114650 −	·035732	·009	·020	·121019	·036579	·009	·020
159	·107595 −	·034644	·009	·020	·113924	·035522	·009	·020	·120253	·036365 −	·009	·020
160	·106918	·034441	·009	·020	·113208	·035314	·009	·020	·119497	·036153	·009	·020
161	·106250	·034240	·009	·020	·112500	·035109	·009	·020	·118750	·035944	·009	·020
162	·105590	·034041	·009	·020	·111801	·034906	·009	·020	·118012	·035737	·009	·020
163	·104938	·033844	·009	·020	·111111	·034705 +	·009	·020	·117284	·035532	·009	·020
164	·104294	·033650 +	·009	·020	·110429	·034507	·009	·020	·116564	·035330	·009	·020
165	·103659	·033458	·009	·020	·109756	·034311	·009	·020	·115854	·035130	·009	·020
166	·103030	·033268	·009$_5$	·020	·109091	·034117	·009	·020	·115152	·034932	·009	·020
167	·102410	·033080	·010	·020	·108434	·033925 −	·009	·020	·114458	·034737	·009	·020
168	·101796	·032895 −	·010	·020	·107784	·033735 −	·009	·020	·113772	·034543	·009	·020
169	·101190	·032711	·010	·020	·107143	·033548	·009	·020	·113095 +	·034352	·009	·020
170	·100592	·032529	·010	·020	·106509	·033362	·009	·020	·112426	·034163	·009	·020
171	·100000	·032350 −	·010	·020	·105882	·033179	·009	·020	·111765 −	·033976	·009	·020
172	·099415 +	·032172	·010	·020	·105263	·032997	·009	·020	·111111	·033790	·009	·020
173	·098837	·031996	·010	·020	·104651	·032818	·009	·020	·110465 +	·033607	·009	·020
174	·098266	·031823	·010	·020	·104046	·032640	·009	·020	·109827	·033426	·009	·020
175	·097701	·031651	·010	·020	·103448	·032464	·009	·020	·109195 +	·033247	·009	·020
176	·097143	·031481	·010	·020	·102857	·032291	·009	·020	·108571	·033070	·009	·020
177	·096591	·031312	·010	·020	·102273	·032119	·009	·020	·107955 −	·032894	·009	·020
178	·096045 +	·031146	·010	·020	·101695 −	·031948	·009	·020	·107345 −	·032721	·009	·020
179	·095506	·030981	·010	·020	·101124	·031780	·009	·020	·106742	·032549	·009	·020
180	·094972	·030818	·010	·020	·100559	·031613	·009$_5$	·020	·106145 +	·032379	·009	·020
181	·094444	·030657	·010	·020	·100000	·031449	·010	·020	·105556	·032210	·009	·020
182	·093923	·030497	·010	·020	·099448	·031285 +	·010	·020	·104972	·032044	·009	·020
183	·093407	·030339	·010	·020	·098901	·031124	·010	·020	·104396	·031879	·009	·020
184	·092896	·030183	·010	·020	·098361	·030964	·010	·020	·103825 +	·031716	·009	·020
185	·092391	·030028	·010	·020	·097826	·030806	·010	·020	·103261	·031554	·009	·020
186	·091892	·029875 −	·010	·020	·097297	·030649	·010	·020	·102703	·031394	·009	·020
187	·091398	·029723	·010	·020	·096774	·030494	·010	·020	·102151	·031236	·009	·020
188	·090909	·029573	·010	·020	·096257	·030340	·010	·020	·101604	·031079	·009$_5$	·020
189	·090426	·029424	·010	·020	·095745 −	·030188	·010	·020	·101064	·030924	·010	·020
190	·089947	·029277	·010	·020	·095238	·030038	·010	·020	·100529	·030771	·010	·020
191	·089474	·029131	·010	·020	·094737	·029889	·010	·020	·100000	·030619	·010	·020
192	·089005 +	·028987	·010	·020	·094241	·029741	·010	·020	·099476	·030468	·010	·020
193	·088542	·028844	·010	·020	·093750	·029595 +	·010	·020	·098958	·030319	·010	·020
194	·088083	·028703	·010	·020	·093264	·029451	·010	·020	·098446	·030171	·010	·020
195	·087629	·028563	·010	·020	·092784	·029307	·010	·020	·097938	·030025 −	·010	·020
196	·087179	·028424	·010	·020	·092308	·029166	·010	·020	·097436	·029880	·010	·020
197	·086735 −	·028286	·010	·020	·091837	·029025 +	·010	·020	·096939	·029736	·010	·020
198	·086294	·028150 +	·010	·020	·091371	·028886	·010	·020	·096447	·029594	·010	·020
199	·085859	·028016	·010	·020	·090909	·028748	·010	·020	·095960	·029454	·010	·020
200	·085427	·027882	·010	·020	·090452	·028611	·010	·020	·095477	·029314	·010	·020

$N=$ size of sample	3				4				5			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}50$	P_2 $\lambda_2=2{\cdot}94$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}20$	P_2 $\lambda_2=2{\cdot}80$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}14$	P_2 $\lambda_2=2{\cdot}68$
201	·010000	·009900	·011	·019	·015000	·012095 −	·012	·019	·020000	·013931	·011	·020
202	·009950 +	·009852	·011	·019	·014925 +	·012036	·012	·019$_5$	·019900	·013862	·011	·020
203	·009901	·009803	·011	·019	·014851	·011977	·012	·020	·019802	·013795 −	·011	·020
204	·009852	·009756	·011	·019	·014778	·011918	·012	·020	·019704	·013728	·011	·020
205	·009804	·009708	·011	·019	·014706	·011861	·012	·020	·019608	·013661	·011	·020
206	·009756	·009661	·011	·019	·014634	·011804	·012	·020	·019512	·013596	·011	·020
207	·009709	·009615 −	·011	·019	·014563	·011747	·012	·020	·019417	·013531	·011	·020
208	·009662	·009569	·011	·019	·014493	·011691	·012	·020	·019324	·013466	·011	·020
209	·009615 +	·009523	·011	·019	·014423	·011635 +	·012	·020	·019231	·013403	·011	·020
210	·009569	·009478	·011	·019	·014354	·011580	·012	·020	·019139	·013339	·011	·020
211	·009524	·009434	·011	·019	·014286	·011526	·012	·020	·019048	·013277	·011	·020
212	·009479	·009389	·011	·019	·014218	·011472	·012	·020	·018957	·013215 −	·011	·020
213	·009434	·009345 +	·011	·019	·014151	·011418	·012	·020	·018868	·013153	·011	·020
214	·009390	·009302	·011	·019	·014085 −	·011365 +	·012	·020	·018779	·013092	·011	·020
215	·009346	·009259	·011	·019	·014019	·011313	·012	·020	·018692	·013032	·011	·020
216	·009302	·009216	·011	·019	·013953	·011261	·012	·020	·018605 −	·012972	·011	·020
217	·009259	·009174	·011	·019	·013889	·011209	·012	·020	·018519	·012913	·011	·020
218	·009217	·009132	·011	·019	·013825 −	·011158	·012	·020	·018433	·012854	·011	·020
219	·009174	·009091	·011	·019	·013761	·011108	·012	·020	·018349	·012796	·011	·020
220	·009132	·009049	·011	·019	·013699	·011058	·012	·020	·018265 −	·012739	·011	·020
221	·009091	·009009	·011	·019	·013636	·011008	·012	·020	·018182	·012682	·011	·020
222	·009050 −	·008968	·011	·019	·013575 −	·010959	·012	·020	·018100	·012625 −	·011	·020
223	·009009	·008928	·011	·019	·013514	·010910	·012	·020	·018018	·012569	·011	·020
224	·008969	·008889	·011	·019	·013453	·010862	·012	·020	·017937	·012513	·011	·020
225	·008929	·008849	·011	·019	·013393	·010814	·012	·020	·017857	·012458	·011	·020
226	·008889	·008810	·011	·019	·013333	·010766	·012	·020	·017778	·012404	·011	·020
227	·008850 −	·008772	·011	·019	·013274	·010719	·012	·020	·017699	·012349	·011	·020
228	·008811	·008733	·011	·019	·013216	·010672	·012	·020	·017621	·012296	·011	·020
229	·008772	·008695 +	·011	·019	·013158	·010626	·012	·020	·017544	·012243	·011	·020
230	·008734	·008658	·011	·019	·013100	·010580	·012	·020	·017467	·012190	·011	·020
231	·008696	·008620	·011	·019	·013043	·010535 −	·012	·020	·017391	·012137	·011	·020
232	·008658	·008583	·011	·019	·012987	·010489	·012	·020	·017316	·012086	·011	·020
233	·008621	·008547	·011	·019	·012931	·010445 −	·012	·020	·017241	·012034	·011	·020
234	·008584	·008510	·011	·019	·012876	·010400	·012	·020	·017167	·011983	·011	·020
235	·008547	·008474	·011	·019	·012821	·010356	·012	·020	·017094	·011933	·011	·020
236	·008511	·008439	·011	·019	·012766	·010313	·012	·020	·017021	·011883	·011	·020
237	·008475	·008403	·011	·019	·012712	·010270	·012	·020	·016949	·011833	·011	·020
238	·008439	·008368	·011	·019	·012658	·010227	·012	·020	·016878	·011784	·011	·020
239	·008403	·008333	·011	·019	·012605 +	·010184	·012	·020	·016807	·011735 −	·011	·020
240	·008368	·008298	·011	·019	·012552	·010142	·012	·020	·016736	·011686	·011	·020
241	·008333	·008264	·011	·019	·012500	·010100	·012	·020	·016667	·011638	·011	·020
242	·008299	·008230	·011	·019	·012448	·010059	·012	·020	·016598	·011590	·011$_5$	·020
243	·008264	·008196	·011	·019	·012397	·010018	·012	·020	·016529	·011543	·012	·020
244	·008230	·008163	·011	·019	·012346	·009977	·012	·020	·016461	·011496	·012	·020
245	·008197	·008130	·011	·019	·012295 +	·009936	·012	·020	·016393	·011450 −	·012	·020
246	·008163	·008097	·011	·019	·012245 −	·009896	·012	·020	·016327	·011404	·012	·020
247	·008130	·008064	·011	·019	·012195 +	·009856	·012	·020	·016260	·011358	·012	·020
248	·008097	·008032	·011	·019	·012146	·009817	·012	·020	·016194	·011312	·012	·020
249	·008065 −	·008000	·011	·019	·012097	·009778	·012	·020	·016129	·011267	·012	·020
250	·008032	·007968	·011	·019	·012048	·009739	·012	·020	·016064	·011223	·012	·020

$N=$ size of sample	6				7				8			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}11$	P_2 $\lambda_2=2{\cdot}63$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}08$	P_2 $\lambda_2=2{\cdot}58$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}02$	P_2 $\lambda_2=2{\cdot}54$
201	·025000	·015535 +	·011	·020	·030000	·016974	·010	·020	·035000	·018287	·010	·020
202	·024876	·015459	·011	·020	·029851	·016891	·010	·020	·034826	·018198	·010	·020
203	·024752	·015384	·011	·020	·029703	·016809	·010	·020	·034653	·018110	·010	·020
204	·024631	·015309	·011	·020	·029557	·016728	·010	·020	·034483	·018023	·010	·020
205	·024510	·015236	·011	·020	·029412	·016648	·010	·020	·034314	·017936	·010	·020
206	·024390	·015163	·011	·020	·029268	·016568	·010	·020	·034146	·017851	·010	·020
207	·024272	·015090	·011	·020	·029126	·016489	·010	·020	·033981	·017766	·010	·020
208	·024155 −	·015019	·011	·020	·028986	·016411	·010	·020	·033816	·017682	·010	·020
209	·024038	·014948	·011	·020	·028846	·016334	·010	·020	·033654	·017599	·010	·020
210	·023923	·014877	·011	·020	·028708	·016257	·010	·020	·033493	·017517	·010	·020
211	·023810	·014808	·011	·020	·028571	·016181	·010	·020	·033333	·017435 +	·010	·020
212	·023697	·014739	·011	·020	·028436	·016106	·010	·020	·033175 +	·017354	·010	·020
213	·023585 −	·014670	·011	·020	·028302	·016032	·010	·020	·033019	·017274	·010	·020
214	·023474	·014603	·011	·020	·028169	·015958	·010	·020	·032864	·017195 −	·010	·020
215	·023364	·014536	·011	·020	·028037	·015885 −	·010	·020	·032710	·017116	·010	·020
216	·023256	·014469	·011	·020	·027907	·015812	·010	·020	·032558	·017038	·010	·020
217	·023148	·014403	·011	·020	·027778	·015740	·010	·020	·032407	·016961	·010	·020
218	·023041	·014338	·011	·020	·027650 −	·015669	·010	·020	·032258	·016885 −	·010	·020
219	·022936	·014273	·011	·020	·027523	·015599	·010	·020	·032110	·016809	·010	·020
220	·022831	·014209	·011	·020	·027397	·015529	·010	·020	·031963	·016734	·010	·020
221	·022727	·014146	·011	·020	·027273	·015460	·010	·020	·031818	·016659	·010	·020
222	·022624	·014083	·011	·020	·027149	·015391	·010	·020	·031674	·016585 +	·010	·020
223	·022523	·014020	·011	·020	·027027	·015323	·010	·020	·031532	·016512	·010	·020
224	·022422	·013958	·011	·020	·026906	·015255 +	·010	·020	·031390	·016440	·010	·020
225	·022321	·013897	·011	·020	·026786	·015189	·010	·020	·031250 −	·016368	·010	·020
226	·022222	·013836	·011	·020	·026667	·015122	·010	·020	·031111	·016297	·010	·020
227	·022124	·013776	·011	·020	·026549	·015057	·010	·020	·030973	·016226	·010	·020
228	·022026	·013716	·011	·020	·026432	·014991	·010	·020	·030837	·016156	·010	·020
229	·021930	·013657	·011	·020	·026316	·014927	·010	·020	·030702	·016086	·010	·020
230	·021834	·013598	·011	·020	·026201	·014863	·010	·020	·030568	·016018	·010	·020
231	·021739	·013540	·011	·020	·026087	·014799	·010	·020	·030435 −	·015949	·010	·020
232	·021645 +	·013482	·011	·020	·025974	·014736	·010	·020	·030303	·015882	·010	·020
233	·021552	·013425 +	·011	·020	·025862	·014674	·010	·020	·030172	·015815 −	·010	·020
234	·021459	·013368	·011	·020	·025751	·014612	·010	·020	·030043	·015748	·010	·020
235	·021368	·013312	·011	·020	·025641	·014551	·010	·020	·029915 −	·015682	·010	·020
236	·021277	·013256	·011	·020	·025532	·014490	·010	·020	·029787	·015617	·010	·020
237	·021186	·013201	·011	·020	·025424	·014430	·010	·020	·029661	·015552	·010	·020
238	·021097	·013146	·011	·020	·025316	·014370	·010	·020	·029536	·015487	·010	·020
239	·021008	·013092	·011	·020	·025210	·014310	·010	·020	·029412	·015424	·020	·020
240	·020921	·013038	·011	·020	·025105 −	·014252	·010	·020	·029289	·015360	·010	·020
241	·020833	·012984	·011	·020	·025000	·014193	·010	·020	·029167	·015298	·010	·020
242	·020747	·012931	·011	·020	·024896	·014135 +	·010	·020	·029046	·015235 +	·010	·020
243	·020661	·012878	·011	·020	·024793	·014078	·010	·020	·028926	·015174	·010	·020
244	·020576	·012826	·011	·020	·024691	·014021	·010	·020	·028807	·015112	·010	·020
245	·020492	·012774	·011	·020	·024590	·013964	·010	·020	·028689	·015052	·010	·020
246	·020408	·012723	·011	·020	·024490	·013908	·010	·020	·028571	·014991	·010	·020
247	·020325 +	·012672	·011	·020	·024390	·013853	·010	·020	·028455 +	·014931	·010	·020
248	·020243	·012622	·011	·020	·024291	·013798	·010	·020	·028340	·014872	·010	·020
249	·020161	·012571	·011	·020	·024194	·013743	·010	·020	·028226	·014813	·010	·020
250	·020080	·012522	·011	·020	·024096	·013689	·010	·020	·028112	·014755 −	·010	·020

n = number of arrays

N = size of sample	9: $\bar{\eta}^2$	9: $\sigma_{\bar{\eta}^2}$	9: P_1 $\lambda_1 = 2{\cdot}96$	9: P_2 $\lambda_2 = 2{\cdot}50$	10: $\bar{\eta}^2$	10: $\sigma_{\bar{\eta}^2}$	10: P_1 $\lambda_1 = 2{\cdot}92$	10: P_2 $\lambda_2 = 2{\cdot}47$
201	·040000	·019499	·010	·020	·045000	·020628	·010	·020
202	·039801	·019404	·010	·020	·044776	·020528	·010	·020$_5$
203	·039604	·019311	·010	·020	·044554	·020429	·010	·021
204	·039409	·019218	·010	·020	·044335 −	·020331	·010	·021
205	·039216	·019126	·010	·020	·044118	·020234	·010	·021
206	·039024	·019035 +	·010	·020	·043902	·020138	·010	·021
207	·038835 −	·018945 −	·010	·020	·043689	·020043	·010	·021
208	·038647	·018856	·010	·020	·043478	·019949	·010	·021
209	·038462	·018767	·010	·020	·043269	·019856	·010	·021
210	·038278	·018680	·010	·020	·043062	·019763	·010	·021
211	·038095 +	·018593	·010	·020	·042857	·019672	·010	·021
212	·037915 −	·018507	·010	·020	·042654	·019581	·010	·021
213	·037736	·018422	·010	·020	·042453	·019491	·010	·021
214	·037559	·018337	·010	·020	·042254	·019402	·010	·021
215	·037383	·018254	·010	·020	·042056	·019314	·010	·021
216	·037209	·018171	·010	·020	·041860	·019227	·010	·021
217	·037037	·018089	·010	·020	·041667	·019140	·010	·021
218	·036866	·018007	·010	·020	·041475 −	·019054	·010	·021
219	·036697	·017927	·010	·020	·041284	·018969	·010	·021
220	·036530	·017847	·010	·020	·041096	·018885 −	·010	·021
221	·036364	·017768	·010	·020	·040909	·018801	·010	·021
222	·036199	·017689	·010	·020	·040724	·018718	·010	·021
223	·036036	·017611	·010	·020	·040541	·018636	·010	·021
224	·035874	·017534	·010	·020	·040359	·018554	·010	·021
225	·035714	·017458	·010	·020	·040179	·018474	·010	·021
226	·035556	·017382	·010	·020	·040000	·018394	·010	·021
227	·035398	·017307	·010	·020	·039823	·018314	·010	·021
228	·035242	·017232	·010	·020	·039648	·018236	·010	·021
229	·035088	·017158	·010	·020	·039474	·018158	·010	·021
230	·034934	·017085 −	·010	·020	·039301	·018080	·010	·021
231	·034783	·017012	·010	·020	·039130	·018004	·010	·021
232	·034632	·016940	·010	·020	·038961	·017928	·010	·021
233	·034483	·016869	·010	·020	·038793	·017852	·010	·021
234	·034335 −	·016798	·010	·020	·038627	·017777	·010	·021
235	·034188	·016728	·010	·020	·038462	·017703	·010	·021
236	·034043	·016658	·010	·020$_5$	·038298	·017630	·010	·021
237	·033898	·016589	·010	·021	·038136	·017557	·010	·021
238	·033755 +	·016521	·010	·021	·037975 −	·017485 −	·010	·021
239	·033613	·016453	·010	·021	·037815 +	·017413	·010	·021
240	·033473	·016385 +	·010	·021	·037657	·017342	·010	·021
241	·033333	·016319	·010	·021	·037500	·017271	·010	·021
242	·033195 +	·016252	·010	·021	·037344	·017201	·010	·021
243	·033058	·016187	·010	·021	·037190	·017132	·010	·021
244	·032922	·016121	·010	·021	·037037	·017063	·010	·021
245	·032787	·016057	·010	·021	·036885 +	·016995 −	·010	·021
246	·032653	·015993	·010	·021	·036735 −	·016927	·010	·021
247	·032520	·015929	·010	·021	·036585 +	·016860	·010	·021
248	·032389	·015866	·010	·021	·036437	·016793	·010	·021
249	·032258	·015803	·010	·021	·036290	·016727	·010	·021
250	·032129	·015741	·010	·021	·036145 −	·016661	·010	·021

N = size of sample	11: $\bar{\eta}^2$	11: $\sigma_{\bar{\eta}^2}$	11: P_1 $\lambda_1 = 2{\cdot}88$	11: P_2 $\lambda_2 = 2{\cdot}44$	12: $\bar{\eta}^2$	12: $\sigma_{\bar{\eta}^2}$	12: P_1 $\lambda_1 = 2{\cdot}84$	12: P_2 $\lambda_2 = 2{\cdot}42$
201	·050000	·021686	·010	·021	·055000	·022685 −	·010	·021
202	·049751	·021582	·010	·021	·054726	·022576	·010	·021
203	·049505 −	·021478	·010	·021	·054455 −	·022468	·010	·021
204	·049261	·021376	·010	·021	·054187	·022361	·010	·021
205	·049020	·021274	·010	·021	·053922	·022255 −	·010$_5$	·021
206	·048780	·021174	·010	·021	·053659	·022150 −	·011	·021
207	·048544	·021074	·010	·021	·053398	·022046	·011	·021
208	·048309	·020975 +	·010	·021	·053140	·021943	·011	·021
209	·048077	·020877	·010	·021	·052885 −	·021841	·011	·021
210	·047847	·020780	·010	·021	·052632	·021740	·011	·021
211	·047619	·020684	·010	·021	·052381	·021640	·011	·021
212	·047393	·020589	·010	·021	·052133	·021540	·011	·021
213	·047170	·020495 +	·010	·021	·051887	·021442	·011	·021
214	·046948	·020402	·010	·021	·051643	·021345 −	·011	·021
215	·046729	·020309	·010	·021	·051402	·021248	·011	·021
216	·046512	·020217	·010	·021	·051163	·021152	·011	·021
217	·046296	·020126	·010	·021	·050926	·021057	·011	·021
218	·046083	·020036	·010	·021	·050691	·020963	·011	·021
219	·045872	·019947	·010	·021	·050459	·020870	·011	·021
220	·045662	·019859	·010	·021	·050228	·020778	·011	·021
221	·045455 −	·019771	·010	·021	·050000	·020686	·011	·021
222	·045249	·019684	·010	·021	·049774	·020596	·011	·021
223	·045045 +	·019598	·010	·021	·049550 −	·020506	·011	·021
224	·044843	·019512	·010	·021	·049327	·020417	·011	·021
225	·044643	·019428	·010	·021	·049107	·020328	·011	·021
226	·044444	·019344	·010	·021	·048889	·020241	·011	·021
227	·044248	·019260	·010	·021	·048673	·020154	·011	·021
228	·044053	·019178	·010	·021	·048458	·020068	·011	·021
229	·043860	·019096	·010	·021	·048246	·019982	·011	·021
230	·043668	·019015 −	·010	·021	·048035 −	·019897	·011	·021
231	·043478	·018935 −	·010	·021	·047826	·019814	·011	·021
232	·043290	·018855 −	·010	·021	·047619	·019730	·011	·021
233	·043103	·018776	·010	·021	·047414	·019648	·011	·021
234	·042918	·018697	·010	·021	·047210	·019566	·011	·021
235	·042735 +	·018619	·010	·021	·047009	·019485 −	·011	·021
236	·042553	·018542	·010	·021	·046809	·019404	·011	·021
237	·042373	·018466	·010	·021	·046610	·019324	·011	·021
238	·042194	·018390	·010	·021	·046414	·019245 +	·011	·021
239	·042017	·018315 −	·010	·021	·046218	·019166	·011	·021
240	·041841	·018240	·010	·021	·046025 +	·019089	·011	·021
241	·041667	·018166	·010	·021	·045833	·019011	·011	·021
242	·041494	·018093	·010	·021	·045643	·018935 −	·011	·021
243	·041322	·018020	·010	·021	·045455 −	·018858	·011	·021
244	·041152	·017947	·010	·021	·045267	·018783	·011	·021
245	·040984	·017876	·010	·021	·045082	·018708	·011	·021
246	·040816	·017805 −	·010	·021	·044898	·018634	·011	·021
247	·040650 +	·017734	·010	·021	·044715 +	·018560	·011	·021
248	·040486	·017664	·010	·021	·044534	·018487	·011	·021
249	·040323	·017595 −	·010	·021	·044355 −	·018415 −	·011	·021
250	·040161	·017526	·010	·021	·044177	·018343	·011	·021

N = size of sample	13: $\bar{\eta}^2$	13: $\sigma_{\bar{\eta}^2}$	13: P_1 $\lambda_1 = 2{\cdot}80$	13: P_2 $\lambda_2 = 2{\cdot}40$	14: $\bar{\eta}^2$	14: $\sigma_{\bar{\eta}^2}$	14: P_1 $\lambda_1 = 2{\cdot}79$	14: P_2 $\lambda_2 = 2{\cdot}39$
201	·060000	·023631	·011	·021	·065000	·024530	·010	·021
202	·059701	·023518	·011	·021	·064677	·024413	·010	·021
203	·059406	·023405 +	·011	·021	·064356	·024297	·010	·021
204	·059113	·023294	·011	·021	·064039	·024182	·010	·021
205	·058824	·023184	·011	·021	·063725 +	·024068	·010	·021
206	·058537	·023075 +	·011	·021	·063415 −	·023955 +	·010	·021
207	·058252	·022967	·011	·021	·063107	·023843	·010	·021
208	·057971	·022860	·011	·021	·062802	·023733	·010	·021
209	·057692	·022754	·011	·021	·062500	·023623	·010	·021
210	·057416	·022649	·011	·021	·062201	·023514	·010	·021
211	·057143	·022545 +	·011	·021	·061905 −	·023406	·010	·021
212	·056872	·022442	·011	·021	·061611	·023300	·010	·021
213	·056604	·022340	·011	·021	·061321	·023194	·010	·021
214	·056338	·022238	·011	·021	·061033	·023089	·010	·021
215	·056075 −	·022138	·011	·021	·060748	·022985 −	·010	·021
216	·055814	·022039	·011	·021	·060465 +	·022882	·010$_5$	·021
217	·055556	·021940	·011	·021	·060185 +	·022780	·011	·021
218	·055300	·021842	·011	·021	·059908	·022679	·011	·021
219	·055046	·021746	·011	·021	·059633	·022579	·011	·021
220	·054795 −	·021650 −	·011	·021	·059361	·022479	·011	·021
221	·054545 +	·021555 −	·011	·021	·059091	·022381	·011	·021
222	·054299	·021460	·011	·021	·058824	·022283	·011	·021
223	·054054	·021367	·011	·021	·058559	·022186	·011	·021
224	·053812	·021274	·011	·021	·058296	·022090	·011	·021
225	·053571	·021182	·011	·021	·058036	·021995 +	·011	·021
226	·053333	·021091	·011	·021	·057778	·021901	·011	·021
227	·053097	·021001	·011	·021	·057522	·021807	·011	·021
228	·052863	·020911	·011	·021	·057269	·021715 −	·011	·021
229	·052632	·020823	·011	·021	·057018	·021623	·011	·021
230	·052402	·020735 −	·011	·021	·056769	·021531	·011	·021
231	·052174	·020647	·011	·021	·056522	·021441	·011	·021
232	·051948	·020561	·011	·021	·056277	·021351	·011	·021
233	·051724	·020475 −	·011	·021	·056034	·021262	·011	·021
234	·051502	·020390	·011	·021	·055794	·021174	·011	·021
235	·051282	·020305 +	·011	·021	·055556	·021087	·011	·021
236	·051064	·020222	·011	·021	·055319	·021000	·011	·021
237	·050847	·020139	·011	·021	·055085 −	·020914	·011	·021
238	·050633	·020056	·011	·021	·054852	·020829	·011	·021
239	·050420	·019975 −	·011	·021	·054622	·020744	·011	·021
240	·050209	·019894	·011	·021	·054393	·020660	·011	·021
241	·050000	·019813	·011	·021	·054167	·020577	·011	·021
242	·049793	·019733	·011	·021	·053942	·020494	·011	·021
243	·049587	·019654	·011	·021	·053719	·020412	·011	·021
244	·049383	·019576	·011	·021	·053498	·020331	·011	·021
245	·049180	·019498	·011	·021	·053279	·020250 +	·011	·021
246	·048980	·019421	·011	·021	·053061	·020170	·011	·021
247	·048780	·019344	·011	·021	·052846	·020091	·011	·021
248	·048583	·019268	·011	·021	·052632	·020012	·011	·021
249	·048387	·019193	·011	·021	·052419	·019934	·011	·021
250	·048193	·019118	·011	·021	·052209	·019857	·011	·021

N = size of sample	15				16			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}78$	P_2 $\lambda_2 = 2{\cdot}38$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}77$	P_2 $\lambda_2 = 2{\cdot}37$
201	·070000	·025388	·010	·020	·075000	·026208	·010	·020
202	·069652	·025267	·010	·020	·074627	·026084	·010	·020
203	·069307	·025147	·010	·020	·074257	·025961	·010	·020
204	·068966	·025029	·010	·020	·073892	·025838	·010	·020
205	·068627	·024911	·010	·020	·073529	·025717	·010	·020
206	·068293	·024795 −	·010	·020	·073171	·025598	·010	·020
207	·067961	·024679	·010	·020	·072816	·025479	·010	·020
208	·067633	·024565 −	·010	·020	·072464	·025361	·010	·020
209	·067308	·024452	·010	·020	·072115 +	·025244	·010	·020
210	·066986	·024339	·010	·020	·071770	·025129	·010	·020
211	·066667	·024228	·010	·020	·071429	·025014	·010	·020
212	·066351	·024118	·010	·020	·071090	·024901	·010	·020
213	·066038	·024009	·010	·020	·070755 −	·024789	·010	·020
214	·065728	·023900	·010	·020	·070423	·024677	·010	·020
215	·065421	·023793	·010	·020	·070093	·024567	·010	·020
216	·065116	·023687	·010	·020	·069767	·024457	·010	·020
217	·064815 −	·023582	·010	·020	·069444	·024349	·010	·020
218	·064516	·023477	·010	·020	·069124	·024241	·010	·020
219	·064220	·023374	·010	·020	·068807	·024135 −	·010	·020
220	·063927	·023271	·010	·020	·068493	·024029	·010	·020
221	·063636	·023169	·010	·020	·068182	·023924	·010	·020
222	·063348	·023069	·010	·020	·067873	·023820	·010	·020
223	·063063	·022969	·010	·020	·067568	·023717	·010	·020
224	·062780	·022869	·010	·020	·067265 −	·023615 +	·010	·020
225	·062500	·022771	·010	·020	·066964	·023514	·010	·020
226	·062222	·022674	·010	·020$_5$	·066667	·023414	·010	·020
227	·061947	·022577	·010	·021	·066372	·023314	·010	·020
228	·061674	·022481	·010	·021	·066079	·023216	·010	·020
229	·061404	·022387	·010	·021	·065789	·023118	·010	·020
230	·061135 +	·022292	·010	·021	·065502	·023021	·010	·020
231	·060870	·022199	·010	·021	·065217	·022925 −	·010	·020
232	·060606	·022106	·010	·021	·064935 +	·022830	·010	·020
233	·060345 −	·022015 −	·010	·021	·064655 +	·022735 −	·010	·020
234	·060086	·021924	·010	·021	·064378	·022641	·010	·020
235	·059829	·021833	·010	·021	·064103	·022548	·010	·020
236	·059574	·021744	·010	·021	·063830	·022456	·010	·020
237	·059322	·021655 −	·010	·021	·063559	·022364	·010	·020
238	·059072	·021567	·010	·021	·063291	·022274	·010	·020
239	·058824	·021479	·010	·021	·063025 +	·022184	·010	·020
240	·058577	·021393	·010	·021	·062762	·022094	·010	·020$_5$
241	·058333	·021307	·010	·021	·062500	·022006	·010	·021
242	·058091	·021221	·010	·021	·062241	·021918	·010	·021
243	·057851	·021137	·010	·021	·061983	·021830	·010	·021
244	·057613	·021053	·010	·021	·061728	·021744	·010	·021
245	·057377	·020969	·010	·021	·061475 +	·021658	·010	·021
246	·057143	·020887	·010	·021	·061224	·021573	·010	·021
247	·056911	·020805 −	·010	·021	·060976	·021488	·010	·021
248	·056680	·020723	·010	·021	·060729	·021405 −	·010	·021
249	·056452	·020643	·010	·021	·060484	·021321	·010	·021
250	·056225 −	·020563	·010	·021	·060241	·021239	·010	·021

N = size of sample	17				18			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}76$	P_2 $\lambda_2 = 2{\cdot}36$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}75$	P_2 $\lambda_2 = 2{\cdot}35$
201	·080000	·026995 −	·010	·020	·085000	·027750 −	·010	·020
202	·079602	·026867	·010	·020	·084577	·027619	·010	·020
203	·079208	·026740	·010	·020	·084158	·027489	·010	·020
204	·078818	·026615 −	·010	·020	·083744	·027360	·010	·020
205	·078431	·026490	·010	·020	·083333	·027233	·010	·020
206	·078049	·026367	·010	·020	·082927	·027107	·010	·020
207	·077670	·026245 +	·010	·020	·082524	·026982	·010	·020
208	·077295 −	·026125 −	·010	·020	·082126	·026858	·010	·020
209	·076923	·026005 −	·010	·020	·081731	·026735 +	·010	·020
210	·076555 +	·025886	·010	·020	·081340	·026614	·010	·020
211	·076190	·025768	·010	·020	·080952	·026493	·010	·020
212	·075829	·025652	·010	·020	·080569	·026374	·010	·020
213	·075472	·025536	·010	·020	·080189	·026255 +	·010	·020
214	·075117	·025422	·010	·020	·079812	·026138	·010	·020
215	·074766	·025309	·010	·020	·079439	·026021	·010	·020
216	·074419	·025196	·010	·020	·079070	·025906	·010	·020
217	·074074	·025085 −	·010	·020	·078704	·025792	·010	·020
218	·073733	·024974	·010	·020	·078341	·025679	·010	·020
219	·073394	·024865 −	·010	·020	·077982	·025566	·010	·020
220	·073059	·024756	·010	·020	·077626	·025455 +	·010	·020
221	·072727	·024649	·010	·020	·077273	·025345 −	·010	·020
222	·072398	·024542	·010	·020	·076923	·025235 +	·010	·020
223	·072072	·024436	·010	·020	·076577	·025127	·010	·020
224	·071749	·024331	·010	·020	·076233	·025019	·010	·020
225	·071429	·024227	·010	·020	·075893	·024913	·010	·020
226	·071111	·024124	·010	·020	·075556	·024807	·010	·020
227	·070796	·024022	·010	·020	·075221	·024702	·010	·020
228	·070485 −	·023921	·010	·020	·074890	·024598	·010	·020
229	·070175 +	·023820	·010	·020	·074561	·024495 +	·010	·020
230	·069869	·023720	·010	·020	·074236	·024393	·010	·020
231	·069565 +	·023622	·010	·020	·073913	·024292	·010	·020
232	·069264	·023524	·010	·020	·073593	·024191	·010	·020
233	·068966	·023426	·010	·020	·073276	·024091	·010	·020
234	·068670	·023330	·010	·020	·072961	·023993	·010	·020
235	·068376	·023234	·010	·020	·072650 −	·023894	·010	·020
236	·068085 +	·023140	·010	·020	·072340	·023797	·010	·020
237	·067797	·023045 +	·010	·020	·072034	·023701	·010	·020
238	·067511	·022952	·010	·020	·071730	·023605 −	·010	·020
239	·067227	·022860	·010	·020	·071429	·023510	·010	·020
240	·066946	·022768	·010	·020	·071130	·023416	·010	·020
241	·066667	·022677	·010	·020	·070833	·023322	·010	·020
242	·066390	·022586	·010	·020	·070539	·023230	·010	·020
243	·066116	·022497	·010	·020	·070248	·023138	·010	·020
244	·065844	·022408	·010	·020	·069959	·023046	·010	·020
245	·065574	·022320	·010	·020	·069672	·022956	·010	·020
246	·065306	·022232	·010	·020	·069388	·022866	·010	·020
247	·065041	·022145 +	·010	·020	·069106	·022777	·010	·020
248	·064777	·022059	·010	·020	·068826	·022689	·010	·020
249	·064516	·021973	·010	·020	·068548	·022601	·010	·020
250	·064257	·021889	·010	·020	·068273	·022514	·010	·020

N = size of sample	19				20			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}74$	P_2 $\lambda_2 = 2{\cdot}34$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}73$	P_2 $\lambda_2 = 2{\cdot}33$
201	·090000	·028476	·010	·020	·095000	·029176	·010	·020
202	·089552	·028342	·010	·020	·094527	·029039	·010	·020
203	·089109	·028209	·010	·020	·094059	·028904	·010	·020
204	·088670	·028078	·010	·020	·093596	·028769	·010	·020
205	·088235 +	·027948	·010	·020	·093137	·028636	·010	·020
206	·087805 −	·027818	·010	·020	·092683	·028504	·010	·020
207	·087379	·027691	·010	·020	·092233	·028374	·010	·020
208	·086957	·027564	·010	·020	·091787	·028244	·010	·020
209	·086538	·027438	·010	·020	·091346	·028116	·010	·020
210	·086124	·027314	·010	·020	·090909	·027989	·010	·020
211	·085714	·027190	·010	·020	·090476	·027863	·010	·020
212	·085308	·027068	·010	·020	·090047	·027738	·010	·020
213	·084906	·026947	·010	·020	·089623	·027614	·010	·020
214	·084507	·026827	·010	·020	·089202	·027491	·010	·020
215	·084112	·026708	·010	·020	·088785 +	·027370	·010	·020
216	·083721	·026590	·010	·020	·088372	·027249	·010	·020
217	·083333	·026473	·010	·020	·087963	·027130	·010	·020
218	·082949	·026357	·010	·020	·087558	·027011	·010	·020
219	·082569	·026242	·010	·020	·087156	·026894	·010	·020
220	·082192	·026128	·010	·020	·086758	·026777	·010	·020
221	·081818	·026015 +	·010	·020	·086364	·026662	·010	·020
222	·081448	·025903	·010	·020	·085973	·026547	·010	·020
223	·081081	·025792	·010	·020	·085586	·026434	·010	·020
224	·080717	·025682	·010	·020	·085202	·026322	·010	·020
225	·080357	·025573	·010	·020	·084821	·026210	·010	·020
226	·080000	·025465 −	·010	·020	·084444	·026099	·010	·020
227	·079646	·025358	·010	·020	·084071	·025990	·010	·020
228	·079295 +	·025251	·010	·020	·083700	·025881	·010	·020
229	·078947	·025146	·010	·020	·083333	·025773	·010	·020
230	·078603	·025041	·010	·020	·082969	·025666	·010	·020
231	·078261	·024937	·010	·020	·082609	·025560	·010	·020
232	·077922	·024834	·010	·020	·082251	·025455 −	·010	·020
233	·077586	·024732	·010	·020	·081897	·025350 +	·010	·020
234	·077253	·024631	·010	·020	·081545 +	·025247	·010	·020
235	·076923	·024530	·010	·020	·081197	·025144	·010	·020
236	·076596	·024431	·010	·020	·080851	·025042	·010	·020
237	·076271	·024332	·010	·020	·080508	·024941	·010	·020
238	·075949	·024234	·010	·020	·080169	·024841	·010	·020
239	·075630	·024137	·010	·020	·079832	·024742	·010	·020
240	·075314	·024040	·010	·020	·079498	·024643	·010	·020
241	·075000	·023945 −	·010	·020	·079167	·024545 +	·010	·020
242	·074689	·023850 −	·010	·020	·078838	·024448	·010	·020
243	·074380	·023756	·010	·020	·078512	·024352	·010	·020
244	·074074	·023662	·010	·020	·078189	·024256	·010	·020
245	·073770	·023569	·010	·020	·077869	·024162	·010	·020
246	·073469	·023477	·010	·020	·077551	·024068	·010	·020
247	·073171	·023386	·010	·020	·077236	·023974	·010	·020
248	·072874	·023296	·010	·020	·076923	·023882	·010	·020
249	·072581	·023206	·010	·020	·076613	·023790	·010	·020
250	·072289	·023116	·010	·020	·076305 +	·023698	·010	·020

n = number of arrays

N = size of sample	3: $\bar{\eta}^2$	3: $\sigma_{\bar{\eta}^2}$	3: P_1 λ_1 = 3·50	3: P_2 λ_2 = 2·94	4: $\bar{\eta}^2$	4: $\sigma_{\bar{\eta}^2}$	4: P_1 λ_1 = 3·20	4: P_2 λ_2 = 2·80	5: $\bar{\eta}^2$	5: $\sigma_{\bar{\eta}^2}$	5: P_1 λ_1 = 3·14	5: P_2 λ_2 = 2·68
251	·008000	·007936	·011	·019	·012000	·009700	·012	·020	·016000	·011178	·012	·020
252	·007968	·007905 −	·011	·019	·011952	·009662	·012	·020	·015936	·011134	·012	·020
253	·007937	·007874	·011	·019	·011905 −	·009624	·012	·020	·015873	·011091	·012	·020
254	·007905 +	·007843	·011	·019	·011858	·009586	·012	·020	·015810	·011047	·012	·020
255	·007874	·007812	·011	·019	·011810	·009549	·012	·020	·015748	·011004	·012	·020
256	·007843	·007782	·011	·019	·011765 −	·009512	·012	·020	·015686	·010962	·012	·020
257	·007813	·007752	·011	·019	·011719	·009475 −	·012	·020	·015625	·010919	·012	·020
258	·007782	·007722	·011	·019	·011673	·009439	·012	·020	·015564	·010877	·012	·020
259	·007752	·007692	·011	·019	·011628	·009402	·012	·020	·015504	·010836	·012	·020
260	·007722	·007663	·011	·019	·011583	·009366	·012	·020	·015444	·010794	·012	·020
261	·007692	·007633	·011	·019	·011538	·009331	·012	·020	·015385 −	·010753	·012	·020
262	·007663	·007604	·011	·019	·011494	·009295 +	·012	·020	·015326	·010713	·012	·020
263	·007634	·007576	·011	·019	·011450 +	·009260	·012	·020	·015267	·010672	·012	·020
264	·007605 −	·007547	·011	·019	·011407	·009225 +	·012	·020	·015209	·010632	·012	·020
265	·007576	·007519	·011	·019	·011364	·009191	·012	·020	·015152	·010592	·012	·020
266	·007547	·007490	·011	·019	·011321	·009156	·012	·020	·015094	·010553	·012	·020
267	·007519	·007462	·011	·019	·011278	·009122	·012	·020	·015038	·010513	·012	·020
268	·007491	·007435 −	·011	·019	·011236	·009088	·012	·020	·014981	·010475 −	·012	·020
269	·007463	·007407	·011	·019	·011194	·009055 −	·012	·020	·014925 −	·010436	·012	·020
270	·007435 −	·007380	·011	·019	·011152	·009022	·012	·020	·014870	·010398	·012	·020
271	·007407	·007353	·011	·019	·011111	·008988	·012	·020	·014815 −	·010359	·012	·020
272	·007380	·007326	·011	·019	·011070	·008956	·012	·020	·014760	·010322	·012	·020
273	·007353	·007299	·011	·019	·011029	·008923	·012	·020	·014706	·010284	·012	·020
274	·007326	·007273	·011	·019	·010989	·008891	·012	·020	·014652	·010247	·012	·020
275	·007299	·007246	·011	·019	·010949	·008858	·012	·020	·014599	·010210	·012	·020
276	·007273	·007220	·011	·019	·010909	·008826	·012	·020	·014545 +	·010173	·012	·020
277	·007246	·007194	·011	·019	·010870	·008795 −	·012	·020	·014493	·010137	·012	·020
278	·007220	·007168	·011	·019	·010830	·008763	·012	·020	·014440	·010101	·012	·020
279	·007194	·007143	·011	·019	·010791	·008732	·012	·020	·014388	·010065 −	·012	·020
280	·007168	·007117	·011	·019	·010753	·008701	·012	·020	·014337	·010029	·012	·020
281	·007143	·007092	·011	·019	·010714	·008670	·012	·020	·014286	·009993	·012	·020
282	·007117	·007067	·011	·019	·010676	·008640	·012	·020	·014235 −	·009958	·012	·020
283	·007092	·007042	·011	·019	·010638	·008609	·012	·020	·014184	·009923	·012	·020
284	·007067	·007017	·011	·019	·010601	·008579	·012	·020	·014134	·009889	·012	·020
285	·007042	·006993	·011	·019	·010563	·008549	·012	·020	·014085 −	·009854	·012	·020
286	·007018	·006968	·011	·019	·010526	·008520	·012	·020	·014035 +	·009820	·012	·020
287	·006993	·006944	·011	·019	·010490	·008490	·012	·020	·013986	·009786	·012	·020
288	·006969	·006920	·011	·019	·010453	·008461	·012	·020	·013937	·009752	·012	·020
289	·006944	·006896	·011	·019	·010417	·008432	·012	·020	·013889	·009719	·012	·020
290	·006920	·006873	·011	·019	·010381	·008403	·012	·020	·013841	·009686	·012	·020
291	·006897	·006849	·011	·019	·010345 −	·008374	·012	·020	·013793	·009652	·012	·020
292	·006873	·006826	·011	·019	·010309	·008345 +	·012	·020	·013746	·009620	·012	·020
293	·006849	·006803	·011	·019	·010274	·008317	·012	·020	·013699	·009587	·012	·020
294	·006826	·006780	·011	·019	·010239	·008289	·012	·020	·013652	·009555 −	·012	·020
295	·006803	·006757	·011	·019	·010204	·008261	·012	·020	·013605 +	·009522	·012	·020
296	·006780	·006734	·011	·019	·010169	·008233	·012	·020	·013559	·009491	·012	·020
297	·006757	·006711	·011	·019	·010135 +	·008206	·012	·020	·013514	·009459	·012	·020
298	·006734	·006689	·011	·019	·010101	·008178	·012	·020	·013468	·009427	·012	·020
299	·006711	·006667	·011	·019	·010067	·008151	·012	·020	·013423	·009396	·012	·020
300	·006689	·006644	·011	·019	·010033	·008124	·012	·020	·013378	·009365 −	·012	·020

N = size of sample	6: $\bar{\eta}^2$	6: $\sigma_{\bar{\eta}^2}$	6: P_1 λ_1 = 3·11	6: P_2 λ_2 = 2·63	7: $\bar{\eta}^2$	7: $\sigma_{\bar{\eta}^2}$	7: P_1 λ_1 = 3·08	7: P_2 λ_2 = 2·58	8: $\bar{\eta}^2$	8: $\sigma_{\bar{\eta}^2}$	8: P_1 λ_1 = 3·02	8: P_2 λ_2 = 2·54
251	·020000	·012472	·011	·020	·024000	·013635 −	·010	·020	·028000	·014697	·010	·020
252	·019920	·012423	·011	·020	·023904	·013581	·010	·020	·027888	·014639	·010	·020
253	·019841	·012375 −	·011	·020	·023810	·013528	·010	·020	·027778	·014582	·010	·020
254	·019763	·012326	·011	·020	·023715 +	·013476	·010	·020	·027668	·014526	·010	·020
255	·019685 +	·012279	·011	·020	·023622	·013423	·010	·020	·027559	·014470	·010	·020
256	·019608	·012231	·011	·020	·023529	·013372	·010	·020	·027451	·014414	·010	·020
257	·019531	·012184	·011	·020	·023438	·013320	·010	·020	·027344	·014359	·010	·020
258	·019455 +	·012137	·011	·020	·023346	·013269	·010	·020	·027237	·014304	·010	·020
259	·019380	·012091	·011	·020	·023256	·013219	·010	·020	·027132	·014249	·010	·020
260	·019305 +	·012045 −	·011	·020	·023166	·013168	·010	·020	·027027	·014195 +	·010	·020
261	·019231	·011999	·011	·020	·023077	·013118	·010	·020	·026923	·014142	·010	·020
262	·019157	·011954	·011	·020	·022989	·013069	·010	·020	·026820	·014088	·010	·020
263	·019084	·011909	·011	·020	·022901	·013020	·010	·020	·026718	·014036	·010	·020
264	·019011	·011864	·011	·020	·022814	·012971	·010	·020	·026616	·013983	·010	·020
265	·018939	·011820	·011	·020	·022727	·012923	·010	·020	·026515 +	·013931	·010	·020
266	·018868	·011776	·011	·020	·022642	·012875 −	·010	·020	·026415 +	·013879	·010	·020
267	·018797	·011732	·011	·020	·022556	·012827	·010	·020	·026316	·013828	·010	·020
268	·018727	·011689	·011	·020	·022472	·012780	·010	·020	·026217	·013777	·010	·020
269	·018657	·011646	·011	·020	·022388	·012733	·010	·020	·026119	·013727	·010	·020
270	·018587	·011603	·011	·020	·022305 −	·012686	·010	·020	·026022	·013677	·010	·020
271	·018519	·011560	·011	·020	·022222	·012640	·010	·020	·025926	·013627	·010	·020
272	·018450 +	·011518	·011	·020	·022140	·012594	·010	·020	·025830	·013577	·010	·020
273	·018382	·011477	·011	·020	·022059	·012548	·010	·020	·025735 +	·013528	·010	·020
274	·018315 +	·011435 +	·011	·020	·021978	·012503	·010	·020	·025641	·013480	·010	·020
275	·018248	·011394	·011	·020	·021898	·012458	·010	·020	·025547	·013431	·010	·020
276	·018182	·011353	·011	·020	·021818	·012413	·010	·020	·025455 −	·013383	·010	·020
277	·018116	·011312	·011	·020	·021739	·012369	·010	·020	·025362	·013335 +	·010	·020
278	·018051	·011272	·011	·020	·021661	·012325 +	·010	·020	·025271	·013288	·010	·020
279	·017986	·011232	·011	·020	·021583	·012281	·010	·020	·025180	·013241	·010	·020
280	·017921	·011192	·011	·020	·021505 +	·012238	·010	·020	·025090	·013194	·010	·020
281	·017857	·011153	·011	·020	·021429	·012195 +	·010	·020	·025000	·013148	·010	·020
282	·017794	·011114	·011	·020	·021352	·012152	·010	·020	·024911	·013102	·010	·020
283	·017730	·011075 −	·011	·020	·021277	·012110	·010	·020	·024823	·013056	·010	·020
284	·017668	·011036	·011	·020	·021201	·012068	·010	·020	·024735 −	·013011	·010	·020
285	·017606	·010998	·011	·020	·021127	·012026	·010	·020	·024648	·012966	·010	·020
286	·017544	·010960	·011	·020	·021053	·011984	·010	·020	·024561	·012921	·010	·020
287	·017483	·010922	·011	·020	·020979	·011943	·010	·020	·024476	·012877	·010	·020
288	·017422	·010884	·011	·020	·020906	·011902	·010	·020	·024390	·012833	·010	·020
289	·017361	·010847	·011	·020	·020833	·011861	·010	·020	·024306	·012789	·010	·020
290	·017301	·010810	·011	·020	·020761	·011821	·010	·020	·024221	·012745 +	·010	·020
291	·017241	·010773	·011	·020	·020690	·011780	·010	·020	·024138	·012702	·010	·020
292	·017182	·010736	·011	·020	·020619	·011740	·010	·020	·024055 −	·012659	·010	·020
293	·017123	·010700	·011	·020	·020548	·011701	·010	·020	·023973	·012616	·010	·020
294	·017065 −	·010664	·011	·020	·020478	·011661	·010	·020	·023891	·012574	·010	·020
295	·017007	·010628	·011	·020	·020408	·011622	·010	·020	·023810	·012532	·010	·020
296	·016949	·010593	·011	·020	·020339	·011583	·010	·020	·023729	·012490	·010	·020
297	·016892	·010557	·011	·020	·020270	·011545 −	·010	·020	·023649	·012448	·010	·020
298	·016835 −	·010522	·011	·020	·020202	·011507	·010	·020	·023569	·012407	·010	·020
299	·016779	·010487	·011	·020	·020134	·011468	·010	·020	·023490	·012366	·010	·020
300	·016722	·010452	·011	·020	·020067	·011431	·010	·020	·023411	·012325 +	·010	·020

N = size of sample	9				10				11				12				13				14			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·96	P_2 $\lambda_2=$ 2·50	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·92	P_2 $\lambda_2=$ 2·47	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·88	P_2 $\lambda_2=$ 2·44	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·84	P_2 $\lambda_2=$ 2·42	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·80	P_2 $\lambda_2=$ 2·40	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·79	P_2 $\lambda_2=$ 2·39
251	·032000	·015679	·010	·021	·036000	·016596	·010	·021	·040000	·017457	·010	·021	·044000	·018271	·011	·021	·048000	·019044	·011	·021	·052000	·019780	·011	·021
252	·031873	·015618	·010	·021	·035857	·016531	·010	·021	·039841	·017390	·010	·021	·043825 –	·018200	·011	·021	·047809	·018970	·011	·021	·051793	·019703	·011	·021
253	·031746	·015557	·010	·021	·035714	·016467	·010	·021	·039683	·017322	·010	·021	·043651	·018130	·011	·021	·047619	·018897	·011	·021	·051587	·019628	·011	·021
254	·031621	·015497	·010	·021	·035573	·016404	·010	·021	·039526	·017255 +	·010	·021	·043478	·018060	·011	·021	·047431	·018825 –	·011	·021	·051383	·019552	·011	·021
255	·031496	·015437	·010	·021	·035433	·016341	·010	·021	·039370	·017189	·010	·021	·043307	·017991	·011	·021	·047244	·018753	·011	·021	·051181	·019478	·011	·021
256	·031373	·015378	·010	·021	·035294	·016278	·010	·021	·039216	·017123	·010	·021	·043137	·017923	·011	·021	·047059	·018681	·011	·021	·050980	·019404	·011	·021
257	·031250	·015319	·010	·021	·035156	·016216	·010	·021	·039063	·017058	·010	·021	·042969	·017854	·011	·021	·046875	·018610	·011	·021	·050781	·019330	·011	·021
258	·031128	·015261	·010	·021	·035019	·016154	·010	·021	·038911	·016993	·010	·021	·042802	·017787	·011	·021	·046693	·018540	·011	·021	·050584	·019257	·011	·021
259	·031008	·015203	·010	·021	·034884	·016093	·010	·021	·038760	·016929	·010	·021	·042636	·017720	·011	·021	·046512	·018470	·011	·021	·050388	·019185 +	·011	·021
260	·030888	·015145 +	·010	·021	·034749	·016032	·010	·021	·038610	·016865 –	·010	·021	·042471	·017653	·011	·021	·046332	·018401	·011	·021	·050193	·019113	·011	·021
261	·030769	·015088	·010	·021	·034615 +	·015972	·010	·021	·038462	·016802	·010	·021	·042308	·017587	·011	·021	·046154	·018332	·011	·021	·050000	·019042	·011	·021
262	·030651	·015031	·010	·021	·034483	·015912	·010	·021	·038314	·016739	·010	·021	·042146	·017521	·011	·021	·045977	·018264	·011	·021	·049808	·018971	·011	·021
263	·030534	·014975 +	·010	·021	·034351	·015852	·010	·021	·038168	·016677	·010$_5$	·021	·041985 –	·017456	·011	·021	·045802	·018196	·011	·021	·049618	·018901	·011	·021
264	·030418	·014919	·010	·021	·034221	·015793	·010	·021	·038023	·016615 –	·011	·021	·041825 +	·017391	·011	·021	·045627	·018129	·011	·021	·049430	·018831	·011	·021
265	·030303	·014864	·010	·021	·034091	·015735 –	·010	·021	·037879	·016553	·011	·021	·041667	·017327	·011	·021	·045455 –	·018062	·011	·021	·049242	·018762	·011	·021
266	·030189	·014809	·010	·021	·033962	·015677	·010	·021	·037736	·016492	·011	·021	·041509	·017263	·011	·021	·045283	·017996	·011	·021	·049057	·018693	·011	·021
267	·030075 +	·014754	·010	·021	·033835 –	·015619	·010	·021	·037594	·016432	·011	·021	·041353	·017200	·011	·021	·045113	·017930	·011	·021	·048872	·018625 +	·011	·021
268	·029963	·014700	·010	·021	·033708	·015562	·010	·021	·037453	·016372	·011	·021	·041199	·017137	·011	·021	·044944	·017864	·011	·021	·048689	·018557	·011	·021
269	·029851	·014646	·010	·021	·033582	·015505 –	·010	·021	·037313	·016312	·011	·021	·041045 –	·017075 +	·011	·021	·044776	·017800	·011	·021	·048507	·018490	·011	·021
270	·029740	·014593	·010	·021	·033457	·015448	·010	·021	·037175 –	·016253	·011	·021	·040892	·017013	·011	·021	·044610	·017735 +	·011	·021	·048327	·018423	·011	·021
271	·029630	·014540	·010	·021	·033333	·015392	·010	·021	·037037	·016194	·011	·021	·040741	·016952	·011	·021	·044444	·017671	·011	·021	·048148	·018357	·011	·021
272	·029520	·014487	·010	·021	·033210	·015337	·010	·021	·036900	·016136	·011	·021	·040590	·016891	·011	·021	·044280	·017608	·011	·021	·047970	·018291	·011	·021
273	·029412	·014435 +	·010	·021	·033088	·015282	·010	·021	·036765 –	·016078	·011	·021	·040441	·016830	·011	·021	·044118	·017545 –	·011	·021	·047794	·018226	·011	·021
274	·029304	·014383	·010	·021	·032967	·015227	·010	·021	·036630	·016020	·011	·021	·040293	·016770	·011	·021	·043956	·017482	·011	·021	·047619	·018161	·011	·021
275	·029197	·014332	·010	·021	·032847	·015172	·010	·021	·036496	·015963	·011	·021	·040146	·016710	·011	·021	·043796	·017420	·011	·021	·047445 +	·018097	·011	·021
276	·029091	·014280	·010	·021	·032727	·015118	·010	·021	·036364	·015906	·011	·021	·040000	·016651	·011	·021	·043636	·017358	·011	·021	·047273	·018033	·011	·021
277	·028986	·014230	·010	·021	·032609	·015065 –	·010	·021	·036232	·015850 –	·011	·021	·039855 +	·016592	·011	·021	·043478	·017297	·011	·021	·047101	·017969	·011	·021
278	·028881	·014179	·010	·021	·032491	·015011	·010	·021	·036101	·015794	·011	·021	·039711	·016534	·011	·021	·043321	·017236	·011	·021	·046931	·017906	·011	·021
279	·028777	·014129	·010	·021	·032374	·014959	·010	·021	·035971	·015738	·011	·021	·039568	·016476	·011	·021	·043165 +	·017176	·011	·021	·046763	·017844	·011	·021
280	·028674	·014080	·010	·021	·032258	·014906	·010	·021	·035842	·015683	·011	·021	·039427	·016418	·011	·021	·043011	·017116	·011	·021	·046595 –	·017782	·011	·021
281	·028571	·014030	·010	·021	·032143	·014854	·010	·021	·035714	·015628	·011	·021	·039286	·016361	·011	·021	·042857	·017057	·011	·021	·046429	·017720	·011	·021
282	·028470	·013981	·010	·021	·032028	·014802	·010	·021	·035587	·015574	·011	·021	·039146	·016304	·011	·021	·042705 –	·016997	·011	·021	·046263	·017659	·011	·021
283	·028369	·013932	·010	·021	·031915 –	·014750 +	·010	·021	·035461	·015520	·011	·021	·039007	·016248	·011	·021	·042553	·016939	·011	·021	·046099	·017598	·011	·021
284	·028269	·013884	·010	·021	·031802	·014700	·010	·021	·035336	·015466	·011	·021	·038869	·016191	·011	·021	·042403	·016880	·011	·021	·045936	·017537	·011	·021
285	·028169	·013836	·010	·021	·031690	·014649	·010	·021	·035211	·015413	·011	·021	·038732	·016136	·011	·021	·042254	·016822	·011	·021	·045775 –	·017477	·011	·021
286	·028070	·013788	·010	·021	·031579	·014598	·010	·021	·035088	·015360	·011	·021	·038596	·016081	·011	·021	·042105 +	·016765 –	·011	·021	·045614	·017417	·011	·021
287	·027972	·013741	·010	·021	·031469	·014548	·010	·021	·034965 +	·015308	·011	·021	·038462	·016026	·011	·021	·041958	·016708	·011	·021	·045455 –	·017358	·011	·021
288	·027875 –	·013694	·010	·021	·031359	·014499	·010	·021	·034843	·015255 +	·011	·021	·038328	·015971	·011	·021	·041812	·016651	·011	·021	·045296	·017299	·011	·021
289	·027778	·013647	·010	·021	·031250	·014449	·010	·021	·034722	·015204	·011	·021	·038194	·015917	·011	·021	·041667	·016595 –	·011	·021	·045139	·017241	·011	·021
290	·027682	·013601	·010	·021	·031142	·014400	·010$_5$	·021	·034602	·015152	·011	·021	·038062	·015863	·011	·021	·041522	·016539	·011	·021	·044983	·017183	·011	·021
291	·027586	·013555 –	·010	·021	·031034	·014352	·011	·021	·034483	·015101	·011	·021	·037931	·015810	·011	·021	·041379	·016483	·011	·021	·044828	·017125 +	·011	·021
292	·027491	·013509	·010	·021	·030928	·014303	·011	·021	·034364	·015050 +	·011	·021	·037801	·015757	·011	·021	·041237	·016428	·011	·021	·044674	·017068	·011	·021
293	·027397	·013464	·010	·021	·030822	·014255 +	·011	·021	·034247	·015000	·011	·021	·037671	·015704	·011	·021	·041096	·016373	·011	·021	·044521	·017011	·011	·021
294	·027304	·013418	·010	·021	·030717	·014207	·011	·021	·034130	·014950 –	·011	·021	·037543	·015652	·011	·021	·040956	·016318	·011	·021	·044369	·016955 –	·011	·021
295	·027211	·013374	·010	·021	·030612	·014160	·011	·021	·034014	·014900	·011	·021	·037415 –	·015600	·011	·021	·040816	·016264	·011	·021	·044218	·016898	·011	·021
296	·027119	·013329	·010	·021	·030508	·014113	·011	·021	·033898	·014850 +	·011	·021	·037288	·015548	·011	·021	·040678	·016211	·011	·021	·044068	·016843	·011	·021
297	·027027	·013285 –	·010	·021	·030405 +	·014066	·011	·021	·033784	·014801	·011	·021	·037162	·015497	·011	·021	·040541	·016157	·011	·021	·043919	·016787	·011	·021
298	·026936	·013241	·010	·021	·030303	·014020	·011	·021	·033670	·014752	·011	·021	·037037	·015446	·011	·021	·040404	·016104	·011	·021	·043771	·016732	·011	·021
299	·026846	·013197	·010	·021	·030201	·013974	·011	·021	·033557	·014704	·011	·021	·036913	·015395 –	·011	·021	·040268	·016051	·011	·021	·043624	·016678	·011	·021
300	·026756	·013154	·010	·021	·030100	·013928	·011	·021	·033445 –	·014656	·011	·021	·036789	·015345 –	·011	·021	·040134	·015999	·011	·021	·043478	·016623	·011	·021

n = number of arrays

N = size of sample	15				16			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2·78$	P_2 $\lambda_2 = 2·38$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2·77$	P_2 $\lambda_2 = 2·37$
251	·056000	·020483	·010$_5$	·021	·060000	·021157	·010	·021
252	·055777	·020404	·011	·021	·059761	·021076	·010	·021
253	·055556	·020326	·011	·021	·059524	·020995 +	·010	·021
254	·055336	·020248	·011	·021	·059289	·020915 +	·010	·021
255	·055118	·020171	·011	·021	·059055 +	·020836	·010	·021
256	·054902	·020095 −	·011	·021	·058824	·020757	·010	·021
257	·054688	·020019	·011	·021	·058594	·020679	·010	·021
258	·054475 −	·019943	·011	·021	·058366	·020601	·010	·021
259	·054264	·019869	·011	·021	·058140	·020524	·010	·021
260	·054054	·019794	·011	·021	·057915 +	·020447	·010	·021
261	·053846	·019721	·011	·021	·057692	·020371	·010	·021
262	·053640	·019648	·011	·021	·057471	·020296	·010	·021
263	·053435 +	·019575 −	·011	·021	·057252	·020221	·010	·021
264	·053232	·019503	·011	·021	·057034	·020147	·010	·021
265	·053030	·019431	·011	·021	·056818	·020073	·010	·021
266	·052830	·019360	·011	·021	·056604	·020000	·010	·021
267	·052632	·019290	·011	·021	·056391	·019927	·010	·021
268	·052434	·019220	·011	·021	·056180	·019855 +	·010	·021
269	·052239	·019150 +	·011	·021	·055970	·019784	·010	·021
270	·052045 −	·019081	·011	·021	·055762	·019712	·010	·021
271	·051852	·019013	·011	·021	·055556	·019642	·010	·021
272	·051661	·018945 +	·011	·021	·055351	·019572	·010	·021
273	·051471	·018878	·011	·021	·055147	·019502	·010	·021
274	·051282	·018810	·011	·021	·054945 +	·019433	·010	·021
275	·051095 −	·018744	·011	·021	·054745 −	·019364	·010	·021
276	·050909	·018678	·011	·021	·054545 +	·019296	·010	·021
277	·050725 −	·018612	·011	·021	·054348	·019229	·010	·021
278	·050542	·018547	·011	·021	·054152	·019161	·010	·021
279	·050360	·018482	·011	·021	·053957	·019095 −	·010	·021
280	·050179	·018418	·011	·021	·053763	·019029	·010	·021
281	·050000	·018354	·011	·021	·053571	·018963	·010	·021
282	·049822	·018291	·011	·021	·053381	·018897	·010	·021
283	·049645 +	·018228	·011	·021	·053191	·018833	·010	·021
284	·049470	·018165 +	·011	·021	·053004	·018768	·010	·021
285	·049296	·018103	·011	·021	·052817	·018704	·010	·021
286	·049123	·018042	·011	·021	·052632	·018640	·010	·021
287	·048951	·017980	·011	·021	·052448	·018577	·010	·021
288	·048780	·017920	·011	·021	·052265 −	·018515 −	·010	·021
289	·048611	·017859	·011	·021	·052083	·018452	·010	·021
290	·048443	·017799	·011	·021	·051903	·018390	·010	·021
291	·048276	·017740	·011	·021	·051724	·018329	·010	·021
292	·048110	·017680	·011	·021	·051546	·018268	·010	·021
293	·047945 +	·017622	·011	·021	·051370	·018207	·010	·021
294	·047782	·017563	·011	·021	·051195 −	·018147	·010	·021
295	·047619	·017505 +	·011	·021	·051020	·018087	·010$_5$	·021
296	·047458	·017447	·011	·021	·050847	·018028	·011	·021
297	·047297	·017390	·011	·021	·050676	·017969	·011	·021
298	·047138	·017333	·011	·021	·050505 +	·017910	·011	·021
299	·046980	·017277	·011	·021	·050336	·017852	·011	·021
300	·046823	·017221	·011	·021	·050167	·017794	·011	·021

N = size of sample	17				18			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2·76$	P_2 $\lambda_2 = 2·36$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2·75$	P_2 $\lambda_2 = 2·35$
251	·064000	·021804	·010	·020	·068000	·022427	·010	·020
252	·063745 +	·021721	·010	·020	·067729	·022342	·010	·020
253	·063492	·021638	·010	·020	·067460	·022256	·010	·020
254	·063241	·021556	·010	·020	·067194	·022172	·010	·020
255	·062992	·021474	·010	·020	·066929	·022088	·010	·020
256	·062745 +	·021393	·010	·020	·066667	·022005 −	·010	·020$_5$
257	·062500	·021312	·010	·020	·066406	·021922	·010	·021
258	·062257	·021232	·010	·020$_5$	·066148	·021840	·010	·021
259	·062016	·021153	·010	·021	·065891	·021759	·010	·021
260	·061776	·021075 −	·010	·021	·065637	·021678	·010	·021
261	·061538	·020996	·010	·021	·065385 −	·021598	·010	·021
262	·061303	·020919	·010	·021	·065134	·021519	·010	·021
263	·061069	·020842	·010	·021	·064885 +	·021440	·010	·021
264	·060837	·020766	·010	·021	·064639	·021361	·010	·021
265	·060606	·020690	·010	·021	·064394	·021284	·010	·021
266	·060377	·020615 −	·010	·021	·064151	·021206	·010	·021
267	·060150 +	·020540	·010	·021	·063910	·021130	·010	·021
268	·059925 −	·020466	·010	·021	·063670	·021053	·010	·021
269	·059701	·020392	·010	·021	·063433	·020978	·010	·021
270	·059480	·020319	·010	·021	·063197	·020903	·010	·021
271	·059259	·020246	·010	·021	·062963	·020828	·010	·021
272	·059041	·020174	·010	·021	·062731	·020754	·010	·021
273	·058824	·020103	·010	·021	·062500	·020681	·010	·021
274	·058608	·020031	·010	·021	·062271	·020608	·010	·021
275	·058394	·019961	·010	·021	·062044	·020535 +	·010	·021
276	·058182	·019891	·010	·021	·061818	·020463	·010	·021
277	·057971	·019821	·010	·021	·061594	·020392	·010	·021
278	·057762	·019752	·010	·021	·061372	·020321	·010	·021
279	·057554	·019683	·010	·021	·061151	·020251	·010	·021
280	·057348	·019615 +	·010	·021	·060932	·020181	·010	·021
281	·057143	·019548	·010	·021	·060714	·020111	·010	·021
282	·056940	·019480	·010	·021	·060498	·020042	·010	·021
283	·056738	·019414	·010	·021	·060284	·019973	·010	·021
284	·056537	·019347	·010	·021	·060071	·019905 +	·010	·021
285	·056338	·019282	·010	·021	·059859	·019838	·010	·021
286	·056140	·019216	·010	·021	·059649	·019771	·010	·021
287	·055944	·019151	·010	·021	·059441	·019704	·010	·021
288	·055749	·019087	·010	·021	·059233	·019638	·010	·021
289	·055556	·019023	·010	·021	·059028	·019572	·010	·021
290	·055363	·018959	·010	·021	·058824	·019507	·010	·021
291	·055172	·018896	·010	·021	·058621	·019442	·010	·021
292	·054983	·018833	·010	·021	·058419	·019377	·010	·021
293	·054795 −	·018770	·010	·021	·058219	·019313	·010	·021
294	·054608	·018708	·010	·021	·058020	·019249	·010	·021
295	·054422	·018647	·010	·021	·057823	·019186	·010	·021
296	·054237	·018586	·010	·021	·057627	·019123	·010	·021
297	·054054	·018525 −	·010	·021	·057432	·019061	·010	·021
298	·053872	·018464	·010	·021	·057239	·018999	·010	·021
299	·053691	·018404	·010	·021	·057047	·018937	·010	·021
300	·053512	·018345 −	·010	·021	·056856	·018876	·010	·021

N = size of sample	19				20			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2·74$	P_2 $\lambda_2 = 2·34$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2·73$	P_2 $\lambda_2 = 2·33$
251	·072000	·023028	·010	·020	·076000	·023608	·010	·020
252	·071713	·022940	·010	·020	·075697	·023518	·010	·020
253	·071429	·022853	·010	·020	·075397	·023429	·010	·020
254	·071146	·022766	·010	·020	·075099	·023340	·010	·020
255	·070866	·022681	·010	·020	·074803	·023253	·010	·020
256	·070588	·022595 +	·010	·020	·074510	·023165 +	·010	·020
257	·070313	·022511	·010	·020	·074219	·023079	·010	·020$_5$
258	·070039	·022427	·010	·020	·073930	·022993	·010	·021
259	·069767	·022343	·010	·020	·073643	·022908	·010	·021
260	·069498	·022261	·010	·020	·073359	·022823	·010	·021
261	·069231	·022179	·010	·020	·073077	·022739	·010	·021
262	·068966	·022097	·010	·020	·072797	·022656	·010	·021
263	·068702	·022016	·010	·020	·072519	·022573	·010	·021
264	·068441	·021936	·010	·020	·072243	·022491	·010	·021
265	·068182	·021856	·010	·020	·071970	·022409	·010	·021
266	·067925 −	·021777	·010	·020$_5$	·071698	·022328	·010	·021
267	·067669	·021698	·010	·021	·071429	·022248	·010	·021
268	·067416	·021620	·010	·021	·071161	·022168	·010	·021
269	·067164	·021543	·010	·021	·070896	·022089	·010	·021
270	·066914	·021466	·010	·021	·070632	·022010	·010	·021
271	·066667	·021390	·010	·021	·070370	·021932	·010	·021
272	·066421	·021314	·010	·021	·070111	·021855 −	·010	·021
273	·066176	·021239	·010	·021	·069853	·021777	·010	·021
274	·065934	·021164	·010	·021	·069597	·021701	·010	·021
275	·065693	·021089	·010	·021	·069343	·021625 +	·010	·021
276	·065455 −	·021016	·010	·021	·069091	·021550 −	·010	·021
277	·065217	·020943	·010	·021	·068841	·021475 −	·010	·021
278	·064982	·020870	·010	·021	·068592	·021400	·010	·021
279	·064748	·020798	·010	·021	·068345 +	·021326	·010	·021
280	·064516	·020726	·010	·021	·068100	·021253	·010	·021
281	·064286	·020655 −	·010	·021	·067857	·021180	·010	·021
282	·064057	·020584	·010	·021	·067616	·021108	·010	·021
283	·063830	·020514	·010	·021	·067376	·021036	·010	·021
284	·063604	·020444	·010	·021	·067138	·020965 −	·010	·021
285	·063380	·020375 −	·010	·021	·066901	·020894	·010	·021
286	·063158	·020306	·010	·021	·066667	·020823	·010	·021
287	·062937	·020237	·010	·021	·066434	·020753	·010	·021
288	·062718	·020170	·010	·021	·066202	·020684	·010	·021
289	·062500	·020102	·010	·021	·065972	·020615 −	·010	·021
290	·062284	·020035 +	·010	·021	·065744	·020546	·010	·021
291	·062069	·019969	·010	·021	·065517	·020478	·010	·021
292	·061856	·019902	·010	·021	·065292	·020410	·010	·021
293	·061644	·019837	·010	·021	·065068	·020343	·010	·021
294	·061433	·019771	·010	·021	·064846	·020276	·010	·021
295	·061224	·019707	·010	·021	·064626	·020210	·010	·021
296	·061017	·019642	·010	·021	·064407	·020144	·010	·021
297	·060811	·019578	·010	·021	·064189	·020079	·010	·021
298	·060606	·019515 −	·010	·021	·063973	·020013	·010	·021
299	·060403	·019452	·010	·021	·063758	·019949	·010	·021
300	·060201	·019389	·010	·021	·063545 +	·019885 −	·010	·021

N = size of sample	3: $\bar{\eta}^2$	3: $\sigma_{\bar{\eta}^2}$	3: P_1 λ_1 = 3·50	3: P_2 λ_2 = 2·94	4: $\bar{\eta}^2$	4: $\sigma_{\bar{\eta}^2}$	4: P_1 λ_1 = 3·20	4: P_2 λ_2 = 2·80
301	·006667	·006622	·011	·019	·010000	·008097	·012	·020
302	·006645 −	·006601	·011	·019	·009967	·008070	·012	·020
303	·006623	·006579	·011	·019	·009934	·008044	·012	·020
304	·006601	·006557	·011	·019	·009901	·008018	·012	·020
305	·006579	·006536	·011	·019	·009868	·007991	·012	·020
306	·006557	·006515 −	·011	·019	·009836	·007965 +	·012	·020
307	·006536	·006493	·011	·019	·009804	·007940	·012	·020
308	·006515 −	·006472	·011	·019	·009772	·007914	·012	·020
309	·006494	·006451	·011	·019	·009740	·007888	·012	·020
310	·006472	·006431	·011	·019	·009709	·007863	·012	·020
311	·006452	·006410	·011	·019	·009677	·007838	·012	·020
312	·006431	·006390	·011	·019	·009646	·007813	·012	·020
313	·006410	·006369	·011	·019	·009615 +	·007788	·012	·020
314	·006390	·006349	·011	·019	·009585 −	·007763	·012	·020
315	·006369	·006329	·011	·019	·009554	·007739	·012	·020
316	·006349	·006309	·011	·019	·009524	·007715 −	·012	·020
317	·006329	·006289	·011	·019	·009494	·007690	·012	·020
318	·006309	·006269	·011	·019	·009464	·007666	·012	·020
319	·006289	·006250 −	·011	·019	·009434	·007642	·012	·020
320	·006270	·006230	·011	·019	·009404	·007619	·012	·020
321	·006250	·006211	·011	·019	·009375	·007595 −	·012	·020
322	·006231	·006192	·011	·019	·009346	·007572	·012	·020
323	·006211	·006173	·011	·019	·009317	·007548	·012	·020
324	·006192	·006154	·011	·019	·009288	·007525 −	·012	·020
325	·006173	·006135 −	·011	·019	·009259	·007502	·012	·020
326	·006154	·006116	·011	·019	·009231	·007479	·012	·020
327	·006135 −	·006097	·011	·019	·009202	·007456	·012	·020
328	·006116	·006079	·011	·019	·009174	·007434	·012	·020
329	·006098	·006060	·011	·019	·009146	·007411	·012	·020
330	·006079	·006042	·011	·019	·009119	·007389	·012	·020
331	·006061	·006024	·011	·019	·009091	·007367	·012	·020
332	·006042	·006006	·011	·019	·009063	·007345 −	·012	·020
333	·006024	·005988	·011	·019	·009036	·007323	·012	·020
334	·006006	·005970	·011	·019	·009009	·007301	·012	·020
335	·005988	·005952	·011	·019	·008982	·007279	·012	·020
336	·005970	·005935 −	·011	·019	·008955 +	·007257	·012	·020
337	·005952	·005917	·011	·019	·008929	·007236	·012	·020
338	·005935 −	·005900	·011	·019	·008902	·007215 −	·012	·020
339	·005917	·005882	·011	·019	·008876	·007194	·012	·020
340	·005900	·005865 +	·011	·019	·008850 −	·007172	·012	·020
341	·005882	·005848	·011	·019	·008824	·007152	·012	·020
342	·005865 +	·005831	·011	·019	·008798	·007131	·012	·020
343	·005848	·005814	·011	·019	·008772	·007110	·012	·020
344	·005831	·005797	·011	·019	·008746	·007089	·012	·020
345	·005814	·005780	·011	·019	·008721	·007069	·012	·020
346	·005797	·005764	·011	·019	·008696	·007049	·012	·020
347	·005780	·005747	·011	·019	·008671	·007028	·012	·020
348	·005764	·005731	·011	·019	·008646	·007008	·012	·020
349	·005747	·005714	·011	·019	·008621	·006988	·012	·020
350	·005731	·005698	·011	·019	·008596	·006968	·012	·020

N = size of sample	5: $\bar{\eta}^2$	5: $\sigma_{\bar{\eta}^2}$	5: P_1 λ_1 = 3·14	5: P_2 λ_2 = 2·68	6: $\bar{\eta}^2$	6: $\sigma_{\bar{\eta}^2}$	6: P_1 λ_1 = 3·11	6: P_2 λ_2 = 2·63
301	·013333	·009334	·012	·020	·016667	·010418	·011	·020
302	·013289	·009303	·012	·020	·016611	·010384	·011	·020
303	·013245 +	·009273	·012	·020	·016556	·010350 −	·011	·020
304	·013201	·009242	·012	·020	·016502	·010316	·011	·020
305	·013158	·009212	·012	·020	·016447	·010283	·011	·020
306	·013115 −	·009182	·012	·020	·016393	·010249	·011	·020
307	·013072	·009153	·012	·020	·016340	·010216	·011	·020
308	·013029	·009123	·012	·020	·016287	·010183	·011	·020
309	·012987	·009094	·012	·020	·016234	·010151	·011	·020
310	·012945 −	·009065 −	·012	·020	·016181	·010118	·011	·020
311	·012903	·009036	·012	·020	·016129	·010086	·011	·020
312	·012862	·009007	·012	·020	·016077	·010054	·011	·020
313	·012821	·008978	·012	·020	·016026	·010022	·011	·020
314	·012780	·008950 +	·012	·020	·015974	·009990	·011	·020
315	·012739	·008922	·012	·020	·015924	·009959	·011	·020
316	·012698	·008894	·012	·020	·015873	·009928	·011	·020
317	·012658	·008866	·012	·020	·015823	·009896	·011	·020
318	·012618	·008838	·012	·020	·015773	·009866	·011	·020
319	·012579	·008811	·012	·020	·015723	·009835 −	·011	·020
320	·012539	·008783	·012	·020	·015674	·009804	·011	·020
321	·012500	·008756	·012	·020	·015625	·009774	·011	·020
322	·012461	·008729	·012	·020	·015576	·009744	·011	·020
323	·012422	·008702	·012	·020	·015528	·009714	·011	·020
324	·012384	·008676	·012	·020	·015480	·009684	·011	·020
325	·012346	·008649	·012	·020	·015432	·009655 −	·011	·020
326	·012308	·008623	·012	·020	·015385 −	·009625 +	·011	·020
327	·012270	·008596	·012	·020	·015337	·009596	·011	·020
328	·012232	·008570	·012	·020	·015291	·009567	·011	·020
329	·012195 +	·008545 −	·012	·020	·015244	·009538	·011	·020
330	·012158	·008519	·012	·020	·015198	·009510	·011	·020
331	·012121	·008493	·012	·020	·015152	·009481	·011	·020
332	·012085 −	·008468	·012	·020	·015106	·009453	·011	·020
333	·012048	·008442	·012	·020	·015060	·009425 −	·011	·020
334	·012012	·008417	·012	·020	·015015 +	·009397	·011	·020
335	·011976	·008392	·012	·020	·014970	·009369	·011	·020
336	·011940	·008368	·012	·020	·014925 +	·009341	·011	·020
337	·011905 −	·008343	·012	·020	·014881	·009314	·011	·020
338	·011869	·008318	·012	·020	·014837	·009286	·011	·020
339	·011834	·008294	·012	·020	·014793	·009259	·011	·020
340	·011799	·008270	·012	·020	·014749	·009232	·011	·020
341	·011765 −	·008246	·012	·020	·014706	·009205 +	·011	·020
342	·011730	·008222	·012	·020	·014663	·009178	·011	·020
343	·011696	·008198	·012	·020	·014620	·009152	·011	·020
344	·011662	·008174	·012	·020	·014577	·009125 +	·011	·020
345	·011628	·008151	·012	·020	·014535 −	·009099	·011	·020
346	·011594	·008127	·012	·020	·014493	·009073	·011	·020
347	·011561	·008104	·012	·020	·014451	·009047	·011	·020
348	·011527	·008081	·012	·020	·014409	·009021	·011	·020
349	·011494	·008058	·012	·020	·014368	·008996	·011	·020
350	·011461	·008035 −	·012	·020	·014327	·008970	·011	·020

N = size of sample	7: $\bar{\eta}^2$	7: $\sigma_{\bar{\eta}^2}$	7: P_1 λ_1 = 3·08	7: P_2 λ_2 = 2·58	8: $\bar{\eta}^2$	8: $\sigma_{\bar{\eta}^2}$	8: P_1 λ_1 = 3·02	8: P_2 λ_2 = 2·54
301	·020000	·011393	·010	·020	·023333	·012285 −	·010	·020
302	·019934	·011356	·010	·020	·023256	·012245 −	·010	·020
303	·019868	·011319	·010	·020	·023179	·012204	·010	·020
304	·019802	·011282	·010	·020	·023102	·012165 +	·010	·020
305	·019737	·011245 +	·010	·020	·023026	·012126	·010	·020
306	·019672	·011209	·010	·020	·022951	·012087	·010	·020
307	·019608	·011173	·010	·020	·022876	·012048	·010	·020
308	·019544	·011137	·010	·020	·022801	·012009	·010	·020
309	·019481	·011101	·010	·020	·022727	·011971	·010	·020
310	·019417	·011066	·010	·020	·022654	·011932	·010	·020
311	·019355 −	·011030	·010	·020	·022581	·011895 −	·010	·021
312	·019293	·010995 +	·010	·020	·022508	·011857	·010	·021
313	·019231	·010961	·010	·020	·022436	·011819	·010	·021
314	·019169	·010926	·010	·020	·022364	·011782	·010	·021
315	·019108	·010892	·010	·020	·022293	·011745 +	·010	·021
316	·019048	·010857	·010	·020	·022222	·011708	·010	·021
317	·018987	·010824	·010	·020	·022152	·011672	·010	·021
318	·018927	·010790	·010	·020	·022082	·011636	·010	·021
319	·018868	·010756	·010	·020	·022013	·011600	·010	·021
320	·018809	·010723	·010	·020	·021944	·011564	·010	·021
321	·018750	·010690	·010	·020	·021875	·011528	·010	·021
322	·018692	·010657	·010	·020	·021807	·011493	·010	·021
323	·018634	·010624	·010	·020	·021739	·011458	·010	·021
324	·018576	·010592	·010	·020	·021672	·011423	·010	·021
325	·018519	·010560	·010	·020	·021605 −	·011388	·010	·021
326	·018462	·010528	·010	·020	·021538	·011353	·010	·021
327	·018405 −	·010496	·010	·020	·021472	·011319	·010	·021
328	·018349	·010464	·010	·020	·021407	·011285 −	·010	·021
329	·018293	·010432	·010	·020	·021341	·011251	·010	·021
330	·018237	·010401	·010	·020	·021277	·011217	·010	·021
331	·018182	·010370	·010	·020	·021212	·011184	·010	·021
332	·018127	·010339	·010	·020	·021148	·011150 +	·010	·021
333	·018072	·010308	·010	·020	·021084	·011117	·010	·021
334	·018018	·010278	·010	·020	·021021	·011084	·010	·021
335	·017964	·010247	·010	·020	·020958	·011052	·010	·021
336	·017910	·010217	·010	·020	·020896	·011019	·010	·021
337	·017857	·010187	·010	·020	·020833	·010987	·010	·021
338	·017804	·010157	·010	·020	·020772	·010954	·010	·021
339	·017751	·010128	·010	·020	·020710	·010922	·010	·021
340	·017699	·010098	·010	·020	·020649	·010891	·010	·021
341	·017647	·010069	·010	·020	·020588	·010859	·010	·021
342	·017595 +	·010039	·010	·020	·020528	·010828	·010	·021
343	·017544	·010010	·010	·020	·020468	·010796	·010	·021
344	·017493	·009982	·010	·020	·020408	·010765 +	·010	·021
345	·017442	·009953	·010	·020	·020349	·010735 −	·010	·021
346	·017391	·009924	·010	·020	·020290	·010704	·010	·021
347	·017341	·009896	·010	·020	·020231	·010673	·010	·021
348	·017291	·009868	·010	·020	·020173	·010643	·010	·021
349	·017241	·009840	·010	·020	·020115 −	·010613	·010	·021
350	·017192	·009812	·010	·020	·020057	·010583	·010	·021

n = number of arrays

N = size of sample	9: $\bar{\eta}^2$	9: $\sigma_{\bar{\eta}^2}$	9: P_1 $\lambda_1=2\cdot96$	9: P_2 $\lambda_2=2\cdot50$	10: $\bar{\eta}^2$	10: $\sigma_{\bar{\eta}^2}$	10: P_1 $\lambda_1=2\cdot92$	10: P_2 $\lambda_2=2\cdot47$	11: $\bar{\eta}^2$	11: $\sigma_{\bar{\eta}^2}$	11: P_1 $\lambda_1=2\cdot88$	11: P_2 $\lambda_2=2\cdot44$
301	·026667	·013111	·010	·021	·030000	·013882	·011	·021	·033333	·014608	·011	·021
302	·026578	·013068	·010	·021	·029900	·013837	·011	·021	·033223	·014560	·011	·021
303	·026490	·013025+	·010	·021	·029801	·013792	·011	·021	·033113	·014513	·011	·021
304	·026403	·012983	·010	·021	·029703	·013747	·011	·021	·033003	·014466	·011	·021
305	·026316	·012941	·010	·021	·029605+	·013703	·011	·021	·032895−	·014420	·011	·021
306	·026230	·012899	·010	·021	·029508	·013659	·011	·021	·032787	·014373	·011	·021
307	·026144	·012858	·010	·021	·029412	·013615+	·011	·021	·032680	·014327	·011	·021
308	·026059	·012817	·010	·021	·029316	·013571	·011	·021	·032573	·014282	·011	·021
309	·025974	·012776	·010	·021	·029221	·013528	·011	·021	·032468	·014236	·011	·021
310	·025890	·012735+	·010	·021	·029126	·013485+	·011	·021	·032362	·014191	·011	·021
311	·025806	·012695−	·010	·021	·029032	·013443	·011	·021	·032258	·014146	·011	·021
312	·025723	·012655−	·010	·021	·028939	·013400	·011	·021	·032154	·014102	·011	·021
313	·025641	·012615−	·010	·021	·028846	·013358	·011	·021	·032051	·014057	·011	·021
314	·025559	·012575+	·010	·021	·028754	·013316	·011	·021	·031949	·014013	·011	·021
315	·025478	·012536	·010	·021	·028662	·013274	·011	·021	·031847	·013969	·011	·021
316	·025397	·012497	·010	·021	·028571	·013233	·011	·021	·031746	·013926	·011	·021
317	·025316	·012458	·010	·021	·028481	·013192	·011	·021	·031646	·013883	·011	·021
318	·025237	·012419	·010	·021	·028391	·013151	·011	·021	·031546	·013840	·011	·021
319	·025157	·012381	·010	·021	·028302	·013110	·011	·021	·031447	·013797	·011	·021
320	·025078	·012342	·010	·021	·028213	·013070	·011	·021	·031348	·013755−	·011	·021
321	·025000	·012304	·010	·021	·028125	·013030	·011	·021	·031250	·013713	·011	·021
322	·024922	·012267	·010	·021	·028037	·012990	·011	·021	·031153	·013671	·011	·021
323	·024845−	·012229	·010	·021	·027950+	·012950+	·011	·021	·031056	·013629	·011	·021
324	·024768	·012192	·010	·021	·027864	·012911	·011	·021	·030960	·013588	·011	·021
325	·024691	·012155−	·010	·021	·027778	·012872	·011	·021	·030864	·013546	·011	·021
326	·024615+	·012118	·010	·021	·027692	·012833	·011	·021	·030769	·013506	·011	·021
327	·024540	·012081	·010	·021	·027607	·012794	·011	·021	·030675−	·013465−	·011	·021
328	·024465−	·012045+	·010	·021	·027523	·012756	·011	·021	·030581	·013425−	·011	·021
329	·024390	·012009	·010	·021	·027439	·012717	·011	·021	·030488	·013384	·011	·021
330	·024316	·011973	·010	·021	·027356	·012679	·011	·021	·030395+	·013344	·011	·021
331	·024242	·011937	·010	·021	·027273	·012642	·011	·021	·030303	·013305−	·011	·021
332	·024169	·011902	·010	·021	·027190	·012604	·011	·021	·030211	·012265+	·011	·021
333	·024096	·011866	·010	·021	·027108	·012567	·011	·021	·030120	·013226	·011	·021
334	·024024	·011831	·010	·021	·027027	·012530	·011	·021	·030030	·013187	·011	·021
335	·023952	·011796	·010	·021	·026946	·012493	·011	·021	·029940	·013148	·011	·021
336	·023881	·011762	·010	·021	·026866	·012456	·011	·021	·029851	·013110	·011	·021
337	·023810	·011727	·010	·021	·026786	·012420	·011	·021	·029762	·013072	·011	·021
338	·023739	·011693	·010	·021	·026706	·012384	·011	·021	·029674	·013033	·011	·021
339	·023669	·011659	·010	·021	·026627	·012347	·011	·021	·029586	·012996	·011	·021
340	·023599	·011625+	·010	·021	·026549	·012312	·011	·021	·029499	·012958	·011	·021
341	·023529	·011591	·010	·021	·026471	·012276	·011	·021	·029412	·012921	·011	·021
342	·023460	·011558	·010	·021	·026393	·012241	·011	·021	·029326	·012883	·011	·021
343	·023392	·011525−	·010	·021	·026316	·012205+	·011	·021	·029240	·012846	·011	·021
344	·023324	·011492	·010	·021	·026239	·012170	·011	·021	·029155−	·012810	·011	·021
345	·023256	·011459	·010	·021	·026163	·012136	·011	·021	·029070	·012773	·011	·021
346	·023188	·011426	·010	·021	·026087	·012101	·011	·021	·028986	·012737	·011	·021
347	·023121	·011393	·010	·021	·026012	·012067	·011	·021	·028902	·012700	·011	·021
348	·023055−	·011361	·010	·021	·025937	·012032	·011	·021	·028818	·012664	·011	·021
349	·022989	·011329	·010	·021	·025862	·011998	·011	·021	·028736	·012629	·011	·021
350	·022923	·011297	·010$_5$	·021	·025788	·011965−	·011	·021	·028653	·012593	·011	·021

N = size of sample	12: $\bar{\eta}^2$	12: $\sigma_{\bar{\eta}^2}$	12: P_1 $\lambda_1=2\cdot84$	12: P_2 $\lambda_2=2\cdot42$	13: $\bar{\eta}^2$	13: $\sigma_{\bar{\eta}^2}$	13: P_1 $\lambda_1=2\cdot80$	13: P_2 $\lambda_2=2\cdot40$	14: $\bar{\eta}^2$	14: $\sigma_{\bar{\eta}^2}$	14: P_1 $\lambda_1=2\cdot79$	14: P_2 $\lambda_2=2\cdot39$
301	·036667	·015295−	·011	·021	·040000	·015947	·011	·021	·043333	·016569	·011	·021
302	·036545−	·015245−	·011	·021	·039867	·015895+	·011	·021	·043189	·016516	·011	·021
303	·036424	·015195+	·011	·021	·039735+	·015844	·011	·021	·043046	·016462	·011	·021
304	·036304	·015146	·011	·021	·039604	·015793	·011	·021	·042904	·016409	·011	·021
305	·036184	·015098	·011	·021	·039474	·015742	·011	·021	·042763	·016357	·011	·021
306	·036066	·015049	·011	·021	·039344	·015692	·011	·021	·042623	·016305−	·011	·021
307	·035948	·015001	·011	·021	·039216	·015642	·011	·021	·042484	·016253	·011	·021
308	·035831	·014953	·011	·021	·039088	·015592	·011	·021	·042345+	·016201	·011	·021
309	·035714	·014906	·011	·021	·038961	·015542	·011	·021	·042208	·016150−	·011	·021
310	·035599	·014859	·011	·021	·038835−	·015493	·011	·021	·042071	·016099	·011	·021
311	·035484	·014812	·011	·021	·038710	·015445−	·011	·021	·041935+	·016048	·011	·021
312	·035370	·014765+	·011	·021	·038585+	·015396	·011	·021	·041801	·015998	·011	·021
313	·035256	·014719	·011	·021	·038462	·015348	·011	·021	·041667	·015948	·011	·021
314	·035144	·014673	·011	·021	·038339	·015300	·011	·021	·041534	·015898	·011	·021
315	·035032	·014627	·011	·021	·038217	·015252	·011	·021	·041401	·015849	·011	·021
316	·034921	·014582	·011	·021	·038095+	·015205+	·011	·021	·041270	·015800	·011	·021
317	·034810	·014537	·011	·021	·037975−	·015158	·011	·021	·041139	·015751	·011	·021
318	·034700	·014492	·011	·021	·037855−	·015111	·011	·021	·041009	·015703	·011	·021
319	·034591	·014447	·011	·021	·037736	·015065−	·011	·021	·040881	·015654	·011	·021
320	·034483	·014403	·011	·021	·037618	·015019	·011	·021	·040752	·015606	·011	·021
321	·034375	·014359	·011	·021	·037500	·014973	·011	·021	·040625	·015559	·011	·021
322	·034268	·014315−	·011	·021	·037383	·014927	·011	·021	·040498	·015512	·011	·021
323	·034161	·014271	·011	·021	·037267	·014882	·011	·021	·040373	·015465−	·011	·021
324	·034056	·014228	·011	·021	·037152	·014837	·011	·021	·040248	·015418	·011	·021
325	·033951	·014185+	·011	·021	·037037	·014792	·011	·021	·040123	·015371	·011	·021
326	·033846	·014142	·011	·021	·036923	·014748	·011	·021	·040000	·015325+	·011	·021
327	·033742	·014100	·011	·021	·036810	·014703	·011	·021	·039877	·015279	·011	·021
328	·033639	·014058	·011	·021	·036697	·014659	·011	·021	·039755+	·015234	·011	·021
329	·033537	·014016	·011	·021	·036585+	·014616	·011	·021	·039634	·015188	·011	·021
330	·033435−	·013974	·011	·021	·036474	·014572	·011	·021	·039514	·015143	·011	·021
331	·033333	·013932	·011	·021	·036364	·014529	·011	·021	·039394	·015098	·011	·021
332	·033233	·013891	·011	·021	·036254	·014486	·011	·021	·039275−	·015054	·011	·021
333	·033133	·013850+	·011	·021	·036145−	·014443	·011	·021	·039157	·015010	·011	·021
334	·033033	·013809	·011	·021	·036036	·014401	·011	·021	·039039	·014966	·011	·021
335	·032934	·013769	·011	·021	·035928	·014359	·011	·021	·038922	·014922	·011	·021
336	·032836	·013729	·011	·021	·035821	·014317	·011	·021	·038806	·014878	·011	·021
337	·032738	·013688	·011	·021	·035714	·014275+	·011	·021	·038690	·014835+	·011	·021
338	·032641	·013649	·011	·021	·035608	·014234	·011	·021	·038576	·014792	·011	·021
339	·032544	·013609	·011	·021	·035503	·014192	·011	·021	·038462	·014749	·011	·021
340	·032448	·013570	·011	·021	·035398	·014152	·011	·021	·038348	·014707	·011	·021
341	·032353	·013531	·011	·021	·035294	·014111	·011	·021	·038235+	·014665−	·011	·021
342	·032258	·013492	·011	·021	·035191	·014070	·011	·021	·038123	·014623	·011	·021
343	·032164	·013453	·011	·021	·035088	·014030	·011	·021	·038012	·014581	·011	·021
344	·032070	·013415−	·011	·021	·034985+	·013990	·011	·021	·037901	·014539	·011	·021
345	·031977	·013376	·011	·021	·034884	·013950+	·011	·021	·037791	·014498	·011	·021
346	·031884	·013338	·011	·021	·034783	·013911	·011	·021	·037681	·014457	·011	·021
347	·031792	·013301	·011	·021	·034682	·013871	·011	·021	·037572	·014416	·011	·021
348	·031700	·013263	·011	·021	·034582	·013832	·011	·021	·037464	·014375+	·011	·021
349	·031609	·013226	·011	·021	·034483	·013793	·011	·021	·037356	·014335−	·011	·021
350	·031519	·013188	·011	·021	·034384	·013754	·011	·021	·037249	·014295−	·011	·021

N = size of sample	15: $\bar{\eta}^2$	15: $\sigma_{\bar{\eta}^2}$	15: P_1 $\lambda_1 = 2{\cdot}78$	15: P_2 $\lambda_2 = 2{\cdot}38$	16: $\bar{\eta}^2$	16: $\sigma_{\bar{\eta}^2}$	16: P_1 $\lambda_1 = 2{\cdot}77$	16: P_2 $\lambda_2 = 2{\cdot}37$	17: $\bar{\eta}^2$	17: $\sigma_{\bar{\eta}^2}$	17: P_1 $\lambda_1 = 2{\cdot}76$	17: P_2 $\lambda_2 = 2{\cdot}36$
301	·046667	·017165 −	·011	·021	·050000	·017736	·011	·021	·053333	·018286	·010	·021
302	·046512	·017109	·011	·021	·049834	·017679	·011	·021	·053156	·018227	·010	·021
303	·046358	·017054	·011	·021	·049669	·017622	·011	·021	·052980	·018168	·010	·021
304	·046205 −	·016999	·011	·021	·049505 −	·017566	·011	·021	·052805 +	·018110	·010	·021
305	·046053	·016945 +	·011	·021	·049342	·017510	·011	·021	·052632	·018052	·010	·021
306	·045902	·016891	·011	·021	·049180	·017454	·011	·021	·052459	·017995 +	·010	·021
307	·045752	·016837	·011	·021	·049020	·017398	·011	·021	·052288	·017938	·010	·021
308	·045603	·016784	·011	·021	·048860	·017343	·011	·021	·052117	·017881	·010	·021
309	·045455 −	·016731	·011	·021	·048701	·017289	·011	·021	·051948	·017825 +	·010	·021
310	·045307	·016678	·011	·021	·048544	·017234	·011	·021	·051780	·017769	·010	·021
311	·045161	·016626	·011	·021	·048387	·017180	·011	·021	·051613	·017714	·010	·021
312	·045016	·016574	·011	·021	·048232	·017127	·011	·021	·051447	·017658	·010	·021
313	·044872	·016522	·011	·021	·048077	·017073	·011	·021	·051282	·017604	·010	·021
314	·044728	·016471	·011	·021	·047923	·017020	·011	·021	·051118	·017549	·010	·021
315	·044586	·016420	·011	·021	·047771	·016968	·011	·021	·050955 +	·017495 −	·010	·021
316	·044444	·016369	·011	·021	·047619	·016915 +	·011	·021	·050794	·017441	·010	·021
317	·044304	·016319	·011	·021	·047468	·016863	·011	·021	·050633	·017387	·010	·021
318	·044164	·016268	·011	·021	·047319	·016812	·011	·021	·050473	·017334	·010	·021
319	·044025 +	·016219	·011	·021	·047170	·016760	·011	·021	·050314	·017281	·010	·021
320	·043887	·016169	·011	·021	·047022	·016709	·011	·021	·050157	·017229	·010	·021
321	·043750	·016120	·011	·021	·046875	·016658	·011	·021	·050000	·017176	·010	·021
322	·043614	·016071	·011	·021	·046729	·016608	·011	·021	·049844	·017125 −	·010	·021
323	·043478	·016022	·011	·021	·046584	·016558	·011	·021	·049689	·017073	·010	·021
324	·043344	·015974	·011	·021	·046440	·016508	·011	·021	·049536	·017022	·010	·021
325	·043210	·015926	·011	·021	·046296	·016458	·011	·021	·049383	·016971	·010	·021
326	·043077	·015878	·011	·021	·046154	·016409	·011	·021	·049231	·016920	·010	·021
327	·042945 −	·015831	·011	·021	·046012	·016360	·011	·021	·049080	·016869	·010	·021
328	·042813	·015784	·011	·021	·045872	·016311	·011	·021	·048930	·016819	·010	·021
329	·042683	·015737	·011	·021	·045732	·016263	·011	·021	·048780	·016770	·010	·021
330	·042553	·015690	·011	·021	·045593	·016215 −	·011	·021	·048632	·016720	·010	·021
331	·042424	·015644	·011	·021	·045455 −	·016167	·011	·021	·048485 −	·016671	·010	·021
332	·042296	·015598	·011	·021	·045317	·016120	·011	·021	·048338	·016622	·010	·021
333	·042169	·015552	·011	·021	·045181	·016072	·011	·021	·048193	·016573	·010	·021
334	·042042	·015506	·011	·021	·045045 +	·016025 +	·011	·021	·048048	·016525 −	·010	·021
335	·041916	·015461	·011	·021	·044910	·015979	·011	·021	·047904	·016477	·010	·021
336	·041791	·015416	·011	·021	·044776	·015932	·011	·021	·047761	·016429	·010	·021
337	·041667	·015371	·011	·021	·044643	·015886	·011	·021	·047619	·016381	·010	·021
338	·041543	·015327	·011	·021	·044510	·015840	·011	·021	·047478	·016334	·010	·021
339	·041420	·015283	·011	·021	·044379	·015795 −	·011	·021	·047337	·016287	·010	·021
340	·041298	·015239	·011	·021	·044248	·015749	·011	·021	·047198	·016240	·010	·021
341	·041176	·015195 −	·011	·021	·044118	·015704	·011	·021	·047059	·016194	·010	·021
342	·041056	·015151	·011	·021	·043988	·015659	·011	·021	·046921	·016148	·010	·021
343	·040936	·015108	·011	·021	·043860	·015615 −	·011	·021	·046784	·016102	·010	·021
344	·040816	·015065 +	·011	·021	·043732	·015570	·011	·021	·046647	·016056	·010	·021
345	·040698	·015022	·011	·021	·043605 −	·015526	·011	·021	·046512	·016011	·010	·021
346	·040580	·014980	·011	·021	·043478	·015482	·011	·021	·046377	·015966	·010	·021
347	·040462	·014938	·011	·021	·043353	·015439	·011	·021	·046243	·015921	·010	·021
348	·040346	·014896	·011	·021	·043228	·015395 +	·011	·021	·046110	·015876	·010	·021
349	·040230	·014854	·011	·021	·043103	·015352	·011	·021	·045977	·015832	·010	·021
350	·040115 −	·014812	·011	·021	·042980	·015309	·011	·021	·045845 +	·015788	·010	·021

N = size of sample	18: $\bar{\eta}^2$	18: $\sigma_{\bar{\eta}^2}$	18: P_1 $\lambda_1 = 2{\cdot}75$	18: P_2 $\lambda_2 = 2{\cdot}35$	19: $\bar{\eta}^2$	19: $\sigma_{\bar{\eta}^2}$	19: P_1 $\lambda_1 = 2{\cdot}74$	19: P_2 $\lambda_2 = 2{\cdot}34$	20: $\bar{\eta}^2$	20: $\sigma_{\bar{\eta}^2}$	20: P_1 $\lambda_1 = 2{\cdot}73$	20: P_2 $\lambda_2 = 2{\cdot}33$
301	·056667	·018815 +	·010	·021	·060000	·019326	·010	·021	·063333	·019821	·010	·021
302	·056478	·018755 −	·010	·021	·059801	·019264	·010	·021	·063123	·019757	·010	·021
303	·056291	·018695 −	·010	·021	·059603	·019203	·010	·021	·062914	·019694	·010	·021
304	·056106	·018635 +	·010	·021	·059406	·019142	·010	·021	·062706	·019632	·010	·021
305	·055921	·018576	·010	·021	·059211	·019081	·010	·021	·062500	·019570	·010	·021
306	·055738	·018517	·010	·021	·059016	·019021	·010	·021	·062295 +	·019508	·010	·021
307	·055556	·018458	·010	·021	·058824	·018961	·010	·021	·062092	·019446	·010	·021
308	·055375 −	·018400	·010	·021	·058632	·018901	·010	·021	·061889	·019385 +	·010	·021
309	·055195 −	·018342	·010	·021	·058442	·018842	·010	·021	·061688	·019325 −	·010	·021
310	·055016	·018285 −	·010	·021	·058252	·018783	·010	·021	·061489	·019264	·010	·021
311	·054839	·018228	·010	·021	·058065 −	·018724	·010	·021	·061290	·019204	·010	·021
312	·054662	·018171	·010	·021	·057878	·018666	·010	·021	·061093	·019145 −	·010	·021
313	·054487	·018115 −	·010	·021	·057692	·018608	·010	·021	·060897	·019086	·010	·021
314	·054313	·018059	·010	·021	·057508	·018551	·010	·021	·060703	·019027	·010	·021
315	·054140	·018003	·010	·021	·057325 −	·018494	·010	·021	·060510	·018968	·010	·021
316	·053968	·017948	·010	·021	·057143	·018437	·010	·021	·060317	·018910	·010	·021
317	·053797	·017893	·010	·021	·056962	·018381	·010	·021	·060127	·018853	·010	·021
318	·053628	·017838	·010	·021	·056782	·018325 −	·010	·021	·059937	·018795 +	·010	·021
319	·053459	·017784	·010	·021	·056604	·018269	·010	·021	·059748	·018738	·010	·021
320	·053292	·017730	·010	·021	·056426	·018213	·010	·021	·059561	·018681	·010	·021
321	·053125	·017676	·010	·021	·056250	·018158	·010	·021	·059375	·018625 +	·010	·021
322	·052960	·017623	·010	·021	·056075 −	·018104	·010	·021	·059190	·018569	·010	·021
323	·052795 +	·017570	·010	·021	·055901	·018049	·010	·021	·059006	·018513	·010	·021
324	·052632	·017517	·010	·021	·055728	·017995 +	·010	·021	·058824	·018458	·010	·021
325	·052469	·017464	·010	·021	·055556	·017941	·010	·021	·058642	·018403	·010	·021
326	·052308	·017412	·010	·021	·055385 −	·017888	·010	·021	·058462	·018348	·010	·021
327	·052147	·017361	·010	·021	·055215 −	·017835 −	·010	·021	·058282	·018294	·010	·021
328	·051988	·017309	·010	·021	·055046	·017782	·010	·021	·058104	·018240	·010	·021
329	·051829	·017258	·010	·021	·054878	·017730	·010	·021	·057927	·018186	·010	·021
330	·051672	·017207	·010	·021	·054711	·017678	·010	·021	·057751	·018133	·010	·021
331	·051515 +	·017156	·010	·021	·054545 +	·017626	·010	·021	·057576	·018080	·010	·021
332	·051360	·017106	·010	·021	·054381	·017574	·010	·021	·057402	·018027	·010	·021
333	·051205 −	·017056	·010	·021	·054217	·017523	·010	·021	·057229	·017974	·010	·021
334	·051051	·017007	·010	·021	·054054	·017472	·010	·021	·057057	·017922	·010	·021
335	·050898	·016957	·010	·021	·053892	·017421	·010	·021	·056886	·017870	·010	·021
336	·050746	·016908	·010	·021	·053731	·017371	·010	·021	·056716	·017819	·010	·021
337	·050595 +	·016859	·010	·021	·053571	·017321	·010	·021	·056548	·017767	·010	·021
338	·050445 +	·016811	·010	·021	·053412	·017271	·010	·021	·056380	·017716	·010	·021
339	·050296	·016762	·010	·021	·053254	·017221	·010	·021	·056213	·017666	·010	·021
340	·050147	·016714	·010	·021	·053097	·017172	·010	·021	·056047	·017615 +	·010	·021
341	·050000	·016667	·010	·021	·052941	·017123	·010	·021	·055882	·017565 +	·010	·021
342	·049853	·016619	·010	·021	·052786	·017075 −	·010	·021	·055718	·017515 +	·010	·021
343	·049708	·016572	·010	·021	·052632	·017026	·010	·021	·055556	·017466	·010	·021
344	·049563	·016525 +	·010	·021	·052478	·016978	·010	·021	·055394	·017416	·010	·021
345	·049419	·016478	·010	·021	·052326	·016930	·010	·021	·055233	·017367	·010	·021
346	·049275 +	·016432	·010	·021	·052174	·016883	·010	·021	·055072	·017319	·010	·021
347	·049133	·016386	·010	·021	·052023	·016835 +	·010	·021	·054913	·017270	·010	·021
348	·048991	·016340	·010	·021	·051873	·016788	·010	·021	·054755 +	·017222	·010	·021
349	·048851	·016294	·010	·021	·051724	·016742	·010	·021	·054598	·017174	·010	·021
350	·048711	·016249	·010	·021	·051576	·016695 −	·010	·021	·054441	·017127	·010	·021

n = number of arrays

N = size of sample	3: $\bar{\eta}^2$	3: $\sigma_{\bar{\eta}^2}$	3: P_1 λ_1 = 3·50	3: P_2 λ_2 = 2·94	4: $\bar{\eta}^2$	4: $\sigma_{\bar{\eta}^2}$	4: P_1 λ_1 = 3·20	4: P_2 λ_2 = 2·80	5: $\bar{\eta}^2$	5: $\sigma_{\bar{\eta}^2}$	5: P_1 λ_1 = 3·14	5: P_2 λ_2 = 2·68
351	·005714	·005682	·011	·019	·008571	·006949	·012	·020	·011429	·008012	·012	·020
352	·005698	·005666	·011	·019	·008547	·006929	·012	·020	·011396	·007989	·012	·020
353	·005682	·005650 −	·011	·019	·008523	·006909	·012	·020	·011364	·007967	·012	·020
354	·005666	·005634	·011	·019	·008499	·006890	·012	·020	·011331	·007945 −	·012	·020
355	·005650 −	·005618	·011	·019	·008475 −	·006871	·012	·020	·011299	·007922	·012	·020
356	·005634	·005602	·011	·019	·008451	·006851	·012	·020	·011268	·007900	·012	·020
357	·005618	·005587	·011	·019	·008427	·006832	·012	·020	·011236	·007878	·012	·020
358	·005602	·005571	·011	·019	·008403	·006813	·012	·020	·011204	·007856	·012	·020
359	·005587	·005555 +	·011	·019	·008380	·006794	·012	·020	·011173	·007835 −	·012	·020
360	·005571	·005540	·011	·019	·008357	·006776	·012	·020	·011142	·007813	·012	·020
361	·005556	·005525 −	·011	·019	·008333	·006757	·012	·020	·011111	·007791	·012	·021
362	·005540	·005510	·011	·019	·008310	·006738	·012	·020	·011080	·007770	·012	·021
363	·005525 −	·005494	·011	·019	·008287	·006720	·012	·020	·011050 −	·007749	·012	·021
364	·005510	·005479	·011	·019	·008264	·006702	·012	·020	·011019	·007728	·012	·021
365	·005495 −	·005464	·011	·019	·008242	·006683	·012	·020	·010989	·007707	·012	·021
366	·005479	·005450 −	·011	·019	·008219	·006665 +	·012	·020	·010959	·007686	·012	·021
367	·005464	·005435 −	·011	·019	·008197	·006647	·012	·020	·010929	·007665 −	·012	·021
368	·005450 −	·005420	·011	·019	·008174	·006629	·012	·020	·010899	·007644	·012	·021
369	·005435 −	·005405 +	·011	·019	·008152	·006611	·012	·020	·010870	·007623	·012	·021
370	·005420	·005391	·011	·019	·008130	·006593	·012	·020	·010840	·007603	·012	·021
371	·005405 +	·005376	·011	·019	·008108	·006576	·012	·020	·010811	·007582	·012	·021
372	·005391	·005362	·011	·019	·008086	·006558	·012	·020	·010782	·007562	·012	·021
373	·005376	·005348	·011	·019	·008065 −	·006540	·012	·020	·010753	·007542	·012	·021
374	·005362	·005333	·011	·019	·008043	·006523	·012	·020	·010724	·007522	·012	·021
375	·005348	·005319	·011	·019	·008021	·006506	·012	·020	·010695 +	·007502	·012	·021
376	·005333	·005305 −	·011	·019	·008000	·006489	·012	·020	·010667	·007482	·012	·021
377	·005319	·005291	·011	·019	·007979	·006471	·012	·020	·010638	·007462	·012	·021
378	·005305 +	·005277	·011	·019	·007958	·006454	·012	·020	·010610	·007443	·012	·021
379	·005291	·005263	·011	·019	·007937	·006437	·012	·020	·010582	·007423	·012	·021
380	·005277	·005249	·011	·019	·007916	·006420	·012	·020	·010554	·007404	·012	·021
381	·005263	·005236	·011	·019	·007895 −	·006404	·012	·020	·010526	·007385 −	·012	·021
382	·005249	·005222	·011	·019	·007874	·006387	·012	·020	·010499	·007365 +	·012	·021
383	·005236	·005208	·011	·019	·007853	·006370	·012	·020	·010471	·007346	·012	·021
384	·005222	·005195 −	·011	·019	·007833	·006354	·012	·020	·010444	·007327	·012	·021
385	·005208	·005181	·011	·019	·007813	·006337	·012	·020	·010417	·007308	·012	·021
386	·005195 −	·005168	·011	·019	·007792	·006321	·012	·020	·010390	·007289	·012	·021
387	·005181	·005155 −	·011	·019	·007772	·006305 −	·012	·020	·010363	·007271	·012	·021
388	·005168	·005141	·011	·019	·007752	·006289	·012	·020	·010336	·007252	·012	·021
389	·005155 −	·005128	·011	·019	·007732	·006273	·012	·020	·010309	·007233	·012	·021
390	·005141	·005115 +	·011	·019	·007712	·006256	·012	·020	·010283	·007215 +	·012	·021
391	·005128	·005102	·011	·019	·007692	·006241	·012	·020	·010256	·007197	·012	·021
392	·005115 +	·005089	·011	·019	·007673	·006225 −	·012	·020	·010230	·007178	·012	·021
393	·005102	·005076	·011	·019	·007653	·006209	·012	·020	·010204	·007160	·012	·021
394	·005089	·005063	·011	·019	·007634	·006193	·012	·020	·010178	·007142	·012	·021
395	·005076	·005050 +	·011	·019	·007614	·006178	·012	·020	·010152	·007124	·012	·021
396	·005063	·005038	·011	·019	·007595 −	·006162	·012	·020	·010127	·007106	·012	·021
397	·005051	·005025 +	·011	·019	·007576	·006147	·012	·020	·010101	·007088	·012	·021
398	·005038	·005012	·011	·019	·007557	·006131	·012	·020	·010076	·007071	·012	·021
399	·005025 +	·005000 −	·011	·019	·007538	·006116	·012	·020	·010050 +	·007053	·012	·021
400	·005013	·004987	·011	·019	·007519	·006101	·012	·020	·010025 +	·007036	·012	·021

N = size of sample	6: $\bar{\eta}^2$	6: $\sigma_{\bar{\eta}^2}$	6: P_1 λ_1 = 3·11	6: P_2 λ_2 = 2·63	7: $\bar{\eta}^2$	7: $\sigma_{\bar{\eta}^2}$	7: P_1 λ_1 = 3·08	7: P_2 λ_2 = 2·58	8: $\bar{\eta}^2$	8: $\sigma_{\bar{\eta}^2}$	8: P_1 λ_1 = 3·02	8: P_2 λ_2 = 2·54
351	·014286	·008945 −	·011	·020	·017143	·009784	·010	·020	·020000	·010553	·010	·021
352	·014245 +	·008920	·011	·020	·017094	·009757	·010	·020	·019943	·010523	·010	·021
353	·014205 −	·008894	·011	·020	·017045 +	·009729	·010	·020	·019886	·010494	·010	·021
354	·014164	·008870	·011	·020	·016997	·009702	·010	·020	·019830	·010464	·010	·021
355	·014124	·008845 −	·011	·020	·016949	·009675 +	·010	·020	·019774	·010435 +	·010	·021
356	·014085 −	·008820	·011	·020	·016901	·009648	·010	·020	·019718	·010406	·010	·021
357	·014045 −	·008796	·011	·020	·016854	·009621	·010	·020	·019663	·010377	·010	·021
358	·014006	·008771	·011	·020	·016807	·009595 −	·010	·020	·019608	·010349	·010	·021
359	·013966	·008747	·011	·020	·016760	·009568	·010	·020	·019553	·010320	·010	·021
360	·013928	·008723	·011	·020	·016713	·009542	·010	·020	·019499	·010292	·010	·021
361	·013889	·008699	·011	·020	·016667	·009516	·010	·020	·019444	·010263	·010	·021
362	·013850 +	·008675 −	·011	·020	·016620	·009490	·010	·020	·019391	·010235 +	·010	·021
363	·013812	·008651	·011	·020	·016575 −	·009464	·010	·020	·019337	·010207	·010	·021
364	·013774	·008628	·011	·020	·016529	·009438	·010	·020	·019284	·010180	·010	·021
365	·013736	·008604	·011	·020	·016484	·009412	·010	·020	·019231	·010152	·010	·021
366	·013699	·008581	·011	·020	·016438	·009387	·010	·020	·019178	·010125 −	·010	·021
367	·013661	·008558	·011	·020	·016393	·009361	·010	·020	·019126	·010097	·010	·021
368	·013624	·008534	·011	·020	·016349	·009336	·010	·020	·019074	·010070	·010	·021
369	·013587	·008511	·011	·020	·016304	·009311	·010	·020	·019022	·010043	·010	·021
370	·013550 +	·008489	·011	·020	·016260	·009286	·010	·020	·018970	·010016	·010	·021
371	·013514	·008466	·011	·020	·016216	·009261	·010	·020	·018919	·009990	·010	·021
372	·013477	·008443	·011	·020	·016173	·009237	·010	·020	·018868	·009963	·010	·021
373	·013441	·008421	·011	·020	·016129	·009212	·010	·020	·018817	·009936	·010	·021
374	·013405 −	·008398	·011	·020	·016086	·009188	·010	·020	·018767	·009910	·010	·021
375	·013369	·008376	·011	·020	·016043	·009163	·010	·020	·018717	·009884	·010	·021
376	·013333	·008354	·011	·020	·016000	·009139	·010	·020	·018667	·009858	·010	·021
377	·013298	·008332	·011	·020	·015957	·009115 +	·010	·020	·018617	·009832	·010	·021
378	·013263	·008310	·011	·020	·015915 +	·009091	·010	·020	·018568	·009806	·010	·021
379	·013228	·008288	·011	·020	·015873	·009067	·010	·020	·018519	·009781	·010	·021
380	·013193	·008267	·011	·020	·015831	·009044	·010	·020	·018470	·009755 +	·010	·021
381	·013158	·008245 +	·011	·020	·015789	·009020	·010	·020	·018421	·009730	·010	·021
382	·013123	·008224	·011	·020	·015748	·008997	·010	·020	·018373	·009705 −	·010	·021
383	·013089	·008202	·011	·020	·015707	·008973	·010	·020	·018325 −	·009679	·010	·021
384	·013055 −	·008181	·011	·020	·015666	·008950 +	·010	·020	·018277	·009654	·010	·021
385	·013021	·008160	·011	·020	·015625	·008927	·010	·020	·018229	·009630	·010	·021
386	·012987	·008139	·011	·020	·015584	·008904	·010	·020	·018182	·009605 −	·010	·021
387	·012953	·008118	·011	·020	·015544	·008881	·010	·020	·018135 −	·009580	·010	·021
388	·012920	·008097	·011	·020	·015504	·008859	·010	·020	·018088	·009556	·010	·021
389	·012887	·008077	·011	·020	·015464	·008836	·010	·020	·018041	·009532	·010	·021
390	·012853	·008056	·011	·020	·015424	·008814	·010	·020	·017995 −	·009507	·010	·021
391	·012821	·008036	·011	·020	·015385 −	·008791	·010	·020	·017949	·009483	·010	·021
392	·012788	·008015 +	·011	·020	·015345 +	·008769	·010	·020	·017903	·009459	·010	·021
393	·012755 +	·007995 +	·011	·020	·015306	·008747	·010	·020	·017857	·009435 +	·010	·021
394	·012723	·007975 −	·011	·020	·015267	·008725 −	·010	·020	·017812	·009412	·010	·021
395	·012690	·007955 −	·011	·020	·015228	·008703	·010	·020	·017766	·009388	·010	·021
396	·012658	·007935 −	·011	·020	·015190	·008681	·010	·020	·017722	·009365 −	·010	·021
397	·012626	·007915 +	·011	·020	·015152	·008659	·010	·020	·017677	·009341	·010	·021
398	·012594	·007895 +	·011	·020	·015113	·008638	·010	·020	·017632	·009318	·010	·021
399	·012563	·007876	·011	·020	·015075 +	·008616	·010	·020	·017588	·009295 −	·010	·021
400	·012531	·007856	·011	·020	·015038	·008595 −	·010	·020	·017544	·009272	·010	·021

N = size of sample	9: $\overline{\eta}^2$	9: $\sigma_{\overline{\eta}^2}$	9: P_1 λ_1=2·96	9: P_2 λ_2=2·50	10: $\overline{\eta}^2$	10: $\sigma_{\overline{\eta}^2}$	10: P_1 λ_1=2·92	10: P_2 λ_2=2·47	11: $\overline{\eta}^2$	11: $\sigma_{\overline{\eta}^2}$	11: P_1 λ_1=2·88	11: P_2 λ_2=2·44	12: $\overline{\eta}^2$	12: $\sigma_{\overline{\eta}^2}$	12: P_1 λ_1=2·84	12: P_2 λ_2=2·42	13: $\overline{\eta}^2$	13: $\sigma_{\overline{\eta}^2}$	13: P_1 λ_1=2·80	13: P_2 λ_2=2·40	14: $\overline{\eta}^2$	14: $\sigma_{\overline{\eta}^2}$	14: P_1 λ_1=2·79	14: P_2 λ_2=2·39
351	·022857	·011265 +	·011	·021	·025714	·011931	·011	·021	·028571	·012558	·011	·021	·031429	·013151	·011	·021	·034286	·013716	·011	·021	·037143	·014255 −	·011	·021
352	·022792	·011233	·011	·021	·025641	·011897	·011	·021	·028490	·012523	·011	·021	·031339	·013115 −	·011	·021	·034188	·013678	·011	·021	·037037	·014215 +	·011	·021
353	·022727	·011202	·011	·021	·025568	·011864	·011	·021	·028409	·012488	·011	·021	·031250 −	·013078	·011	·021	·034091	·013640	·011	·021	·036932	·014176	·011	·021
354	·022663	·011171	·011	·021	·025496	·011831	·011	·021	·028329	·012453	·011	·021	·031161	·013042	·011	·021	·033994	·013602	·011	·021	·036827	·014136	·011	·021
355	·022599	·011140	·011	·021	·025424	·011798	·011	·021	·028249	·012418	·011	·021	·031073	·013006	·011	·021	·033898	·013564	·011	·021	·036723	·014097	·011	·021
356	·022535 +	·011109	·011	·021	·025352	·011766	·011	·021	·028169	·012384	·011	·021	·030986	·012970	·011	·021	·033803	·013527	·011	·021	·036620	·014058	·011	·021
357	·022472	·011078	·011	·021	·025281	·011733	·011	·021	·028090	·012350 −	·011	·021	·030899	·012934	·011	·021	·033708	·013489	·011	·021	·036517	·014020	·011	·021
358	·022409	·011047	·011	·021	·025210	·011701	·011	·021	·028011	·012316	·011	·021	·030812	·012898	·011	·021	·033613	·013452	·011	·021	·036415 −	·013981	·011	·021
359	·022346	·011017	·011	·021	·025140	·011668	·011	·021	·027933	·012282	·011	·021	·030726	·012863	·011	·021	·033520	·013416	·011	·021	·036313	·013943	·011	·021
360	·022284	·010987	·011	·021	·025070	·011636	·011	·021	·027855 +	·012248	·011	·021	·030641	·012828	·011	·021	·033426	·013379	·011	·021	·036212	·013905 +	·011	·021
361	·022222	·010957	·011	·021	·025000	·011605 −	·011	·021	·027778	·012215 −	·011	·021	·030556	·012793	·011	·021	·033333	·013343	·011	·021	·036111	·013867	·011	·021
362	·022161	·010927	·011	·021	·024931	·011573	·011	·021	·027701	·012182	·011	·021	·030471	·012758	·011	·021	·033241	·013306	·011	·021	·036011	·013830	·011	·021
363	·022099	·010897	·011	·021	·024862	·011542	·011	·021	·027624	·012149	·011	·021	·030387	·012723	·011	·021	·033149	·013270	·011	·021	·035912	·013792	·011	·021
364	·022039	·010867	·011	·021	·024793	·011510	·011	·021	·027548	·012116	·011	·021	·030303	·012689	·011	·021	·033058	·013234	·011	·021	·035813	·013755 +	·011	·021
365	·021978	·010838	·011	·021	·024725 +	·011479	·011	·021	·027473	·012083	·011	·021	·030220	·012655 −	·011	·021	·032967	·013199	·011	·021	·035714	·013718	·011	·021
366	·021918	·010809	·011	·021	·024658	·011448	·011	·021	·027397	·012050 +	·011	·021	·030137	·012621	·011	·021	·032877	·013163	·011	·021	·035616	·013681	·011	·021
367	·021858	·010779	·011	·021	·024590	·011417	·011	·021	·027322	·012018	·011	·021	·030055 −	·012587	·011	·021	·032787	·013128	·011	·021	·035519	·013645 −	·011	·021
368	·021798	·010750 +	·011	·021	·024523	·011387	·011	·021	·027248	·011986	·011	·021	·029973	·012553	·011	·021	·032698	·013093	·011	·021	·035422	·013608	·011	·021
369	·021739	·010722	·011	·021	·024457	·011356	·011	·021	·027174	·011954	·011	·021	·029891	·012520	·011	·021	·032609	·013058	·011	·021	·035326	·013572	·011	·021
370	·021680	·010693	·011	·021	·024390	·011326	·011	·021	·027100	·011922	·011	·021	·029810	·012486	·011	·021	·032520	·013023	·011	·021	·035230	·013536	·011	·021
371	·021622	·010665 −	·011	·021	·024324	·011296	·011	·021	·027027	·011890	·011	·021	·029730	·012453	·011	·021	·032432	·012989	·011	·021	·035135 +	·013500 +	·011	·021
372	·021563	·010636	·011	·021	·024259	·011266	·011	·021	·026954	·011859	·011	·021	·029650 −	·012420	·011	·021	·032345 +	·012955 −	·011	·021	·035040	·013465 −	·011	·021
373	·021505 +	·010608	·011	·021	·024194	·011236	·011	·021	·026882	·011827	·011	·021	·029570	·012388	·011	·021	·032258	·012920	·011	·021	·034946	·013429	·011	·021
374	·021448	·010580	·011	·021	·024129	·011206	·011	·021	·026810	·011796	·011	·021	·029491	·012355 −	·011	·021	·032172	·012886	·011	·021	·034853	·013394	·011	·021
375	·021390	·010552	·011	·021	·024064	·011177	·011	·021	·026738	·011765 +	·011	·021	·029412	·012323	·011	·021	·032086	·012853	·011	·021	·034759	·013359	·011	·021
376	·021333	·010524	·011	·021	·024000	·011147	·011	·021	·026667	·011734	·011	·021	·029333	·012290	·011	·021	·032000	·012819	·011	·021	·034667	·013324	·011	·021
377	·021277	·010497	·011	·021	·023936	·011118	·011	·021	·026596	·011704	·011	·021	·029255 +	·012258	·011	·021	·031915 −	·012786	·011	·021	·034574	·013289	·011	·021
378	·021220	·010469	·011	·021	·023873	·011089	·011	·021	·026525 +	·011673	·011	·021	·029178	·012226	·011	·021	·031830	·012752	·011	·021	·034483	·013255 −	·011	·021
379	·021164	·010442	·011	·021	·023810	·011060	·011	·021	·026455 +	·011643	·011	·021	·029101	·012194	·011	·021	·031746	·012719	·011	·021	·034392	·013221	·011	·021
380	·021108	·010415 −	·011	·021	·023747	·011032	·011	·021	·026385 +	·011613	·011	·021	·029024	·012163	·011	·021	·031662	·012686	·011	·021	·034301	·013186	·011	·021
381	·021053	·010388	·011	·021	·023684	·011003	·011	·021	·026316	·011582	·011	·021	·028947	·012131	·011	·021	·031579	·012654	·011	·021	·034211	·013152	·011	·021
382	·020997	·010361	·011	·021	·023622	·010974	·011	·021	·026247	·011553	·011	·021	·028871	·012100	·011	·021	·031496	·012621	·011	·021	·034121	·013119	·011	·021
383	·020942	·010334	·011	·021	·023560	·010946	·011	·021	·026178	·011523	·011	·021	·028796	·012069	·011	·021	·031414	·012589	·011	·021	·034031	·013085 −	·011	·021
384	·020888	·010307	·011	·021	·023499	·010918	·011	·021	·026110	·011493	·011	·021	·028721	·012038	·011	·021	·031332	·012556	·011	·021	·033943	·013051	·011	·021
385	·020833	·010281	·011	·021	·023438	·010890	·011	·021	·026042	·011464	·011	·021	·028646	·012007	·011	·021	·031250	·012524	·011	·021	·033854	·013018	·011	·021
386	·020779	·010254	·011	·021	·023377	·010862	·011	·021	·025974	·011434	·011	·021	·028571	·011977	·011	·021	·031169	·012492	·011	·021	·033766	·012985 −	·011	·021
387	·020725 +	·010228	·011	·021	·023316	·010834	·011	·021	·025907	·011405 +	·011	·021	·028497	·011946	·011	·021	·031088	·012461	·011	·021	·033679	·012952	·011	·021
388	·020672	·010202	·011	·021	·023256	·010807	·011	·021	·025840	·011376	·011	·021	·028424	·011916	·011	·021	·031008	·012429	·011	·021	·033592	·012919	·011	·021
389	·020619	·010176	·011	·021	·023196	·010779	·011	·021	·025773	·011347	·011	·021	·028351	·011885 +	·011	·021	·030928	·012398	·011	·021	·033505 +	·012887	·011	·021
390	·020566	·010150 +	·011	·021	·023136	·010752	·011	·021	·025707	·011319	·011	·021	·028278	·011855 +	·011	·021	·030848	·012366	·011	·021	·033419	·012854	·011	·021
391	·020513	·010125 −	·011	·021	·023077	·010725 −	·011	·021	·025641	·011290	·011	·021	·028205 +	·011826	·011	·021	·030769	·012335 +	·011	·021	·033333	·012822	·011	·021
392	·020460	·010099	·011	·021	·023018	·010698	·011	·021	·025575 +	·011262	·011	·021	·028133	·011796	·011	·021	·030691	·012304	·011	·021	·033248	·012790	·011	·021
393	·020408	·010074	·011	·021	·022959	·010671	·011	·021	·025510	·011233	·011	·021	·028061	·011766	·011	·021	·030612	·012273	·011	·021	·033163	·012758	·011	·021
394	·020356	·010048	·011	·021	·022901	·010644	·011	·021	·025445 +	·011205 +	·011	·021	·027990	·011737	·011	·021	·030534	·012243	·011	·021	·033079	·012726	·011	·021
395	·020305 −	·010023	·011	·021	·022843	·010618	·011	·021	·025381	·011177	·011	·021	·027919	·011708	·011	·021	·030457	·012212	·011	·021	·032995 −	·012694	·011	·021
396	·020253	·009998	·011	·021	·022785 −	·010591	·011	·021	·025316	·011149	·011	·021	·027848	·011678	·011	·021	·030380	·012182	·011	·021	·032911	·012663	·011	·021
397	·020202	·009973	·011	·021	·022727	·010565 −	·011	·021	·025253	·011122	·011	·021	·027778	·011649	·011	·021	·030303	·012152	·011	·021	·032828	·012631	·011	·021
398	·020151	·009949	·011	·021	·022670	·010538	·011	·021	·025189	·011094	·011	·021	·027708	·011621	·011	·021	·030227	·012122	·011	·021	·032746	·012600	·011	·021
399	·020101	·009924	·011	·021	·022613	·010512	·011	·021	·025126	·011067	·011	·021	·027638	·011592	·011	·021	·030151	·012092	·011	·021	·032663	·012569	·011	·021
400	·020050 +	·009899	·011	·021	·022556	·010486	·011	·021	·025063	·011039	·011	·021	·027569	·011563	·011	·021	·030075 +	·012062	·011	·021	·032581	·012538	·011	·021

n = number of arrays

N = size of sample	15: $\bar{\eta}^2$	15: $\sigma_{\bar{\eta}^2}$	15: P_1 $\lambda_1=2·78$	15: P_2 $\lambda_2=2·38$	16: $\bar{\eta}^2$	16: $\sigma_{\bar{\eta}^2}$	16: P_1 $\lambda_1=2·77$	16: P_2 $\lambda_2=2·37$
351	·040000	·014771	·011	·021	·042857	·015267	·011	·021
352	·039886	·014730	·011	·021	·042735 +	·015224	·011	·021
353	·039773	·014689	·011	·021	·042614	·015182	·011	·021
354	·039660	·014648	·011	·021	·042493	·015140	·011	·021
355	·039548	·014608	·011	·021	·042373	·015098	·011	·021
356	·039437	·014568	·011	·021	·042254	·015057	·011	·021
357	·039326	·014528	·011	·021	·042135 −	·015016	·011	·021
358	·039216	·014488	·011	·021	·042017	·014975 −	·011	·021
359	·039106	·014449	·011	·021	·041899	·014934	·011	·021
360	·038997	·014409	·011	·021	·041783	·014893	·011	·021
361	·038889	·014370	·011	·021	·041667	·014853	·011	·021
362	·038781	·014331	·011	·021	·041551	·014813	·011	·021
363	·038674	·014293	·011	·021	·041436	·014773	·011	·021
364	·038567	·014254	·011	·021	·041322	·014733	·011	·021
365	·038462	·014216	·011	·021	·041209	·014694	·011	·021
366	·038356	·014178	·011	·021	·041096	·014654	·011	·021
367	·038251	·014140	·011	·021	·040984	·014615 +	·011	·021
368	·038147	·014102	·011	·021	·040872	·014576	·011	·021
369	·038043	·014065 −	·011	·021	·040761	·014538	·011	·021
370	·037940	·014027	·011	·021	·040650 +	·014499	·011	·021
371	·037838	·013990	·011	·021	·040541	·014461	·011	·021
372	·037736	·013954	·011	·021	·040431	·014423	·011	·021
373	·037634	·013917	·011	·021	·040323	·014385 +	·011	·021
374	·037534	·013880	·011	·021	·040214	·014348	·011	·021
375	·037433	·013844	·011	·021	·040107	·014310	·011	·021
376	·037333	·013808	·011	·021	·040000	·014273	·011	·021
377	·037234	·013772	·011	·021	·039894	·014236	·011	·021
378	·037135 +	·013736	·011	·021	·039788	·014199	·011	·021
379	·037037	·013701	·011	·021	·039683	·014162	·011	·021
380	·036939	·013665 +	·011	·021	·039578	·014126	·011	·021
381	·036842	·013630	·011	·021	·039474	·014089	·011	·021
382	·036745 +	·013595 +	·011	·021	·039370	·014053	·011	·021
383	·036649	·013560	·011	·021	·039267	·014017	·011	·021
384	·036554	·013526	·011	·021	·039164	·013982	·011	·021
385	·036458	·013491	·011	·021	·039063	·013946	·011	·021
386	·036364	·013457	·011	·021	·038961	·013911	·011	·021
387	·036269	·013423	·011	·021	·038860	·013875 +	·011	·021
388	·036176	·013389	·011	·021	·038760	·013840	·011	·021
389	·036082	·013355 +	·011	·021	·038660	·013805 +	·011	·021
390	·035990	·013322	·011	·021	·038560	·013771	·011	·021
391	·035897	·013288	·011	·021	·038462	·013736	·011	·021
392	·035806	·013255 −	·011	·021	·038363	·013702	·011	·021
393	·035714	·013222	·011	·021	·038265 +	·013668	·011	·021
394	·035623	·013189	·011	·021	·038168	·013634	·011	·021
395	·035533	·013156	·011	·021	·038071	·013600	·011	·021
396	·035443	·013123	·011	·021	·037975 −	·013566	·011	·021
397	·035354	·013091	·011	·021	·037879	·013533	·011	·021
398	·035264	·013059	·011	·021	·037783	·013499	·011	·021
399	·035176	·013027	·011	·021	·037688	·013466	·011	·021
400	·035088	·012995 −	·011	·021	·037594	·013433	·011	·021

N = size of sample	17: $\bar{\eta}^2$	17: $\sigma_{\bar{\eta}^2}$	17: P_1 $\lambda_1=2·76$	17: P_2 $\lambda_2=2·36$	18: $\bar{\eta}^2$	18: $\sigma_{\bar{\eta}^2}$	18: P_1 $\lambda_1=2·75$	18: P_2 $\lambda_2=2·35$
351	·045714	·015744	·010	·021	·048571	·016204	·010	·021
352	·045584	·015700	·010	·021	·048433	·016159	·010	·021
353	·045455 −	·015657	·010	·021	·048295 +	·016115 −	·010	·021
354	·045326	·015614	·010	·021	·048159	·016070	·010	·021
355	·045198	·015571	·011	·021	·048023	·016026	·010	·021
356	·045070	·015528	·011	·021	·047887	·015982	·010	·021
357	·044944	·015485 +	·011	·021	·047753	·015939	·010	·021
358	·044818	·015443	·011	·021	·047619	·015895 +	·010	·021
359	·044693	·015401	·011	·021	·047486	·015852	·010	·021
360	·044568	·015359	·011	·021	·047354	·015809	·010	·021
361	·044444	·015318	·011	·021	·047222	·015766	·010	·021
362	·044321	·015277	·011	·021	·047091	·015724	·010	·021
363	·044199	·015235 +	·011	·021	·046961	·015682	·010	·021
364	·044077	·015195 −	·011	·021	·046832	·015640	·010	·021
365	·043956	·015154	·011	·021	·046703	·015598	·010	·021
366	·043836	·015113	·011	·021	·046575 +	·015556	·010	·021
367	·043716	·015073	·011	·021	·046448	·015515 −	·010	·021
368	·043597	·015033	·011	·021	·046322	·015474	·010	·021
369	·043478	·014993	·011	·021	·046196	·015433	·010	·021
370	·043360	·014954	·011	·021	·046070	·015392	·010	·021
371	·043243	·014914	·011	·021	·045946	·015352	·010	·021
372	·043127	·014875 +	·011	·021	·045822	·015311	·010	·021
373	·043011	·014836	·011	·021	·045699	·015271	·010	·021
374	·042895 +	·014797	·011	·021	·045576	·015231	·010	·021
375	·042781	·014759	·011	·021	·045455 −	·015192	·010	·021
376	·042667	·014720	·011	·021	·045333	·015152	·010	·021
377	·042553	·014682	·011	·021	·045213	·015113	·010	·021
378	·042440	·014644	·011	·021	·045093	·015074	·010	·021
379	·042328	·014606	·011	·021	·044974	·015035 +	·010	·021
380	·042216	·014569	·011	·021	·044855 −	·014997	·010	·021
381	·042105 +	·014531	·011	·021	·044737	·014958	·010	·021
382	·041995 −	·014494	·011	·021	·044619	·014920	·010	·021
383	·041885 −	·014457	·011	·021	·044503	·014882	·010	·021
384	·041775 +	·014420	·011	·021	·044386	·014844	·010	·021
385	·041667	·014384	·011	·021	·044271	·014806	·010	·021
386	·041558	·014347	·011	·021	·044156	·014769	·010	·021
387	·041451	·014311	·011	·021	·044041	·014732	·010	·021
388	·041344	·014275 +	·011	·021	·043928	·014694	·010	·021
389	·041237	·014239	·011	·021	·043814	·014658	·010	·021
390	·041131	·014203	·011	·021	·043702	·014621	·010	·021
391	·041026	·014168	·011	·021	·043590	·014584	·010	·021
392	·040921	·014132	·011	·021	·043478	·014548	·010	·021
393	·040816	·014097	·011	·021	·043367	·014512	·010	·021
394	·040712	·014062	·011	·021	·043257	·014476	·010	·021
395	·040609	·014027	·011	·021	·043147	·014440	·010	·021
396	·040506	·013993	·011	·021	·043038	·014404	·010	·021
397	·040404	·013958	·011	·021	·042929	·014369	·010	·021
398	·040302	·013924	·011	·021	·042821	·014334	·010	·021
399	·040201	·013890	·011	·021	·042714	·014298	·010	·021
400	·040100	·013856	·011	·021	·042607	·014263	·010	·021

N = size of sample	19: $\bar{\eta}^2$	19: $\sigma_{\bar{\eta}^2}$	19: P_1 $\lambda_1=2·74$	19: P_2 $\lambda_2=2·34$	20: $\bar{\eta}^2$	20: $\sigma_{\bar{\eta}^2}$	20: P_1 $\lambda_1=2·73$	20: P_2 $\lambda_2=2·33$
351	·051429	·016649	·010	·021	·054286	·017079	·010	·021
352	·051282	·016603	·010	·021	·054131	·017032	·010	·021
353	·051136	·016557	·010	·021	·053977	·016985 +	·010	·021
354	·050992	·016511	·010	·021	·053824	·016939	·010	·021
355	·050847	·016466	·010	·021	·053672	·016892	·010	·021
356	·050704	·016421	·010	·021	·053521	·016846	·010	·021
357	·050562	·016376	·010	·021	·053371	·016800	·010	·021
358	·050420	·016332	·010	·021	·053221	·016755 −	·010	·021
359	·050279	·016288	·010	·021	·053073	·016709	·010	·021
360	·050139	·016244	·010	·021	·052925 −	·016664	·010	·021
361	·050000	·016200	·010	·021	·052778	·016619	·010	·021
362	·049861	·016156	·010	·021	·052632	·016575 −	·010	·021
363	·049724	·016113	·010	·021	·052486	·016530	·010	·021
364	·049587	·016070	·010	·021	·052342	·016486	·010	·021
365	·049451	·016027	·010	·021	·052198	·016442	·010	·021
366	·049315 +	·015984	·010	·021	·052055 −	·016398	·010	·021
367	·049180	·015942	·010	·021	·051913	·016355 +	·010	·021
368	·049046	·015900	·010	·021	·051771	·016312	·010	·021
369	·048913	·015858	·010	·021	·051630	·016269	·010	·021
370	·048780	·015816	·010	·021	·051491	·016226	·010	·021
371	·048649	·015774	·010	·021	·051351	·016183	·010	·021
372	·048518	·015733	·010	·021	·051213	·016141	·010	·021
373	·048387	·015692	·010	·021	·051075 +	·016099	·010	·021
374	·048257	·015651	·010	·021	·050938	·016057	·010	·021
375	·048128	·015610	·010	·021	·050802	·016016	·010	·021
376	·048000	·015570	·010	·021	·050667	·015974	·010	·021
377	·047872	·015530	·010	·021	·050532	·015933	·010	·021
378	·047745 +	·015490	·010	·021	·050398	·015892	·010	·021
379	·047619	·015450 −	·010	·021	·050265 −	·015851	·010	·021
380	·047493	·015410	·010	·021	·050132	·015810	·010	·021
381	·047368	·015371	·010	·021	·050000	·015770	·010	·021
382	·047244	·015331	·010	·021	·049869	·015730	·010	·021
383	·047120	·015292	·010	·021	·049738	·015690	·010	·021
384	·046997	·015254	·010	·021	·049608	·015650 −	·010	·021
385	·046875	·015215 −	·010	·021	·049479	·015610	·010	·021
386	·046753	·015176	·010	·021	·049351	·015571	·010	·021
387	·046632	·015138	·010	·021	·049223	·015532	·010	·021
388	·046512	·015100	·010	·021	·049096	·015493	·010	·021
389	·046392	·015062	·010	·021	·048969	·015454	·010	·021
390	·046272	·015025 −	·010	·021	·048843	·015415 +	·010	·021
391	·046154	·014987	·010	·021	·048718	·015377	·010	·021
392	·046036	·014950 −	·010	·021	·048593	·015339	·010	·021
393	·045918	·014913	·010	·021	·048469	·015301	·010	·021
394	·045802	·014876	·010	·021	·048346	·015263	·010	·021
395	·045685 +	·014839	·010	·021	·048223	·015225 +	·010	·021
396	·045570	·014802	·010	·021	·048101	·015188	·010	·021
397	·045455 −	·014766	·010	·021	·047980	·015150 +	·010	·021
398	·045340	·014730	·010	·021	·047859	·015113	·010	·021
399	·045226	·014694	·010	·021	·047739	·015076	·010	·021
400	·045113	·014658	·010	·021	·047619	·015040	·010	·021

N = size of sample	3: $\bar{\eta}^2$	3: $\sigma_{\bar{\eta}^2}$	3: P_1 $\lambda_1 = 3.50$	3: P_2 $\lambda_2 = 2.94$	4: $\bar{\eta}^2$	4: $\sigma_{\bar{\eta}^2}$	4: P_1 $\lambda_1 = 3.20$	4: P_2 $\lambda_2 = 2.80$
401	.005000	.004975+	.011	.019	.007500	.006086	.012	.020
402	.004988	.004963	.011	.019	.007481	.006070	.012	.020
403	.004975+	.004950+	.011	.019	.007463	.006055+	.012	.020
404	.004963	.004938	.011	.019	.007444	.006041	.012	.020
405	.004950+	.004926	.011	.019	.007426	.006026	.012	.020
406	.004938	.004914	.011	.019	.007407	.006011	.012	.020
407	.004926	.004902	.011	.019	.007389	.005996	.012	.020
408	.004914	.004890	.011	.019	.007371	.005982	.012	.020
409	.004902	.004878	.011	.019	.007353	.005967	.012	.020
410	.004890	.004866	.011	.019	.007335−	.005952	.012	.020
411	.004878	.004854	.011	.019	.007317	.005938	.012	.020
412	.004866	.004843	.011	.019	.007299	.005924	.012	.020
413	.004854	.004831	.011	.019	.007282	.005909	.012	.020
414	.004843	.004819	.011	.019	.007264	.005895+	.012	.020
415	.004831	.004808	.011	.019	.007246	.005881	.012	.020
416	.004819	.004796	.011	.019	.007229	.005867	.012	.020
417	.004808	.004785−	.011	.019	.007212	.005853	.012	.020
418	.004796	.004773	.011	.019	.007194	.005839	.012	.020
419	.004785−	.004762	.011	.019	.007177	.005825+	.012	.020
420	.004773	.004751	.011	.019	.007160	.005811	.012	.020
421	.004762	.004739	.011	.019	.007143	.005797	.012	.020
422	.004751	.004728	.011	.019	.007126	.005784	.012	.020
423	.004739	.004717	.011	.019	.007109	.005770	.012	.020
424	.004728	.004706	.011	.019	.007092	.005757	.012	.020
425	.004717	.004695−	.011	.019	.007075+	.005743	.012	.020
426	.004706	.004684	.011	.019	.007059	.005730	.012	.020
427	.004695−	.004673	.011	.019	.007042	.005716	.012	.020
428	.004684	.004662	.011	.019	.007026	.005703	.012	.020
429	.004673	.004651	.011	.019	.007009	.005690	.012	.020
430	.004662	.004640	.011	.019	.006993	.005677	.012	.020
431	.004651	.004630	.011	.019	.006977	.005663	.012	.020
432	.004640	.004619	.011	.019	.006961	.005650+	.012	.020
433	.004630	.004608	.011	.019	.006944	.005637	.012	.020
434	.004619	.004598	.011	.019	.006928	.005624	.012	.020
435	.004608	.004587	.011	.019	.006912	.005612	.012	.020
436	.004598	.004577	.011	.019	.006897	.005599	.012	.020
437	.004587	.004566	.011	.019	.006881	.005586	.012	.020
438	.004577	.004556	.011	.019	.006865−	.005573	.012	.020
439	.004566	.004545+	.011	.019	.006849	.005561	.012	.020
440	.004556	.004535+	.011	.019	.006834	.005548	.012	.020
441	.004545+	.004525−	.011	.019	.006818	.005535+	.012	.020
442	.004535+	.004515−	.011	.019	.006803	.005523	.012	.020
443	.004525+	.004504	.011	.019	.006787	.005511	.012	.020
444	.004515−	.004494	.011	.019	.006772	.005498	.012	.020
445	.004505−	.004484	.011	.019	.006757	.005486	.012	.020
446	.004494	.004474	.011	.019	.006742	.005474	.012	.020
447	.004484	.004464	.011	.019	.006726	.005461	.012	.020
448	.004474	.004454	.011	.019	.006711	.005449	.012	.020
449	.004464	.004444	.011	.019	.006696	.005437	.012	.020
450	.004454	.004435−	.011	.019	.006682	.005425+	.012	.020

N = size of sample	5: $\bar{\eta}^2$	5: $\sigma_{\bar{\eta}^2}$	5: P_1 $\lambda_1 = 3.14$	5: P_2 $\lambda_2 = 2.68$	6: $\bar{\eta}^2$	6: $\sigma_{\bar{\eta}^2}$	6: P_1 $\lambda_1 = 3.11$	6: P_2 $\lambda_2 = 2.63$
401	.010000	.007018	.012	.021	.012500	.007837	.011	.020
402	.009975+	.007001	.012	.021	.012469	.007817	.011	.020
403	.009950+	.006983	.012	.021	.012438	.007798	.011	.020
404	.009926	.006966	.012	.021	.012407	.007779	.011	.020
405	.009901	.006949	.012	.021	.012376	.007760	.011	.020
406	.009877	.006932	.012	.021	.012346	.007741	.011	.020
407	.009852	.006915+	.012	.021	.012315+	.007722	.011	.020
408	.009828	.006898	.012	.021	.012285+	.007703	.011	.020
409	.009804	.006882	.012	.021	.012255−	.007684	.011	.020
410	.009780	.006865−	.012	.021	.012225−	.007666	.011	.020
411	.009756	.006848	.012	.021	.012195+	.007647	.011	.020
412	.009732	.006832	.012	.021	.012165+	.007629	.011	.020
413	.009709	.006815+	.012	.021	.012136	.007610	.011	.020
414	.009685+	.006799	.012	.021	.012107	.007592	.011	.020
415	.009662	.006783	.012	.021	.012077	.007574	.011	.020
416	.009639	.006766	.012	.021	.012048	.007556	.011	.020
417	.009615+	.006750+	.012	.021	.012019	.007538	.011	.020
418	.009592	.006734	.012	.021	.011990	.007520	.011	.020
419	.009569	.006718	.012	.021	.011962	.007502	.011	.020
420	.009547	.006702	.012	.021	.011933	.007484	.011	.020
421	.009524	.006686	.012	.021	.011905−	.007467	.011	.020
422	.009501	.006671	.012	.021	.011876	.007449	.011	.020
423	.009479	.006655−	.012	.021	.011848	.007431	.011	.020
424	.009456	.006639	.012	.021	.011820	.007414	.011	.020
425	.009434	.006624	.012	.021	.011792	.007397	.011	.020
426	.009412	.006608	.012	.021	.011765−	.007379	.011	.020
427	.009390	.006593	.012	.021	.011737	.007362	.011	.020
428	.009368	.006577	.012	.021	.011710	.007345+	.011	.020
429	.009346	.006562	.012	.021	.011682	.007328	.011	.020
430	.009324	.006547	.012	.021	.011655+	.007311	.011	.020
431	.009302	.006532	.012	.021	.011628	.007294	.011	.020
432	.009281	.006517	.012	.021	.011601	.007278	.011	.020
433	.009259	.006502	.012	.021	.011574	.007261	.011	.020
434	.009238	.006487	.012	.021	.011547	.007244	.011	.020
435	.009217	.006472	.012	.021	.011521	.007228	.011	.020
436	.009195+	.006457	.012	.021	.011494	.007211	.011	.020
437	.009174	.006443	.012	.021	.011468	.007195−	.011	.020
438	.009153	.006428	.012	.021	.011442	.007178	.011	.020
439	.009132	.006413	.012	.021	.011416	.007162	.011	.020
440	.009112	.006399	.012	.021	.011390	.007146	.011	.020
441	.009091	.006384	.012	.021	.011364	.007130	.011	.020
442	.009070	.006370	.012	.021	.011338	.007114	.011	.020
443	.009050−	.006356	.012	.021	.011312	.007098	.011	.020
444	.009029	.006342	.012	.021	.011287	.007082	.011	.020
445	.009009	.006327	.012	.021	.011261	.007066	.011	.020
446	.008989	.006313	.012	.021	.011236	.007050+	.011	.020
447	.008969	.006299	.012	.021	.011211	.007035−	.011	.020
448	.008949	.006285+	.012	.021	.011186	.007019	.011	.020
449	.008929	.006271	.012	.021	.011161	.007004	.011	.020
450	.008909	.006257	.012	.021	.011136	.006988	.011	.020

N = size of sample	7: $\bar{\eta}^2$	7: $\sigma_{\bar{\eta}^2}$	7: P_1 $\lambda_1 = 3.08$	7: P_2 $\lambda_2 = 2.58$	8: $\bar{\eta}^2$	8: $\sigma_{\bar{\eta}^2}$	8: P_1 $\lambda_1 = 3.02$	8: P_2 $\lambda_2 = 2.54$
401	.015000	.008574	.010	.020	.017500	.009249	.010	.021
402	.014963	.008552	.010	.020	.017456	.009226	.010	.021
403	.014925+	.008531	.010	.020	.017413	.009203	.010	.021
404	.014888	.008510	.010	.020	.017370	.009181	.010	.021
405	.014851	.008490	.010	.020	.017327	.009158	.010	.021
406	.014815−	.008469	.010	.020	.017284	.009136	.010	.021
407	.014778	.008448	.010	.020	.017241	.009114	.010	.021
408	.014742	.008428	.010	.020	.017199	.009092	.010	.021
409	.014706	.008407	.010	.020	.017157	.009070	.010	.021
410	.014670	.008387	.010	.020	.017115−	.009048	.010	.021
411	.014634	.008367	.010	.020	.017073	.009026	.010	.021
412	.014599	.008346	.010	.020	.017032	.009004	.010	.021
413	.014563	.008326	.010	.020	.016990	.008982	.010	.021
414	.014528	.008306	.010	.020	.016949	.008961	.010	.021
415	.014493	.008287	.010	.020	.016908	.008940	.010	.021
416	.014458	.008267	.010	.020	.016867	.008918	.010	.021
417	.014423	.008247	.010	.020	.016827	.008897	.010	.021
418	.014388	.008228	.010	.020	.016787	.008876	.010	.021
419	.014354	.008208	.010	.020	.016746	.008855−	.010	.021
420	.014320	.008189	.010	.020	.016706	.008834	.010	.021
421	.014286	.008169	.010	.020	.016667	.008813	.010	.021
422	.014252	.008150+	.010	.020	.016627	.008792	.010	.021
423	.014218	.008131	.010	.020	.016588	.008772	.010	.021
424	.014184	.008112	.010	.020	.016548	.008751	.010	.021
425	.014151	.008093	.010	.020	.016509	.008731	.010	.021
426	.014118	.008074	.010	.020	.016471	.008711	.010	.021
427	.014085−	.008055+	.010	.020	.016432	.008690	.010	.021
428	.014052	.008037	.010	.020	.016393	.008670	.010	.021
429	.014019	.008018	.010	.020	.016355+	.008650+	.010	.021
430	.013986	.008000	.010	.020	.016317	.008630	.010	.021
431	.013953	.007981	.010	.020	.016279	.008610	.010	.021
432	.013921	.007963	.010	.020	.016241	.008591	.010	.021
433	.013889	.007945−	.010	.020	.016204	.008571	.010	.021
434	.013857	.007926	.010	.020	.016166	.008551	.010	.021
435	.013825−	.007908	.010	.020	.016129	.008532	.010	.021
436	.013793	.007890	.010	.020	.016092	.008512	.010	.021
437	.013761	.007872	.010	.020	.016055+	.008493	.010	.021
438	.013730	.007854	.010	.020	.016018	.008474	.010	.021
439	.013699	.007837	.010	.020	.015982	.008455−	.010	.021
440	.013667	.007819	.010	.020	.015945+	.008436	.010	.021
441	.013636	.007801	.010	.020	.015909	.008417	.010	.021
442	.013605+	.007784	.010	.020	.015873	.008398	.010	.021
443	.013575−	.007766	.010	.020	.015837	.008379	.010	.021
444	.013544	.007749	.010	.020	.015801	.008360	.010	.021
445	.013514	.007732	.010	.020	.015766	.008342	.010	.021
446	.013483	.007715−	.010	.020	.015730	.008323	.010	.021
447	.013453	.007697	.010	.020	.015695+	.008305−	.010	.021
448	.013423	.007680	.010	.020	.015660	.008286	.010	.021
449	.013393	.007663	.010	.020	.015625	.008268	.010	.021
450	.013363	.007646	.010	.020	.015590	.008250−	.010	.021

n = number of arrays

N = size of sample	9: $\bar{\eta}^2$	9: $\sigma_{\bar{\eta}^2}$	9: P_1 $\lambda_1=2·96$	9: P_2 $\lambda_2=2·50$	10: $\bar{\eta}^2$	10: $\sigma_{\bar{\eta}^2}$	10: P_1 $\lambda_1=2·92$	10: P_2 $\lambda_2=2·47$
401	·020000	·009875 −	·011	·021	·022500	·010460	·011	·021
402	·019950 +	·009851	·011	·021	·022444	·010435 −	·011	·021
403	·019900	·009826	·011	·021	·022388	·010409	·011	·021
404	·019851	·009802	·011	·021	·022333	·010384	·011	·021
405	·019802	·009778	·011	·021	·022277	·010358	·011	·021
406	·019753	·009754	·011	·021	·022222	·010333	·011	·021
407	·019704	·009731	·011	·021	·022167	·010308	·011	·021
408	·019656	·009707	·011	·021	·022113	·010283	·011	·021
409	·019608	·009684	·011	·021	·022059	·010258	·011	·021
410	·019560	·009660	·011	·021	·022005 −	·010233	·011	·021
411	·019512	·009637	·011	·021	·021951	·010209	·011	·021
412	·019465 −	·009614	·011	·021	·021898	·010184	·011	·021
413	·019417	·009591	·011	·021	·021845 −	·010160	·011	·021
414	·019370	·009568	·011	·021	·021792	·010136	·011	·021
415	·019324	·009545 +	·011	·021	·021739	·010112	·011	·021
416	·019277	·009522	·011	·021	·021687	·010087	·011	·021
417	·019231	·009500 −	·011	·021	·021635 −	·010064	·011	·021
418	·019185 −	·009477	·011	·021	·021583	·010040	·011	·021
419	·019139	·009455 −	·011	·021	·021531	·010016	·011	·021
420	·019093	·009432	·011	·021	·021480	·009992	·011	·021
421	·019048	·009410	·011	·021	·021429	·009969	·011	·021
422	·019002	·009388	·011	·021	·021378	·009946	·011	·021
423	·018957	·009366	·011	·021	·021327	·009922	·011	·021
424	·018913	·009344	·011	·021	·021277	·009899	·011	·021
425	·018868	·009323	·011	·021	·021226	·009876	·011	·021
426	·018824	·009301	·011	·021	·021176	·009853	·011	·021
427	·018779	·009279	·011	·021	·021127	·009830	·011	·021
428	·018735 +	·009258	·011	·021	·021077	·009808	·011	·021
429	·018692	·009236	·011	·021	·021028	·009785 +	·011	·021
430	·018648	·009215 +	·011	·021	·020979	·009763	·011	·021
431	·018605 −	·009194	·011	·021	·020930	·009740	·011	·021
432	·018561	·009173	·011	·021	·020882	·009718	·011	·021
433	·018519	·009152	·011	·021	·020833	·009696	·011	·021
434	·018476	·009131	·011	·021	·020785 +	·009674	·011	·021
435	·018433	·009110	·011	·021	·020737	·009652	·011	·021
436	·018391	·009090	·011	·021	·020690	·009630	·011	·021
437	·018349	·009069	·011	·021	·020642	·009608	·011	·021
438	·018307	·009048	·011	·021	·020595 −	·009586	·011	·021
439	·018265 −	·009028	·011	·021	·020548	·009565 −	·011	·021
440	·018223	·009008	·011	·021	·020501	·009543	·011	·021
441	·018182	·008987	·011	·021	·020455 −	·009522	·011	·021
442	·018141	·008967	·011	·021	·020408	·009500 +	·011	·021
443	·018100	·008947	·011	·021	·020362	·009479	·011	·021
444	·018059	·008927	·011	·021	·020316	·009458	·011	·021
445	·018018	·008907	·011	·021	·020270	·009437	·011	·021
446	·017978	·008888	·011	·021	·020225 −	·009416	·011	·021
447	·017937	·008868	·011	·021	·020179	·009395 +	·011	·021
448	·017897	·008848	·011	·021	·020134	·009374	·011	·021
449	·017857	·008829	·011	·021	·020089	·009354	·011	·021
450	·017817	·008809	·011	·021	·020045 −	·009333	·011	·021

N = size of sample	11: $\bar{\eta}^2$	11: $\sigma_{\bar{\eta}^2}$	11: P_1 $\lambda_1=2·88$	11: P_2 $\lambda_2=2·44$	12: $\bar{\eta}^2$	12: $\sigma_{\bar{\eta}^2}$	12: P_1 $\lambda_1=2·84$	12: P_2 $\lambda_2=2·42$
401	·025000	·011012	·011	·021	·027500	·011535 −	·011	·021
402	·024938	·010985 +	·011	·021	·027431	·011507	·011	·021
403	·024876	·010958	·011	·021	·027363	·011478	·011	·021
404	·024814	·010931	·011	·021	·027295 +	·011450 +	·011	·021
405	·024752	·010905 −	·011	·021	·027228	·011423	·011	·021
406	·024691	·010878	·011	·021	·027160	·011395 −	·011	·021
407	·024631	·010852	·011	·021	·027094	·011367	·011	·021
408	·024570	·010826	·011	·021	·027027	·011340	·011	·021
409	·024510	·010800	·011	·021	·026961	·011312	·011	·021
410	·024450 −	·010774	·011	·021	·026895 −	·011285 +	·011	·021
411	·024390	·010748	·011	·021	·026829	·011258	·011	·021
412	·024331	·010722	·011	·021	·026764	·011231	·011	·021
413	·024272	·010696	·011	·021	·026699	·011204	·011	·021
414	·024213	·010671	·011	·021	·026634	·011178	·011	·021
415	·024155 −	·010645 +	·011	·021	·026570	·011151	·011	·021
416	·024096	·010620	·011	·021	·026506	·011125 −	·011	·021
417	·024038	·010595 −	·011	·021	·026442	·011098	·011	·021
418	·023981	·010570	·011	·021	·026279	·011072	·011	·021
419	·023923	·010545 −	·011	·021	·026316	·011046	·011	·021
420	·023866	·010520	·011	·021	·026253	·011020	·011	·021
421	·023810	·010495 +	·011	·021	·026190	·010994	·011	·021
422	·023753	·010471	·011	·021	·026128	·010969	·011	·021
423	·023697	·010446	·011	·021	·026066	·010943	·011	·021
424	·023641	·010422	·011	·021	·026005 −	·010918	·011	·021
425	·023585 −	·010398	·011	·021	·025943	·010892	·011	·021
426	·023529	·010374	·011	·021	·025882	·010867	·011	·021
427	·023474	·010350 −	·011	·021	·025822	·010842	·011	·021
428	·023419	·010326	·011	·021	·025761	·010817	·011	·021
429	·023364	·010302	·011	·021	·025701	·010792	·011	·021
430	·023310	·010278	·011	·021	·025641	·010767	·011	·021
431	·023256	·010255 −	·011	·021	·025581	·010743	·011	·021
432	·023202	·010231	·011	·021	·025522	·010718	·011	·021
433	·023148	·010208	·011	·021	·025463	·010694	·011	·021
434	·023095 −	·010185 −	·011	·021	·025404	·010669	·011	·021
435	·023041	·010162	·011	·021	·025346	·010645 +	·011	·021
436	·022989	·010139	·011	·021	·025287	·010621	·011	·021
437	·022936	·010116	·011	·021	·025229	·010597	·011	·021
438	·022883	·010093	·011	·021	·025172	·010573	·011	·021
439	·022831	·010070	·011	·021	·025114	·010549	·011	·021
440	·022779	·010048	·011	·021	·025057	·010526	·011	·021
441	·022727	·010025 +	·011	·021	·025000	·010502	·011	·021
442	·022676	·010003	·011	·021	·024943	·010479	·011	·021
443	·022624	·009980	·011	·021	·024887	·010455 +	·011	·021
444	·022573	·009958	·011	·021	·024831	·010432	·011	·021
445	·022523	·009936	·011	·021	·024775 −	·010409	·011	·021
446	·022472	·009914	·011	·021	·024719	·010386	·011	·021
447	·022422	·009892	·011	·021	·024664	·010363	·011	·021
448	·022371	·009870	·011	·021	·024609	·010340	·011	·021
449	·022321	·009848	·011	·021	·024554	·010317	·011	·021
450	·022272	·009827	·011	·021	·024499	·010295 −	·011	·021

N = size of sample	13: $\bar{\eta}^2$	13: $\sigma_{\bar{\eta}^2}$	13: P_1 $\lambda_1=2·80$	13: P_2 $\lambda_2=2·40$	14: $\bar{\eta}^2$	14: $\sigma_{\bar{\eta}^2}$	14: P_1 $\lambda_1=2·79$	14: P_2 $\lambda_2=2·39$
401	·030000	·012032	·011	·021	·032500	·012507	·011	·021
402	·029925 +	·012003	·011	·021	·032419	·012477	·011	·021
403	·029851	·011974	·011	·021	·032338	·012446	·011	·021
404	·029777	·011944	·011	·021	·032258	·012416	·011	·021
405	·029703	·011915 +	·011	·021	·032178	·012386	·011	·021
406	·029630	·011886	·011	·021	·032099	·012356	·011	·021
407	·029557	·011858	·011	·021	·032020	·012326	·011	·021
408	·029484	·011829	·011	·021	·031941	·012296	·011	·021
409	·029412	·011801	·011	·021	·031863	·012267	·011	·021
410	·029340	·011772	·011	·021	·031785 −	·012237	·011	·021
411	·029268	·011744	·011	·021	·031707	·012208	·011	·021
412	·029197	·011716	·011	·021	·031630	·012179	·011	·021
413	·029126	·011688	·011	·021	·031553	·012150 −	·011	·021
414	·029056	·011660	·011	·021	·031477	·012121	·011	·021
415	·028986	·011632	·011	·021	·031401	·012092	·011	·021
416	·028916	·011605 −	·011	·021	·031325 +	·012064	·011	·021
417	·028846	·011578	·011	·021	·031250 −	·012035 +	·011	·021
418	·028777	·011550 +	·011	·021	·031175 +	·012007	·011	·021
419	·028708	·011523	·011	·021	·031100	·011979	·011	·021
420	·028640	·011496	·011	·021	·031026	·011951	·011	·021
421	·028571	·011469	·011	·021	·030952	·011923	·011	·021
422	·028504	·011442	·011	·021	·030879	·011895 −	·011	·021
423	·028436	·011416	·011	·021	·030806	·011867	·011	·021
424	·028369	·011389	·011	·021	·030733	·011840	·011	·021
425	·028302	·011363	·011	·021	·030660	·011812	·011	·021
426	·028235 +	·011336	·011	·021	·030588	·011785 +	·011	·021
427	·028169	·011310	·011	·021	·030516	·011758	·011	·021
428	·028103	·011284	·011	·021	·030445 −	·011731	·011	·021
429	·028037	·011258	·011	·021	·030374	·011704	·011	·021
430	·027972	·011233	·011	·021	·030303	·011677	·011	·021
431	·027907	·011207	·011	·021	·030233	·011650 +	·011	·021
432	·027842	·011181	·011	·021	·030162	·011624	·011	·021
433	·027778	·011156	·011	·021	·030093	·011598	·011	·021
434	·027714	·011130	·011	·021	·030023	·011571	·011	·021
435	·027650 −	·011105 +	·011	·021	·029954	·011545 +	·011	·021
436	·027586	·011080	·011	·021	·029885 +	·011519	·011	·021
437	·027523	·011055 +	·011	·021	·029817	·011493	·011	·021
438	·027460	·011030	·011	·021	·029748	·011467	·011	·021
439	·027397	·011006	·011	·021	·029680	·011441	·011	·021
440	·027335 −	·010981	·011	·021	·029613	·011416	·011	·021
441	·027273	·010956	·011	·021	·029545 +	·011390	·011	·021
442	·027211	·010932	·011	·021	·029478	·011365 −	·011	·021
443	·027149	·010908	·011	·021	·029412	·011340	·011	·021
444	·027088	·010883	·011	·021	·029345 +	·011315 −	·011	·021
445	·027027	·010859	·011	·021	·029279	·011290	·011	·021
446	·026966	·010835 +	·011	·021	·029213	·011265 −	·011	·021
447	·026906	·010811	·011	·021	·029148	·011240	·011	·021
448	·026846	·010787	·011	·021	·029083	·011215 +	·011	·021
449	·026786	·010764	·011	·021	·029018	·011190	·011	·021
450	·026726	·010740	·011	·021	·028953	·011166	·011	·021

N = size of sample	15 $\bar{\eta}^2$	15 $\sigma_{\bar{\eta}^2}$	15 P_1 $\lambda_1 = 2{\cdot}78$	15 P_2 $\lambda_2 = 2{\cdot}38$	16 $\bar{\eta}^2$	16 $\sigma_{\bar{\eta}^2}$	16 P_1 $\lambda_1 = 2{\cdot}77$	16 P_2 $\lambda_2 = 2{\cdot}37$
401	·035000	·012963	·011	·021	·037500	·013400	·011	·021
402	·034913	·012931	·011	·021	·037406	·013368	·011	·021
403	·034826	·012900	·011	·021	·037313	·013335 +	·011	·021
404	·034739	·012868	·011	·021	·037221	·013303	·011	·021
405	·034653	·012837	·011	·021	·037129	·013271	·011	·021
406	·034568	·012806	·011	·021	·037037	·013239	·011	·021
407	·034483	·012775 +	·011	·021	·036946	·013207	·011	·021
408	·034398	·012744	·011	·021	·036855 +	·013175 −	·011	·021
409	·034314	·012714	·011	·021	·036765 −	·013143	·011	·021
410	·034230	·012683	·011	·021	·036675 −	·013112	·011	·021
411	·034146	·012653	·011	·021	·036585 +	·013081	·011	·021
412	·034063	·012623	·011	·021	·036496	·013049	·011	·021
413	·033981	·012593	·011	·021	·036408	·013018	·011	·021
414	·033898	·012563	·011	·021	·036320	·012988	·011	·021
415	·033816	·012533	·011	·021	·036232	·012957	·011	·021
416	·033735 −	·012504	·011	·021	·036145 −	·012926	·011	·021
417	·033654	·012474	·011	·021	·036058	·012896	·011	·021
418	·033573	·012445 −	·011	·021	·035971	·012866	·011	·021
419	·033493	·012416	·011	·021	·035885 +	·012835 +	·011	·021
420	·033413	·012387	·011	·021	·035800	·012805 +	·011	·021
421	·033333	·012358	·011	·021	·035714	·012776	·011	·021
422	·033254	·012329	·011	·021	·035629	·012746	·011	·021
423	·033175 +	·012300	·011	·021	·035545 +	·012716	·011	·021
424	·033097	·012272	·011	·021	·035461	·012687	·011	·021
425	·033019	·012243	·011	·021	·035377	·012658	·011	·021
426	·032941	·012215 +	·011	·021	·035294	·012628	·011	·021
427	·032864	·012187	·011	·021	·035211	·012599	·011	·021
428	·032787	·012159	·011	·021	·035129	·012571	·011	·021
429	·032710	·012131	·011	·021	·035047	·012542	·011	·021
430	·032634	·012103	·011	·021	·034965 +	·012513	·011	·021
431	·032558	·012076	·011	·021	·034884	·012485 −	·011	·021
432	·032483	·012048	·011	·021	·034803	·012456	·011	·021
433	·032407	·012021	·011	·021	·034722	·012428	·011	·021
434	·032333	·011994	·011	·021	·034642	·012400	·011	·021
435	·032258	·011967	·011	·021	·034562	·012372	·011	·021
436	·032184	·011940	·011	·021	·034483	·012344	·011	·021
437	·032110	·011913	·011	·021	·034404	·012316	·011	·021
438	·032037	·011886	·011	·021	·034325 −	·012289	·011	·021
439	·031963	·011859	·011	·021	·034247	·012261	·011	·021
440	·031891	·011833	·011	·021	·034169	·012234	·011	·021
441	·031818	·011806	·011	·021	·034091	·012207	·011	·021
442	·031746	·011780	·011	·021	·034014	·012179	·011	·021
443	·031674	·011754	·011	·021	·033937	·012152	·011	·021
444	·031603	·011728	·011	·021	·033860	·012125 +	·011	·021
445	·031532	·011702	·011	·021	·033784	·012099	·011	·021
446	·031461	·011676	·011	·021	·033708	·012072	·011	·021
447	·031390	·011651	·011	·021	·033632	·012046	·011	·021
448	·031320	·011625 −	·011	·021	·033557	·012019	·011	·021
449	·031250	·011600	·011	·021	·033482	·011993	·011	·021
450	·031180	·011574	·011	·021	·033408	·011967	·011	·021

N = size of sample	17 $\bar{\eta}^2$	17 $\sigma_{\bar{\eta}^2}$	17 P_1 $\lambda_1 = 2{\cdot}76$	17 P_2 $\lambda_2 = 2{\cdot}36$	18 $\bar{\eta}^2$	18 $\sigma_{\bar{\eta}^2}$	18 P_1 $\lambda_1 = 2{\cdot}75$	18 P_2 $\lambda_2 = 2{\cdot}35$
401	·040000	·013822	·011	·021	·042500	·014229	·010$_5$	·021
402	·039900	·013788	·011	·021	·042394	·014194	·011	·021
403	·039801	·013755 −	·011	·021	·042289	·014160	·011	·021
404	·039702	·013721	·011	·021	·042184	·014125 +	·011	·021
405	·039604	·013688	·011	·021	·042079	·014091	·011	·021
406	·039506	·013655 +	·011	·021	·041975 +	·014057	·011	·021
407	·039409	·013622	·011	·021	·041872	·014024	·011	·021
408	·039312	·013590	·011	·021	·041769	·013990	·011	·021
409	·039216	·013557	·011	·021	·041667	·013956	·011	·021
410	·039120	·013525 −	·011	·021	·041565 −	·013923	·011	·021
411	·039024	·013492	·011	·021	·041463	·013890	·011	·021
412	·038929	·013460	·011	·021	·041363	·013857	·011	·021
413	·038835 −	·013428	·011	·021	·041262	·013824	·011	·021
414	·038741	·013397	·011	·021	·041162	·013792	·011	·021
415	·038647	·013365 +	·011	·021	·041063	·013759	·011	·021
416	·038554	·013334	·011	·021	·040964	·013727	·011	·021
417	·038462	·013302	·011	·021	·040865 +	·013694	·011	·021
418	·038369	·013271	·011	·021	·040767	·013662	·011	·021
419	·038278	·013240	·011	·021	·040670	·013630	·011	·021
420	·038186	·013209	·011	·021	·040573	·013599	·011	·021
421	·038095 +	·013178	·011	·021	·040476	·013567	·011	·021
422	·038005 −	·013148	·011	·021	·040380	·013536	·011	·021
423	·037915 −	·013117	·011	·021	·040284	·013504	·011	·021
424	·037825 +	·013087	·011	·021	·040189	·013473	·011	·021
425	·037736	·013057	·011	·021	·040094	·013442	·011	·021
426	·037647	·013027	·011	·021	·040000	·013411	·011	·021
427	·037559	·012997	·011	·021	·039906	·013380	·011	·021
428	·037471	·012967	·011	·021	·039813	·013350 −	·011	·021
429	·037383	·012937	·011	·021	·039720	·013319	·011	·021
430	·037296	·012908	·011	·021	·039627	·013289	·011	·021
431	·037209	·012878	·011	·021	·039535 −	·013259	·011	·021
432	·037123	·012849	·011	·021	·039443	·013229	·011	·021
433	·037037	·012820	·011	·021	·039352	·013199	·011	·021
434	·036952	·012791	·011	·021	·039261	·013169	·011	·021
435	·036866	·012762	·011	·021	·039171	·013139	·011	·021
436	·036782	·012734	·011	·021	·039080	·013110	·011	·021
437	·036697	·012705 +	·011	·021	·038991	·013080	·011	·021
438	·036613	·012677	·011	·021	·038902	·013051	·011	·021
439	·036530	·012648	·011	·021	·038813	·013022	·011	·021
440	·036446	·012620	·011	·021	·038724	·012993	·011	·021
441	·036364	·012592	·011	·021	·038636	·012964	·011	·021
442	·036281	·012564	·011	·021	·038549	·012935 +	·011	·021
443	·036199	·012536	·011	·021	·038462	·012907	·011	·021
444	·036117	·012508	·011	·021	·038375 −	·012878	·011	·021
445	·036036	·012481	·011	·021	·038288	·012850 −	·011	·021
446	·035955 +	·012453	·011	·021	·038202	·012822	·011	·021
447	·035874	·012426	·011	·021	·038117	·012794	·011	·021
448	·035794	·012399	·011	·021	·038031	·012766	·011	·021
449	·035714	·012372	·011	·021	·037946	·012738	·011	·021
450	·035635 −	·012345 −	·011	·021	·037862	·012710	·011	·021

N = size of sample	19 $\bar{\eta}^2$	19 $\sigma_{\bar{\eta}^2}$	19 P_1 $\lambda_1 = 2{\cdot}74$	19 P_2 $\lambda_2 = 2{\cdot}34$	20 $\bar{\eta}^2$	20 $\sigma_{\bar{\eta}^2}$	20 P_1 $\lambda_1 = 2{\cdot}73$	20 P_2 $\lambda_2 = 2{\cdot}33$
401	·045000	·014622	·010	·021	·047500	·015003	·010	·021
402	·044888	·014587	·010	·021	·047382	·014967	·010	·021
403	·044776	·014551	·010	·021	·047264	·014931	·010	·021
404	·044665 +	·014516	·010	·021	·047146	·014894	·010	·021
405	·044554	·014481	·010	·021	·047030	·014859	·010	·021
406	·044444	·014446	·010	·021	·046914	·014823	·010	·021
407	·044335 −	·014412	·010	·021	·046798	·014787	·010	·021
408	·044226	·014377	·010	·021	·046683	·014752	·010	·021
409	·044118	·014343	·010	·021	·046569	·014717	·010	·021
410	·044010	·014309	·010	·021	·046455 −	·014682	·010	·021
411	·043902	·014275 −	·010	·021	·046341	·014647	·010	·021
412	·043796	·014241	·010	·021	·046229	·014612	·010	·021
413	·043689	·014207	·010	·021	·046117	·014578	·010	·021
414	·043584	·014174	·010	·021	·046005 −	·014543	·010	·021
415	·043478	·014140	·010	·021	·045894	·014509	·010	·021
416	·043373	·014107	·010	·021	·045783	·014475 +	·010	·021
417	·043269	·014074	·010	·021	·045673	·014441	·010	·021
418	·043165 +	·014041	·010	·021	·045564	·014408	·010	·021
419	·043062	·014008	·010	·021	·045455 −	·014374	·010	·021
420	·042959	·013976	·010	·021	·045346	·014341	·010	·021
421	·042857	·013943	·010	·021	·045238	·014307	·010	·021
422	·042755 +	·013911	·010	·021	·045131	·014274	·010	·021
423	·042654	·013879	·010	·021	·045024	·014241	·010	·021
424	·042553	·013847	·010	·021	·044917	·014208	·010	·021
425	·042453	·013815 −	·010	·021	·044811	·014176	·010	·021
426	·042353	·013783	·010	·021	·044706	·014143	·010	·021
427	·042254	·013752	·010	·021	·044601	·014111	·010	·021
428	·042155 −	·013720	·010	·021	·044496	·014079	·010	·021
429	·042056	·013689	·010	·021	·044393	·014047	·010	·021
430	·041958	·013658	·010	·021	·044289	·014015 −	·010	·021
431	·041860	·013627	·010	·021	·044186	·013983	·010	·021
432	·041763	·013596	·010	·021	·044084	·013951	·010	·021
433	·041667	·013565 +	·010	·021	·043981	·013920	·010	·021
434	·041570	·013535 −	·010	·021	·043880	·013889	·010	·021
435	·041475 −	·013504	·010	·021	·043779	·013857	·010	·021
436	·041379	·013474	·010	·021	·043678	·013826	·010	·021
437	·041284	·013444	·010	·021	·043578	·013795 +	·010	·021
438	·041190	·013414	·010	·021	·043478	·013765 −	·010	·021
439	·041096	·013384	·010	·021	·043379	·013734	·010	·021
440	·041002	·013354	·010	·021	·043280	·013704	·010	·021
441	·040909	·013324	·010	·021	·043182	·013673	·010	·021
442	·040816	·013295 −	·010	·021	·043084	·013643	·010	·021
443	·040724	·013265 +	·010	·021	·042986	·013613	·010	·021
444	·040632	·013236	·010	·021	·042889	·013583	·010	·021
445	·040541	·013207	·010	·021	·042793	·013553	·010	·021
446	·040449	·013178	·010	·021	·042697	·013523	·010	·021
447	·040359	·013149	·010	·021	·042601	·013494	·010	·021
448	·040268	·013121	·010	·021	·042506	·013464	·010	·021
449	·040179	·013092	·010	·021	·042411	·013435 −	·010	·021
450	·040089	·013063	·010	·021	·042316	·013406	·010	·021

n = number of arrays

N = size of sample	3: $\bar\eta^2$	3: $\sigma_{\bar\eta^2}$	3: P_1 $\lambda_1 = 3{\cdot}50$	3: P_2 $\lambda_2 = 2{\cdot}94$	4: $\bar\eta^2$	4: $\sigma_{\bar\eta^2}$	4: P_1 $\lambda_1 = 3{\cdot}20$	4: P_2 $\lambda_2 = 2{\cdot}80$	5: $\bar\eta^2$	5: $\sigma_{\bar\eta^2}$	5: P_1 $\lambda_1 = 3{\cdot}14$	5: P_2 $\lambda_2 = 2{\cdot}68$	6: $\bar\eta^2$	6: $\sigma_{\bar\eta^2}$	6: P_1 $\lambda_1 = 3{\cdot}11$	6: P_2 $\lambda_2 = 2{\cdot}63$	7: $\bar\eta^2$	7: $\sigma_{\bar\eta^2}$	7: P_1 $\lambda_1 = 3{\cdot}08$	7: P_2 $\lambda_2 = 2{\cdot}58$	8: $\bar\eta^2$	8: $\sigma_{\bar\eta^2}$	8: P_1 $\lambda_1 = 3{\cdot}02$	8: P_2 $\lambda_2 = 2{\cdot}54$
451	·004444	·004425 –	·011	·019	·006667	·005413	·012	·020	·008889	·006244	·012	·021	·011111	·006973	·011	·021	·013333	·007630	·010	·020	·015556	·008232	·010	·021
452	·004435 –	·004415 –	·011	·019	·006652	·005401	·012	·020	·008869	·006230	·012	·021	·011086	·006957	·011	·021	·013304	·007613	·010	·020	·015521	·008214	·010	·021
453	·004425 –	·004405 +	·011	·019	·006637	·005389	·012	·020	·008850 –	·006216	·012	·021	·011062	·006942	·011	·021	·013274	·007596	·010	·020	·015487	·008196	·010	·021
454	·004415 +	·004396	·011	·019	·006623	·005377	·012	·020	·008830	·006202	·012	·021	·011038	·006927	·011	·021	·013245 +	·007579	·010	·020	·015453	·008178	·010	·021
455	·004405 +	·004386	·011	·019	·006608	·005366	·012	·020	·008811	·006189	·012	·021	·011013	·006912	·011	·021	·013216	·007563	·010	·020	·015419	·008160	·010	·021
456	·004396	·004376	·011	·019	·006593	·005354	·012	·020	·008791	·006175 +	·012	·021	·010989	·006897	·011	·021	·013187	·007546	·010	·020	·015385 –	·008142	·010	·021
457	·004386	·004367	·011	·019	·006579	·005342	·012	·020	·008772	·006162	·012	·021	·010965 –	·006882	·011	·021	·013158	·007530	·010	·020	·015351	·008124	·010	·021
458	·004376	·004357	·011	·019	·006565 –	·005331	·012	·020	·008753	·006149	·012	·021	·010941	·006867	·011	·021	·013129	·007514	·010	·020	·015317	·008107	·010	·021
459	·004367	·004348	·011	·019	·006550 +	·005319	·012	·020	·008734	·006135 +	·012	·021	·010917	·006852	·011	·021	·013100	·007497	·010	·020	·015284	·008089	·010	·021
460	·004357	·004338	·011	·019	·006536	·005308	·012	·020	·008715 –	·006122	·012	·021	·010893	·006837	·011	·021	·013072	·007481	·010	·020	·015251	·008072	·010	·021
461	·004348	·004329	·011	·019	·006522	·005296	·012	·020	·008696	·006109	·012	·021	·010870	·006822	·011	·021	·013043	·007465 +	·010	·020	·015217	·008054	·010	·021
462	·004338	·004320	·011	·019	·006508	·005285 –	·012	·020	·008677	·006096	·012	·021	·010846	·006808	·011	·021	·013015 +	·007449	·010	·020	·015184	·008037	·010	·021
463	·004329	·004310	·011	·019	·006494	·005273	·012	·020	·008658	·006082	·012	·021	·010823	·006793	·011	·021	·012987	·007433	·010	·020	·015152	·008020	·010	·021
464	·004320	·004301	·011	·019	·006479	·005262	·012	·020	·008639	·006069	·012	·021	·010799	·006778	·011	·021	·012959	·007417	·010	·020	·015119	·008003	·010	·021
465	·004310	·004292	·011	·019	·006466	·005251	·012	·020	·008621	·006056	·012	·021	·010776	·006764	·011	·021	·012931	·007401	·010	·020	·015086	·007986	·010	·021
466	·004301	·004283	·011	·019	·006452	·005239	·012	·020	·008602	·006043	·012	·021	·010753	·006749	·011	·021	·012903	·007386	·010	·020	·015054	·007969	·010	·021
467	·004292	·004273	·011	·019	·006438	·005228	·012	·020	·008584	·006031	·012	·021	·010730	·006735 +	·011	·021	·012876	·007370	·010	·020	·015021	·007952	·010	·021
468	·004283	·004264	·011	·019	·006424	·005217	·012	·020	·008565 –	·006018	·012	·021	·010707	·006721	·011	·021	·012848	·007354	·010	·020	·014989	·007935 –	·010	·021
469	·004274	·004255 +	·011	·019	·006410	·005206	·012	·020	·008547	·006005 –	·012	·021	·010684	·006706	·011	·021	·012821	·007339	·010	·020	·014957	·007918	·010	·021
470	·004264	·004246	·011	·019	·006397	·005195 –	·012	·020	·008529	·005992	·012	·021	·010661	·006692	·011	·021	·012793	·007323	·010	·020	·014925 +	·007901	·010	·021
471	·004255 +	·004237	·011	·019	·006383	·005184	·012	·020	·008511	·005980	·012	·021	·010638	·006678	·011	·021	·012766	·007308	·010	·020	·014894	·007885 –	·010	·021
472	·004246	·004228	·011	·019	·006369	·005173	·012	·020	·008493	·005967	·012	·021	·010616	·006664	·011	·021	·012739	·007292	·010	·020	·014862	·007868	·010	·021
473	·004237	·004219	·011	·019	·006356	·005162	·012	·020	·008475 –	·005954	·012	·021	·010593	·006650 +	·011	·021	·012712	·007277	·010	·020	·014831	·007852	·010	·021
474	·004228	·004210	·011	·019	·006342	·005151	·012	·020	·008457	·005942	·012	·021	·010571	·006636	·011	·021	·012685 –	·007262	·010	·020	·014799	·007835 +	·010	·021
475	·004219	·004202	·011	·019	·006329	·005140	·012	·020	·008439	·005929	·012	·021	·010549	·006622	·011	·021	·012658	·007247	·010	·020	·014768	·007819	·010	·021
476	·004211	·004193	·011	·019	·006316	·005130	·012	·020	·008421	·005917	·012	·021	·010526	·006608	·011	·021	·012632	·007231	·010	·020	·014737	·007803	·010	·021
477	·004202	·004184	·011	·019	·006303	·005119	·012	·020	·008403	·005905 –	·012	·021	·010504	·006595 –	·011	·021	·012605 +	·007216	·010	·026	·014706	·007786	·010	·021
478	·004193	·004175 +	·011	·019	·006289	·005108	·012	·020	·008386	·005892	·012	·021	·010482	·006581	·011	·021	·012579	·007201	·010	·020	·014675 +	·007770	·010	·021
479	·004184	·004167	·011	·019	·006276	·005098	·012	·020	·008368	·005880	·012	·021	·010460	·006567	·011	·021	·012552	·007186	·010	·020	·014644	·007754	·010	·021
480	·004175 +	·004158	·011	·019	·006263	·005087	·012	·020	·008351	·005868	·012	·021	·010438	·006554	·011	·021	·012526	·007172	·010	·020	·014614	·007738	·010	·021
481	·004167	·004149	·011	·019	·006250	·005077	·012	·020	·008333	·005856	·012	·021	·010417	·006540	·011	·021	·012500	·007157	·010	·020	·014583	·007722	·010	·021
482	·004158	·004141	·011	·019	·006237	·005066	·012	·020	·008316	·005844	·012	·021	·010395 –	·006527	·011	·021	·012474	·007142	·010	·020	·014553	·007706	·010	·021
483	·004149	·004132	·011	·019	·006224	·005056	·012	·020	·008299	·005832	·012	·021	·010373	·006513	·011	·021	·012448	·007127	·010	·020	·014523	·007690	·010	·021
484	·004141	·004124	·011	·019	·006211	·005045 +	·012	·020	·008282	·005820	·012	·021	·010352	·006500	·011	·021	·012422	·007113	·010	·020	·014493	·007674	·010	·021
485	·004132	·004115 +	·011	·019	·006198	·005035 –	·012	·020	·008264	·005808	·012	·021	·010331	·006486	·011	·021	·012397	·007098	·010	·020	·014463	·007659	·010	·021
486	·004124	·004107	·011	·019	·006186	·005025 –	·012	·020	·008247	·005796	·012	·021	·010309	·006473	·011	·021	·012371	·007084	·010	·020	·014433	·007643	·010	·021
487	·004115 +	·004098	·011	·019	·006173	·005014	·012	·020	·008230	·005784	·012	·021	·010288	·006460	·011	·021	·012346	·007069	·010	·020	·014403	·007628	·010	·021
488	·004107	·004090	·011	·019	·006160	·005004	·012	·020	·008214	·005772	·012	·021	·010267	·006447	·011	·021	·012320	·007055 –	·010	·020	·014374	·007612	·010	·021
489	·004098	·004082	·011	·019	·006148	·004994	·012	·020	·008197	·005760	·012	·021	·010246	·006434	·011	·021	·012295 +	·007040	·010	·020	·014344	·007597	·010	·021
490	·004090	·004073	·011	·019	·006135 –	·004984	·012	·020	·008180	·005749	·012	·021	·010225 –	·006421	·011	·021	·012270	·007026	·010	·020	·014315 –	·007581	·010	·021
491	·004082	·004065 +	·011	·019	·006122	·004973	·012	·020	·008163	·005737	·012	·021	·010204	·006408	·011	·021	·012245 –	·007012	·010	·020	·014286	·007566	·010	·021
492	·004073	·004057	·011	·019	·006110	·004963	·012	·020	·008147	·005725 +	·012	·021	·010183	·006395 –	·011	·021	·012220	·006998	·010	·020	·014257	·007551	·010	·021
493	·004065 +	·004049	·011	·019	·006098	·004953	·012	·020	·008130	·005714	·012	·021	·010163	·006382	·011	·021	·012195 +	·006984	·010	·020	·014228	·007535 +	·010	·021
494	·004057	·004040	·011	·019	·006085 +	·004943	·012	·020	·008114	·005702	·012	·021	·010142	·006369	·011	·021	·012170	·006970	·010	·020	·014199	·007520	·010	·021
495	·004049	·004032	·011	·019	·006073	·004933	·012	·020	·008097	·005691	·012	·021	·010121	·006356	·011	·021	·012146	·006956	·010	·020	·014170	·007505 +	·010	·021
496	·004040	·004024	·011	·019	·006061	·004924	·012	·020	·008081	·005679	·012	·021	·010101	·006343	·011	·021	·012121	·006942	·010	·020	·014141	·007490	·010	·021
497	·004032	·004016	·011	·019	·006048	·004914	·012	·020	·008065 –	·005668	·012	·021	·010081	·006331	·011	·021	·012097	·006928	·010	·020	·014113	·007475 +	·010	·021
498	·004024	·004008	·011	·019	·006036	·004904	·012	·020	·008048	·005657	·012	·021	·010060	·006318	·011	·021	·012072	·006914	·010	·020	·014085 –	·007460	·010	·021
499	·004016	·004000	·011	·019	·006024	·004894	·012	·020	·008032	·005645 +	·012	·021	·010040	·006305 +	·011	·021	·012048	·006900	·010	·020	·014056	·007445 +	·010	·021
500	·004008	·003992	·011	·019	·006012	·004884	·012	·020	·008016	·005634	·012	·021	·010020	·006293	·011	·021	·012024	·006886	·010	·020	·014028	·007431	·010	·021

N = size of sample	9: $\bar{\eta}^2$	9: $\sigma_{\bar{\eta}^2}$	9: P_1 $\lambda_1=2{\cdot}96$	9: P_2 $\lambda_2=2{\cdot}50$	10: $\bar{\eta}^2$	10: $\sigma_{\bar{\eta}^2}$	10: P_1 $\lambda_1=2{\cdot}92$	10: P_2 $\lambda_2=2{\cdot}47$
451	·017778	·008790	·011	·021	·020000	·009313	·011	·021
452	·017738	·008771	·011	·021	·019956	·009292	·011	·021
453	·017699	·008752	·011	·021	·019912	·009272	·011	·021
454	·017660	·008732	·011	·021	·019868	·009252	·011	·021
455	·017621	·008713	·011	·021	·019824	·009232	·011	·021
456	·017582	·008694	·011	·021	·019780	·009212	·011	·021
457	·017544	·008676	·011	·021	·019737	·009192	·011	·021
458	·017505 +	·008657	·011	·021	·019694	·009172	·011	·021
459	·017467	·008638	·011	·021	·019651	·009152	·011	·021
460	·017429	·008620	·011	·021	·019608	·009132	·011	·021
461	·017391	·008601	·011	·021	·019565 +	·009113	·011	·021
462	·017354	·008583	·011	·021	·019523	·009093	·011	·021
463	·017316	·008564	·011	·021	·019481	·009074	·011	·021
464	·017279	·008546	·011	·021	·019438	·009054	·011	·021
465	·017241	·008528	·011	·021	·019397	·009035 +	·011	·021
466	·017204	·008510	·011	·021	·019355 −	·009016	·011	·021
467	·017167	·008491	·011	·021	·019313	·008997	·011	·021
468	·017131	·008474	·011	·021	·019272	·008978	·011	·021
469	·017094	·008456	·011	·021	·019231	·008959	·011	·021
470	·017058	·008438	·011	·021	·019190	·008940	·011	·021
471	·017021	·008420	·011	·021	·019149	·008921	·011	·021
472	·016985 +	·008402	·011	·021	·019108	·008902	·011	·021
473	·016949	·008385 −	·011	·021	·019068	·008884	·011	·021
474	·016913	·008367	·011	·021	·019027	·008865 +	·011	·021
475	·016878	·008350 −	·011	·021	·018987	·008847	·011	·021
476	·016842	·008332	·011	·021	·018947	·008828	·011	·021
477	·016807	·008315 −	·011	·021	·018908	·008810	·011	·021
478	·016771	·008298	·011	·021	·018868	·008792	·011	·021
479	·016736	·008281	·011	·021	·018828	·008774	·011	·021
480	·016701	·008263	·011	·021	·018789	·008755 +	·011	·021
481	·016667	·008246	·011	·021	·018750	·008737	·011	·021
482	·016632	·008229	·011	·021	·018711	·008719	·011	·021
483	·016598	·008213	·011	·021	·018672	·008702	·011	·021
484	·016563	·008196	·011	·021	·018634	·008684	·011	·021
485	·016529	·008179	·011	·021	·018595 +	·008666	·011	·021
486	·016495 −	·008162	·011	·021	·018557	·008648	·011	·021
487	·016461	·008146	·011	·021	·018519	·008631	·011	·021
488	·016427	·008129	·011	·021	·018480	·008613	·011	·021
489	·016393	·008113	·011	·021	·018443	·008596	·011	·021
490	·016360	·008096	·011	·021	·018405 −	·008578	·011	·021
491	·016327	·008080	·011	·021	·018367	·008561	·011	·021
492	·016293	·008064	·011	·021	·018330	·008544	·011	·021
493	·016260	·008047	·011	·021	·018293	·008527	·011	·021
494	·016227	·008031	·011	·021	·018256	·008510	·011	·021
495	·016194	·008015 +	·011	·021	·018219	·008493	·011	·021
496	·016162	·007999	·011	·021	·018182	·008476	·011	·021
497	·016129	·007983	·011	·021	·018145 +	·008459	·011	·021
498	·016097	·007967	·011	·021	·018109	·008442	·011	·021
499	·016064	·007951	·011	·021	·018072	·008425 +	·011	·021
500	·016032	·007936	·011	·021	·018036	·008408	·011	·021

N = size of sample	11: $\bar{\eta}^2$	11: $\sigma_{\bar{\eta}^2}$	11: P_1 $\lambda_1=2{\cdot}88$	11: P_2 $\lambda_2=2{\cdot}44$	12: $\bar{\eta}^2$	12: $\sigma_{\bar{\eta}^2}$	12: P_1 $\lambda_1=2{\cdot}84$	12: P_2 $\lambda_2=2{\cdot}42$
451	·022222	·009805 +	·011	·021	·024444	·010272	·011	·021
452	·022173	·009784	·011	·021	·024390	·010250 −	·011	·021
453	·022124	·009762	·011	·021	·024336	·010227	·011	·021
454	·022075 +	·009741	·011	·021	·024283	·010205 +	·011	·021
455	·022026	·009720	·011	·021	·024229	·010183	·011	·021
456	·021978	·009699	·011	·021	·024176	·010161	·011	·021
457	·021930	·009678	·011	·021	·024123	·010139	·011	·021
458	·021882	·009657	·011	·021	·024070	·010117	·011	·021
459	·021834	·009636	·011	·021	·024017	·010095 +	·011	·021
460	·021786	·009616	·011	·021	·023965 +	·010074	·011	·021
461	·021739	·009595 −	·011	·021	·023913	·010052	·011	·021
462	·021692	·009574	·011	·021	·023861	·010031	·011	·021
463	·021645 +	·009554	·011	·021	·023810	·010009	·011	·021
464	·021598	·009534	·011	·021	·023758	·009988	·011	·021
465	·021552	·009513	·011	·021	·023707	·009967	·011	·021
466	·021505 +	·009493	·011	·021	·023656	·009946	·011	·021
467	·021459	·009473	·011	·021	·023605 +	·009924	·011	·021
468	·021413	·009453	·011	·021	·023555 −	·009904	·011	·021
469	·021368	·009433	·011	·021	·023504	·009883	·011	·021
470	·021322	·009413	·011	·021	·023454	·009862	·011	·021
471	·021277	·009393	·011	·021	·023404	·009841	·011	·021
472	·021231	·009374	·011	·021	·023355 −	·009821	·011	·021
473	·021186	·009354	·011	·021	·023305 +	·009800	·011	·021
474	·021142	·009335 −	·011	·021	·023256	·009780	·011	·021
475	·021097	·009315 +	·011	·021	·023207	·009759	·011	·021
476	·021053	·009296	·011	·021	·023158	·009739	·011	·021
477	·021008	·009277	·011	·021	·023109	·009719	·011	·021
478	·020964	·009257	·011	·021	·023061	·009699	·011	·021
479	·020921	·009238	·011	·021	·023013	·009679	·011	·021
480	·020877	·009219	·011	·021	·022965 −	·009659	·011	·021
481	·020833	·009200	·011	·021	·022917	·009639	·011	·021
482	·020790	·009181	·011	·021	·022869	·009619	·011	·021
483	·020747	·009163	·011	·021	·022822	·009600	·011	·021
484	·020704	·009144	·011	·021	·022774	·009580	·011	·021
485	·020661	·009125 +	·011	·021	·022727	·009560	·011	·021
486	·020619	·009107	·011	·021	·022680	·009541	·011	·021
487	·020576	·009088	·011	·021	·022634	·009522	·011	·021
488	·020534	·009070	·011	·021	·022587	·009502	·011	·021
489	·020492	·009051	·011	·021	·022541	·009483	·011	·021
490	·020450 −	·009033	·011	·021	·022495 −	·009464	·011	·021
491	·020408	·009015 −	·011	·021	·022449	·009445 −	·011	·021
492	·020367	·008997	·011	·021	·022403	·009426	·011	·021
493	·020325 +	·008979	·011	·021	·022358	·009407	·011	·021
494	·020284	·008961	·011	·021	·022312	·009388	·011	·021
495	·020243	·008943	·011	·021	·022267	·009370	·011	·021
496	·020202	·008925 −	·011	·021	·022222	·009351	·011	·021
497	·020161	·008907	·011	·021	·022177	·009332	·011	·021
498	·020121	·008889	·011	·021	·022133	·009314	·011	·021
499	·020080	·008872	·011	·021	·022088	·009295 +	·011	·021
500	·020040	·008854	·011	·021	·022044	·009277	·011	·021

N = size of sample	13: $\bar{\eta}^2$	13: $\sigma_{\bar{\eta}^2}$	13: P_1 $\lambda_1=2{\cdot}80$	13: P_2 $\lambda_2=2{\cdot}40$	14: $\bar{\eta}^2$	14: $\sigma_{\bar{\eta}^2}$	14: P_1 $\lambda_1=2{\cdot}79$	14: P_2 $\lambda_2=2{\cdot}39$
451	·026667	·010717	·011	·021	·028889	·011142	·011	·021
452	·026608	·010693	·011	·021	·028825 −	·011117	·011	·021
453	·026549	·010670	·011	·021	·028761	·011093	·011	·021
454	·026490	·010647	·011	·021	·028698	·011069	·011	·021
455	·026432	·010624	·011	·021	·028634	·011045 +	·011	·021
456	·026374	·010601	·011	·021	·028571	·011021	·011	·021
457	·026316	·010578	·011	·021	·028509	·010997	·011	·021
458	·026258	·010555 +	·011	·021	·028446	·010974	·011	·021
459	·026201	·010532	·011	·021	·028384	·010950 +	·011	·021
460	·026144	·010510	·011	·021	·028322	·010927	·011	·021
461	·026087	·010487	·011	·021	·028261	·010903	·011	·021
462	·026030	·010465 −	·011	·021	·028200	·010880	·011	·021
463	·025974	·010443	·011	·021	·028139	·010857	·011	·021
464	·025918	·010420	·011	·021	·028078	·010834	·011	·021
465	·025862	·010398	·011	·021	·028017	·010811	·011	·021
466	·025806	·010376	·011	·021	·027957	·010788	·011	·021
467	·025751	·010354	·011	·021	·027897	·010765 +	·011	·021
468	·025696	·010333	·011	·021	·027837	·010743	·011	·021
469	·025641	·010311	·011	·021	·027778	·010720	·011	·021
470	·025586	·010289	·011	·021	·027719	·010698	·011	·021
471	·025532	·010268	·011	·021	·027660	·010675 +	·011	·021
472	·025478	·010246	·011	·021	·027601	·010653	·011	·021
473	·025424	·010225 −	·011	·021	·027542	·010631	·011	·021
474	·025370	·010203	·011	·021	·027484	·010609	·011	·021
475	·025316	·010182	·011	·021	·027426	·010587	·011	·021
476	·025263	·010161	·011	·021	·027368	·010565 −	·011	·021
477	·025210	·010140	·011	·021	·027311	·010543	·011	·021
478	·025157	·010119	·011	·021	·027254	·010521	·011	·021
479	·025105 −	·010098	·011	·021	·027197	·010499	·011	·021
480	·025052	·010078	·011	·021	·027140	·010478	·011	·021
481	·025000	·010057	·011	·021	·027083	·010456	·011	·021
482	·024948	·010036	·011	·021	·027027	·010435 −	·011	·021
483	·024896	·010016	·011	·021	·026971	·010414	·011	·021
484	·024845 −	·009995 +	·011	·021	·026915 +	·010392	·011	·021
485	·024793	·009975 +	·011	·021	·026860	·010371	·011	·021
486	·024742	·009955 −	·011	·021	·026804	·010350 +	·011	·021
487	·024691	·009935 −	·011	·021	·026749	·010329	·011	·021
488	·024641	·009914	·011	·021	·026694	·010308	·011	·021
489	·024590	·009894	·011	·021	·026639	·010288	·011	·021
490	·024540	·009875 −	·011	·021	·026585 −	·010267	·011	·021
491	·024490	·009855 −	·011	·021	·026531	·010246	·011	·021
492	·024440	·009835 −	·011	·021	·026477	·010226	·011	·021
493	·024390	·009815 +	·011	·021	·026423	·010205 +	·011	·021
494	·024341	·009796	·011	·021	·026369	·010185 −	·011	·021
495	·024291	·009776	·011	·021	·026316	·010165 −	·011	·021
496	·024242	·009757	·011	·021	·026263	·010144	·011	·021
497	·024194	·009737	·011	·021	·026210	·010124	·011	·021
498	·024145 −	·009718	·011	·021	·026157	·010104	·011	·021
499	·024096	·009699	·011	·021	·026104	·010084	·011	·021
500	·024048	·009679	·011	·021	·026052	·010064	·011	·021

n = number of arrays

N = size of sample	15				16				17			
	$\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1 $\lambda_1=2·78$	P_2 $\lambda_2=2·38$	$\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1 $\lambda_1=2·77$	P_2 $\lambda_2=2·37$	$\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1 $\lambda_1=2·76$	P_2 $\lambda_2=2·36$
451	·031111	·011549	·011	·021	·033333	·011941	·011	·021	·035556	·012318	·011	·021
452	·031042	·011524	·011	·021	·033259	·011915 −	·011	·021	·035477	·012291	·011	·021
453	·030973	·011499	·011	·021	·033186	·011889	·011	·021	·035398	·012265 −	·011	·021
454	·030905 +	·011474	·011	·021	·033113	·011863	·011	·021	·035320	·012238	·011	·021
455	·030837	·011449	·011	·021	·033040	·011837	·011	·021	·035242	·012212	·011	·021
456	·030769	·011424	·011	·021	·032967	·011812	·011	·021	·035165 −	·012185 +	·011	·021
457	·030702	·011400	·011	·021	·032895 −	·011786	·011	·021	·035088	·012159	·011	·021
458	·030635 −	·011375 +	·011	·021	·032823	·011761	·011	·021	·035011	·012133	·011	·021
459	·030568	·011351	·011	·021	·032751	·011736	·011	·021	·034934	·012107	·011	·021
460	·030501	·011327	·011	·021	·032680	·011711	·011	·021	·034858	·012081	·011	·021
461	·030435 −	·011302	·011	·021	·032609	·011686	·011	·021	·034783	·012056	·011	·021
462	·030369	·011278	·011	·021	·032538	·011661	·011	·021	·034707	·012030	·011	·021
463	·030303	·011254	·011	·021	·032468	·011636	·011	·021	·034632	·012004	·011	·021
464	·030238	·011230	·011	·021	·032397	·011612	·011	·021	·034557	·011979	·011	·021
465	·030172	·011207	·011	·021	·032328	·011587	·011	·021	·034483	·011954	·011	·021
466	·030108	·011183	·011	·021	·032258	·011563	·011	·021	·034409	·011929	·011	·021
467	·030043	·011159	·011	·021	·032189	·011538	·011	·021	·034335 −	·011903	·011	·021
468	·029979	·011136	·011	·021	·032120	·011514	·011	·021	·034261	·011878	·011	·021
469	·029915 −	·011113	·011	·021	·032051	·011490	·011	·021	·034188	·011854	·011	·021
470	·029851	·011089	·011	·021	·031983	·011466	·011	·021	·034115 −	·011829	·011	·021
471	·029787	·011066	·011	·021	·031915 −	·011442	·011	·021	·034043	·011804	·011	·021
472	·029724	·011043	·011	·021	·031847	·011418	·011	·021	·033970	·011780	·011	·021
473	·029661	·011020	·011	·021	·031780	·011394	·011	·021	·033898	·011755 +	·011	·021
474	·029598	·010997	·011	·021	·031712	·011371	·011	·021	·033827	·011731	·011	·021
475	·029536	·010974	·011	·021	·031646	·011347	·011	·021	·033755 +	·011706	·011	·021
476	·029474	·010952	·011	·021	·031579	·011324	·011	·021	·033684	·011682	·011	·021
477	·029412	·010929	·011	·021	·031513	·011300	·011	·021	·033613	·011658	·011	·021
478	·029350 +	·010906	·011	·021	·031447	·011277	·011	·021	·033543	·011634	·011	·021
479	·029289	·010884	·011	·021	·031381	·011254	·011	·021	·033473	·011610	·011	·021
480	·029228	·010862	·011	·021	·031315 +	·011231	·011	·021	·033403	·011587	·011	·021
481	·029167	·010839	·011	·021	·031250	·011208	·011	·021	·033333	·011563	·011	·021
482	·029106	·010817	·011	·021	·031185 +	·011185 −	·011	·021	·033264	·011539	·011	·021
483	·029046	·010795 +	·011	·021	·031120	·011162	·011	·021	·033195 +	·011516	·011	·021
484	·028986	·010773	·011	·021	·031056	·011139	·011	·021	·033126	·011493	·011	·021
485	·028926	·010751	·011	·021	·030992	·011117	·011	·021	·033058	·011469	·011	·021
486	·028866	·010730	·011	·021	·030928	·011094	·011	·021	·032990	·011446	·011	·021
487	·028807	·010708	·011	·021	·030864	·011072	·011	·021	·032922	·011423	·011	·021
488	·028747	·010686	·011	·021	·030801	·011050 −	·011	·021	·032854	·011400	·011	·021
489	·028689	·010665 −	·011	·021	·030738	·011027	·011	·021	·032787	·011377	·011	·021
490	·028630	·010643	·011	·021	·030675 −	·011005 +	·011	·021	·032720	·011354	·011	·021
491	·028571	·010622	·011	·021	·030612	·010983	·011	·021	·032653	·011331	·011	·021
492	·028513	·010601	·011	·021	·030550 −	·010961	·011	·021	·032587	·011309	·011	·021
493	·028455 +	·010579	·011	·021	·030488	·010939	·011	·021	·032520	·011286	·011	·021
494	·028398	·010558	·011	·021	·030426	·010918	·011	·021	·032454	·011264	·011	·021
495	·028340	·010537	·011	·021	·030364	·010896	·011	·021	·032389	·011241	·011	·021
496	·028283	·010516	·011	·021	·030303	·010874	·011	·021	·032323	·011219	·011	·021
497	·028226	·010496	·011	·021	·030242	·010853	·011	·021	·032258	·011197	·011	·021
498	·028169	·010475 −	·011	·021	·030181	·010831	·011	·021	·032193	·011175 −	·011	·021
499	·028112	·010454	·011	·021	·030120	·010810	·011	·021	·032129	·011153	·011	·021
500	·028056	·010434	·011	·021	·030060	·010789	·011	·021	·032064	·011131	·011	·021

N = size of sample	18				19				20			
	$\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1 $\lambda_1=2·75$	P_2 $\lambda_2=2·35$	$\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1 $\lambda_1=2·74$	P_2 $\lambda_2=2·34$	$\bar\eta^2$	$\sigma_{\bar\eta^2}$	P_1 $\lambda_1=2·73$	P_2 $\lambda_2=2·33$
451	·037778	·012682	·011	·021	·040000	·013035 +	·010	·021	·042222	·013377	·010	·021
452	·037694	·012655 −	·011	·021	·039911	·013007	·010	·021	·042129	·013348	·010	·021
453	·037611	·012628	·011	·021	·039823	·012979	·010	·021	·042035 +	·013319	·010	·021
454	·037528	·012600	·011	·021	·039735 +	·012951	·010	·021	·041943	·013290	·010	·021
455	·037445 −	·012573	·011	·021	·039648	·012923	·010	·021	·041850 +	·013262	·010	·021
456	·037363	·012546	·011	·021	·039560	·012895 +	·010	·021	·041758	·013233	·010	·021
457	·037281	·012519	·011	·021	·039474	·012867	·010	·021	·041667	·013205 −	·010	·021
458	·037199	·012492	·011	·021	·039387	·012840	·010	·021	·041575 +	·013177	·010	·021
459	·037118	·012466	·011	·021	·039301	·012813	·010	·021	·041485 −	·013149	·010	·021
460	·037037	·012439	·011	·021	·039216	·012785 +	·010	·021	·041394	·013121	·010	·021
461	·036957	·012413	·011	·021	·039130	·012758	·011	·021	·041304	·013093	·010	·021
462	·036876	·012385	·011	·021	·039046	·012731	·011	·021	·041215 −	·013065 +	·010	·021
463	·036797	·012360	·011	·021	·038961	·012704	·011	·021	·041126	·013037	·010	·021
464	·036717	·012334	·011	·021	·038877	·012677	·011	·021	·041037	·013010	·010	·021
465	·036638	·012308	·011	·021	·038793	·012651	·011	·021	·040948	·012983	·010	·021
466	·036559	·012282	·011	·021	·038710	·012624	·011	·021	·040860	·012955 +	·010	·021
467	·036481	·012256	·011	·021	·038627	·012597	·011	·021	·040773	·012928	·010	·021
468	·036403	·012230	·011	·021	·038544	·012571	·011	·021	·040685 +	·012901	·010	·021
469	·036325 −	·012205 −	·011	·021	·038462	·012545 −	·011	·021	·040598	·012874	·010	·021
470	·036247	·012179	·011	·021	·038380	·012519	·011	·021	·040512	·012847	·010	·021
471	·036170	·012154	·011	·021	·038298	·012493	·011	·021	·040426	·012821	·010	·021
472	·036093	·012129	·011	·021	·038217	·012467	·011	·021	·040340	·012794	·010	·021
473	·036017	·012104	·011	·021	·038136	·012441	·011	·021	·040254	·012768	·010	·021
474	·035941	·012079	·011	·021	·038055 −	·012415 +	·011	·021	·040169	·012741	·010	·021
475	·035865 −	·012054	·011	·021	·037975 −	·012390	·011	·021	·040084	·012715 −	·010	·021
476	·035789	·012029	·011	·021	·037895 −	·012364	·011	·021	·040000	·012689	·010	·021
477	·035714	·012004	·011	·021	·037815 +	·012339	·011	·021	·039916	·012663	·010	·021
478	·035639	·011979	·011	·021	·037736	·012313	·011	·021	·039832	·012637	·010	·021
479	·035565 −	·011955 −	·011	·021	·037657	·012288	·011	·021	·039749	·012611	·010	·021
480	·035491	·011930	·011	·021	·037578	·012263	·011	·021	·039666	·012585 +	·010	·021
481	·035417	·011906	·011	·021	·037500	·012238	·011	·021	·039583	·012560	·010	·021
482	·035343	·011882	·011	·021	·037422	·012213	·011	·021	·039501	·012534	·010	·021
483	·035270	·011858	·011	·021	·037344	·012188	·011	·021	·039419	·012509	·010	·021
484	·035197	·011834	·011	·021	·037267	·012164	·011	·021	·039337	·012483	·010	·021
485	·035124	·011810	·011	·021	·037190	·012139	·011	·021	·039256	·012458	·010	·021
486	·035052	·011786	·011	·021	·037113	·012114	·011	·021	·039175 +	·012433	·010	·021
487	·034979	·011762	·011	·021	·037037	·012090	·011	·021	·039095 −	·012408	·010	·021
488	·034908	·011738	·011	·021	·036961	·012066	·011	·021	·039014	·012383	·010	·021
489	·034836	·011715 −	·011	·021	·036885 +	·012042	·011	·021	·038934	·012358	·010	·021
490	·034765 −	·011691	·011	·021	·036810	·012017	·011	·021	·038855 −	·012334	·010	·021
491	·034694	·011668	·011	·021	·036735 −	·011993	·011	·021	·038776	·012309	·010	·021
492	·034623	·011645 −	·011	·021	·036660	·011970	·011	·021	·038697	·012285 −	·010	·021
493	·034553	·011621	·011	·021	·036585 +	·011946	·011	·021	·038618	·012260	·010	·021
494	·034483	·011598	·011	·021	·036511	·011922	·011	·021	·038540	·012236	·010	·021
495	·034413	·011575 +	·011	·021	·036437	·011898	·011	·021	·038462	·012212	·010	·021
496	·034343	·011552	·011	·021	·036364	·011875 −	·011	·021	·038384	·012187	·010	·021
497	·034274	·011530	·011	·021	·036290	·011851	·011	·021	·038306	·012163	·010	·021
498	·034205 +	·011507	·011	·021	·036217	·011828	·011	·021	·038229	·012139	·010	·021
499	·034137	·011484	·011	·021	·036145 −	·011805 −	·011	·021	·038153	·012116	·010	·021
500	·034068	·011462	·011	·021	·036072	·011782	·011	·021	·038076	·012092	·010	·021

N = size of sample	3: $\bar{\eta}^2$	3: $\sigma_{\bar{\eta}^2}$	3: P_1 $\lambda_1=3·50$	3: P_2 $\lambda_2=2·94$	4: $\bar{\eta}^2$	4: $\sigma_{\bar{\eta}^2}$	4: P_1 $\lambda_1=3·20$	4: P_2 $\lambda_2=2·80$	5: $\bar{\eta}^2$	5: $\sigma_{\bar{\eta}^2}$	5: P_1 $\lambda_1=3·12$	5: P_2 $\lambda_2=2·68$
501	·004000	·003984	·011	·019	·006000	·004875 −	·012$_5$	·020	·008000	·005623	·012	·021
502	·003992	·003976	·011	·019	·005988	·004865 −	·013	·020	·007984	·005612	·012	·021
503	·003984	·003968	·011	·019	·005976	·004855 +	·013	·020	·007968	·005601	·012	·021
504	·003976	·003960	·011	·019	·005964	·004846	·013	·020	·007952	·005590	·012	·021
505	·003968	·003953	·011	·019	·005952	·004836	·013	·020	·007937	·005579	·012	·021
506	·003960	·003945 −	·011	·019	·005941	·004826	·013	·020	·007921	·005568	·012	·021
507	·003953	·003937	·011	·019	·005929	·004817	·013	·020	·007905 +	·005557	·012	·021
508	·003945 −	·003929	·011	·019	·005917	·004808	·013	·020	·007890	·005546	·012	·021
509	·003937	·003922	·011	·019	·005906	·004798	·013	·020	·007874	·005535 −	·012	·021
510	·003929	·003914	·011	·019	·005894	·004789	·013	·020	·007859	·005524	·012	·021
511	·003922	·003906	·011	·019	·005882	·004779	·013	·020	·007843	·005513	·012	·021
512	·003914	·003899	·011	·019	·005871	·004770	·013	·020	·007828	·005503	·012	·021
513	·003906	·003891	·011	·019	·005859	·004761	·013	·020	·007813	·005492	·012	·021
514	·003899	·003883	·011	·019	·005848	·004752	·013	·020	·007797	·005481	·012	·021
515	·003891	·003876	·011	·019	·005837	·004742	·013	·020	·007782	·005471	·012	·021
516	·003883	·003868	·011	·019	·005825 +	·004733	·013	·020	·007767	·005460	·012	·021
517	·003876	·003861	·011	·019	·005814	·004724	·013	·020	·007752	·005450 −	·012	·021
518	·003868	·003854	·011	·019	·005803	·004715 +	·013	·020	·007737	·005439	·012	·021
519	·003861	·003846	·011	·019	·005792	·004706	·013	·020	·007722	·005429	·012	·021
520	·003854	·003839	·011	·019	·005780	·004697	·013	·020	·007707	·005418	·012	·021
521	·003846	·003831	·011	·019	·005769	·004688	·013	·020	·007692	·005408	·012	·021
522	·003839	·003824	·011	·019	·005758	·004679	·013	·020	·007678	·005398	·012	·021
523	·003831	·003817	·011	·019	·005747	·004670	·013	·020	·007663	·005387	·012	·021
524	·003824	·003809	·011	·019	·005736	·004661	·013	·020	·007648	·005377	·012	·021
525	·003817	·003802	·011	·019	·005725 +	·004652	·013	·020	·007634	·005367	·012	·021
526	·003810	·003795 +	·011	·019	·005714	·004644	·013	·020	·007619	·005357	·012	·021
527	·003802	·003788	·011	·019	·005703	·004635 −	·013	·020	·007605 −	·005347	·012	·021
528	·003795 +	·003781	·011	·019	·005693	·004626	·013	·020	·007590	·005337	·012	·021
529	·003788	·003774	·011	·019	·005682	·004617	·013	·020	·007576	·005326	·012	·021
530	·003781	·003766	·011	·019	·005671	·004609	·013	·020	·007561	·005316	·012	·021
531	·003774	·003759	·011	·019	·005660	·004600	·013	·020	·007547	·005306	·012	·021
532	·003766	·003752	·011	·019	·005650 −	·004591	·013	·020	·007533	·005297	·012	·021
533	·003759	·003745 +	·011	·019	·005639	·004583	·013	·020	·007519	·005287	·012	·021
534	·003752	·003738	·011	·019	·005629	·004574	·013	·020	·007505 −	·005277	·012	·021
535	·003745 +	·003731	·011	·019	·005618	·004566	·013	·020	·007491	·005267	·012	·021
536	·003738	·003724	·011	·019	·005607	·004557	·013	·020	·007477	·005257	·012	·021
537	·003731	·003717	·011	·019	·005597	·004549	·013	·020	·007463	·005247	·012	·021
538	·003724	·003711	·011	·019	·005587	·004540	·013	·020	·007449	·005238	·012	·021
539	·003717	·003704	·011	·019	·005576	·004532	·013	·020	·007435 −	·005228	·012	·021
540	·003711	·003697	·011	·019	·005566	·004523	·013	·020	·007421	·005218	·012	·021
541	·003704	·003690	·011	·019	·005556	·004515 +	·013	·020	·007407	·005209	·012	·021
542	·003697	·003683	·011	·019	·005545 +	·004507	·013	·020	·007394	·005199	·012	·021
543	·003690	·003676	·011	·019	·005535 +	·004499	·013	·020	·007380	·005190	·012	·021
544	·003683	·003670	·011	·019	·005525 −	·004490	·013	·020	·007366	·005180	·012	·021
545	·003676	·003663	·011	·019	·005515 −	·004482	·013	·020	·007353	·005171	·012	·021
546	·003670	·003656	·011	·019	·005505 −	·004474	·013	·020	·007339	·005161	·012	·021
547	·003663	·003650 −	·011	·019	·005495 −	·004466	·013	·020	·007326	·005152	·012	·021
548	·003656	·003643	·011	·019	·005484	·004458	·013	·020	·007313	·005142	·012	·021
549	·003650 −	·003636	·011	·019	·005474	·004450 −	·013	·020	·007299	·005133	·012	·021
550	·003643	·003630	·011	·019	·005464	·004441	·013	·020	·007286	·005124	·012	·021

N = size of sample	6: $\bar{\eta}^2$	6: $\sigma_{\bar{\eta}^2}$	6: P_1 $\lambda_1=3·11$	6: P_2 $\lambda_2=2·63$	7: $\bar{\eta}^2$	7: $\sigma_{\bar{\eta}^2}$	7: P_1 $\lambda_1=3·08$	7: P_2 $\lambda_2=2·58$	8: $\bar{\eta}^2$	8: $\sigma_{\bar{\eta}^2}$	8: P_1 $\lambda_1=3·02$	8: P_2 $\lambda_2=2·54$
501	·010000	·006280	·011	·021	·012000	·006873	·010	·020	·014000	·007416	·010	·021
502	·009980	·006268	·011	·021	·011976	·006859	·010	·020	·013972	·007401	·010	·021
503	·009960	·006255 +	·011	·021	·011952	·006846	·010	·020	·013944	·007387	·010	·021
504	·009940	·006243	·011	·021	·011928	·006832	·010	·020	·013917	·007372	·010	·021
505	·009921	·006231	·011	·021	·011905 −	·006819	·010	·020	·013889	·007358	·010	·021
506	·009901	·006219	·011	·021	·011881	·006805 +	·010	·020	·013861	·007343	·010	·021
507	·009881	·006206	·011	·021	·011858	·006792	·010	·020	·013834	·007329	·010	·021
508	·009862	·006194	·011	·021	·011834	·006779	·010	·020	·013807	·007314	·010	·021
509	·009843	·006182	·011	·021	·011811	·006765 +	·010	·020	·013780	·007300	·010	·021
510	·009823	·006170	·011	·021	·011788	·006752	·010	·020	·013752	·007286	·010	·021
511	·009804	·006158	·011	·021	·011765 −	·006739	·010	·020	·013725 +	·007272	·010	·021
512	·009785 −	·006146	·011	·021	·011742	·006726	·010	·020	·013699	·007258	·010	·021
513	·009766	·006134	·011	·021	·011719	·006713	·010	·020	·013672	·007244	·010	·021
514	·009747	·006122	·011	·021	·011696	·006700	·010	·020	·013645 +	·007230	·010	·021
515	·009728	·006110	·011	·021	·011673	·006687	·010	·020	·013619	·007216	·010	·021
516	·009709	·006099	·011	·021	·011650 +	·006674	·010	·020	·013592	·007202	·010	·021
517	·009690	·006087	·011	·021	·011628	·006661	·010	·020	·013566	·007188	·010	·021
518	·009671	·006075 +	·011	·021	·011605 +	·006649	·010	·020	·013540	·007174	·010	·021
519	·009653	·006064	·011	·021	·011583	·006636	·010	·020	·013514	·007160	·010	·021
520	·009634	·006052	·011	·021	·011561	·006623	·010	·020$_5$	·013487	·007147	·010	·021
521	·009615 +	·006040	·011	·021	·011538	·006610	·010	·021	·013462	·007133	·010	·021
522	·009597	·006029	·011	·021	·011516	·006598	·010	·021	·013436	·007120	·010	·021
523	·009579	·006017	·011	·021	·011494	·006585 +	·010	·021	·013410	·007106	·010	·021
524	·009560	·006006	·011	·021	·011472	·006573	·010	·021	·013384	·007093	·010	·021
525	·009542	·005995 −	·011	·021	·011450 +	·006560	·010	·021	·013359	·007079	·010	·021
526	·009524	·005983	·011	·021	·011429	·006548	·010	·021	·013333	·007066	·010	·021
527	·009506	·005972	·011	·021	·011407	·006536	·010	·021	·013308	·007053	·010	·021
528	·009488	·005961	·011	·021	·011385 +	·006523	·010	·021	·013283	·007039	·010	·021
529	·009470	·005949	·011	·021	·011364	·006511	·010	·021	·013258	·007026	·010	·021
530	·009452	·005938	·011	·021	·011342	·006499	·010	·021	·013233	·007013	·010$_5$	·021
531	·009434	·005927	·011	·021	·011321	·006487	·010	·021	·013208	·007000	·011	·021
532	·009416	·005916	·011	·021	·011299	·006475 −	·010	·021	·013183	·006987	·011	·021
533	·009398	·005905 +	·011	·021	·011278	·006463	·010	·021	·013158	·006974	·011	·021
534	·009381	·005894	·011	·021	·011257	·006450 +	·010	·021	·013133	·006961	·011	·021
535	·009363	·005883	·011	·021	·011236	·006438	·010	·021	·013109	·006948	·011	·021
536	·009346	·005872	·011	·021	·011215 −	·006427	·010	·021	·013084	·006935 −	·011	·021
537	·009328	·005861	·011	·021	·011194	·006415 −	·010	·021	·013060	·006922	·011	·021
538	·009311	·005850 +	·011	·021	·011173	·006403	·010	·021	·013035 +	·006909	·011	·021
539	·009294	·005840	·011	·021	·011152	·006391	·010	·021	·013011	·006897	·011	·021
540	·009276	·005829	·011	·021	·011132	·006379	·010	·021	·012987	·006884	·011	·021
541	·009259	·005818	·011	·021	·011111	·006367	·010	·021	·012963	·006871	·011	·021
542	·009242	·005807	·011	·021	·011091	·006356	·010	·021	·012939	·006859	·011	·021
543	·009225 +	·005797	·011	·021	·011070	·006344	·010	·021	·012915 +	·006846	·011	·021
544	·009208	·005786	·011	·021	·011050 −	·006333	·010	·021	·012891	·006834	·011	·021
545	·009191	·005776	·011	·021	·011029	·006321	·010	·021	·012868	·006821	·011	·021
546	·009174	·005765 +	·011	·021	·011009	·006309	·010	·021	·012844	·006809	·011	·021
547	·009158	·005755 −	·011	·021	·010989	·006298	·010	·021	·012821	·006796	·011	·021
548	·009141	·005744	·011	·021	·010969	·006287	·010	·021	·012797	·006784	·011	·021
549	·009124	·005734	·011	·021	·010949	·006275 +	·010	·021	·012774	·006772	·011	·021
550	·009107	·005723	·011	·021	·010929	·006264	·010	·021	·012750 +	·006760	·011	·021

n = number of arrays

N = size of sample	9: $\bar{\eta}^2$	9: $\sigma_{\bar{\eta}^2}$	9: P_1 λ_1 = 2·96	9: P_2 λ_2 = 2·50	10: $\bar{\eta}^2$	10: $\sigma_{\bar{\eta}^2}$	10: P_1 λ_1 = 2·92	10: P_2 λ_2 = 2·47	11: $\bar{\eta}^2$	11: $\sigma_{\bar{\eta}^2}$	11: P_1 λ_1 = 2·88	11: P_2 λ_2 = 2·44
501	·016000	·007920	·011	·021	·018000	·008392	·011	·021	·020000	·008837	·011	·021
502	·015968	·007904	·011	·021	·017964	·008375 +	·011	·021	·019960	·008819	·011	·021
503	·015936	·007889	·011	·021	·017928	·008359	·011	·021	·019920	·008802	·011	·021
504	·015905 −	·007873	·011	·021	·017893	·008342	·011	·021	·019881	·008785 −	·011	·021
505	·015873	·007858	·011	·021	·017857	·008326	·011	·021	·019841	·008767	·011	·021
506	·015842	·007842	·011	·021	·017822	·008310	·011	·021	·019802	·008750 +	·011	·021
507	·015810	·007827	·011	·021	·017787	·008293	·011	·021	·019763	·008733	·011	·021
508	·015779	·007812	·011	·021	·017751	·008277	·011	·021	·019724	·008716	·011	·021
509	·015748	·007796	·011	·021	·017717	·008261	·011	·021	·019685 +	·008699	·011	·021
510	·015717	·007781	·011	·021	·017682	·008245 +	·011	·021	·019646 −	·008682	·011	·021
511	·015686	·007766	·011	·021	·017647	·008229	·011	·021	·019608	·008666	·011	·021
512	·015656	·007751	·011	·021	·017613	·008213	·011	·021	·019569	·008649	·011	·021
513	·015625	·007736	·011	·021	·017578	·008197	·011	·021	·019531	·008632	·011	·021
514	·015595 −	·007721	·011	·021	·017544	·008181	·011	·021	·019493	·008615 +	·011	·021
515	·015564	·007706	·011	·021	·017510	·008166	·011	·021	·019455 +	·008599	·011	·021
516	·015534	·007692	·011	·021	·017476	·008150 +	·011	·021	·019417	·008582	·011	·021
517	·015504	·007677	·011	·021	·017442	·008134	·011	·021	·019380	·008566	·011	·021
518	·015474	·007662	·011	·021	·017408	·008119	·011	·021	·019342	·008550 −	·011	·021
519	·015444	·007647	·011	·021	·017375 −	·008103	·011	·021	·019305 +	·008533	·011	·021
520	·015414	·007633	·011	·021	·017341	·008088	·011	·021	·019268	·008517	·011	·021
521	·015385 −	·007618	·011	·021	·017308	·008073	·011	·021	·019231	·008501	·011	·021
522	·015355 +	·007604	·011	·021	·017274	·008057	·011	·021	·019194	·008485 −	·011	·021
523	·015326	·007589	·011	·021	·017241	·008042	·011	·021	·019157	·008469	·011	·021
524	·015296	·007575 −	·011	·021	·017208	·008027	·011	·021	·019120	·008453	·011	·021
525	·015267	·007561	·011	·021	·017176	·008012	·011	·021	·019084	·008437	·011	·021
526	·015238	·007546	·011	·021	·017143	·007996	·011	·021	·019048	·008421	·011	·021
527	·015209	·007532	·011	·021	·017110	·007981	·011	·021	·019011	·008405 −	·011	·021
528	·015180	·007518	·011	·021	·017078	·007966	·011	·021	·018975 +	·008389	·011	·021
529	·015152	·007504	·011	·021	·017045 +	·007951	·011	·021	·018939	·008374	·011	·021
530	·015123	·007490	·011	·021	·017013	·007937	·011	·021	·018904	·008358	·011	·021
531	·015094	·007476	·011	·021	·016981	·007922	·011	·021	·018868	·008342	·011	·021
532	·015066	·007462	·011	·021	·016949	·007907	·011	·021	·018832	·008327	·011	·021
533	·015038	·007448	·011	·021	·016917	·007892	·011	·021	·018797	·008311	·011	·021
534	·015009	·007434	·011	·021	·016886	·007878	·011	·021	·018762	·008296	·011	·021
535	·014981	·007420	·011	·021	·016854	·007863	·011	·021	·018727	·008281	·011	·021
536	·014953	·007407	·011	·021	·016822	·007849	·011	·021	·018692	·008265 +	·011	·021
537	·014925 +	·007393	·011	·021	·016791	·007834	·011	·021	·018657	·008250 −	·011	·021
538	·014898	·007379	·011	·021	·016760	·007820	·011	·021	·018622	·008235 −	·011	·021
539	·014870	·007366	·011	·021	·016729	·007805 +	·011	·021	·018587	·008220	·011	·021
540	·014842	·007352	·011	·021	·016698	·007791	·011	·021	·018553	·008205 −	·011	·021
541	·014815 −	·007339	·011	·021	·016667	·007777	·011	·021	·018519	·008190	·011	·021
542	·014787	·007325 +	·011	·021	·016636	·007762	·011	·021	·018484	·008175 −	·011	·021
543	·014760	·007312	·011	·021	·016605 +	·007748	·011	·021	·018450 +	·008160	·011	·021
544	·014733	·007299	·011	·021	·016575 −	·007734	·011	·021	·018416	·008145 −	·011	·021
545	·014706	·007285 +	·011	·021	·016544	·007720	·011	·021	·018382	·008130	·011	·021
546	·014679	·007272	·011	·021	·016514	·007706	·011	·021	·018349	·008115 +	·011	·021
547	·014652	·007259	·011	·021	·016484	·007692	·011	·021	·018315 +	·008101	·011	·021
548	·014625 +	·007246	·011	·021	·016453	·007678	·011	·021	·018282	·008086	·011	·021
549	·014599	·007233	·011	·021	·016423	·007664	·011	·021	·018248	·008071	·011	·021
550	·014572	·007220	·011	·021	·016393	·007650 +	·011	·021	·018215 −	·008057	·011	·021

N = size of sample	12: $\bar{\eta}^2$	12: $\sigma_{\bar{\eta}^2}$	12: P_1 λ_1 = 2·84	12: P_2 λ_2 = 2·42	13: $\bar{\eta}^2$	13: $\sigma_{\bar{\eta}^2}$	13: P_1 λ_1 = 2·80	13: P_2 λ_2 = 2·40	14: $\bar{\eta}^2$	14: $\sigma_{\bar{\eta}^2}$	14: P_1 λ_1 = 2·79	14: P_2 λ_2 = 2·39
501	·022000	·009259	·011	·021	·024000	·009660	·011	·021	·026000	·010045 −	·011	·021
502	·021956	·009240	·011	·021	·023952	·009641	·011	·021	·025948	·010025 −	·011	·021
503	·021912	·009222	·011	·021	·023904	·009622	·011	·021	·025896	·010005 +	·011	·021
504	·021869	·009204	·011	·021	·023857	·009604	·011	·021	·025845 −	·009986	·011	·021
505	·021825 +	·009186	·011	·021	·023810	·009585 −	·011	·021	·025794	·009966	·011	·021
506	·021782	·009168	·011	·021	·023762	·009566	·011	·021	·025743	·009947	·011	·021
507	·021739	·009150 +	·011	·021	·023715 +	·009547	·011	·021	·025692	·009927	·011	·021
508	·021696	·009132	·011	·021	·023669	·009529	·011	·021	·025641	·009908	·011	·021
509	·021654	·009115 −	·011	·021	·023622	·009510	·011	·021	·025591	·009889	·011	·021
510	·021611	·009097	·011	·021	·023576	·009492	·011	·021	·025540	·009870	·011	·021
511	·021569	·009079	·011	·021	·023529	·009474	·011	·021	·025490	·009851	·011	·021
512	·021526	·009062	·011	·021	·023483	·009455 +	·011	·021	·025440	·009832	·011	·021
513	·021484	·009044	·011	·021	·023438	·009437	·011	·021	·025391	·009813	·011	·021
514	·021442	·009027	·011	·021	·023392	·009419	·011	·021	·025341	·009794	·011	·021
515	·021401	·009010	·011	·021	·023346	·009401	·011	·021	·025292	·009775 +	·011	·021
516	·021359	·008992	·011	·021	·023301	·009383	·011	·021	·025243	·009756	·011	·021
517	·021318	·008975 +	·011	·021	·023256	·009365 −	·011	·021	·025194	·009738	·011	·021
518	·021277	·008958	·011	·021	·023211	·009347	·011	·021	·025145 +	·009719	·011	·021
519	·021236	·008941	·011	·021	·023166	·009329	·011	·021	·025097	·009701	·011	·021
520	·021195 −	·008924	·011	·021	·023121	·009312	·011	·021	·025048	·009682	·011	·021
521	·021154	·008907	·011	·021	·023077	·009294	·011	·021	·025000	·009664	·011	·021
522	·021113	·008890	·011	·021	·023033	·009276	·011	·021	·024952	·009646	·011	·021
523	·021073	·008873	·011	·021	·022989	·009259	·011	·021	·024904	·009627	·011	·021
524	·021033	·008857	·011	·021	·022945 −	·009241	·011	·021	·024857	·009609	·011	·021
525	·020992	·008840	·011	·021	·022901	·009224	·011	·021	·024809	·009591	·011	·021
526	·020952	·008823	·011	·021	·022857	·009207	·011	·021	·024762	·009573	·011	·021
527	·020913	·008807	·011	·021	·022814	·009189	·011	·021	·024715 −	·009555 +	·011	·021
528	·020873	·008790	·011	·021	·022770	·009172	·011	·021	·024668	·009537	·011	·021
529	·020833	·008774	·011	·021	·022727	·009155 +	·011	·021	·024621	·009520	·011	·021
530	·020794	·008757	·011	·021	·022684	·009138	·011	·021	·024575 −	·009502	·011	·021
531	·020755 −	·008741	·011	·021	·022642	·009121	·011	·021	·024528	·009484	·011	·021
532	·020716	·008725 −	·011	·021	·022599	·009104	·011	·021	·024482	·009467	·011	·021
533	·020677	·008709	·011	·021	·022556	·009087	·011	·021	·024436	·009449	·011	·021
534	·020638	·008692	·011	·021	·022514	·009070	·011	·021	·024390	·009432	·011	·021
535	·020599	·008676	·011	·021	·022472	·009054	·011	·021	·024345 −	·009414	·011	·021
536	·020561	·008660	·011	·021	·022430	·009037	·011	·021	·024299	·009397	·011	·021
537	·020522	·008644	·011	·021	·022388	·009020	·011	·022	·024254	·009380	·011	·021
538	·020484	·008629	·011	·021	·022346	·009004	·011	·022	·024209	·009362	·011	·021
539	·020446	·008613	·011	·021	·022305 −	·008987	·011	·022	·024164	·009345 +	·011	·021
540	·020408	·008597	·011	·021	·022263	·008971	·011	·022	·024119	·009328	·011	·021
541	·020370	·008581	·011	·021	·022222	·008954	·011	·022	·024074	·009311	·011	·021
542	·020333	·008565 +	·011	·021	·022181	·008938	·011	·022	·024030	·009294	·011	·021
543	·020295 +	·008550 −	·011	·021	·022140	·008922	·011	·022	·023985 +	·009277	·011	·021
544	·020258	·008534	·011	·021	·022099	·008905 +	·011	·022	·023941	·009260	·011	·021
545	·020221	·008519	·011	·021	·022059	·008889	·011	·022	·023897	·009244	·011	·021
546	·020183	·008503	·011	·021	·022018	·008873	·011	·022	·023853	·009227	·011	·021
547	·020147	·008488	·011	·021	·021978	·008857	·011	·022	·023810	·009210	·011	·021
548	·020110	·008473	·011	·021	·021938	·008841	·011	·022	·023766	·009194	·011	·021
549	·020073	·008457	·011	·021	·021898	·008825 +	·011	·022	·023723	·009177	·011	·021
550	·020036	·008442	·011	·021	·021858	·008809	·011	·022	·023679	·009161	·011	·021

N = size of sample	15 $\bar{\eta}^2$	15 $\sigma_{\bar{\eta}^2}$	15 P_1 $\lambda_1=2·78$	15 P_2 $\lambda_2=2·38$	16 $\bar{\eta}^2$	16 $\sigma_{\bar{\eta}^2}$	16 P_1 $\lambda_1=2·77$	16 P_2 $\lambda_2=2·37$	17 $\bar{\eta}^2$	17 $\sigma_{\bar{\eta}^2}$	17 P_1 $\lambda_1=2·76$	17 P_2 $\lambda_2=2·36$
501	·028000	·010413	·011	·021	·030000	·010767	·011	·021	·032000	·011109	·011	·021
502	·027944	·010393	·011	·021	·029940	·010746	·011	·021	·031936	·011087	·011	·021
503	·027888	·010372	·011	·021	·029880	·010725 +	·011	·021	·031873	·011066	·011	·021
504	·027833	·010352	·011	·021	·029821	·010704	·011	·021	·031809	·011044	·011	·021
505	·027778	·010332	·011	·021	·029762	·010683	·011	·021	·031746	·011022	·011	·021
506	·027723	·010312	·011	·021	·029703	·010663	·011	·021	·031683	·011001	·011	·021
507	·027668	·010292	·011	·021	·029644	·010642	·011	·021	·031621	·010980	·011	·021
508	·027613	·010272	·011	·021	·029586	·010621	·011	·021	·031558	·010958	·011	·021
509	·027559	·010252	·011	·021	·029528	·010601	·011	·021	·031496	·010937	·011	·021
510	·027505 −	·010232	·011	·021	·029470	·010580	·011	·021	·031434	·010916	·011	·021
511	·027451	·010212	·011	·021	·029412	·010560	·011	·021	·031373	·010895 +	·011	·021
512	·027397	·010192	·011	·021	·029354	·010540	·011	·021	·031311	·010874	·011	·021
513	·027344	·010173	·011	·021	·029297	·010519	·011	·021	·031250	·010853	·011	·021
514	·027290	·010153	·011	·021	·029240	·010499	·011	·021	·031189	·010833	·011	·021
515	·027237	·010134	·011	·021	·029183	·010479	·011	·021	·031128	·010812	·011	·021
516	·027184	·010115 −	·011	·021	·029126	·010459	·011	·021	·031068	·010791	·011	·021
517	·027132	·010095 +	·011	·021	·029070	·010439	·011	·021	·031008	·010771	·011	·021
518	·027079	·010076	·011	·021	·029014	·010419	·011	·021	·030948	·010750 +	·011	·021
519	·027027	·010057	·011	·021	·028958	·010400	·011	·021	·030888	·010730	·011	·021
520	·026975 −	·010038	·011	·021	·028902	·010380	·011	·021	·030829	·010710	·011	·021
521	·026923	·010019	·011	·021	·028846	·010360	·011	·021	·030769	·010689	·011	·021
522	·026871	·010000	·011	·021	·028791	·010341	·011	·021	·030710	·010669	·011	·021
523	·026820	·009981	·011	·021	·028736	·010321	·011	·021	·030651	·010649	·011	·021
524	·026769	·009962	·011	·021	·028681	·010302	·011	·021	·030593	·010629	·011	·021
525	·026718	·009944	·011	·021	·028626	·010282	·011	·021	·030534	·010609	·011	·021
526	·026667	·009925 −	·011	·021	·028571	·010263	·011	·021	·030476	·010589	·011	·021
527	·026616	·009906	·011	·021	·028517	·010244	·011	·021	·030418	·010570	·011	·021
528	·026565 +	·009888	·011	·021	·028463	·010225 −	·011	·021	·030361	·010550 −	·011	·021
529	·026515 +	·009869	·011	·021	·028409	·010206	·011	·021	·030303	·010530	·011	·021
530	·026465 +	·009851	·011	·021	·028355 +	·010187	·011	·021	·030246	·010511	·011	·021
531	·026415 +	·009833	·011	·021	·028302	·010168	·011	·021	·030189	·010491	·011	·021
532	·026365 +	·009814	·011	·021	·028249	·010149	·011	·021	·030132	·010472	·011	·021
533	·026316	·009796	·011	·021	·028195 +	·010130	·011	·021	·030075 +	·010452	·011	·021
534	·026266	·009778	·011	·021	·028143	·010112	·011	·021	·030019	·010433	·011	·021
535	·026217	·009760	·011	·021	·028090	·010093	·011	·021	·029963	·010414	·011	·021
536	·026168	·009742	·011	·021	·028037	·010074	·011	·021	·029907	·010395 −	·011	·021
537	·026119	·009724	·011	·021	·027985 +	·010056	·011	·021	·029851	·010376	·011	·021
538	·026071	·009706	·011	·021	·027933	·010038	·011	·021	·029795 +	·010357	·011	·021
539	·026022	·009689	·011	·021	·027881	·010019	·011	·021	·029740	·010338	·011	·021
540	·025974	·009671	·011	·021	·027829	·010001	·011	·021	·029685 −	·010319	·011	·021
541	·025926	·009653	·011	·021	·027778	·009983	·011	·021	·029630	·010300	·011	·021
542	·025878	·009636	·011	·021	·027726	·009965 −	·011	·021	·029575 −	·010282	·011	·021
543	·025830	·009618	·011	·021	·027675 +	·009946	·011	·021	·029520	·010263	·011	·021
544	·025783	·009601	·011	·021	·027624	·009928	·011	·021	·029466	·010244	·011	·021
545	·025735 +	·009583	·011	·021	·027574	·009910	·011	·021	·029412	·010226	·011	·021
546	·025688	·009566	·011	·021	·027523	·009893	·011	·021	·029358	·010207	·011	·021
547	·025641	·009549	·011	·021	·027473	·009875 −	·011	·021	·029304	·010189	·011	·021
548	·025594	·009532	·011	·021	·027422	·009857	·011	·021	·029250 +	·010171	·011	·021
549	·025547	·009515 −	·011	·021	·027372	·009839	·011	·021	·029197	·010152	·011	·021
550	·025501	·009497	·011	·021	·027322	·009822	·011	·021	·029144	·010134	·011	·021

N = size of sample	18 $\bar{\eta}^2$	18 $\sigma_{\bar{\eta}^2}$	18 P_1 $\lambda_1=2·75$	18 P_2 $\lambda_2=2·35$	19 $\bar{\eta}^2$	19 $\sigma_{\bar{\eta}^2}$	19 P_1 $\lambda_1=2·74$	19 P_2 $\lambda_2=2·34$	20 $\bar{\eta}^2$	20 $\sigma_{\bar{\eta}^2}$	20 P_1 $\lambda_1=2·73$	20 P_2 $\lambda_2=2·33$
501	·034000	·011439	·011	·021	·036000	·011759	·011	·021	·038000	·012068	·010	·021
502	·033932	·011417	·011	·021	·035928	·011736	·011	·021	·037924	·012045 −	·010	·021
503	·033865 −	·011394	·011	·021	·035857	·011713	·011	·021	·037849	·012021	·011	·021
504	·033797	·011372	·011	·021	·035785 +	·011690	·011	·021	·037773	·011998	·011	·021
505	·033730	·011350 +	·011	·021	·035714	·011667	·011	·021	·037698	·011974	·011	·021
506	·033663	·011328	·011	·021	·035644	·011645 −	·011	·021	·037624	·011951	·011	·021
507	·033597	·011306	·011	·021	·035573	·011622	·011	·021	·037549	·011928	·011	·021
508	·033531	·011284	·011	·021	·035503	·011600	·011	·021	·037475 +	·011905 +	·011	·021
509	·033465 −	·011262	·011	·021	·035433	·011577	·011	·021	·037402	·011882	·011	·021
510	·033399	·011241	·011	·021	·035363	·011555 −	·011	·021	·037328	·011859	·011	·021
511	·033333	·011219	·011	·021	·035294	·011533	·011	·021	·037255 −	·011837	·011	·021
512	·033268	·011198	·011	·021	·035225 +	·011511	·011	·021	·037182	·011814	·011	·021
513	·033203	·011176	·011	·021	·035156	·011489	·011	·021	·037109	·011791	·011	·021
514	·033138	·011155 −	·011	·021	·035088	·011467	·011	·021	·037037	·011769	·011	·021
515	·033074	·011133	·011	·021	·035019	·011445 −	·011	·021	·036965 −	·011746	·011	·021
516	·033010	·011112	·011	·021	·034951	·011423	·011	·021	·036893	·011724	·011	·021
517	·032946	·011091	·011	·021	·034884	·011401	·011	·021	·036822	·011702	·011	·021
518	·032882	·011070	·011	·021	·034816	·011380	·011	·021	·036750 +	·011680	·011	·021
519	·032819	·011049	·011	·021	·034749	·011358	·011	·021	·036680	·011658	·011	·021
520	·032755 +	·011028	·011	·021	·034682	·011337	·011	·021	·036609	·011636	·011	·021
521	·032692	·011007	·011	·021	·034615 +	·011315 +	·011	·021	·036538	·011614	·011	·021
522	·032630	·010987	·011	·021	·034549	·011294	·011	·021	·036468	·011592	·011	·021
523	·032567	·010966	·011	·021	·034483	·011273	·011	·021	·036398	·011570	·011	·021
524	·032505 −	·010945 +	·011	·021	·034417	·011252	·011	·021	·036329	·011549	·011	·021
525	·032443	·010925 −	·011	·021	·034351	·011231	·011	·021	·036260	·011527	·011	·021
526	·032381	·010905 −	·011	·021	·034286	·011210	·011	·021	·036190	·011505 +	·011	·021
527	·032319	·010884	·011	·021	·034221	·011189	·011	·021	·036122	·011484	·011	·021
528	·032258	·010864	·011	·021	·034156	·011168	·011	·021	·036053	·011463	·011	·021
529	·032197	·010844	·011	·021	·034091	·011147	·011	·021	·035985 −	·011441	·011	·021
530	·032136	·010824	·011	·021	·034026	·011127	·011	·021	·035917	·011420	·011	·021
531	·032075 +	·010804	·011	·021	·033962	·011106	·011	·021	·035849	·011399	·011	·021
532	·032015 +	·010784	·011	·021	·033898	·011085 +	·011	·021	·035782	·011378	·011	·021
533	·031955 −	·010764	·011	·021	·033835 −	·011065 −	·011	·021	·035714	·011357	·011	·021
534	·031895 −	·010744	·011	·021	·033771	·011045 −	·011	·021	·035647	·011336	·011	·021
535	·031835 +	·010724	·011	·021	·033708	·011024	·011	·021	·035581	·011315 +	·011	·021
536	·031776	·010704	·011	·021	·033645 −	·011004	·011	·021	·035514	·011295 −	·011	·021
537	·031716	·010685 −	·011	·021	·033582	·010984	·011	·021	·035448	·011274	·011	·021
538	·031657	·010665 +	·011	·021	·033520	·010964	·011	·021	·035382	·011254	·011	·021
539	·031599	·010646	·011	·021	·033457	·010944	·011	·021	·035316	·011233	·011	·021
540	·031540	·010626	·011	·021	·033395 +	·010924	·011	·021	·035250 +	·011213	·011	·021
541	·031481	·010607	·011	·021	·033333	·010904	·011	·021	·035185 +	·011192	·011	·021
542	·031423	·010588	·011	·021	·033272	·010884	·011	·021	·035120	·011172	·011	·021
543	·031365 +	·010569	·011	·021	·033210	·010865 −	·011	·021	·035055 +	·011152	·011	·021
544	·031308	·010550 −	·011	·021	·033149	·010845 +	·011	·021	·034991	·011132	·011	·021
545	·031250 −	·010531	·011	·021	·033088	·010826	·011	·021	·034926	·011112	·011	·021
546	·031193	·010512	·011	·021	·033028	·010806	·011	·021	·034862	·011092	·011	·021
547	·031136	·010493	·011	·021	·032967	·010787	·011	·021	·034799	·011072	·011	·021
548	·031079	·010474	·011	·021	·032907	·010767	·011	·021	·034735 −	·011052	·011	·021
549	·031022	·010455 +	·011	·021	·032847	·010748	·011	·021	·034672	·011032	·011	·021
550	·030965 +	·010436	·011	·021	·032787	·010729	·011	·021	·034608	·011012	·011	·021

n = number of arrays

N = size of sample	3: $\bar{\eta}^2$	3: $\sigma_{\bar{\eta}^2}$	3: P_1 λ_1 = 3·50	3: P_2 λ_2 = 2·94	4: $\bar{\eta}^2$	4: $\sigma_{\bar{\eta}^2}$	4: P_1 λ_1 = 3·20	4: P_2 λ_2 = 2·80	5: $\bar{\eta}^2$	5: $\sigma_{\bar{\eta}^2}$	5: P_1 λ_1 = 3·14	5: P_2 λ_2 = 2·68
551	·003636	·003623	·011	·019	·005455 −	·004433	·013	·020	·007273	·005115 −	·012	·021
552	·003630	·003617	·011	·019	·005445 −	·004425 +	·013	·020	·007260	·005105 +	·012	·021
553	·003623	·003610	·011	·019	·005435 −	·004417	·013	·020	·007246	·005096	·012	·021
554	·003617	·003604	·011	·019	·005425 −	·004409	·013	·020	·007233	·005087	·012	·021
555	·003610	·003597	·011	·019	·005415 +	·004402	·013	·020	·007220	·005078	·012	·021
556	·003604	·003591	·011	·019	·005405 +	·004394	·013	·020	·007207	·005069	·012	·021
557	·003597	·003584	·011	·019	·005396	·004386	·013	·020	·007194	·005060	·012	·021
558	·003591	·003578	·011	·019	·005386	·004378	·013	·020	·007181	·005051	·012	·021
559	·003584	·003571	·011	·019	·005376	·004370	·013	·020	·007168	·005042	·012	·021
560	·003578	·003565 +	·011	·019	·005367	·004362	·013	·020	·007156	·005033	·012	·021
561	·003571	·003559	·011	·019	·005357	·004355 −	·013	·020	·007143	·005024	·012	·021
562	·003565 +	·003552	·011	·019	·005348	·004347	·013	·020	·007130	·005015 −	·012	·021
563	·003559	·003546	·011	·019	·005338	·004339	·013	·020	·007117	·005006	·012	·021
564	·003552	·003540	·011	·019	·005329	·004331	·013	·020	·007105 −	·004997	·012	·021
565	·003546	·003534	·011	·019	·005319	·004324	·013	·020	·007092	·004988	·012	·021
566	·003540	·003527	·011	·019	·005310	·004316	·013	·020	·007080	·004980	·012	·021
567	·003534	·003521	·011	·019	·005300	·004309	·013	·020	·007067	·004971	·012	·021
568	·003527	·003515 −	·011	·019	·005291	·004301	·013	·020	·007055 −	·004962	·012	·021
569	·003521	·003509	·011	·019	·005282	·004294	·013	·020	·007042	·004953	·012	·021
570	·003515 −	·003503	·011	·019	·005272	·004286	·013	·020	·007030	·004945 −	·012	·021
571	·003509	·003496	·011	·019	·005263	·004279	·013	·020	·007018	·004936	·012	·021
572	·003503	·003490	·011	·019	·005254	·004271	·013	·020	·007005 +	·004927	·012	·021
573	·003497	·003484	·011	·019	·005245 −	·004264	·013	·020	·006993	·004919	·012	·021
574	·003490	·003478	·011	·019	·005236	·004256	·013	·020	·006981	·004910	·012	·021
575	·003484	·003472	·011	·019	·005226	·004249	·013	·020	·006969	·004902	·012	·021
576	·003478	·003466	·011	·019	·005217	·004241	·013	·020	·006957	·004893	·012	·021
577	·003472	·003460	·011	·019	·005208	·004234	·013	·020	·006944	·004885 −	·012	·021
578	·003466	·003454	·011	·019	·005199	·004227	·013	·020	·006932	·004876	·012	·021
579	·003460	·003448	·011	·019	·005190	·004220	·013	·020	·006920	·004868	·012	·021
580	·003454	·003442	·011	·019	·005181	·004212	·013	·020	·006908	·004860	·012	·021
581	·003448	·003436	·011	·019	·005172	·004205 +	·013	·020	·006897	·004851	·012	·021
582	·003442	·003431	·011	·019	·005164	·004198	·013	·020	·006885 −	·004843	·012	·021
583	·003436	·003425 −	·011	·019	·005155 −	·004191	·013	·020	·006873	·004835 −	·012	·021
584	·003431	·003419	·011	·019	·005146	·004184	·013	·020	·006861	·004827	·012	·021
585	·003425 −	·003413	·011	·019	·005137	·004176	·013	·020	·006849	·004818	·012	·021
586	·003419	·003407	·011	·019	·005128	·004169	·013	·020	·006838	·004810	·012	·021
587	·003413	·003401	·011	·019	·005119	·004162	·013	·020	·006826	·004802	·012	·021
588	·003407	·003396	·011	·019	·005111	·004155 +	·013	·020	·006814	·004794	·012	·021
589	·003401	·003390	·011	·019	·005102	·004148	·013	·020	·006803	·004786	·012	·021
590	·003396	·003384	·011	·019	·005093	·004141	·013	·020	·006791	·004778	·012	·021
591	·003390	·003378	·011	·019	·005085 −	·004134	·013	·020	·006780	·004770	·012	·021
592	·003384	·003373	·011	·019	·005076	·004127	·013	·020	·006768	·004762	·012	·021
593	·003378	·003367	·011	·019	·005068	·004120	·013	·020	·006757	·004754	·012	·021
594	·003373	·003361	·011	·019	·005059	·004113	·013	·020	·006745 +	·004746	·012	·021
595	·003367	·003356	·011	·019	·005051	·004106	·013	·020	·006734	·004738	·012	·021
596	·003361	·003350 +	·011	·019	·005042	·004100	·013	·020	·006723	·004730	·012	·021
597	·003356	·003344	·011	·019	·005034	·004093	·013	·020	·006711	·004722	·012	·021
598	·003350 +	·003339	·011	·019	·005025 +	·004086	·013	·020	·006700	·004714	·012	·021
599	·003344	·003333	·011	·019	·005017	·004079	·013	·020	·006689	·004706	·012	·021
600	·003339	·003328	·011	·019	·005008	·004072	·013	·020	·006678	·004698	·012	·021

N = size of sample	6: $\bar{\eta}^2$	6: $\sigma_{\bar{\eta}^2}$	6: P_1 λ_1 = 3·11	6: P_2 λ_2 = 2·63	7: $\bar{\eta}^2$	7: $\sigma_{\bar{\eta}^2}$	7: P_1 λ_1 = 3·08	7: P_2 λ_2 = 2·58	8: $\bar{\eta}^2$	8: $\sigma_{\bar{\eta}^2}$	8: P_1 λ_1 = 3·02	8: P_2 λ_2 = 2·54
551	·009091	·005713	·011	·021	·010909	·006253	·010	·021	·012727	·006747	·011	·021
552	·009074	·005703	·011	·021	·010889	·006241	·010	·021	·012704	·006735 +	·011	·021
553	·009058	·005692	·011	·021	·010870	·006230	·010	·021	·012681	·006723	·011	·021
554	·009042	·005682	·011	·021	·010850 −	·006219	·010	·021	·012658	·006711	·011	·021
555	·009025 +	·005672	·011	·021	·010830	·006208	·010	·021	·012635 +	·006699	·011	·021
556	·009009	·005662	·011	·021	·010811	·006197	·010	·021	·012613	·006687	·011	·021
557	·008993	·005652	·011	·021	·010791	·006186	·010	·021	·012590	·006675 +	·011	·021
558	·008977	·005642	·011	·021	·010772	·006175 −	·010	·021	·012567	·006663	·011	·021
559	·008961	·005632	·011	·021	·010753	·006164	·010	·021	·012545 −	·006651	·011	·021
560	·008945 −	·005622	·011	·021	·010733	·006153	·010	·021	·012522	·006640	·011	·021
561	·008929	·005612	·011	·021	·010714	·006142	·010	·021	·012500	·006628	·011	·021
562	·008913	·005602	·011	·021	·010695 +	·006131	·010	·021	·012478	·006616	·011	·021
563	·008897	·005592	·011	·021	·010676	·006120	·010	·021	·012456	·006604	·011	·021
564	·008881	·005582	·011	·021	·010657	·006109	·010	·021	·012433	·006593	·011	·021
565	·008865 +	·005572	·011	·021	·010638	·006098	·010	·021	·012411	·006581	·011	·021
566	·008850 −	·005562	·011	·021	·010619	·006088	·010	·021	·012389	·006570	·011	·021
567	·008834	·005553	·011	·021	·010601	·006077	·010	·021	·012367	·006558	·011	·021
568	·008818	·005543	·011	·021	·010582	·006066	·010	·021	·012346	·006547	·011	·021
569	·008803	·005533	·011	·021	·010563	·006056	·010	·021	·012324	·006535 +	·011	·021
570	·008787	·005523	·011	·021	·010545 −	·006045 +	·010	·021	·012302	·006524	·011	·021
571	·008772	·005514	·011	·021	·010526	·006035 −	·010	·021	·012281	·006512	·011	·021
572	·008757	·005504	·011	·021	·010508	·006024	·010	·021	·012259	·006501	·011	·021
573	·008741	·005495 −	·011	·021	·010490	·006014	·010	·021	·012238	·006490	·011	·021
574	·008726	·005485 +	·011	·021	·010471	·006003	·010	·021	·012216	·006479	·011	·021
575	·008711	·005476	·011	·021	·010453	·005993	·010	·021	·012195 +	·006467	·011	·021
576	·008696	·005466	·011	·021	·010435 −	·005983	·010	·021	·012174	·006456	·011	·021
577	·008681	·005457	·011	·021	·010417	·005972	·010	·021	·012153	·006445 +	·011	·021
578	·008666	·005447	·011	·021	·010399	·005962	·010	·021	·012132	·006434	·011	·021
579	·008651	·005438	·011	·021	·010381	·005952	·010	·021	·012111	·006423	·011	·021
580	·008636	·005429	·011	·021	·010363	·005942	·010	·021	·012090	·006412	·011	·021
581	·008621	·005419	·011	·021	·010345 −	·005931	·010	·021	·012069	·006401	·011	·021
582	·008606	·005410	·011	·021	·010327	·005921	·010	·021	·012048	·006390	·011	·021
583	·008591	·005401	·011	·021	·010309	·005911	·010	·021	·012027	·006379	·011	·021
584	·008576	·005392	·011	·021	·010292	·005901	·010	·021	·012007	·006368	·011	·021
585	·008562	·005382	·011	·021	·010274	·005891	·010	·021	·011986	·006358	·011	·021
586	·008547	·005372	·011	·021	·010256	·005881	·010	·021	·011966	·006347	·011	·021
587	·008532	·005364	·011	·021	·010239	·005871	·010	·021	·011945 +	·006336	·011	·021
588	·008518	·005355 +	·011	·021	·010221	·005861	·010	·021	·011925 +	·006325 +	·011	·021
589	·008503	·005346	·011	·021	·010204	·005851	·010	·021	·011905 −	·006315 −	·011	·021
590	·008489	·005337	·011	·021	·010187	·005841	·010	·021	·011885 −	·006304	·011	·021
591	·008475 −	·005328	·011	·021	·010169	·005832	·010	·021	·011864	·006293	·011	·021
592	·008460	·005319	·011	·021	·010152	·005822	·010	·021	·011844	·006283	·011	·021
593	·008446	·005310	·011	·021	·010135 +	·005812	·010	·021	·011824	·006272	·011	·021
594	·008432	·005301	·011	·021	·010118	·005802	·010	·021	·011804	·006262	·011	·021
595	·008418	·005292	·011	·021	·010101	·005793	·010	·021	·011785 −	·006251	·011	·021
596	·008403	·005284	·011	·021	·010084	·005783	·010	·021	·011765 −	·006241	·011	·021
597	·008389	·005275 −	·011	·021	·010067	·005773	·010	·021	·011745 −	·006231	·011	·021
598	·008375 +	·005266	·011	·021	·010050 +	·005764	·010	·021	·011725 +	·006220	·011	·021
599	·008361	·005257	·011	·021	·010033	·005754	·010	·021	·011706	·006210	·011	·021
600	·008347	·005248	·011	·021	·010017	·005745 −	·010	·021	·011686	·006200	·011	·021

N = size of sample	9				10			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}96$	P_2 $\lambda_2 = 2{\cdot}50$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}92$	P_2 $\lambda_2 = 2{\cdot}47$
551	·014545 +	·007207	·011	·021	·016364	·007637	·011	·021
552	·014519	·007194	·011	·021	·016334	·007623	·011	·021
553	·014493	·007181	·011	·021	·016304	·007609	·011	·021
554	·014467	·007168	·011	·021	·016275 −	·007596	·011	·021
555	·014440	·007155 −	·011	·021	·016245 +	·007582	·011	·021
556	·014414	·007142	·011	·021	·016216	·007569	·011	·021
557	·014388	·007129	·011	·021	·016187	·007555 +	·011	·021
558	·014363	·007117	·011	·021	·016158	·007542	·011	·021
559	·014337	·007104	·011	·021	·016129	·007528	·011	·021
560	·014311	·007092	·011	·021	·016100	·007515 −	·011	·021
561	·014286	·007079	·011	·021	·016071	·007502	·011	·021
562	·014260	·007067	·011	·021	·016043	·007488	·011	·021
563	·014235 −	·007054	·011	·021	·016014	·007475 +	·011	·021
564	·014210	·007042	·011	·021	·015986	·007462	·011	·021
565	·014184	·007029	·011	·021	·015957	·007449	·011	·021
566	·014159	·007017	·011	·021	·015929	·007436	·011	·021
567	·014134	·007005 −	·011	·021	·015901	·007423	·011	·021
568	·014109	·006992	·011	·021	·015873	·007410	·011	·021
569	·014085 −	·006980	·011	·021	·015845 +	·007397	·011	·021
570	·014060	·006968	·011	·021	·015817	·007384	·011	·021
571	·014035 +	·006956	·011	·021	·015789	·007371	·011	·021
572	·014011	·006944	·011	·021	·015762	·007359	·011	·021
573	·013986	·006932	·011	·021	·015734	·007346	·011	·021
574	·013962	·006920	·011	·021	·015707	·007333	·011	·021
575	·013937	·006908	·011	·021	·015679	·007320	·011	·021
576	·013913	·006896	·011	·021	·015652	·007308	·011	·021
577	·013889	·006884	·011	·021	·015625 −	·007295 +	·011	·021
578	·013865 −	·006872	·011	·021	·015598	·007283	·011	·021
579	·013841	·006860	·011	·021	·015571	·007270	·011	·021
580	·013817	·006849	·011	·021	·015544	·007258	·011	·021
581	·013793	·006837	·011	·021	·015517	·007245 +	·011	·021
582	·013769	·006825 +	·011	·021	·015491	·007233	·011	·021
583	·013746	·006814	·011	·021	·015464	·007221	·011	·021
584	·013722	·006802	·011	·021	·015437	·007209	·011	·021
585	·013699	·006791	·011	·021	·015411	·007196	·011	·021
586	·013675 +	·006779	·011	·021	·015385 −	·007184	·011	·021
587	·013652	·006768	·011	·021	·015358	·007172	·011	·021
588	·013629	·006756	·011	·021	·015332	·007160	·011	·021
589	·013605 +	·006745 −	·011	·021	·015306	·007148	·011	·021
590	·013582	·006733	·011	·021	·015280	·007136	·011	·021
591	·013559	·006722	·011	·021	·015254	·007124	·011	·021
592	·013536	·006711	·011	·021	·015228	·007112	·011	·021
593	·013514	·006700	·011	·021	·015203	·007100	·011	·021
594	·013491	·006688	·011	·021	·015177	·007088	·011	·021
595	·013468	·006677	·011	·021	·015152	·007076	·011	·021
596	·013445 +	·006666	·011	·021	·015126	·007064	·011	·021
597	·013423	·006655 +	·011	·021	·015101	·007053	·011	·021
598	·013400	·006644	·011	·021	·015075 +	·007041	·011	·021
599	·013378	·006633	·011	·021	·015050 +	·007029	·011	·021
600	·013356	·006622	·011	·021	·015025 +	·007018	·011	·021

N = size of sample	11				12			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}88$	P_2 $\lambda_2 = 2{\cdot}44$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}84$	P_2 $\lambda_2 = 2{\cdot}42$
551	·018182	·008042	·011	·021	·020000	·008427	·011	·021
552	·018149	·008028	·011	·021	·019964	·008412	·011	·021
553	·018116	·008013	·011	·021	·019928	·008397	·011	·021
554	·018083	·007999	·011	·021	·019892	·008382	·011	·021
555	·018051	·007985 −	·011	·021	·019856	·008367	·011	·021
556	·018018	·007971	·011	·021	·019820	·008352	·011	·021
557	·017986	·007956	·011	·021	·019784	·008337	·011	·021
558	·017953	·007942	·011	·021	·019749	·008322	·011	·021
559	·017921	·007928	·011	·021	·019713	·008308	·011	·021
560	·017889	·007914	·011	·021	·019678	·008293	·011	·021
561	·017857	·007900	·011	·021	·019643	·008278	·011	·021
562	·017825 +	·007886	·011	·021	·019608	·008264	·011	·021
563	·017794	·007872	·011	·021	·019573	·008249	·011	·021
564	·017762	·007859	·011	·021	·019538	·008235 −	·011	·021
565	·017730	·007845 −	·011	·021	·019504	·008220	·011	·021
566	·017699	·007831	·011	·021	·019469	·008206	·011	·021
567	·017668	·007817	·011	·021	·019435 −	·008192	·011	·021
568	·017637	·007804	·011	·021	·019400	·008177	·011	·021
569	·017606	·007790	·011	·021	·019366	·008163	·011	·021
570	·017575 −	·007777	·011	·021	·019332	·008149	·011	·021
571	·017544	·007763	·011	·021	·019298	·008135 −	·011	·021
572	·017513	·007750 −	·011	·021	·019264	·008121	·011	·021
573	·017483	·007736	·011	·021	·019231	·008107	·011	·021
574	·017452	·007723	·011	·021	·019197	·008093	·011	·021
575	·017422	·007710	·011	·021	·019164	·008079	·011	·021
576	·017391	·007696	·011	·021	·019130	·008065 −	·011	·021
577	·017361	·007683	·011	·021	·019097	·008051	·011	·021
578	·017331	·007670	·011	·021	·019064	·008037	·011	·021
579	·017301	·007657	·011	·021	·019031	·008023	·011	·021
580	·017271	·007644	·011	·021	·018998	·008010	·011	·021
581	·017241	·007631	·011	·021	·018966	·007996	·011	·021
582	·017212	·007618	·011	·021	·018933	·007982	·011	·021
583	·017182	·007605 −	·011	·021	·018900	·007969	·011	·021
584	·017153	·007592	·011	·021	·018868	·007955 +	·011	·021
585	·017123	·007579	·011	·021	·018836	·007942	·011	·021
586	·017094	·007566	·011	·021	·018803	·007929	·011	·021
587	·017065 −	·007553	·011	·021	·018771	·007915 +	·011	·021
588	·017036	·007541	·011	·021	·018739	·007902	·011	·021
589	·017007	·007528	·011	·021	·018707	·007889	·011	·021
590	·016978	·007515 +	·011	·021	·018676	·007875 +	·011	·021
591	·016949	·007503	·011	·021	·018644	·007862	·011	·021
592	·016920	·007490	·011	·021	·018613	·007849	·011	·021
593	·016892	·007478	·011	·021	·018581	·007836	·011	·021
594	·016863	·007465 +	·011	·021	·018550	·007823	·011	·021
595	·016835 +	·007453	·011	·021	·018519	·007810	·011	·021
596	·016807	·007440	·011	·021	·018487	·007797	·011	·021
597	·016779	·007428	·011	·021	·018456	·007784	·011	·021
598	·016750 +	·007416	·011	·021	·018425 +	·007771	·011	·021
599	·016722	·007403	·011	·021	·018395 −	·007758	·011	·021
600	·016694	·007391	·011	·021	·018364	·007745 +	·011	·021

N = size of sample	13				14			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}80$	P_2 $\lambda_2 = 2{\cdot}40$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2{\cdot}79$	P_2 $\lambda_2 = 2{\cdot}39$
551	·021818	·008794	·011	·022	·023636	·009144	·011	·021
552	·021779	·008778	·011	·022	·023593	·009128	·011	·021
553	·021739	·008762	·011	·022	·023551	·009111	·011	·021
554	·021700	·008746	·011	·022	·023508	·009095 +	·011	·021
555	·021661	·008731	·011	·022	·023466	·009079	·011	·021
556	·021622	·008715 +	·011	·022	·023423	·009063	·011	·021
557	·021583	·008700	·011	·022	·023381	·009047	·011	·021
558	·021544	·008684	·011	·022	·023339	·009031	·011	·021
559	·021505 +	·008669	·011	·022	·023297	·009015 −	·011	·021
560	·021467	·008654	·011	·022	·023256	·008999	·011	·021
561	·021429	·008639	·011	·022	·023214	·008983	·011	·021
562	·021390	·008623	·011	·022	·023173	·008967	·011	·021
563	·021352	·008608	·011	·022	·023132	·008952	·011	·021
564	·021314	·008593	·011	·022	·023091	·008936	·011	·021
565	·021277	·008578	·011	·022	·023050 −	·008920	·011	·021
566	·021239	·008563	·011	·022	·023009	·008905 −	·011	·021
567	·021201	·008548	·011	·022	·022968	·008889	·011	·021
568	·021164	·008533	·011	·022	·022928	·008874	·011	·021
569	·021127	·008518	·011	·022	·022887	·008858	·011	·021
570	·021090	·008504	·011	·022	·022847	·008843	·011	·021
571	·021053	·008489	·011	·022	·022807	·008828	·011	·021
572	·021016	·008474	·011	·022	·022767	·008812	·011	·021
573	·020979	·008460	·011	·022	·022727	·008797	·011	·021
574	·020942	·008445 −	·011	·022	·022688	·008782	·011	·021
575	·020906	·008430	·011	·022	·022648	·008767	·011	·021
576	·020870	·008416	·011	·022	·022609	·008752	·011	·021
577	·020833	·008402	·011	·022	·022569	·008737	·011	·021
578	·020797	·008387	·011	·022	·022530	·008722	·011	·021
579	·020761	·008373	·011	·022	·022491	·008707	·011	·021
580	·020725 +	·008359	·011	·022	·022453	·008692	·011	·021
581	·020690	·008344	·011	·022	·022414	·008677	·011	·021
582	·020654	·008330	·011	·022	·022375 +	·008663	·011	·021
583	·020619	·008316	·011	·022	·022337	·008648	·011	·021
584	·020583	·008302	·011	·022	·022298	·008633	·011	·021
585	·020548	·008288	·011	·022	·022260	·008619	·011	·021
586	·020513	·008274	·011	·022	·022222	·008604	·011	·021
587	·020478	·008260	·011	·022	·022184	·008590	·011	·021
588	·020443	·008246	·011	·022	·022147	·008575 +	·011	·021
589	·020408	·008232	·011	·022	·022109	·008561	·011	·021
590	·020374	·008218	·011	·022	·022071	·008547	·011	·021
591	·020339	·008205 −	·011	·022	·022034	·008532	·011	·021
592	·020305 −	·008191	·011	·022	·021997	·008518	·011	·021
593	·020270	·008177	·011	·022	·021959	·008504	·011	·021
594	·020236	·008164	·011	·022	·021922	·008490	·011	·021
595	·020202	·008150 −	·011	·022	·021886	·008475 +	·011	·021
596	·020168	·008136	·011	·022	·021849	·008461	·011	·021
597	·020134	·008123	·011	·022	·021812	·008447	·011	·021
598	·020101	·008110	·011	·022	·021776	·008433	·011	·021
599	·020067	·008096	·011	·022	·021739	·008420	·011	·021
600	·020033	·008083	·011	·022	·021703	·008406	·011	·021

n = number of arrays

N = size of sample	15				16				17				18				19				20			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·78	P_2 λ_2 = 2·38	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·77	P_2 λ_2 = 2·37	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·76	P_2 λ_2 = 2·36	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·75	P_2 λ_2 = 2·35	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·74	P_2 λ_2 = 2·34	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·73	P_2 λ_2 = 2·33
551	·025455 −	·009480	·011	·021	·027273	·009804	·011	·021	·029091	·010116	·011	·021	·030909	·010418	·011	·021	·032727	·010710	·011	·021	·034545 +	·010993	·011	·021
552	·025408	·009464	·011	·021	·027223	·009787	·011	·021	·029038	·010098	·011	·021	·030853	·010399	·011	·021	·032668	·010691	·011	·021	·034483	·010973	·011	·021
553	·025362	·009447	·011	·021	·027174	·009769	·011	·021	·028986	·010080	·011	·021	·030797	·010381	·011	·021	·032609	·010672	·011	·021	·034420	·010954	·011	·021
554	·025316	·009430	·011	·021	·027125 −	·009752	·011	·021	·028933	·010062	·011	·021	·030741	·010362	·011	·021	·032550 −	·010653	·011	·021	·034358	·010934	·011	·021
555	·025271	·009413	·011	·021	·027076	·009734	·011	·021	·028881	·010044	·011	·021	·030686	·010344	·011	·021	·032491	·010634	·011	·021	·034296	·010915 −	·011	·021
556	·025225 +	·009396	·011	·021	·027027	·009717	·011	·021	·028829	·010026	·011	·021	·030631	·010325 +	·011	·021	·032432	·010615 −	·011	·021	·034234	·010896	·011	·021
557	·025180	·009380	·011	·021	·026978	·009700	·011	·021	·028777	·010009	·011	·021	·030576	·010307	·011	·021	·032374	·010596	·011	·021	·034173	·010876	·011	·021
558	·025135 −	·009363	·011	·021	·026930	·009683	·011	·021	·028725 +	·009991	·011	·021	·030521	·010289	·011	·021	·032316	·010578	·011	·021	·034111	·010857	·011	·021
559	·025090	·009347	·011	·021	·026882	·009666	·011	·021	·028674	·009973	·011	·021	·030466	·010271	·011	·021	·032258	·010559	·011	·021	·034050 +	·010838	·011	·021
560	·025045 −	·009330	·011	·021	·026834	·009649	·011	·021	·028623	·009956	·011	·021	·030411	·010253	·011	·021	·032200	·010540	·011	·021	·033989	·010819	·011	·021
561	·025000	·009314	·011	·021	·026786	·009632	·011	·021	·028571	·009938	·011	·021	·030357	·010235 −	·011	·021	·032143	·010522	·011	·021	·033929	·010800	·011	·021
562	·024955 +	·009297	·011	·021	·026738	·009615 −	·011	·021	·028520	·009921	·011	·021	·030303	·010217	·011	·021	·032086	·010504	·011	·021	·033868	·010781	·011	·021
563	·024911	·009281	·011	·021	·026690	·009598	·011	·021	·028470	·009904	·011	·021	·030249	·010199	·011	·021	·032028	·010485 +	·011	·021	·033808	·010763	·011	·021
564	·024867	·009265 −	·011	·021	·026643	·009581	·011	·021	·028419	·009886	·011	·021	·030195 +	·010181	·011	·021	·031972	·010467	·011	·021	·033748	·010744	·011	·021
565	·024823	·009249	·011	·021	·026596	·009564	·011	·021	·028369	·009869	·011	·021	·030142	·010164	·011	·021	·031915 −	·010449	·011	·021	·033688	·010725 +	·011	·021
566	·024779	·009232	·011	·021	·026549	·009548	·011	·021	·028319	·009852	·011	·021	·030088	·010146	·011	·021	·031858	·010431	·011	·021	·033628	·010707	·011	·021
567	·024735 −	·009216	·011	·021	·026502	·009531	·011	·021	·028269	·009835 −	·011	·021	·030035 +	·010128	·011	·021	·031802	·010412	·011	·021	·033569	·010688	·011	·021
568	·024691	·009200	·011	·021	·026455 +	·009515 −	·011	·021	·028219	·009818	·011	·021	·029982	·010111	·011	·021	·031746	·010394	·011	·021	·033510	·010669	·011	·021
569	·024648	·009184	·011	·021	·026408	·009498	·011	·021	·028169	·009801	·011	·021	·029930	·010093	·011	·021	·031690	·010376	·011	·021	·033451	·010651	·011	·021
570	·024605 −	·009168	·011	·021	·026362	·009482	·011	·021	·028120	·009784	·011	·021	·029877	·010076	·011	·021	·031634	·010359	·011	·021	·033392	·010633	·011	·021
571	·024561	·009153	·011	·021	·026316	·009465 +	·011	·021	·028070	·009767	·011	·021	·029825 −	·010058	·011	·021	·031579	·010341	·011	·021	·033333	·010614	·011	·021
572	·024518	·009137	·011	·021	·026270	·009449	·011	·021	·028021	·009750 +	·011	·021	·029772	·010041	·011	·021	·031524	·010323	·011	·021	·033275 −	·010596	·011	·021
573	·024476	·009121	·011	·021	·026224	·009433	·011	·021	·027972	·009733	·011	·021	·029720	·010024	·011	·021	·031469	·010305 +	·011	·021	·033217	·010578	·011	·021
574	·024433	·009105 +	·011	·021	·026178	·009416	·011	·021	·027923	·009717	·011	·021	·029668	·010007	·011	·021	·031414	·010288	·011	·021	·033159	·010560	·011	·021
575	·024390	·009090	·011	·021	·026132	·009400	·011	·021	·027875 −	·009700	·011	·021	·029617	·009990	·011	·021	·031359	·010270	·011	·021	·033101	·010542	·011	·021
576	·024348	·009074	·011	·021	·026087	·009384	·011	·021	·027826	·009683	·011	·021	·029565 +	·009972	·011	·021	·031304	·010252	·011	·021	·033043	·010524	·011	·021
577	·024306	·009059	·011	·021	·026042	·009368	·011	·021	·027778	·009667	·011	·021	·029514	·009955 +	·011	·021	·031250 −	·010235 −	·011	·021	·032986	·010506	·011	·021
578	·024263	·009043	·011	·021	·025997	·009352	·011	·021	·027730	·009650 +	·011	·021	·029463	·009938	·011	·021	·031196	·010217	·011	·021	·032929	·010488	·011	·021
579	·024221	·009028	·011	·021	·025952	·009336	·011	·021	·027682	·009634	·011	·021	·029412	·009922	·011	·021	·031142	·010200	·011	·021	·032872	·010470	·011	·021
580	·024180	·009012	·011	·021	·025907	·009320	·011	·021	·027634	·009618	·011	·021	·029361	·009905 −	·011	·021	·031088	·010183	·011	·021	·032815 +	·010452	·011	·021
581	·024138	·008997	·011	·021	·025862	·009305 −	·011	·021	·027586	·009601	·011	·021	·029310	·009888	·011	·021	·031034	·010166	·011	·021	·032759	·010435 −	·011	·021
582	·024096	·008982	·011	·021	·025818	·009289	·011	·021	·027539	·009585 −	·011	·021	·029260	·009871	·011	·021	·030981	·010148	·011	·021	·032702	·010417	·011	·021
583	·024055 −	·008967	·011	·021	·025773	·009273	·011	·021	·027491	·009569	·011	·021	·029210	·009855 −	·011	·021	·030928	·010131	·011	·021	·032646	·010400	·011	·021
584	·024014	·008951	·011	·021	·025729	·009257	·011	·021	·027444	·009553	·011	·021	·029160	·009838	·011	·021	·030875 −	·010114	·011	·021	·032590	·010382	·011	·021
585	·023973	·008936	·011	·021	·025685 −	·009242	·011	·021	·027397	·009536	·011	·021	·029110	·009821	·011	·021	·030822	·010097	·011	·021	·032534	·010365 −	·011	·021
586	·023932	·008921	·011	·021	·025641	·009226	·011	·021	·027350 +	·009520	·011	·021	·029060	·009805 −	·011	·021	·030769	·010080	·011	·021	·032479	·010347	·011	·021
587	·023891	·008906	·011	·021	·025597	·009211	·011	·021	·027304	·009504	·011	·021	·029010	·009788	·011	·021	·030717	·010063	·011	·021	·032423	·010330	·011	·021
588	·023850 +	·008891	·011	·021	·025554	·009195 +	·011	·021	·027257	·009488	·011	·021	·028961	·009772	·011	·021	·030664	·010046	·011	·021	·032368	·010313	·011	·021
589	·023810	·008876	·011	·021	·025510	·009180	·011	·021	·027211	·009473	·011	·021	·028912	·009756	·011	·021	·030612	·010030	·011	·021	·032313	·010295 +	·011	·021
590	·023769	·008861	·011	·021	·025467	·009164	·011	·021	·027165 −	·009457	·011	·021	·028862	·009739	·011	·021	·030560	·010013	·011	·021	·032258	·010278	·011	·021
591	·023729	·008847	·011	·021	·025424	·009149	·011	·021	·027119	·009441	·011	·021	·028814	·009723	·011	·021	·030508	·009996	·011	·021	·032203	·010261	·011	·021
592	·023689	·008832	·011	·021	·025381	·009134	·011	·021	·027073	·009425 +	·011	·021	·028765 −	·009707	·011	·021	·030457	·009980	·011	·021	·032149	·010244	·011	·021
593	·023649	·008817	·011	·021	·025338	·009119	·011	·021	·027027	·009410	·011	·021	·028716	·009691	·011	·021	·030405 +	·009963	·011	·021	·032095 −	·010227	·011	·021
594	·023609	·008802	·011	·021	·025295 +	·009104	·011	·021	·026981	·009394	·011	·021	·028668	·009675 −	·011	·021	·030354	·009947	·011	·021	·032040	·010210	·011	·021
595	·023569	·008788	·011	·021	·025253	·009088	·011	·021	·026936	·009378	·011	·021	·028620	·009659	·011	·021	·030303	·009930	·011	·021	·031987	·010193	·011	·021
596	·023529	·008773	·011	·021	·025210	·009073	·011	·021	·026891	·009363	·011	·021	·028571	·009643	·011	·021	·030252	·009914	·011	·021	·031933	·010177	·011	·021
597	·023490	·008759	·011	·021	·025168	·009058	·011	·021	·026846	·009347	·011	·021	·028523	·009627	·011	·021	·030201	·009897	·011	·021	·031879	·010160	·011	·021
598	·023451	·008744	·011	·021	·025126	·009043	·011	·021	·026801	·009332	·011	·021	·028476	·009611	·011	·021	·030151	·009881	·011	·021	·031826	·010143	·011	·021
599	·023411	·008730	·011	·021	·025084	·009028	·011	·021	·026756	·009317	·011	·021	·028428	·009595 +	·011	·021	·030100	·009865 −	·011	·021	·031773	·010126	·011	·021
600	·023372	·008716	·011	·021	·025042	·009014	·011	·021	·026711	·009301	·011	·021	·028381	·009579	·011	·021	·030050 +	·009849	·011	·021	·031720	·010110	·011	·021

N = size of sample	3: $\bar{\eta}^2$	3: $\sigma_{\bar{\eta}^2}$	3: P_1 λ_1 = 3·50	3: P_2 λ_2 = 2·94	4: $\bar{\eta}^2$	4: $\sigma_{\bar{\eta}^2}$	4: P_1 λ_1 = 3·20	4: P_2 λ_2 = 2·80
601	·003333	·003322	·011	·019	·005000	·004065 +	·013	·020
602	·003328	·003317	·011	·019	·004992	·004059	·013	·020
603	·003322	·003311	·011	·019	·004983	·004052	·013	·020
604	·003317	·003306	·011	·019	·004975 +	·004045 +	·013	·020
605	·003311	·003300	·011	·019	·004967	·004039	·013	·020
606	·003306	·003295 −	·011	·019	·004959	·004032	·013	·020
607	·003300	·003289	·011	·019	·004950 +	·004025 +	·013	·020
608	·003295 −	·003284	·011	·019	·004942	·004019	·013	·020
609	·003289	·003279	·011	·019	·004934	·004012	·013	·020
610	·003284	·003273	·011	·019	·004926	·004006	·013	·020
611	·003279	·003268	·011	·019	·004918	·003999	·013	·020
612	·003273	·003263	·011	·019	·004910	·003993	·013	·020
613	·003268	·003257	·011	·019	·004902	·003986	·013	·020
614	·003263	·003252	·011	·019	·004894	·003980	·013	·020
615	·003257	·003247	·011	·019	·004886	·003973	·013	·020
616	·003252	·003241	·011	·019	·004878	·003967	·013	·020
617	·003247	·003236	·011	·019	·004870	·003960	·013	·020
618	·003241	·003231	·011	·019	·004862	·003954	·013	·020
619	·003236	·003226	·011	·019	·004854	·003948	·013	·020
620	·003231	·003221	·011	·019	·004847	·003941	·013	·020
621	·003226	·003215 +	·011	·019	·004839	·003935 −	·013	·020
622	·003221	·003210	·011	·019	·004831	·003929	·013	·020
623	·003215 +	·003205 +	·011	·019	·004823	·003922	·013	·020
624	·003210	·003200	·011	·019	·004815 +	·003916	·013	·020
625	·003205 +	·003195 −	·011	·019	·004808	·003910	·013	·020
626	·003200	·003190	·011	·019	·004800	·003904	·013	·020
627	·003195 −	·003185 −	·011	·019	·004792	·003897	·013	·020
628	·003190	·003180	·011	·019	·004785 −	·003891	·013	·020
629	·003185 −	·003175 −	·011	·019	·004777	·003885 −	·013	·020
630	·003180	·003170	·011	·019	·004769	·003879	·013	·020
631	·003175 −	·003165 −	·011	·019	·004762	·003873	·013	·020
632	·003170	·003160	·011	·019	·004754	·003867	·013	·020
633	·003165 −	·003155 −	·011	·019	·004747	·003860	·013	·020
634	·003160	·003150 −	·011	·019	·004739	·003854	·013	·020
635	·003155 −	·003145 −	·011	·019	·004732	·003848	·013	·020
636	·003150 −	·003140	·011	·019	·004724	·003842	·013	·020
637	·003145 −	·003135 −	·011	·019	·004717	·003836	·013	·020
638	·003140	·003130	·011	·019	·004710	·003830	·013	·020
639	·003135 −	·003125 −	·011	·019	·004702	·003824	·013	·020
640	·003130	·003120	·011	·019	·004695 −	·003818	·013	·020
641	·003125	·003115 +	·011	·019	·004688	·003812	·013	·020
642	·003120	·003110	·011	·019	·004680	·003806	·013	·020
643	·003115 +	·003106	·011	·019	·004673	·003801	·013	·020
644	·003110	·003101	·011	·019	·004666	·003795 −	·013	·020
645	·003106	·003096	·011	·019	·004658	·003789	·013	·020
646	·003101	·003091	·011	·019	·004651	·003783	·013	·020
647	·003096	·003086	·011	·019	·004644	·003777	·013	·020
648	·003091	·003082	·011	·019	·004637	·003771	·013	·020
649	·003086	·003077	·011	·019	·004630	·003766	·013	·020
650	·003082	·003072	·011	·019	·004622	·003760	·013	·020

N = size of sample	5: $\bar{\eta}^2$	5: $\sigma_{\bar{\eta}^2}$	5: P_1 λ_1 = 3·14	5: P_2 λ_2 = 2·68	6: $\bar{\eta}^2$	6: $\sigma_{\bar{\eta}^2}$	6: P_1 λ_1 = 3·11	6: P_2 λ_2 = 2·63
601	·006667	·004690	·012	·021	·008333	·005240	·011	·021
602	·006656	·004683	·012	·021	·008319	·005231	·011	·021
603	·006645 −	·004675 −	·012	·021	·008306	·005222	·011	·021
604	·006633	·004667	·012	·021	·008292	·005214	·011	·021
605	·006623	·004660	·012	·021	·008278	·005205 +	·011	·021
606	·006612	·004652	·012	·021	·008264	·005197	·011	·021
607	·006601	·004644	·012	·021	·008251	·005188	·011	·021
608	·006590	·004637	·012	·021	·008237	·005180	·011	·021
609	·006579	·004629	·012	·021	·008224	·005171	·011	·021
610	·006568	·004622	·012	·021	·008210	·005163	·011	·021
611	·006557	·004614	·012	·021	·008197	·005154	·011	·021
612	·006547	·004606	·012	·021	·008183	·005146	·011	·021
613	·006536	·004599	·012	·021	·008170	·005138	·011	·021
614	·006525 +	·004592	·012	·021	·008157	·005129	·011	·021
615	·006515 −	·004584	·012	·021	·008143	·005121	·011	·021
616	·006504	·004577	·012	·021	·008130	·005113	·011	·021
617	·006494	·004569	·012	·021	·008117	·005104	·011	·021
618	·006483	·004562	·012	·021	·008104	·005096	·011	·021
619	·006472	·004555 −	·012	·021	·008091	·005088	·011	·021
620	·006462	·004547	·012	·021	·008078	·005080	·011	·021
621	·006452	·004540	·012	·021	·008065 −	·005072	·011	·021
622	·006441	·004533	·012	·021	·008052	·005064	·011	·021
623	·006431	·004525 +	·012	·021	·008039	·005055 +	·011	·021
624	·006421	·004518	·012	·021	·008026	·005047	·011	·021
625	·006410	·004511	·012	·021	·008013	·005039	·011	·021
626	·006400	·004504	·012	·021	·008000	·005031	·011	·021
627	·006390	·004497	·012	·021	·007987	·005023	·011	·021
628	·006380	·004489	·012	·021	·007974	·005015 +	·011	·021
629	·006369	·004482	·012	·021	·007962	·005007	·011	·021
630	·006359	·004475 +	·012	·021	·007949	·005000	·011	·021
631	·006349	·004468	·012	·021	·007937	·004992	·011	·021
632	·006339	·004461	·012	·021	·007924	·004984	·011	·021
633	·006329	·004454	·012	·021	·007911	·004976	·011	·021
634	·006319	·004447	·012	·021	·007899	·004968	·011	·021
635	·006309	·004440	·012	·021	·007886	·004960	·011	·021
636	·006299	·004433	·012	·021	·007874	·004953	·011	·021
637	·006289	·004426	·012	·021	·007862	·004945 −	·011	·021
638	·006279	·004419	·012	·021	·007849	·004937	·011	·021
639	·006270	·004412	·012	·021	·007837	·004929	·011	·021
640	·006260	·004406	·012	·021	·007825 −	·004922	·011	·021
641	·006250	·004399	·012	·021	·007813	·004914	·011	·021
642	·006240	·004392	·012	·021	·007800	·004906	·011	·021
643	·006231	·004385 +	·012	·021	·007788	·004899	·011	·021
644	·006221	·004378	·012	·021	·007776	·004891	·011	·021
645	·006211	·004372	·012	·021	·007764	·004884	·011	·021
646	·006202	·004365 −	·012	·021	·007752	·004876	·011	·021
647	·006192	·004358	·012	·021	·007740	·004869	·011	·021
648	·006182	·004351	·012	·021	·007728	·004861	·011	·021
649	·006173	·004345 −	·012	·021	·007716	·004854	·011	·021
650	·006163	·004338	·012	·021	·007704	·004846	·011	·021

N = size of sample	7: $\bar{\eta}^2$	7: $\sigma_{\bar{\eta}^2}$	7: P_1 λ_1 = 3·08	7: P_2 λ_2 = 2·58	8: $\bar{\eta}^2$	8: $\sigma_{\bar{\eta}^2}$	8: P_1 λ_1 = 3·02	8: P_2 λ_2 = 2·54
601	·010000	·005735 +	·010	·021	·011667	·006189	·011	·021
602	·009983	·005726	·010	·021	·011647	·006179	·011	·021
603	·009967	·005716	·010	·021	·011628	·006169	·011	·021
604	·009950 +	·005707	·010	·021	·011609	·006159	·011	·021
605	·009934	·005697	·010	·021	·011589	·006149	·011	·021
606	·009917	·005688	·010	·021	·011570	·006138	·011	·021
607	·009901	·005679	·010	·021	·011551	·006128	·011	·021
608	·009885 −	·005669	·010	·021	·011532	·006118	·011	·021
609	·009868	·005660	·010	·021	·011513	·006108	·011	·021
610	·009852	·005651	·010	·021	·011494	·006099	·011	·021
611	·009836	·005642	·010	·021	·011475 +	·006089	·011	·021
612	·009820	·005632	·010	·021	·011457	·006079	·011	·021
613	·009804	·005623	·010	·021	·011438	·006069	·011	·021
614	·009788	·005614	·010	·021	·011419	·006059	·011	·021
615	·009772	·005605 +	·010	·021	·011401	·006049	·011	·021
616	·009756	·005596	·010	·021	·011382	·006039	·011	·021
617	·009740	·005587	·010	·021	·011364	·006030	·011	·021
618	·009724	·005578	·010	·021	·011345 +	·006020	·011	·021
619	·009709	·005569	·010	·021	·011327	·006010	·011	·021
620	·009693	·005560	·010	·021	·011309	·006001	·011	·021
621	·009677	·005551	·010	·021	·011290	·005991	·011	·021
622	·009662	·005542	·010	·021	·011272	·005982	·011	·021
623	·009646	·005533	·010	·021	·011254	·005972	·011	·021
624	·009631	·005525 −	·010	·021	·011236	·005962	·011	·021
625	·009615 +	·005516	·010	·021	·011218	·005953	·011	·021
626	·009600	·005507	·010	·021	·011200	·005944	·011	·021
627	·009585 −	·005498	·010	·021	·011182	·005934	·011	·021
628	·009569	·005490	·010	·021	·011164	·005925 −	·011	·021
629	·009554	·005481	·010	·021	·011146	·005915 +	·011	·021
630	·009539	·005472	·010	·021	·011129	·005906	·011	·021
631	·009524	·005464	·010	·021	·011111	·005897	·011	·021
632	·009509	·005455 +	·010	·021	·011094	·005887	·011	·021
633	·009494	·005446	·010	·021	·011076	·005878	·011	·021
634	·009479	·005438	·010	·021	·011058	·005869	·011	·021
635	·009464	·005429	·010	·021	·011041	·005860	·011	·021
636	·009449	·005421	·010	·021	·011024	·005851	·011	·021
637	·009434	·005412	·010	·021	·011006	·005841	·011	·021
638	·009419	·005404	·010	·021	·010989	·005832	·011	·021
639	·009404	·005396	·010	·021	·010972	·005823	·011	·021
640	·009390	·005387	·010	·021	·010955 −	·005814	·011	·021
641	·009375	·005379	·010	·021	·010938	·005805 +	·011	·021
642	·009360	·005370	·010	·021	·010920	·005796	·011	·021
643	·009346	·005362	·010	·021	·010903	·005787	·011	·021
644	·009331	·005354	·010	·021	·010886	·005778	·011	·021
645	·009317	·005346	·010	·021	·010870	·005769	·011	·021
646	·009302	·005337	·010	·021	·010853	·005761	·011	·021
647	·009288	·005329	·010	·021	·010836	·005752	·011	·021
648	·009274	·005321	·010	·021	·010819	·005743	·011	·021
649	·009259	·005313	·010	·021	·010802	·005734	·011	·021
650	·009245 −	·005305 −	·010	·021	·010786	·005725 +	·011	·021

n = number of arrays

N = size of sample	9				10				11			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·96$	P_2 $\lambda_2=2·50$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·92$	P_2 $\lambda_2=2·47$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·88$	P_2 $\lambda_2=2·44$
601	·013333	·006611	·011	·021	·015000	·007006	·011	·021	·016667	·007379	·011	·021
602	·013311	·006600	·011	·021	·014975 +	·006995 −	·011	·021	·016639	·007367	·011	·021
603	·013289	·006589	·011	·021	·014950 +	·006983	·011	·021	·016611	·007355 −	·011	·021
604	·013267	·006578	·011	·021	·014925 +	·006972	·011	·021	·016584	·007343	·011	·021
605	·013245 +	·006568	·011	·021	·014901	·006960	·011	·021	·016556	·007331	·011	·021
606	·013223	·006557	·011	·021	·014876	·006949	·011	·021	·016529	·007319	·011	·021
607	·013201	·006546	·011	·021	·014851	·006937	·011	·021	·016502	·007307	·011	·021
608	·013180	·006535 +	·011	·021	·014827	·006926	·011	·021	·016474	·007295 −	·011	·021
609	·013158	·006525 −	·011	·021	·014803	·006915 −	·011	·021	·016447	·007283	·011	·021
610	·013136	·006514	·011	·021	·014778	·006904	·011	·021	·016420	·007271	·011	·021
611	·013115 −	·006504	·011	·021	·014754	·006892	·011	·021	·016393	·007259	·011	·021
612	·013093	·006493	·011	·021	·014730	·006881	·011	·021	·016367	·007247	·011	·021
613	·013072	·006483	·011	·021	·014706	·006870	·011	·021	·016340	·007236	·011	·021
614	·013051	·006472	·011	·021	·014682	·006859	·011	·021	·016313	·007224	·011	·021
615	·013029	·006462	·011	·021	·014658	·006848	·011	·021	·016287	·007212	·011	·021
616	·013008	·006451	·011	·021	·014634	·006837	·011	·021	·016260	·007201	·011	·021
617	·012987	·006441	·011	·021	·014610	·006826	·011	·021	·016234	·007189	·011	·021
618	·012966	·006430	·011	·021	·014587	·006815 −	·011	·021	·016207	·007178	·011	·021
619	·012945 −	·006420	·011	·021	·014563	·006804	·011	·021	·016181	·007166	·011	·021
620	·012924	·006410	·011	·021	·014540	·006793	·011	·021	·016155 +	·007155 −	·011	·021
621	·012903	·006400	·011	·021	·014516	·006782	·011	·021	·016129	·007143	·011	·021
622	·012882	·006389	·011	·021	·014493	·006771	·011	·021	·016103	·007132	·011	·021
623	·012862	·006379	·011	·021	·014469	·006761	·011	·021	·016077	·007120	·011	·021
624	·012841	·006369	·011	·021	·014446	·006750 −	·011	·021	·016051	·007109	·011	·021
625	·012821	·006359	·011	·021	·014423	·006739	·011	·021	·016026	·007098	·011	·021
626	·012800	·006349	·011	·021	·014400	·006728	·011	·021	·016000	·007087	·011	·021
627	·012780	·006339	·011	·021	·014377	·006718	·011	·021	·015974	·007075 +	·011	·021
628	·012759	·006329	·011	·021	·014354	·006707	·011	·021	·015949	·007064	·011	·021
629	·012739	·006319	·011	·021	·014331	·006697	·011	·021	·015924	·007053	·011	·021
630	·012719	·006309	·011	·021	·014308	·006686	·011	·021	·015898	·007042	·011	·021
631	·012698	·006299	·011	·021	·014286	·006675 +	·011	·021	·015873	·007031	·011	·021
632	·012678	·006289	·011	·021	·014263	·006665 +	·011	·021	·015848	·007020	·011	·021
633	·012658	·006279	·011	·021	·014241	·006655 −	·011	·021	·015823	·007009	·011	·021
634	·012638	·006269	·011	·021	·014218	·006644	·011	·021	·015798	·006998	·011	·021
635	·012618	·006259	·011	·021	·014196	·006634	·011	·021	·015773	·006987	·011	·021
636	·012598	·006250 −	·011	·021	·014173	·006623	·011	·021	·015748	·006976	·011	·021
637	·012579	·006240	·011	·021	·014151	·006613	·011	·021	·015723	·006965 +	·011	·021
638	·012559	·006230	·011	·021	·014129	·006603	·011	·021	·015699	·006954	·011	·021
639	·012539	·006220	·011	·021	·014107	·006593	·011	·021	·015674	·006944	·011	·021
640	·012520	·006211	·011	·021	·014085 −	·006582	·011	·021	·015649	·006933	·011	·021
641	·012500	·006201	·011	·021	·014063	·006572	·011	·021	·015625	·006922	·011	·021
642	·012480	·006192	·011	·021	·014041	·006562	·011	·021	·015601	·006911	·011	·021
643	·012461	·006182	·011	·021	·014019	·006552	·011	·021	·015576	·006901	·011	·021
644	·012442	·006172	·011	·021	·013997	·006542	·011	·021	·015552	·006890	·011	·021
645	·012422	·006163	·011	·021	·013975 +	·006532	·011	·021	·015528	·006880	·011	·021
646	·012403	·006153	·011	·021	·013953	·006522	·011	·021	·015504	·006869	·011	·021
647	·012384	·006144	·011	·021	·013932	·006512	·011	·021	·015480	·006858	·011	·021
648	·012365 −	·006135 −	·011	·021	·013910	·006502	·011	·021	·015456	·006848	·011	·021
649	·012346	·006125 +	·011	·021	·013889	·006492	·011	·021	·015432	·006837	·011	·021
650	·012327	·006116	·011	·021	·013867	·006482	·011	·021	·015408	·006827	·011	·021

N = size of sample	12				13				14			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·84$	P_2 $\lambda_2=2·42$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·80$	P_2 $\lambda_2=2·40$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·79$	P_2 $\lambda_2=2·39$
601	·018333	·007732	·011	·021	·020000	·008069	·011	·022	·021667	·008392	·011	·021
602	·018303	·007720	·011	·021	·019967	·008056	·011	·022	·021631	·008378	·011	·021
603	·018272	·007707	·011	·021	·019934	·008043	·011	·022	·021595 −	·008364	·011	·021
604	·018242	·007694	·011	·021	·019900	·008030	·011	·022	·021559	·008351	·011	·021
605	·018212	·007682	·011	·021	·019868	·008017	·011	·022	·021523	·008337	·011	·021
606	·018182	·007669	·011	·021	·019835 −	·008004	·011	·022	·021488	·008323	·011	·021
607	·018152	·007657	·011	·021	·019802	·007991	·011	·022	·021452	·008310	·011	·021
608	·018122	·007644	·011	·021	·019769	·007978	·011	·022	·021417	·008296	·011	·021
609	·018092	·007632	·011	·021	·019737	·007965 −	·011	·022	·021382	·008283	·011	·021
610	·018062	·007619	·011	·021	·019704	·007952	·011	·022	·021346	·008269	·011	·021
611	·018033	·007607	·011	·021	·019672	·007939	·011	·022	·021311	·008256	·011	·021
612	·018003	·007595 −	·011	·021	·019640	·007926	·011	·022	·021277	·008243	·011	·021
613	·017974	·007583	·011	·021	·019608	·007913	·011	·022	·021242	·008229	·011	·021
614	·017945 −	·007570	·011	·021	·019576	·007900	·011	·022	·021207	·008216	·011	·021
615	·017915 +	·007558	·011	·021	·019544	·007888	·011	·022	·021173	·008203	·011	·021
616	·017886	·007546	·011	·021	·019512	·007875 −	·011	·022	·021138	·008190	·011	·021
617	·017857	·007534	·011	·021	·019481	·007862	·011	·022	·021104	·008177	·011	·021
618	·017828	·007522	·011	·021	·019449	·007850 −	·011	·022	·021070	·008163	·011	·021
619	·017799	·007510	·011	·021	·019417	·007837	·011	·022	·021036	·008150 +	·011	·021
620	·017771	·007498	·011	·021	·019386	·007825 −	·011	·022	·021002	·008137	·011	·021
621	·017742	·007486	·011	·021	·019355 −	·007812	·011	·022	·020968	·008124	·011	·021
622	·017713	·007474	·011	·021	·019324	·007800	·011	·022	·020934	·008112	·011	·021
623	·017685 −	·007462	·011	·021	·019293	·007787	·011	·022	·020900	·008099	·011	·021
624	·017657	·007450 +	·011	·021	·019262	·007775 −	·011	·022	·020867	·008086	·011	·021
625	·017628	·007438	·011	·021	·019231	·007763	·011	·022	·020833	·008073	·011	·021
626	·017600	·007426	·011	·021	·019200	·007750 +	·011	·022	·020800	·008060	·011	·021
627	·017572	·007415 −	·011	·021	·019169	·007738	·011	·022	·020767	·008048	·011	·021
628	·017544	·007403	·011	·021	·019139	·007726	·011	·022	·020734	·008035 −	·011	·021
629	·017516	·007391	·011	·021	·019108	·007714	·011	·022	·020701	·008022	·011	·021
630	·017488	·007380	·011	·021	·019078	·007702	·011	·022	·020668	·008010	·011	·021
631	·017460	·007368	·011	·021	·019048	·007690	·011	·022	·020635 −	·007997	·011	·021
632	·017433	·007357	·011	·021	·019017	·007677	·011	·022	·020602	·007985 −	·011	·021
633	·017405 +	·007345 +	·011	·021	·018987	·007665 +	·011	·022	·020570	·007972	·011	·021
634	·017378	·007334	·011	·021	·018957	·007654	·011	·022	·020537	·007960	·011	·021
635	·017350 +	·007322	·011	·021	·018927	·007642	·011	·022	·020505 −	·007947	·011	·021
636	·017323	·007311	·011	·021	·018898	·007630	·011	·022	·020472	·007935 −	·011	·021
637	·017296	·007299	·011	·021	·018868	·007618	·011	·022	·020440	·007923	·011	·021
638	·017268	·007288	·011	·021	·018838	·007606	·011	·022	·020408	·007910	·011	·021
639	·017241	·007277	·011	·021	·018809	·007594	·011	·022	·020376	·007898	·011	·021
640	·017214	·007265 +	·011	·021	·018779	·007582	·011	·022	·020344	·007886	·011	·021
641	·017188	·007254	·011	·021	·018750	·007571	·011	·022	·020313	·007874	·011	·021
642	·017161	·007243	·011	·021	·018721	·007559	·011	·022	·020281	·007861	·011	·021
643	·017134	·007232	·011	·021	·018692	·007547	·011	·022	·020249	·007849	·011	·021
644	·017107	·007221	·011	·021	·018663	·007536	·011	·022	·020218	·007837	·011	·021
645	·017081	·007210	·011	·021	·018634	·007524	·011	·022	·020186	·007825 +	·011	·021
646	·017054	·007199	·011	·021	·018605 −	·007513	·011	·022	·020155 +	·007813	·011	·021
647	·017028	·007188	·011	·021	·018576	·007501	·011	·022	·020124	·007801	·011	·021
648	·017002	·007177	·011	·021	·018547	·007490	·011	·022	·020093	·007789	·011	·021
649	·016975 +	·007165 +	·011	·021	·018519	·007478	·011	·022	·020062	·007778	·011	·021
650	·016949	·007155 −	·011	·021	·018490	·007467	·011	·022	·020031	·007766	·011	·021

N = size of sample	15				16				17			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·78$	P_2 $\lambda_2=2·38$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·77$	P_2 $\lambda_2=2·37$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·76$	P_2 $\lambda_2=2·36$
601	·023333	·008701	·011	·021	·025000	·008999	·011	·021	·026667	·009286	·011	·021
602	·023295 −	·008687	·011	·021	·024958	·008984	·011	·021	·026622	·009271	·011	·021
603	·023256	·008673	·011	·021	·024917	·008969	·011	·021	·026578	·009256	·011	·021
604	·023217	·008658	·011	·021	·024876	·008955 −	·011	·021	·026534	·009241	·011	·021
605	·023179	·008644	·011	·021	·024834	·008940	·011	·021	·026490	·009226	·011	·021
606	·023140	·008630	·011	·021	·024793	·008926	·011	·021	·026446	·009211	·011	·021
607	·023102	·008616	·011	·021	·024752	·008911	·011	·021	·026403	·009196	·011	·021
608	·023064	·008602	·011	·021	·024712	·008897	·011	·021	·026359	·009181	·011	·021
609	·023026	·008588	·011	·021	·024671	·008882	·011	·021	·026316	·009166	·011	·021
610	·022989	·008574	·011	·021	·024631	·008868	·011	·021	·026273	·009151	·011	·021
611	·022951	·008560	·011	·021	·024590	·008853	·011	·021	·026230	·009136	·011	·021
612	·022913	·008547	·011	·021	·024550 −	·008839	·011	·021	·026187	·009121	·011	·021
613	·022876	·008533	·011	·021	·024510	·008825 −	·011	·021	·026144	·009107	·011	·021
614	·022838	·008519	·011	·021	·024470	·008811	·011	·021	·026101	·009092	·011	·021
615	·022801	·008505 +	·011	·021	·024430	·008797	·011	·021	·026059	·009078	·011	·021
616	·022764	·008492	·011	·021	·024390	·008783	·011	·021	·026016	·009063	·011	·021
617	·022727	·008478	·011	·021	·024351	·008768	·011	·021	·025974	·009048	·011	·021
618	·022690	·008465 −	·011	·021	·024311	·008754	·011	·021	·025932	·009034	·011	·021
619	·022654	·008451	·011	·021	·024272	·008740	·011	·021	·025890	·009020	·011	·021
620	·022617	·008438	·011	·021	·024233	·008727	·011	·021	·025848	·009005 +	·011	·021
621	·022581	·008424	·011	·021	·024194	·008713	·011	·021	·025806	·008991	·011	·021
622	·022544	·008411	·011	·021	·024155 −	·008699	·011	·021	·025765 −	·008977	·011	·021
623	·022508	·008397	·011	·021	·024116	·008685 +	·011	·021	·025723	·008962	·011	·021
624	·022472	·008384	·011	·021	·024077	·008671	·011	·021	·025682	·008948	·011	·021
625	·022436	·008371	·011	·021	·024038	·008658	·011	·021	·025641	·008934	·011	·021
626	·022400	·008358	·011	·021	·024000	·008644	·011	·021	·025600	·008920	·011	·021
627	·022364	·008345 −	·011	·021	·023962	·008630	·011	·021	·025559	·008906	·011	·021
628	·022329	·008331	·011	·021	·023923	·008617	·011	·021	·025518	·008892	·011	·021
629	·022293	·008318	·011	·021	·023885 +	·008603	·011	·021	·025478	·008878	·011	·021
630	·022258	·008305 +	·011	·021	·023847	·008590	·011	·021	·025437	·008864	·011	·021
631	·022222	·008292	·011	·021	·023810	·008576	·011	·021	·025397	·008850 +	·011	·021
632	·022187	·008279	·011	·021	·023772	·008563	·011	·021	·025357	·008837	·011	·021
633	·022152	·008266	·011	·021	·023734	·008550 −	·011	·021	·025316	·008823	·011	·021
634	·022117	·008253	·011	·021	·023697	·008536	·011	·021	·025276	·008809	·011	·021
635	·022082	·008241	·011	·021	·023659	·008523	·011	·021	·025237	·008795 +	·011	·021
636	·022047	·008228	·011	·021	·023622	·008510	·011	·021	·025197	·008782	·011	·021
637	·022013	·008215 −	·011	·021	·023585 −	·008496	·011	·021	·025157	·008768	·011	·021
638	·021978	·008202	·011	·021	·023548	·008483	·011	·021	·025118	·008755 −	·011	·021
639	·021944	·008190	·011	·021	·023511	·008470	·011	·021	·025078	·008741	·011	·021
640	·021909	·008177	·011	·021	·023474	·008457	·011	·021	·025039	·008727	·011	·021
641	·021875	·008164	·011	·021	·023438	·008444	·011	·021	·025000	·008714	·011	·021
642	·021841	·008152	·011	·021	·023401	·008431	·011	·021	·024961	·008701	·011	·021
643	·021807	·008139	·011	·021	·023364	·008418	·011	·021	·024922	·008687	·011	·021
644	·021773	·008127	·011	·021	·023328	·008405 +	·011	·021	·024883	·008674	·011	·021
645	·021739	·008114	·011	·021	·023292	·008392	·011	·021	·024845 −	·008661	·011	·021
646	·021705 +	·008102	·011	·021	·023256	·008380	·011	·021	·024806	·008647	·011	·021
647	·021672	·008089	·011	·021	·023220	·008367	·011	·021	·024768	·008634	·011	·021
648	·021638	·008077	·011	·021	·023184	·008354	·011	·021	·024730	·008621	·011	·021
649	·021605 −	·008065 −	·011	·021	·023148	·008341	·011	·021	·024691	·008608	·011	·021
650	·021572	·008052	·011	·021	·023112	·008329	·011	·021	·024653	·008595 −	·011	·021

N = size of sample	18				19				20			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·75$	P_2 $\lambda_2=2·35$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·74$	P_2 $\lambda_2=2·34$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·73$	P_2 $\lambda_2=2·33$
601	·028333	·009564	·011	·021	·030000	·009832	·011	·021	·031667	·010093	·011	·021
602	·028286	·009548	·011	·021	·029950 +	·009816	·011	·021	·031614	·010077	·011	·021
603	·028239	·009532	·011	·021	·029900	·009800	·011	·021	·031561	·010060	·011	·021
604	·028192	·009517	·011	·021	·029851	·009784	·011	·021	·031509	·010044	·011	·021
605	·028146	·009501	·011	·021	·029801	·009768	·011	·021	·031457	·010028	·011	·021
606	·028099	·009486	·011	·021	·029752	·009753	·011	·021	·031405 −	·010011	·011	·021
607	·028053	·009471	·011	·021	·029703	·009737	·011	·021	·031353	·009995 +	·011	·021
608	·028007	·009455 +	·011	·021	·029654	·009721	·011	·021	·031301	·009979	·011	·021
609	·027961	·009440	·011	·021	·029605 +	·009705 +	·011	·021	·031250 −	·009963	·011	·021
610	·027915 −	·009425 −	·011	·021	·029557	·009690	·011	·021	·031199	·009947	·011	·021
611	·027869	·009409	·011	·021	·029508	·009674	·011	·021	·031148	·009931	·011	·021
612	·027823	·009394	·011	·021	·029460	·009658	·011	·021	·031097	·009915 −	·011	·021
613	·027778	·009379	·011	·021	·029412	·009643	·011	·021	·031046	·009899	·011	·021
614	·027732	·009364	·011	·021	·029364	·009627	·011	·021	·030995 +	·009883	·011	·021
615	·027687	·009349	·011	·021	·029316	·009612	·011	·021	·030945 −	·009867	·011	·021
616	·027642	·009334	·011	·021	·029268	·009597	·011	·021	·030894	·009851	·011	·021
617	·027597	·009319	·011	·021	·029221	·009581	·011	·021	·030844	·009836	·011	·021
618	·027553	·009304	·011	·021	·029173	·009566	·011	·021	·030794	·009820	·011	·021
619	·027508	·009290	·011	·021	·029126	·009551	·011	·021	·030744	·009804	·011	·021
620	·027464	·009275 −	·011	·021	·029079	·009536	·011	·021	·030695 −	·009789	·011	·021
621	·027419	·009260	·011	·021	·029032	·009521	·011	·021	·030645 +	·009773	·011	·021
622	·027375 +	·009245 +	·011	·021	·028986	·009505 +	·011	·021	·030596	·009758	·011	·021
623	·027331	·009231	·011	·021	·028939	·009490	·011	·021	·030547	·009742	·011	·021
624	·027287	·009216	·011	·021	·028892	·009475 +	·011	·021	·030498	·009727	·011	·021
625	·027244	·009202	·011	·021	·028846	·009461	·011	·021	·030449	·009712	·011	·021
626	·027200	·009187	·011	·021	·028800	·009446	·011	·021	·030400	·009696	·011	·021
627	·027157	·009173	·011	·021	·028754	·009431	·011	·021	·030351	·009681	·011	·021
628	·027113	·009158	·011	·021	·028708	·009416	·011	·021	·030303	·009666	·011	·021
629	·027070	·009144	·011	·021	·028662	·009401	·011	·021	·030255 −	·009651	·011	·021
630	·027027	·009130	·011	·021	·028617	·009387	·011	·021	·030207	·009636	·011	·021
631	·026984	·009115 +	·011	·021	·028571	·009372	·011	·021	·030159	·009621	·011	·021
632	·026941	·009101	·011	·021	·028526	·009357	·011	·021	·030111	·009606	·011	·021
633	·026899	·009087	·011	·021	·028481	·009343	·011	·021	·030063	·009591	·011	·021
634	·026856	·009073	·011	·021	·028436	·009328	·011	·021	·030016	·009576	·011	·021
635	·026814	·009059	·011	·021	·028391	·009314	·011	·021	·029968	·009561	·011	·021
636	·026772	·009045 −	·011	·021	·028346	·009299	·011	·021	·029921	·009546	·011	·021
637	·026730	·009031	·011	·021	·028302	·009285 −	·011	·021	·029874	·009532	·011	·021
638	·026688	·009017	·011	·021	·028257	·009271	·011	·021	·029827	·009517	·011	·021
639	·026646	·009003	·011	·021	·028213	·009256	·011	·021	·029781	·009502	·011	·021
640	·026604	·008989	·011	·021	·028169	·009242	·011	·021	·029734	·009488	·011	·021
641	·026563	·008975 +	·011	·021	·028125	·009228	·011	·021	·029688	·009473	·011	·021
642	·026521	·008961	·011	·021	·028081	·009214	·011	·021	·029641	·009459	·011	·021
643	·026480	·008948	·011	·021	·028037	·009200	·011	·021	·029595 +	·009444	·011	·021
644	·026439	·008934	·011	·021	·027994	·009185 +	·011	·021	·029549	·009430	·011	·021
645	·026398	·008920	·011	·021	·027950 +	·009171	·011	·021	·029503	·009415 +	·011	·021
646	·026357	·008907	·011	·021	·027907	·009157	·011	·021	·029457	·009401	·011	·021
647	·026316	·008893	·011	·021	·027864	·009143	·011	·021	·029412	·009387	·011	·021
648	·026275 +	·008879	·011	·021	·027821	·009130	·011	·021	·029366	·009372	·011	·021
649	·026235 −	·008866	·011	·021	·027778	·009116	·011	·021	·029321	·009358	·011	·021
650	·026194	·008852	·011	·021	·027735 −	·009102	·011	·021	·029276	·009344	·011	·021

n = number of arrays

N = size of sample	3: $\bar{\eta}^2$	3: $\sigma_{\bar{\eta}^2}$	3: P_1 $\lambda_1 = 3{\cdot}50$	3: P_2 $\lambda_2 = 2{\cdot}94$	4: $\bar{\eta}^2$	4: $\sigma_{\bar{\eta}^2}$	4: P_1 $\lambda_1 = 3{\cdot}20$	4: P_2 $\lambda_2 = 2{\cdot}80$	5: $\bar{\eta}^2$	5: $\sigma_{\bar{\eta}^2}$	5: P_1 $\lambda_1 = 3{\cdot}14$	5: P_2 $\lambda_2 = 2{\cdot}68$
651	·003077	·003067	·011	·019	·004615 +	·003754	·013	·020	·006154	·004331	·012	·021
652	·003072	·003063	·011	·019	·004608	·003748	·013	·020	·006144	·004325 −	·012	·021
653	·003067	·003058	·011	·019	·004601	·003742	·013	·020	·006135 −	·004318	·012	·021
654	·003063	·003053	·011	·019	·004594	·003737	·013	·020	·006126	·004312	·012	·021
655	·003058	·003049	·011	·019	·004587	·003731	·013	·020	·006116	·004305 −	·012	·021
656	·003053	·003044	·011	·019	·004580	·003725 +	·013	·020	·006107	·004298	·012	·021
657	·003049	·003039	·011	·019	·004573	·003720	·013	·020	·006098	·004292	·012	·021
658	·003044	·003035 −	·011	·019	·004566	·003714	·013	·020	·006088	·004285 +	·012	·021
659	·003040	·003030	·011	·019	·004559	·003709	·013	·020	·006079	·004279	·012	·021
660	·003035 −	·003026	·011	·019	·004552	·003703	·013	·020	·006070	·004272	·012	·021
661	·003030	·003021	·011	·019	·004545 +	·003697	·013	·020	·006061	·004266	·012	·021
662	·003026	·003017	·011	·019	·004539	·003692	·013	·020	·006051	·004260	·012	·021
663	·003021	·003012	·011	·019	·004532	·003686	·013	·020	·006042	·004253	·012	·021
664	·003017	·003008	·011	·019	·004525 −	·003681	·013	·020	·006033	·004247	·012	·021
665	·003012	·003003	·011	·019	·004518	·003675 +	·013	·020	·006024	·004240	·012	·021
666	·003008	·002998	·011	·019	·004511	·003670	·013	·020	·006015 +	·004234	·012	·021
667	·003003	·002994	·011	·019	·004505 −	·003664	·013	·020	·006006	·004228	·012	·021
668	·002999	·002990	·011	·019	·004498	·003659	·013	·020	·005997	·004221	·012	·021
669	·002994	·002985 +	·011	·019	·004491	·003653	·013	·020	·005988	·004215 +	·012	·021
670	·002990	·002981	·011	·019	·004484	·003648	·013	·020	·005979	·004209	·012	·021
671	·002985 +	·002976	·011	·019	·004478	·003642	·013	·020	·005970	·004203	·012	·021
672	·002981	·002972	·011	·019	·004471	·003637	·013	·020	·005961	·004196	·012	·021
673	·002976	·002967	·011	·019	·004464	·003632	·013	·020	·005952	·004190	·012	·021
674	·002972	·002963	·011	·019	·004458	·003626	·013	·020	·005944	·004184	·012	·021
675	·002967	·002959	·011	·019	·004451	·003621	·013	·020	·005935 −	·004178	·012	·021
676	·002963	·002954	·011	·019	·004444	·003615 +	·013	·020	·005926	·004172	·012	·021
677	·002959	·002950 −	·011	·019	·004438	·003610	·013	·020	·005917	·004166	·012	·021
678	·002954	·002945 +	·011	·019	·004431	·003605 −	·013	·020	·005908	·004159	·012	·021
679	·002950 −	·002941	·011	·019	·004425 −	·003600	·013	·020	·005900	·004153	·012	·021
680	·002946	·002937	·011	·019	·004418	·003594	·013	·020	·005891	·004147	·012	·021
681	·002941	·002933	·011	·019	·004412	·003589	·013	·020	·005882	·004141	·012	·021
682	·002937	·002928	·011	·019	·004405 +	·003584	·013	·020	·005874	·004135 +	·012	·021
683	·002933	·002924	·011	·019	·004399	·003578	·013	·020	·005865 −	·004129	·012	·021
684	·002928	·002920	·011	·019	·004392	·003573	·013	·020	·005857	·004123	·012	·021
685	·002924	·002915 +	·011	·019	·004386	·003568	·013	·020	·005848	·004117	·012	·021
686	·002920	·002911	·011	·019	·004380	·003563	·013	·020	·005839	·004111	·012	·021
687	·002915 +	·002907	·011	·019	·004373	·003558	·013	·020	·005831	·004105 +	·012	·021
688	·002911	·002903	·011	·019	·004367	·003553	·013	·020	·005822	·004099	·012	·021
689	·002907	·002899	·011	·019	·004360	·003547	·013	·020	·005814	·004093	·012	·021
690	·002903	·002894	·011	·019	·004354	·003542	·013	·020	·005806	·004087	·012	·021
691	·002899	·002890	·011	·019	·004348	·003537	·013	·020	·005797	·004081	·012	·021
692	·002894	·002886	·011	·019	·004342	·003532	·013	·020	·005789	·004075 +	·012	·021
693	·002890	·002882	·011	·019	·004335 +	·003527	·013	·020	·005780	·004070	·012	·021
694	·002886	·002878	·011	·019	·004329	·003522	·013	·020	·005772	·004064	·012	·021
695	·002882	·002874	·011	·019	·004323	·003517	·013	·020	·005764	·004058	·012	·021
696	·002878	·002869	·011	·019	·004317	·003512	·013	·020	·005755 +	·004052	·012	·021
697	·002874	·002865 +	·011	·019	·004310	·003507	·013	·020	·005747	·004046	·012	·021
698	·002869	·002861	·011	·019	·004304	·003502	·013	·020	·005739	·004041	·012	·021
699	·002865 +	·002857	·011	·019	·004298	·003497	·013	·020	·005731	·004035 −	·012	·021
700	·002861	·002853	·011	·019	·004292	·003492	·013	·020	·005722	·004029	·012	·021

N = size of sample	6: $\bar{\eta}^2$	6: $\sigma_{\bar{\eta}^2}$	6: P_1 $\lambda_1 = 3{\cdot}11$	6: P_2 $\lambda_2 = 2{\cdot}63$	7: $\bar{\eta}^2$	7: $\sigma_{\bar{\eta}^2}$	7: P_1 $\lambda_1 = 3{\cdot}08$	7: P_2 $\lambda_2 = 2{\cdot}58$	8: $\bar{\eta}^2$	8: $\sigma_{\bar{\eta}^2}$	8: P_1 $\lambda_1 = 3{\cdot}02$	8: P_2 $\lambda_2 = 2{\cdot}54$
651	·007692	·004839	·011	·021	·009231	·005297	·010	·021	·010769	·005717	·011	·021
652	·007680	·004831	·011	·021	·009217	·005289	·010	·021	·010753	·005708	·011	·021
653	·007669	·004824	·011	·021	·009202	·005280	·010	·021	·010736	·005699	·011	·021
654	·007657	·004817	·011	·021	·009188	·005272	·010	·021	·010720	·005690	·011	·021
655	·007645 +	·004809	·011	·021	·009174	·005264	·010	·021	·010703	·005682	·011	·021
656	·007634	·004802	·011	·021	·009160	·005256	·010	·021	·010687	·005673	·011	·021
657	·007622	·004795 −	·011	·021	·009146	·005248	·010	·021	·010671	·005665 −	·011	·021
658	·007610	·004788	·011	·021	·009132	·005241	·010	·021	·010654	·005656	·011	·021
659	·007599	·004780	·011	·021	·009119	·005233	·010	·021	·010638	·005648	·011	·021
660	·007587	·004773	·011	·021	·009105 −	·005225 −	·010	·021	·010622	·005639	·011	·021
661	·007576	·004766	·011	·021	·009091	·005217	·010	·021	·010606	·005631	·011	·021
662	·007564	·004759	·011	·021	·009077	·005209	·010	·021	·010590	·005622	·011	·021
663	·007553	·004752	·011	·021	·009063	·005201	·010	·021	·010574	·005614	·011	·021
664	·007541	·004744	·011	·021	·009050 −	·005193	·010	·021	·010558	·005605 +	·011	·021
665	·007530	·004737	·011	·021	·009036	·005186	·010	·021	·010542	·005597	·011	·021
666	·007519	·004730	·011	·021	·009023	·005178	·010	·021	·010526	·005588	·011	·021
667	·007508	·004723	·011	·021	·009009	·005170	·010	·021	·010511	·005580	·011	·021
668	·007496	·004716	·011	·021	·008996	·005162	·010	·021	·010495 −	·005572	·011	·021
669	·007485 +	·004709	·011	·021	·008982	·005155 −	·010	·021	·010479	·005564	·011	·021
670	·007474	·004702	·011	·021	·008969	·005147	·010	·021	·010463	·005555 +	·011	·021
671	·007463	·004695 +	·011	·021	·008955 +	·005139	·010	·021	·010448	·005547	·011	·021
672	·007452	·004688	·011	·021	·008942	·005132	·010	·021	·010432	·005539	·011	·021
673	·007440	·004681	·011	·021	·008929	·005124	·010	·021	·010417	·005531	·011	·021
674	·007429	·004674	·011	·021	·008915 +	·005117	·010	·021	·010401	·005522	·011	·021
675	·007418	·004667	·011	·021	·008902	·005109	·010	·021	·010386	·005514	·011	·021
676	·007407	·004661	·011	·021	·008889	·005102	·010	·021	·010370	·005506	·011	·021
677	·007396	·004654	·011	·021	·008876	·005094	·010	·021	·010355 +	·005498	·011	·021
678	·007386	·004647	·011	·021	·008863	·005087	·010	·021	·010340	·005490	·011	·021
679	·007375 −	·004640	·011	·021	·008850 −	·005079	·010	·021	·010324	·005482	·011	·021
680	·007364	·004633	·011	·021	·008837	·005072	·010	·021	·010309	·005474	·011	·021
681	·007353	·004626	·011	·021	·008824	·005064	·010	·021	·010294	·005466	·011	·021
682	·007342	·004620	·011	·021	·008811	·005057	·010	·021	·010279	·005458	·011	·021
683	·007331	·004613	·011	·021	·008798	·005050 −	·010	·021	·010264	·005450 +	·011	·021
684	·007321	·004606	·011	·021	·008785 −	·005042	·010	·021	·010249	·005442	·011	·021
685	·007310	·004600	·011	·021	·008772	·005035 −	·010	·021	·010234	·005434	·011	·021
686	·007299	·004593	·011	·021	·008759	·005028	·010	·021	·010219	·005426	·011	·021
687	·007289	·004586	·011	·021	·008746	·005020	·010	·021	·010204	·005419	·011	·021
688	·007278	·004580	·011	·021	·008734	·005013	·010	·021	·010189	·005411	·011	·021
689	·007267	·004573	·011	·021	·008721	·005006	·010	·021	·010174	·005403	·011	·021
690	·007257	·004566	·011	·021	·008708	·004999	·010	·021	·010160	·005395 +	·011	·021
691	·007246	·004560	·011	·021	·008696	·004991	·010	·021	·010145 −	·005387	·011	·021
692	·007236	·004553	·011	·021	·008683	·004984	·010	·021	·010130	·005380	·011	·021
693	·007225 +	·004547	·011	·021	·008671	·004977	·010	·021	·010116	·005372	·011	·021
694	·007215 +	·004540	·011	·021	·008658	·004970	·010	·021	·010101	·005364	·011	·021
695	·007205 −	·004544	·011	·021	·008646	·004963	·010	·021	·010086	·005356	·011	·021
696	·007194	·004527	·011	·021	·008633	·004956	·010	·021	·010072	·005349	·011	·021
697	·007184	·004521	·011	·021	·008621	·004949	·010	·021	·010057	·005341	·011	·021
698	·007174	·004514	·011	·021	·008608	·004941	·010	·021	·010043	·005334	·011	·021
699	·007163	·004508	·011	·021	·008596	·004934	·010	·021	·010029	·005326	·011	·021
700	·007153	·004501	·011	·021	·008584	·004927	·010	·021	·010014	·005318	·011	·021

N = size of sample	9				10				11			
	$\overline{\eta}^2$	$\sigma_{\overline{\eta}^2}$	P_1 $\lambda_1=2{\cdot}96$	P_2 $\lambda_2=2{\cdot}50$	$\overline{\eta}^2$	$\sigma_{\overline{\eta}^2}$	P_1 $\lambda_1=2{\cdot}92$	P_2 $\lambda_2=2{\cdot}47$	$\overline{\eta}^2$	$\sigma_{\overline{\eta}^2}$	P_1 $\lambda_1=2{\cdot}88$	P_2 $\lambda_2=2{\cdot}44$
651	·012308	·006106	·011	·021	·013846	·006472	·011	·021	·015385 −	·006817	·011	·021
652	·012289	·006097	·011	·021	·013825 −	·006462	·011	·021	·015361	·006806	·011	·021
653	·012270	·006088	·011	·021	·013804	·006452	·011	·021	·015337	·006796	·011	·021
654	·012251	·006079	·011	·021	·013783	·006442	·011	·021	·015314	·006786	·011	·021
655	·012232	·006069	·011	·021	·013761	·006433	·011	·021	·015291	·006775 +	·011	·021
656	·012214	·006060	·011	·021	·013740	·006423	·011	·021	·015267	·006765 +	·011	·021
657	·012195 +	·006051	·011	·021	·013720	·006413	·011	·021	·015244	·006755 −	·011	·021
658	·012177	·006042	·011	·021	·013699	·006403	·011	·021	·015221	·006745 −	·011	·021
659	·012158	·006033	·011	·021	·013678	·006394	·011	·021	·015198	·006734	·011	·021
660	·012140	·006024	·011	·021	·013657	·006384	·011	·021	·015175 −	·006724	·011	·021
661	·012121	·006015 −	·011	·021	·013636	·006375 −	·011	·021	·015152	·006714	·011	·021
662	·012103	·006006	·011	·021	·013616	·006365 +	·011	·021	·015129	·006704	·011	·021
663	·012085 −	·005997	·011	·021	·013595 +	·006356	·011	·021	·015106	·006694	·011	·021
664	·012066	·005988	·011	·021	·013575 −	·006346	·011	·021	·015083	·006684	·011	·021
665	·012048	·005979	·011	·021	·013554	·006337	·011	·021	·015060	·006674	·011	·021
666	·012030	·005970	·011	·021	·013534	·006327	·011	·021	·015038	·006664	·011	·021
667	·012012	·005961	·011	·021	·013514	·006318	·011	·021	·015015 +	·006654	·011	·021
668	·011994	·005952	·011	·021	·013493	·006308	·011	·021	·014993	·006644	·011	·021
669	·011976	·005943	·011	·021	·013473	·006299	·011	·021	·014970	·006635 −	·011	·021
670	·011958	·005934	·011	·021	·013453	·006290	·011	·021	·014948	·006625 −	·011	·021
671	·011940	·005926	·011	·021	·013433	·006280	·011	·021	·014925 +	·006615 −	·011	·021
672	·011923	·005917	·011	·021	·013413	·006271	·011	·021	·014903	·006605 +	·011	·021
673	·011905 −	·005908	·011	·021	·013393	·006262	·011	·021	·014881	·006595 +	·011	·021
674	·011887	·005899	·011	·021	·013373	·006252	·011	·021	·014859	·006586	·011	·021
675	·011869	·005891	·011	·021	·013353	·006243	·011	·021	·014837	·006576	·011	·021
676	·011852	·005882	·011	·021	·013333	·006234	·011	·021	·014815 −	·006566	·011	·021
677	·011834	·005873	·011	·021	·013314	·006225 −	·011	·021	·014793	·006557	·011	·021
678	·011817	·005865 −	·011	·021	·013294	·006216	·011	·021	·014771	·006547	·011	·021
679	·011799	·005856	·011	·021	·013274	·006207	·011	·021	·014749	·006538	·011	·021
680	·011782	·005848	·011	·021	·013255 −	·006198	·011	·021	·014728	·006528	·011	·021
681	·011765 −	·005839	·011	·021	·013235 +	·006189	·011	·021	·014706	·006519	·011	·021
682	·011747	·005831	·011	·021	·013216	·006180	·011	·021	·014684	·006509	·011	·021
683	·011730	·005822	·011	·021	·013196	·006171	·011	·021	·014663	·006500 −	·011	·021
684	·011713	·005814	·011	·021	·013177	·006162	·011	·021	·014641	·006490	·011	·021
685	·011696	·005805 +	·011	·021	·013158	·006153	·011	·021	·014620	·006481	·011	·021
686	·011679	·005797	·011	·021	·013139	·006144	·011	·021	·014599	·006471	·011	·021
687	·011662	·005788	·011	·021	·013120	·006135 −	·011	·021	·014577	·006462	·011	·021
688	·011645 −	·005780	·011	·021	·013100	·006126	·011	·021	·014556	·006453	·011	·021
689	·011628	·005772	·011	·021	·013081	·006117	·011	·021	·014535 −	·006443	·011	·021
690	·011611	·005763	·011	·021	·013062	·006108	·011	·021	·014514	·006434	·011	·021
691	·011594	·005755 +	·011	·021	·013043	·006100	·011	·021	·014493	·006425 −	·011	·021
692	·011577	·005747	·011	·021	·013025 −	·006091	·011	·021	·014472	·006416	·011	·021
693	·011561	·005739	·011	·021	·013006	·006082	·011	·021	·014451	·006407	·011	·021
694	·011544	·005730	·011	·021	·012987	·006074	·011	·021	·014430	·006397	·011	·021
695	·011527	·005722	·011	·021	·012968	·006065 −	·011	·021	·014409	·006388	·011	·021
696	·011511	·005714	·011	·021	·012950 −	·006056	·011	·021	·014388	·006379	·011	·021
697	·011494	·005706	·011	·021	·012931	·006048	·011	·021	·014368	·006370	·011	·021
698	·011478	·005698	·011	·021	·012912	·006039	·011	·021	·014347	·006361	·011	·021
699	·011461	·005690	·011	·021	·012894	·006030	·011	·021	·014327	·006352	·011	·021
700	·011445 −	·005681	·011	·021	·012876	·006022	·011	·021	·014306	·006343	·011	·021

N = size of sample	12				13				14			
	$\overline{\eta}^2$	$\sigma_{\overline{\eta}^2}$	P_1 $\lambda_1=2{\cdot}84$	P_2 $\lambda_2=2{\cdot}42$	$\overline{\eta}^2$	$\sigma_{\overline{\eta}^2}$	P_1 $\lambda_1=2{\cdot}80$	P_2 $\lambda_2=2{\cdot}40$	$\overline{\eta}^2$	$\sigma_{\overline{\eta}^2}$	P_1 $\lambda_1=2{\cdot}79$	P_2 $\lambda_2=2{\cdot}39$
651	·016923	·007144	·011	·021	·018462	·007456	·011	·022	·020000	·007754	·011	·021
652	·016897	·007133	·011	·021	·018433	·007444	·011	·022	·019969	·007742	·011	·021
653	·016871	·007122	·011	·021	·018405 −	·007433	·011	·022	·019939	·007730	·011	·021
654	·016845 +	·007111	·011	·021	·018377	·007422	·011	·022	·019908	·007719	·011	·021
655	·016820	·007100	·011	·021	·018349	·007410	·011	·022	·019878	·007707	·011	·021
656	·016794	·007090	·011	·021	·018321	·007399	·011	·022	·019847	·007695 +	·011	·021
657	·016768	·007079	·011	·021	·018293	·007388	·011	·022	·019817	·007684	·011	·021
658	·016743	·007068	·011	·021	·018265 −	·007377	·011	·022	·019787	·007672	·011	·021
659	·016717	·007058	·011	·021	·018237	·007366	·011	·022	·019757	·007661	·011	·021
660	·016692	·007047	·011	·021	·018209	·007355 −	·011	·022	·019727	·007649	·011	·021
661	·016667	·007037	·011	·021	·018182	·007344	·011	·022	·019697	·007638	·011	·021
662	·016641	·007026	·011	·021	·018154	·007333	·011	·022	·019667	·007626	·011	·021
663	·016616	·007016	·011	·021	·018127	·007322	·011	·022	·019637	·007615 −	·011	·021
664	·016591	·007005 +	·011	·021	·018100	·007311	·011	·022	·019608	·007604	·011	·021
665	·016566	·006995 −	·011	·021	·018072	·007300	·011	·022	·019578	·007592	·011	·021
666	·016541	·006984	·011	·021	·018045 +	·007289	·011	·022	·019549	·007581	·011	·021
667	·016517	·006974	·011	·021	·018018	·007278	·011	·022	·019520	·007570	·011	·021
668	·016492	·006963	·011	·021	·017991	·007268	·011	·022	·019490	·007559	·011	·021
669	·016467	·006953	·011	·021	·017964	·007257	·011	·022	·019461	·007547	·011	·021
670	·016442	·006943	·011	·021	·017937	·007246	·011	·022	·019432	·007536	·011	·021
671	·016418	·006933	·011	·021	·017910	·007235 +	·011	·022	·019403	·007525 +	·011	·021
672	·016393	·006922	·011	·021	·017884	·007225 −	·011	·022	·019374	·007514	·011	·021
673	·016369	·006912	·011	·021	·017857	·007214	·011	·022	·019345 +	·007503	·011	·021
674	·016345 −	·006902	·011	·021	·017831	·007203	·011	·022	·019316	·007492	·011	·021
675	·016320	·006892	·011	·021	·017804	·007193	·011	·022	·019288	·007481	·011	·021
676	·016296	·006882	·011	·021	·017778	·007182	·011	·022	·019259	·007470	·011	·021
677	·016272	·006872	·011	·021	·017751	·007172	·011	·022	·019231	·007459	·011	·021
678	·016248	·006862	·011	·021	·017725 +	·007161	·011	·022	·019202	·007448	·011	·021
679	·016224	·006852	·011	·021	·017699	·007151	·011	·022	·019174	·007437	·011	·021
680	·016200	·006842	·011	·021	·017673	·007140	·011	·022	·019146	·007426	·011	·021
681	·016176	·006832	·011	·021	·017647	·007130	·011	·022	·019118	·007416	·011	·021
682	·016153	·006822	·011	·021	·017621	·007120	·011	·022	·019090	·007405 −	·011	·021
683	·016129	·006812	·011	·021	·017595 +	·007109	·011	·022	·019062	·007394	·011	·021
684	·016105 +	·006802	·011	·021	·017570	·007099	·011	·022	·019034	·007383	·011	·021
685	·016082	·006792	·011	·021	·017544	·007089	·011	·022	·019006	·007373	·011	·021
686	·016058	·006782	·011	·021	·017518	·007079	·011	·022	·018978	·007362	·011	·021
687	·016035 −	·006772	·011	·021	·017493	·007068	·011	·022	·018950 +	·007351	·011	·021
688	·016012	·006763	·011	·021	·017467	·007058	·011	·022	·018923	·007341	·011	·021
689	·015988	·006753	·011	·021	·017442	·007048	·011	·022	·018895 +	·007330	·011	·021
690	·015965 +	·006743	·011	·021	·017417	·007038	·011	·022	·018868	·007320	·011	·021
691	·015942	·006734	·011	·021	·017391	·007028	·011	·022	·018841	·007309	·011	·021
692	·015919	·006724	·011	·021	·017366	·007018	·011	·022	·018813	·007299	·011	·021
693	·015896	·006714	·011	·021	·017341	·007008	·011	·022	·018786	·007288	·011	·021
694	·015873	·006705 −	·011	·021	·017316	·006998	·011	·022	·018759	·007278	·011	·021
695	·015850 +	·006695 +	·011	·021	·017291	·006988	·011	·022	·018732	·007268	·011	·021
696	·015827	·006686	·011	·021	·017266	·006978	·011	·022	·018705 +	·007257	·011	·021
697	·015805 −	·006676	·011	·021	·017241	·006968	·011	·022	·018678	·007247	·011	·021
698	·015782	·006667	·011	·021	·017217	·006958	·011	·022	·018651	·007237	·011	·021
699	·015759	·006657	·011	·021	·017192	·006948	·011	·022	·018625 −	·007226	·011	·021
700	·015737	·006648	·011	·021	·017167	·006938	·011	·022	·018598	·007216	·011	·021

n = number of arrays

N = size of sample	15: $\bar{\eta}^2$	15: $\sigma_{\bar{\eta}^2}$	15: P_1 $\lambda_1=2{\cdot}78$	15: P_2 $\lambda_2=2{\cdot}38$	16: $\bar{\eta}^2$	16: $\sigma_{\bar{\eta}^2}$	16: P_1 $\lambda_1=2{\cdot}77$	16: P_2 $\lambda_2=2{\cdot}37$	17: $\bar{\eta}^2$	17: $\sigma_{\bar{\eta}^2}$	17: P_1 $\lambda_1=2{\cdot}76$	17: P_2 $\lambda_2=2{\cdot}36$
651	·021538	·008040	·011	·021	·023077	·008316	·011	·021	·024615 +	·008582	·011	·021
652	·021505 +	·008028	·011	·021	·023041	·008303	·011	·021	·024578	·008569	·011	·021
653	·021472	·008016	·011	·021	·023006	·008291	·011	·021	·024540	·008556	·011	·021
654	·021440	·008004	·011	·021	·022971	·008278	·011	·021	·024502	·008543	·011	·021
655	·021407	·007992	·011	·021	·022936	·008266	·011	·021	·024465 −	·008530	·011	·021
656	·021374	·007980	·011	·021	·022901	·008253	·011	·021	·024427	·008517	·011	·021
657	·021341	·007968	·011	·021	·022866	·008241	·011	·021	·024390	·008504	·011	·021
658	·021309	·007956	·011	·021	·022831	·008228	·011	·021	·024353	·008492	·011	·021
659	·021277	·007944	·011	·021	·022796	·008216	·011	·021	·024316	·008479	·011	·021
660	·021244	·007932	·011	·021	·022762	·008204	·011	·021	·024279	·008466	·011	·021
661	·021212	·007920	·011	·021	·022727	·008192	·011	·021	·024242	·008454	·011	·021
662	·021180	·007908	·011	·021	·022693	·008179	·011	·021	·024206	·008441	·011	·021
663	·021148	·007896	·011	·021	·022659	·008167	·011	·021	·024169	·008428	·011	·021
664	·021116	·007885 −	·011	·021	·022624	·008155 +	·011	·021	·024133	·008416	·011	·021
665	·021084	·007873	·011	·021	·022590	·008143	·011	·021	·024096	·008403	·011	·021
666	·021053	·007861	·011	·021	·022556	·008131	·011	·021	·024060	·008391	·011	·021
667	·021021	·007849	·011	·021	·022523	·008119	·011	·021	·024024	·008379	·011	·021
668	·020990	·007838	·011	·021	·022489	·008107	·011	·021	·023988	·008366	·011	·021
669	·020958	·007826	·011	·021	·022455 +	·008095 −	·011	·021	·023952	·008354	·011	·021
670	·020927	·007815 −	·011	·021	·022422	·008083	·011	·021	·023916	·008342	·011	·021
671	·020896	·007803	·011	·021	·022388	·008071	·011	·021	·023881	·008329	·011	·021
672	·020864	·007792	·011	·021	·022355 −	·008059	·011	·021	·023845 +	·008317	·011	·021
673	·020833	·007780	·011	·021	·022321	·008047	·011	·021	·023810	·008305 −	·011	·021
674	·020802	·007769	·011	·021	·022288	·008035 +	·011	·021	·023774	·008293	·011	·021
675	·020772	·007757	·011	·021	·022255 +	·008024	·011	·021	·023739	·008280	·011	·021
676	·020741	·007746	·011	·021	·022222	·008012	·011	·021	·023704	·008268	·011	·021
677	·020710	·007735 −	·011	·021	·022189	·008000	·011	·021	·023669	·008256	·011	·021
678	·020679	·007723	·011	·021	·022157	·007989	·011	·021	·023634	·008244	·011	·021
679	·020649	·007712	·011	·021	·022124	·007977	·011	·021	·023599	·008232	·011	·021
680	·020619	·007701	·011	·021	·022091	·007965 +	·011	·021	·023564	·008220	·011	·021
681	·020588	·007690	·011	·021	·022059	·007954	·011	·021	·023529	·008208	·011	·021
682	·020558	·007679	·011	·021	·022026	·007942	·011	·021	·023495 −	·008197	·011	·021
683	·020528	·007668	·011	·021	·021994	·007931	·011	·021	·023460	·008185 −	·011	·021
684	·020498	·007656	·011	·021	·021962	·007919	·011	·021	·023426	·008173	·011	·021
685	·020468	·007645 +	·011	·021	·021930	·007908	·011	·021	·023392	·008161	·011	·021
686	·020438	·007634	·011	·021	·021898	·007896	·011	·021	·023358	·008149	·011	·021
687	·020408	·007623	·011	·021	·021866	·007885 +	·011	·021	·023324	·008138	·011	·021
688	·020378	·007612	·011	·021	·021834	·007874	·011	·021	·023290	·008126	·011	·021
689	·020349	·007601	·011	·021	·021802	·007862	·011	·021	·023256	·008114	·011	·021
690	·020319	·007591	·011	·021	·021771	·007851	·011	·021	·023222	·008103	·011	·021
691	·020290	·007580	·011	·021	·021739	·007840	·011	·021	·023188	·008091	·011	·021
692	·020260	·007569	·011	·021	·021708	·007829	·011	·021	·023155 −	·008079	·011	·021
693	·020231	·007558	·011	·021	·021676	·007818	·011	·021	·023121	·008068	·011	·021
694	·020202	·007547	·011	·021	·021645 +	·007806	·011	·021	·023088	·008056	·011	·021
695	·020173	·007536	·011	·021	·021614	·007795 +	·011	·021	·023055 −	·008045 +	·011	·021
696	·020144	·007526	·011	·021	·021583	·007784	·011	·021	·023022	·008034	·011	·021
697	·020115 −	·007515 +	·011	·021	·021552	·007773	·011	·021	·022989	·008022	·011	·021
698	·020086	·007504	·011	·021	·021521	·007762	·011	·021	·022956	·008011	·011	·021
699	·020057	·007494	·011	·021	·021490	·007751	·011	·021	·022923	·008000	·011	·021
700	·020029	·007483	·011	·021	·021459	·007740	·011	·021	·022890	·007988	·011	·021

N = size of sample	18: $\bar{\eta}^2$	18: $\sigma_{\bar{\eta}^2}$	18: P_1 $\lambda_1=2{\cdot}75$	18: P_2 $\lambda_2=2{\cdot}35$	19: $\bar{\eta}^2$	19: $\sigma_{\bar{\eta}^2}$	19: P_1 $\lambda_1=2{\cdot}74$	19: P_2 $\lambda_2=2{\cdot}34$	20: $\bar{\eta}^2$	20: $\sigma_{\bar{\eta}^2}$	20: P_1 $\lambda_1=2{\cdot}73$	20: P_2 $\lambda_2=2{\cdot}33$
651	·026154	·008839	·011	·021	·027692	·009088	·011	·021	·029231	·009330	·011	·021
652	·026114	·008826	·011	·021	·027650 −	·009074	·011	·021	·029186	·009316	·011	·021
653	·026074	·008812	·011	·021	·027607	·009061	·011	·021	·029141	·009302	·011	·021
654	·026034	·008799	·011	·021	·027565 +	·009047	·011	·021	·029096	·009288	·011	·021
655	·025994	·008786	·011	·021	·027523	·009033	·011	·021	·029052	·009274	·011	·021
656	·025954	·008773	·011	·021	·027481	·009020	·011	·021	·029008	·009260	·011	·021
657	·025915 −	·008759	·011	·021	·027439	·009006	·011	·021	·028963	·009246	·011	·021
658	·025875 +	·008746	·011	·021	·027397	·008993	·011	·021	·028919	·009232	·011	·021
659	·025836	·008733	·011	·021	·027356	·008979	·011	·021	·028875 +	·009218	·011	·021
660	·025797	·008720	·011	·021	·027314	·008966	·011	·021	·028832	·009204	·011	·021
661	·025758	·008707	·011	·021	·027273	·008953	·011	·021	·028788	·009191	·011	·021
662	·025719	·008694	·011	·021	·027231	·008939	·011	·021	·028744	·009177	·011	·021
663	·025680	·008681	·011	·021	·027190	·008926	·011	·021	·028701	·009163	·011	·021
664	·025641	·008668	·011	·021	·027149	·008913	·011	·021	·028658	·009150 −	·011	·021
665	·025602	·008655 +	·011	·021	·027108	·008899	·011	·021	·028614	·009136	·011	·021
666	·025564	·008643	·011	·021	·027068	·008886	·011	·021	·028571	·009123	·011	·021
667	·025526	·008630	·011	·021	·027027	·008873	·011	·021	·028529	·009109	·011	·021
668	·025487	·008617	·011	·021	·026987	·008860	·011	·021	·028486	·009096	·011	·021
669	·025449	·008604	·011	·021	·026946	·008847	·011	·021	·028443	·009082	·011	·021
670	·025411	·008592	·011	·021	·026906	·008834	·011	·021	·028401	·009069	·011	·021
671	·025373	·008579	·011	·021	·026866	·008821	·011	·021	·028358	·009056	·011	·021
672	·025335 +	·008566	·011	·021	·026826	·008808	·011	·021	·028316	·009042	·011	·021
673	·025298	·008554	·011	·021	·026786	·008795 +	·011	·021	·028274	·009029	·011	·021
674	·025260	·008541	·011	·021	·026746	·008782	·011	·021	·028232	·009016	·011	·021
675	·025223	·008529	·011	·021	·026706	·008769	·011	·021	·028190	·009003	·011	·021
676	·025185 +	·008516	·011	·021	·026667	·008757	·011	·021	·028148	·008990	·011	·021
677	·025148	·008504	·011	·021	·026627	·008744	·011	·021	·028107	·008977	·011	·021
678	·025111	·008492	·011	·021	·026588	·008731	·011	·021	·028065 −	·008964	·011	·021
679	·025074	·008479	·011	·021	·026549	·008718	·011	·021	·028024	·008951	·011	·021
680	·025037	·008467	·011	·021	·026510	·008706	·011	·021	·027982	·008938	·011	·021
681	·025000	·008455 −	·011	·021	·026471	·008693	·011	·021	·027941	·008925 −	·011	·021
682	·024963	·008442	·011	·021	·026432	·008681	·011	·021	·027900	·008912	·011	·021
683	·024927	·008430	·011	·021	·026393	·008668	·011	·021	·027859	·008899	·011	·021
684	·024890	·008418	·011	·021	·026354	·008656	·011	·021	·027818	·008886	·011	·021
685	·024854	·008406	·011	·021	·026316	·008643	·011	·021	·027778	·008873	·011	·021
686	·024818	·008394	·011	·021	·026277	·008631	·011	·021	·027737	·008861	·011	·021
687	·024781	·008382	·011	·021	·026239	·008618	·011	·021	·027697	·008848	·011	·021
688	·024745 +	·008370	·011	·021	·026201	·008606	·011	·021	·027656	·008835 +	·011	·021
689	·024709	·008358	·011	·021	·026163	·008594	·011	·021	·027616	·008823	·011	·021
690	·024673	·008346	·011	·021	·026125 −	·008581	·011	·021	·027576	·008810	·011	·021
691	·024638	·008334	·011	·021	·026087	·008569	·011	·021	·027536	·008797	·011	·021
692	·024602	·008322	·011	·021	·026049	·008557	·011	·021	·027496	·008785 −	·011	·021
693	·024566	·008310	·011	·021	·026012	·008545 −	·011	·021	·027457	·008772	·011	·021
694	·024531	·008298	·011	·021	·025974	·008533	·011	·021	·027417	·008760	·011	·021
695	·024496	·008286	·011	·021	·025937	·008520	·011	·021	·027378	·008747	·011	·021
696	·024460	·008275 −	·011	·021	·025899	·008508	·011	·021	·027338	·008735 +	·011	·021
697	·024425 +	·008263	·011	·021	·025862	·008496	·011	·021	·027299	·008723	·011	·021
698	·024390	·008251	·011	·021	·025825 −	·008484	·011	·021	·027260	·008710	·011	·021
699	·024355 +	·008240	·011	·021	·025788	·008472	·011	·021	·027221	·008698	·011	·021
700	·024320	·008228	·011	·021	·025751	·008460	·011	·021	·027182	·008686	·011	·021

N = size of sample	3: $\bar{\eta}^2$	3: $\sigma_{\bar{\eta}^2}$	3: P_1 λ_1 = 3·50	3: P_2 λ_2 = 2·94	4: $\bar{\eta}^2$	4: $\sigma_{\bar{\eta}^2}$	4: P_1 λ_1 = 3·20	4: P_2 λ_2 = 2·80
701	·002857	·002849	·011	·019	·004286	·003487	·013	·020
702	·002853	·002845 −	·011	·019	·004280	·003482	·013	·020
703	·002849	·002841	·011	·019	·004274	·003477	·013	·020
704	·002845 −	·002837	·011	·019	·004267	·003472	·013	·020
705	·002841	·002833	·011	·019	·004261	·003467	·013	·020
706	·002837	·002829	·011	·019	·004255 +	·003462	·013	·020
707	·002833	·002825 −	·011	·019	·004249	·003457	·013	·020
708	·002829	·002821	·011	·019	·004243	·003452	·013	·020
709	·002825 −	·002817	·011	·019	·004237	·003448	·013	·020
710	·002821	·002813	·011	·019	·004231	·003443	·013	·020
711	·002817	·002809	·011	·019	·004225 +	·003438	·013	·020
712	·002813	·002805 +	·011	·019	·004219	·003433	·013	·020
713	·002809	·002801	·011	·019	·004213	·003428	·013	·020
714	·002805 +	·002797	·011	·019	·004208	·003423	·013	·020
715	·002801	·002793	·011	·019	·004202	·003419	·013	·020
716	·002797	·002789	·011	·019	·004196	·003414	·013	·020
717	·002793	·002786	·011	·019	·004190	·003409	·013	·020
718	·002789	·002782	·011	·019	·004184	·003404	·013	·020
719	·002786	·002778	·011	·019	·004178	·003400	·013	·020
720	·002782	·002774	·011	·019	·004172	·003395 −	·013	·020
721	·002778	·002770	·011	·019	·004167	·003390	·013	·020
722	·002774	·002766	·011	·019	·004161	·003386	·013	·020
723	·002770	·002762	·011	·019	·004155 +	·003381	·013	·020
724	·002766	·002759	·011	·019	·004149	·003376	·013	·020
725	·002762	·002755 −	·011	·019	·004144	·003372	·013	·020
726	·002759	·002751	·011	·019	·004138	·003367	·013	·020
727	·002755 −	·002747	·011	·019	·004132	·003362	·013	·020
728	·002751	·002743	·011	·019	·004127	·003358	·013	·020
729	·002747	·002740	·011	·019	·004121	·003353	·013	·020
730	·002743	·002736	·011	·019	·004115 +	·003349	·013	·020
731	·002740	·002732	·011	·019	·004110	·003344	·013	·020
732	·002736	·002729	·011	·019	·004104	·003339	·013	·020
733	·002732	·002725 −	·011	·019	·004098	·003335 −	·013	·020
734	·002729	·002721	·011	·019	·004093	·003330	·013	·020
735	·002725 −	·002717	·011	·019	·004087	·003326	·013	·020
736	·002721	·002714	·011	·019	·004082	·003321	·013	·020
737	·002717	·002710	·011	·019	·004076	·003317	·013	·020
738	·002714	·002706	·011	·019	·004071	·003312	·013	·020
739	·002710	·002703	·011	·019	·004065 +	·003308	·013	·020
740	·002706	·002699	·011	·019	·004060	·003303	·013	·020
741	·002703	·002695 +	·011	·019	·004054	·003299	·013	·020
742	·002699	·002692	·011	·019	·004049	·003295 −	·013	·020
743	·002695 +	·002688	·011	·019	·004043	·003290	·013	·020
744	·002692	·002685 −	·011	·019	·004038	·003286	·013	·020
745	·002688	·002681	·011	·019	·004032	·003281	·013	·020
746	·002685 −	·002677	·011	·019	·004027	·003277	·013	·020
747	·002681	·002674	·011	·019	·004021	·003273	·013	·020
748	·002677	·002670	·011	·019	·004016	·003268	·013	·020
749	·002674	·002667	·011	·019	·004011	·003264	·013	·020
750	·002670	·002663	·011	·019	·004005 +	·003259	·013	·020

N = size of sample	5: $\bar{\eta}^2$	5: $\sigma_{\bar{\eta}^2}$	5: P_1 λ_1 = 3·14	5: P_2 λ_2 = 2·68	6: $\bar{\eta}^2$	6: $\sigma_{\bar{\eta}^2}$	6: P_1 λ_1 = 3·11	6: P_2 λ_2 = 2·63
701	·005714	·004023	·012	·021	·007143	·004495 −	·011	·021
702	·005706	·004018	·012	·021	·007133	·004489	·011	·021
703	·005698	·004012	·012	·021	·007123	·004482	·011	·021
704	·005690	·004006	·012	·021	·007112	·004476	·011	·021
705	·005682	·004001	·012	·021	·007102	·004470	·011	·021
706	·005674	·003995 −	·012	·021	·007092	·004463	·011	·021
707	·005666	·003989	·012	·021	·007082	·004457	·011	·021
708	·005658	·003984	·012	·021	·007072	·004451	·011	·021
709	·005650 −	·003978	·012	·021	·007062	·004444	·011	·021
710	·005642	·003972	·012	·021	·007052	·004438	·011	·021
711	·005634	·003967	·012	·021	·007042	·004432	·011	·021
712	·005626	·003961	·012	·021	·007032	·004426	·011	·021
713	·005618	·003956	·012	·021	·007022	·004420	·011	·021
714	·005610	·003950 +	·012	·021	·007013	·004413	·011	·021
715	·005602	·003945 −	·012	·021	·007003	·004407	·011	·021
716	·005594	·003939	·012	·021	·006993	·004401	·011	·021
717	·005587	·003934	·012	·021	·006983	·004395 +	·011	·021
718	·005579	·003928	·012	·021	·006974	·004389	·011	·021
719	·005571	·003923	·012	·021	·006964	·004383	·011	·021
720	·005563	·003917	·012	·021	·006954	·004377	·011	·021
721	·005556	·003912	·012	·021	·006944	·004371	·011	·021
722	·005548	·003907	·012	·021	·006935 −	·004365 −	·011	·021
723	·005540	·003901	·012	·021	·006925 +	·004359	·011	·021
724	·005533	·003896	·012	·021	·006916	·004353	·011	·021
725	·005525 −	·003890	·012	·021	·006906	·004347	·011	·021
726	·005517	·003885 +	·012	·021	·006897	·004341	·011	·021
727	·005510	·003880	·012	·021	·006887	·004335 −	·011	·021
728	·005502	·003875 −	·012	·021	·006878	·004329	·011	·021
729	·005495 −	·003869	·012	·021	·006868	·004323	·011	·021
730	·005487	·003864	·012	·021	·006859	·004317	·011	·021
731	·005479	·003859	·012	·021	·006849	·004311	·011	·021
732	·005472	·003853	·012	·021	·006840	·004305 +	·011	·021
733	·005464	·003848	·012	·021	·006831	·004299	·011	·021
734	·005457	·003843	·012	·021	·006821	·004294	·011	·021
735	·005450 −	·003838	·012	·021	·006812	·004288	·011	·021
736	·005442	·003833	·012	·021	·006803	·004282	·011	·021
737	·005435 −	·003827	·012	·021	·006793	·004276	·011	·021
738	·005427	·003822	·012	·021	·006784	·004270	·011	·021
739	·005420	·003817	·012	·021	·006775 +	·004265 −	·011	·021
740	·005413	·003812	·012	·021	·006766	·004259	·011	·021
741	·005405 +	·003807	·012	·021	·006757	·004253	·011	·021
742	·005398	·003802	·012	·021	·006748	·004247	·011	·021
743	·005391	·003796	·012	·021	·006739	·004242	·011	·021
744	·005384	·003791	·012	·021	·006729	·004236	·011	·021
745	·005376	·003786	·012	·021	·006720	·004230	·011	·021
746	·005369	·003781	·012	·021	·006711	·004225 −	·011	·021
747	·005362	·003776	·012	·021	·006702	·004219	·011	·021
748	·005355 −	·003771	·012	·021	·006693	·004213	·011	·021
749	·005348	·003766	·012	·021	·006684	·004208	·011	·021
750	·005340	·003761	·012	·021	·006676	·004202	·011	·021

N = size of sample	7: $\bar{\eta}^2$	7: $\sigma_{\bar{\eta}^2}$	7: P_1 λ_1 = 3·08	7: P_2 λ_2 = 2·58	8: $\bar{\eta}^2$	8: $\sigma_{\bar{\eta}^2}$	8: P_1 λ_1 = 3·02	8: P_2 λ_2 = 2·54
701	·008571	·004920	·010	·021	·010000	·005311	·011	·021
702	·008559	·004913	·010	·021	·009986	·005303	·011	·021
703	·008547	·004906	·010	·021	·009972	·005296	·011	·021
704	·008535 −	·004900	·010	·021	·009957	·005288	·011	·021
705	·008523	·004893	·010	·021	·009943	·005281	·011	·021
706	·008511	·004886	·010	·021	·009929	·005273	·011	·021
707	·008499	·004879	·010	·021	·009915 +	·005266	·011	·021
708	·008487	·004872	·010	·021	·009901	·005259	·011	·021
709	·008475 −	·004865 +	·010	·021	·009887	·005251	·011	·021
710	·008463	·004858	·010	·021	·009873	·005244	·011	·021
711	·008451	·004852	·010	·021	·009859	·005237	·011	·021
712	·008439	·004845 −	·010	·021	·009845 +	·005229	·011	·021
713	·008427	·004838	·010	·021	·009831	·005222	·011	·021
714	·008415 +	·004831	·010	·021	·009818	·005215 −	·011	·021
715	·008403	·004825 −	·010	·021	·009804	·005207	·011	·021
716	·008392	·004818	·010	·021	·009790	·005200	·011	·021
717	·008380	·004811	·010	·021	·009777	·005193	·011	·021
718	·008368	·004804	·010	·021	·009763	·005186	·011	·021
719	·008357	·004798	·010	·021	·009749	·005179	·011	·021
720	·008345 −	·004791	·010	·021	·009736	·005171	·011	·021
721	·008333	·004785 −	·010	·021	·009722	·005164	·011	·021
722	·008322	·004778	·010	·021	·009709	·005157	·011	·021
723	·008310	·004771	·010	·021	·009695 +	·005150 +	·011	·021
724	·008299	·004765 −	·010	·021	·009682	·005143	·011	·021
725	·008287	·004758	·010	·021	·009669	·005136	·011	·021
726	·008276	·004752	·010	·021	·009655 +	·005129	·011	·021
727	·008264	·004745 +	·010	·021	·009642	·005122	·011	·021
728	·008253	·004739	·010	·021	·009629	·005115 −	·011	·021
729	·008242	·004732	·010	·021	·009615 +	·005108	·011	·021
730	·008230	·004726	·010	·021	·009602	·005101	·011	·021
731	·008219	·004719	·010	·021	·009589	·005094	·011	·021
732	·008208	·004713	·010	·021	·009576	·005087	·011	·021
733	·008197	·004707	·010	·021	·009563	·005080	·011	·021
734	·008186	·004700	·010	·021	·009550 −	·005073	·011	·021
735	·008174	·004694	·010	·021	·009537	·005066	·011	·021
736	·008163	·004687	·010	·021	·009524	·005060	·011	·021
737	·008152	·004681	·010	·021	·009511	·005053	·011	·021
738	·008141	·004675 −	·010	·021	·009498	·005046	·011	·021
739	·008130	·004668	·010	·021	·009485 +	·005039	·011	·021
740	·008119	·004662	·010	·021	·009472	·005032	·011	·021
741	·008108	·004656	·010	·021	·009459	·005026	·011	·021
742	·008097	·004650 −	·010	·021	·009447	·005019	·011	·021
743	·008086	·004643	·010	·021	·009434	·005012	·011	·021
744	·008075 +	·004637	·010	·021	·009421	·005005 +	·011	·021
745	·008065 −	·004631	·010	·021	·009409	·004999	·011	·021
746	·008054	·004625 −	·010	·021	·009396	·004992	·011	·021
747	·008043	·004619	·010	·021	·009383	·004985 +	·011	·021
748	·008032	·004613	·010	·021	·009371	·004979	·011	·021
749	·008021	·004606	·010	·021	·009358	·004972	·011	·021
750	·008011	·004600	·010	·021	·009346	·004966	·011	·021

N size			P_2 $\lambda_1 = 2·96$	$\lambda_2 = 2·50$			P_1 $\lambda_1 = 2·92$	P_2 $\lambda_2 = 2·47$			P_1 $\lambda_1 = 2·88$	P_2 $\lambda_2 = 2·44$			λ_1 2·[illegible]	P_1 $\lambda_2 = 2·42$			P_1 $\lambda_1 = 2·80$	P_1 λ_2 2·40			P_1 $\lambda_1 = 2·79$	P_2 $\lambda_2 = 2·39$
	·011429	·005673	·011	·021	·012857	·006[illegible]3	·01	·02	·014286	·006334	·01	·021	·015714	·006638	·01	·021	·017143	·006	·011	·022	·018571	·00720	·011	·021
	·011412	·005665	·011	·021	·012839	·006[illegible]5 −	·01	·02	·014265 −	·006325	·01	·021	·015692	·006629	·01	·021	·017118	·006	·011	·022	·018545 −	·00719	·011	·021
	·011396	·005657	·011	·021	·012821	·005996	·01	·02	·014245 −	·006316	·01	·021	·015670	·006620	·01	·021	·017094	·006	·011	·022	·018519	·00718	·011	·021
	·011380	·005649	·011	·021	·012802	·005988	·01	·02	·014225 −	·006307	·01	·021	·015647	·006610	·01	·021	·017070	·006	·011	·022	·018492	·00717	·011	·021
[illegible]	·011364	·005641	·011	·021	·012784	·005979	·01	·02	·014205 −	·006298	·01	·021	·015625	·006601	·01	·021	·017045 +	·006	·011	·022	·018466	·00716	·011	·021
706	·011348	·005633	·011	·021	·012766	·005971	·01	·02	·014184	·006289	·01	·021	·015603	·006592	·01	·021	·017021	·006	·011	·022	·018440	·00715	·011	·021
707	·011331	·005626	·011	·021	·012748	·005963	·01	·02	·014164	·006281	·01	·021	·015581	·006582	·01	·021	·016997	·006	·011	·022	·018414	·00714 −	·011	·021
708	·011315	·005618	·011	·021	·012730	·005954	·01	·02	·014144	·006272	·01	·02	·015559	·006573	·01	·021	·016973	·006	·011	·022	·018388	·007135 +	·011	·021
709	·011299	·005610	·011	·021	·012712	·005946	·0	·02	·014124	·006263	·0	·02	·015537	·006564	·01	·021	·016949	·006	·01	·022	·018362	·007126	·011	·021
710	·011283	·005602	·011	·021	·012694	·005938	·01	·02	·014104	·006254	·0	·02	·015515	·006555 −	·01	·021	·016925 +	·006	·01	·022	·018336	·007116	·011	·021
711	·011268	·005594	·011	·021	·012676	·005929	·0	·02	·014085 −	·006245	·0	·02	·015493	·006546	·01	·021	·016901	·006832	·01	·022	·018310	·007106	·011	·021
712	·011252	·005586	·011	·021	·012658	·005921	·0	·02	·014065 −	·006237	·0	·02	·015471	·006537	·01	·021	·016878	·006822	·01	·022	·018284	·007096	·011	·021
713	·011236	·005578	·011	·021	·012640	·005913	·0	·02	·014045 −	·006228	·0	·02	·015449	·006527	·01	·021	·016854	·006813	·01	·022	·018258	·007086	·011	·021
714	·011220	·005571	·011	·021	·012623	·005904	·0	·02	·014025 +	·006219	·0	·02	·015428	·006518	·01	·02	·016830	·006803	·01	·022	·018233	·007076	·011	·021
715	·011204	·005563	·011	·021	·012605	·005896	·0	·02	·014006	·006211	·0	·02	·015406	·006509	·01	·02	·016807	·006794	·01	·022	·018207	·007066	·011	·021
716	·011189	·005555	·011	·021	·012587	·005888	·0	·02	·013986	·006202	·0	·02	·015385	·006500 +	·01	·021	·016783	·006784	·01	·022	·018182	·007057	·011	·021
717	·011173	·005548	·011	·021	·012570	·005880	·0	·02	·013966	·006194	·0	·02	·015363	·006491	·01	·021	·016760	·006775 +	·01	·022	·018156	·007047	·011	·021
718	·011158	·005540	·011	·021	·012552	·005872	·0	·02	·013947	·006185 +	·0	·02	·015342	·006482	·01	·02	·016736	·006766	·01	·022	·018131	·007037	·011	·021
719	·011142	·005532	·011	·021	·012535	·005864	·0	·02	·013928	·006176	·0	·02	·015320	·006473	·01	·02	·016713	·006756	·01	·022	·018106	·007027	·011	·021
720	·011127	·005525	·011	·021	·012517	·005856	·01	·02	·013908	·006168	·0	·02	·015299	·006464	·01	·02	·016690	·006747	·01	·022	·018081	·007018	·011	·021
721	·011111	·005517	·011	·021	·012500	·005847	·01	·02	·013889	·006159	·0	·02	·015278	·006456	·01	·02	·016667	·006738	·01	·022	·018056	·007008	·011	·021
722	·011096	·005509	·011	·021	·012483	·005839	·01	·02	·013870	·006151	·0	·02	·015257	·006447	·01	·02	·016644	·006729	·01	·022	·018031	·006998	·011	·021
723	·011080	·005502	·011	·021	·012465	·005831	·01	·02	·013850 +	·006143	·0	·02	·015235	·006438	·01	·02	·016620	·006719	·01	·022	·018006	·006989	·011	·021
724	·011065 +	·005494	·011	·021	·012448	·005823	·01	·02	·013831	·006134	·0	·02	·015214	·006429	·01	·02	·016598	·006710	·01	·022	·017981	·006979	·011	·021
725	·011050 −	·005487	·01	·021	·012431	·005815	·01	·02	·013812	·006126	·0	·02	·015193	·006420	·01	·02	·016575 −	·006701	·01	·022	·017956	·006970	·011	·021
726	·011034	·005479	·01	·021	·012414	·005807	·01	·02	·013793	·006117	·0	·02	·015172	·006411	·01	·021	·016552	·006692	·01	·022	·017931	·006960	·011	·021
727	·011019	·005472	·01	·021	·012397	·005800	·01	·02	·013774	·006109	·0	·02	·015152	·006403	·01	·02	·016529	·006683	·01	·022	·017906	·006951	·011	·021
728	·011004	·005464	·01	·021	·012380	·005792	·01	·02	·013755 −	·006101	·0	·02	·015131	·006394	·01	·02	·016506	·006674	·01	·022	·017882	·006941	·011	·021
729	·010989	·005457	·01	·021	·012363	·005784	·01	·02	·013736	·006092	·0	·02	·015110	·006385 +	·01	·02	·016484	·006665 −	·01	·022	·017857	·006932	·011	·021
730	·010974	·005449	·01	·021	·012346	·005776	·01	·02	·013717	·006084	·0	·02	·015089	·006377	·01	·02	·016461	·006655 +	·01	·022	·017833	922	·011	·021
731	·010959	·005442	·01	·021	·012329	·005768	·01	·021	·013699	·006076	·0	·02	·015068	·006368	·01	·02	·016438	·006646	·01	·022	·017808	913	·011	·021
732	·010944	·005435 −	·01	·021	·012312	·005760	·01	·021	·013680	·006068	·0	·02	·015048	·006359	·01	·02	·016416	·006637	·01	·022	·017784	904	·011	·021
733	·010929	·005427	·01	·021	·012295 +	·005752	·01	·021	·013661	·006059	·0	·02	·015027	·006351	·01	·02	·016393	·006628	·01	·022	·017760	894	·011	·021
734	·010914	·005420	·01	·021	·012278	·005745	·01	·021	·013643	·006051	·0	·02	·015007	·006342	·01	·02	·016371	·006620	·01	·022	·017735 +	885 +	·011	·021
735	·010899	·005412	·01	·021	·012262	·005737	·01	·02	·013624	·006043	·0	·02	·014986	·006334	·01	·02	·016349	·006611	·01	·022	·017711	876	·011	·021
736	·010884	·005405 +	·01	·021	·012245 −	·005729	·01	·021	·013605	·006035 −	·0	·02	·014966	·006325 −	·01	·02	·016327	·006602	·01	·022	·017687	866	·011	·021
737	·010870	·005398	·01	·021	·012228	·005721	·01	·021	·013587	·006027	·01	·02	·014946	·006316	·01	·02	·016304	·006593	·01	·022	·017663	857	·011	·021
738	·010855 −	·005391	·01	·021	·012212	·005714	·01	·021	·013569	·006019	·0	·02	·014925 +	·006308	·01	·02	·016282	·006584	·01	·022	·017639	848	·011	·021
739	·010840	·005383	·01	·021	·012195 +	·005706	·01	·021	·013550	·006010	·0	·02	·014905 +	·006300	·01	·02	·016260	·006575 +	·01	·022	·017615 +	839	·011	·021
740	·010825 +	·005376	·01	·021	·012179	·005698	·01	·021	·013532	·006002	·0	·02	·014885 −	·006291	·01	·02	·016238	·006566	·01	·022	·017591	830	·011	·021
741	·010811	·005369	·01	·021	·012162	·005691	·01	·021	·013514	·005994	·0	·02	·014865 −	·006283	·01	·02	·016216	·006557	·01	·022	·017568	821	·011	·021
742	·010796	·005362	·01	·021	·012146	·005683	·01	·02	·013495	·005986	·0	·02	·014845 −	·006274	·01	·02	·016194	·006549	·01	·022	·017544	·006811	·011	·021
743	·010782	·005354	·01	·021	·012129	·005675	·01	·02	·013477	·005978	·0	·02	·014825 −	·006266	·01	·02	·016173	·006540	·01	·022	·017520	·006802	·011	·021
744	·010767	·005347	·01	·021	·012113	·005668	·01	·02	·013459	·005970	·0	·02	·014805 −	·006257	·01	·02	·016151	·006531	·01	·022	·017497	·006793	·011	·021
745	·010753	·005340	·01	·021	·012097	·005660	·01	·02	·013441	·005962	·0	·02	·014785 −	·006249	·01	·02	·016129	·006523	·01	·022	·017473	·006784	·011	·021
746	·010738	·005333	·01	·021	·012081	·005653	·01	·02	·013423	·005954	·0	·02	·014765 +	·006241	·01	·02	·016107	·006514	·01	·022	·017450 −	·006775 +	·011	·021
747	·010724	·005326	·01	·021	·012064	·005645	·01	·02	·013405	·005947	·0	·02	·014745 +	·006233	·01	·02	·016086	·006505 +	·01	·022	·017426	·006766	·011	·021
748	[illegible]0	·005319	·01	·021	·012048	·005638	·01	·02	·013387	·005939	·0	·02	·014726	·006224	·01	·02	·016064	·006497	·01	·022	·017403	757	·011	·021
749	·01[illegible]5 +	·005312	·01	·021	·012032	·005630	·01	·02	·013369	·005931	·0	·02	·014706	·006216	·0	·02	·016043	·006488	·01	·022	·017380	748	·011	·021
750	·01[illegible]1	·005305 −	·01	·021	·012016	·005623	·01	·02	·013351	·005923	·0	·02	·014686	·006208	·0	·02	·016021	·006479	·01	[illegible]	·017356	739	·011	·021

N = size of sample	15				16			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2\cdot78$	P_2 $\lambda_2=2\cdot38$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2\cdot77$	P_2 $\lambda_2=2\cdot37$
701	·020000	·007473	·011	·021	·021429	·007729	·011	·021
702	·019971	·007462	·011	·021	·021398	·007718	·011	·021
703	·019943	·007452	·011	·021	·021368	·007708	·011	·021
704	·019915 −	·007441	·011	·021	·021337	·007697	·011	·021
705	·019886	·007431	·011	·021	·021307	·007686	·011	·021
706	·019858	·007420	·011	·021	·021277	·007675 +	·011	·021
707	·019830	·007410	·011	·021	·021246	·007664	·011	·021
708	·019802	·007400	·011	·021	·021216	·007654	·011	·021
709	·019774	·007389	·011	·021	·021186	·007643	·011	·021
710	·019746	·007379	·011	·021	·021157	·007632	·011	·021
711	·019718	·007369	·011	·021	·021127	·007622	·011	·021
712	·019691	·007358	·011	·021	·021097	·007611	·011	·021
713	·019663	·007348	·011	·021	·021067	·007601	·011	·021
714	·019635 +	·007338	·011	·021	·021038	·007590	·011	·021
715	·019608	·007328	·011	·021	·021008	·007580	·011	·021
716	·019580	·007318	·011	·021	·020979	·007569	·011	·021
717	·019553	·007308	·011	·021	·020950 −	·007559	·011	·021
718	·019526	·007297	·011	·021	·020921	·007548	·011	·021
719	·019499	·007287	·011	·021	·020891	·007538	·011	·021
720	·019471	·007277	·011	·021	·020862	·007527	·011	·021
721	·019444	·007267	·011	·021	·020833	·007517	·011	·021
722	·019417	·007257	·011	·021	·020804	·007507	·011	·021
723	·019391	·007248	·011	·021	·020776	·007497	·011	·021
724	·019364	·007238	·011	·021	·020747	·007486	·011	·021
725	·019337	·007228	·011	·021	·020718	·007476	·011	·021
726	·019310	·007218	·011	·021	·020690	·007466	·011	·021
727	·019284	·007208	·011	·021	·020661	·007456	·011	·021
728	·019257	·007198	·011	·021	·020633	·007446	·011	·021
729	·019231	·007188	·011	·021	·020604	·007436	·011	·021
730	·019204	·007179	·011	·021	·020576	·007425 +	·011	·021
731	·019178	·007169	·011	·021	·020548	·007415 +	·011	·021
732	·019152	·007159	·011	·021	·020520	·007405 +	·011	·021
733	·019126	·007150 −	·011	·021	·020492	·007395 +	·011	·021
734	·019100	·007140	·011	·021	·020464	·007385 +	·011	·021
735	·019074	·007130	·011	·021	·020436	·007375 +	·011	·021
736	·019048	·007121	·011	·021	·020408	·007366	·011	·021
737	·019022	·007111	·011	·021	·020380	·007356	·011	·021
738	·018996	·007102	·011	·021	·020353	·007346	·011	·021
739	·018970	·007092	·011	·021	·020325 +	·007336	·011	·021
740	·018945 −	·007083	·011	·021	·020298	·007326	·011	·021
741	·018919	·007073	·011	·021	·020270	·007316	·011	·021
742	·018893	·007064	·011	·021	·020243	·007307	·011	·021
743	·018868	·007054	·011	·021	·020216	·007297	·011	·021
744	·018843	·007045 −	·011	·021	·020188	·007287	·011	·021
745	·018817	·007036	·011	·021	·020161	·007277	·011	·021
746	·018792	·007026	·011	·021	·020134	·007268	·011	·021
747	·018767	·007017	·011	·021	·020107	·007258	·011	·021
748	·018742	·007008	·011	·021	·020080	·007249	·011	·021
749	·018717	·006998	·011	·021	·020053	·007239	·011	·021
750	·018692	·006989	·011	·021	·020027	·007229	·011	·021

N = size of sample	17				18			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2\cdot76$	P_2 $\lambda_2=2\cdot36$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2\cdot75$	P_2 $\lambda_2=2\cdot35$
701	·022857	·007977	·011	·021	·024286	·008216	·011	·021
702	·022825 −	·007966	·011	·021	·024251	·008205 −	·011	·021
703	·022792	·007955 −	·011	·021	·024217	·008193	·011	·021
704	·022760	·007943	·011	·021	·024182	·008182	·011	·021
705	·022727	·007932	·011	·021	·024148	·008170	·011	·021
706	·022695 +	·007921	·011	·021	·024113	·008159	·011	·021
707	·022663	·007910	·011	·021	·024079	·008148	·011	·021
708	·022631	·007899	·011	·021	·024045 +	·008136	·011	·021
709	·022599	·007888	·011	·021	·024011	·008125 −	·011	·021
710	·022567	·007877	·011	·021	·023977	·008114	·011	·021
711	·022535 +	·007866	·011	·021	·023944	·008102	·011	·021
712	·022504	·007855 +	·011	·021	·023910	·008091	·011	·021
713	·022472	·007844	·011	·021	·023876	·008080	·011	·021
714	·022440	·007833	·011	·021	·023843	·008069	·011	·021
715	·022409	·007823	·011	·021	·023810	·008058	·011	·021
716	·022378	·007812	·011	·021	·023776	·008046	·011	·021
717	·022346	·007801	·011	·021	·023743	·008035 +	·011	·021
718	·022315 +	·007790	·011	·021	·023710	·008024	·011	·021
719	·022284	·007780	·011	·021	·023677	·008013	·011	·021
720	·022253	·007769	·011	·021	·023644	·008002	·011	·021
721	·022222	·007758	·011	·021	·023611	·007991	·011	·021
722	·022191	·007748	·011	·021	·023578	·007980	·011	·021
723	·022161	·007737	·011	·021	·023546	·007969	·011	·021
724	·022130	·007726	·011	·021	·023513	·007959	·011	·021
725	·022099	·007716	·011	·021	·023481	·007948	·011	·021
726	·022069	·007705 +	·011	·021	·023448	·007937	·011	·021
727	·022039	·007695 −	·011	·021	·023416	·007926	·011	·021
728	·022008	·007684	·011	·021	·023384	·007915 +	·011	·021
729	·021978	·007674	·011	·021	·023352	·007905 −	·011	·021
730	·021948	·007664	·011	·021	·023320	·007894	·011	·021
731	·021918	·007653	·011	·021	·023288	·007883	·011	·021
732	·021888	·007643	·011	·021	·023256	·007873	·011	·021
733	·021858	·007633	·011	·021	·023224	·007862	·011	·021
734	·021828	·007622	·011	·021	·023192	·007851	·011	·021
735	·021798	·007612	·011	·021	·023161	·007841	·011	·021
736	·021769	·007602	·011	·021	·023129	·007830	·011	·021
737	·021739	·007592	·011	·021	·023098	·007820	·011	·021
738	·021710	·007581	·011	·021	·023066	·007809	·011	·021
739	·021680	·007571	·011	·021	·023035 +	·007799	·011	·021
740	·021651	·007561	·011	·021	·023004	·007789	·011	·021
741	·021622	·007551	·011	·021	·022973	·007778	·011	·021
742	·021592	·007541	·011	·021	·022942	·007768	·011	·021
743	·021563	·007531	·011	·021	·022911	·007757	·011	·021
744	·021534	·007521	·011	·021	·022880	·007747	·011	·021
745	·021505 +	·007511	·011	·021	·028849	·007737	·011	·021
746	·021477	·007501	·011	·021	·022819	·007727	·011	·021
747	·021448	·007491	·011	·021	·022788	·007716	·011	·021
748	·021419	·007481	·011	·021	·022758	·007706	·011	·021
749	·021390	·007471	·011	·021	·022727	·007696	·011	·021
750	·021362	·007461	·011	·021	·022697	·007686	·011	·021

N = size of sample	19				20			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2\cdot74$	P_2 $\lambda_2=2\cdot34$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2\cdot73$	P_2 $\lambda_2=2\cdot33$
701	·025714	·008448	·011	·021	·027143	·008674	·011	·021
702	·025678	·008437	·011	·021	·027104	·008661	·011	·021
703	·025641	·008425 −	·011	·021	·027066	·008649	·011	·021
704	·025605 −	·008413	·011	·021	·027027	·008637	·011	·021
705	·025568	·008401	·011	·021	·026989	·008625 +	·011	·021
706	·025532	·008389	·011	·021	·026950 +	·008613	·011	·021
707	·025496	·008378	·011	·021	·026912	·008601	·011	·021
708	·025460	·008366	·011	·021	·026874	·008589	·011	·021
709	·025424	·008354	·011	·021	·026836	·008577	·011	·021
710	·025388	·008343	·011	·021	·026798	·008565 +	·011	·021
711	·025352	·008331	·011	·021	·026761	·008553	·011	·021
712	·025316	·008320	·011	·021	·026723	·008541	·011	·021
713	·025281	·008308	·011	·021	·026685 +	·008530	·011	·021
714	·025245 +	·008297	·011	·021	·026648	·008518	·011	·021
715	·025210	·008285 +	·011	·021	·026611	·008506	·011	·021
716	·025175 −	·008274	·011	·021	·026573	·008494	·011	·021
717	·025140	·008262	·011	·021	·026536	·008483	·011	·021
718	·025105 −	·008251	·011	·021	·026499	·008471	·011	·021
719	·025070	·008240	·011	·021	·026462	·008459	·011	·021
720	·025035 −	·008228	·011	·021	·026426	·008448	·011	·021
721	·025000	·008217	·011	·021	·026389	·008436	·011	·021
722	·024965 +	·008206	·011	·021	·026352	·008425 −	·011	·021
723	·024931	·008195 −	·011	·021	·026316	·008413	·011	·021
724	·024896	·008183	·011	·021	·026279	·008402	·011	·021
725	·024862	·008172	·011	·021	·026243	·008390	·011	·021
726	·024828	·008161	·011	·021	·026207	·008379	·011	·021
727	·024793	·008150 +	·011	·021	·026171	·008368	·011	·021
728	·024759	·008139	·011	·021	·026135 −	·008356	·011	·021
729	·024725 +	·008128	·011	·021	·026099	·008345 −	·011	·021
730	·024691	·008117	·011	·021	·026063	·008334	·011	·021
731	·024658	·008106	·011	·021	·026027	·008322	·011	·021
732	·024624	·008095 +	·011	·021	·025992	·008311	·011	·021
733	·024590	·008084	·011	·021	·025956	·008300	·011	·021
734	·024557	·008073	·011	·021	·025921	·008289	·011	·021
735	·024523	·008063	·011	·021	·025886	·008278	·011	·021
736	·024490	·008052	·011	·021	·025850 +	·008267	·011	·021
737	·024457	·008041	·011	·021	·025815 +	·008256	·011	·021
738	·024423	·008030	·011	·021	·025780	·008244	·011	·021
739	·024390	·008019	·011	·021	·025745 +	·008233	·011	·021
740	·024357	·008009	·011	·021	·025710	·008223	·011	·021
741	·024324	·007998	·011	·021	·025676	·008212	·011	·021
742	·024291	·007987	·011	·021	·025641	·008201	·011	·021
743	·024259	·007977	·011	·021	·025606	·008190	·011	·021
744	·024226	·007966	·011	·021	·025572	·008179	·011	·021
745	·024194	·007956	·011	·021	·025538	·008168	·011	·021
746	·024161	·007945 +	·011	·021	·025503	·008157	·011	·021
747	·024129	·007935 −	·011	·021	·025469	·008146	·011	·021
748	·024096	·007924	·011	·021	·025435 +	·008136	·011	·021
749	·024064	·007914	·011	·021	·025401	·008125 −	·011	·021
750	·024032	·007903	·011	·021	·025367	·008114	·011	·021

n = number of arrays

N = size of sample	3				4				5				6				7				8			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}50$	P_2 $\lambda_2=2{\cdot}94$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}20$	P_2 $\lambda_2=2{\cdot}80$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}14$	P_2 $\lambda_2=2{\cdot}68$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}11$	P_2 $\lambda_2=2{\cdot}63$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}08$	P_2 $\lambda_2=2{\cdot}58$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}02$	P_2 $\lambda_2=2{\cdot}54$
751	·002667	·002660	·011	·019	·004000	·003255 −	·013	·020	·005333	·003756	·012	·021	·006667	·004197	·011	·021	·008000	·004594	·010	·021	·009333	·004959	·011	·021
752	·002663	·002656	·011	·019	·003995 −	·003251	·013	·020	·005326	·003751	·012	·021	·006658	·004191	·011	·021	·007989	·004588	·010	·021	·009321	·004952	·011	·021
753	·002660	·002653	·011	·019	·003989	·003246	·013	·020	·005319	·003746	·012	·021	·006649	·004186	·011	·021	·007979	·004582	·010	·021	·009309	·004946	·011	·021
754	·002656	·002649	·011	·019	·003984	·003242	·013	·020	·005312	·003741	·012	·021	·006640	·004180	·011	·021	·007968	·004576	·010	·021	·009296	·004939	·011	·021
755	·002653	·002645 +	·011	·019	·003979	·003238	·013	·020	·005305 +	·003736	·012	·021	·006631	·004175 −	·011	·021	·007958	·004570	·010	·021	·009284	·004933	·011	·021
756	·002649	·002642	·011	·019	·003974	·003234	·013	·020	·005298	·003731	·012	·021	·006623	·004169	·011	·021	·007947	·004564	·010	·021	·009272	·004926	·011	·021
757	·002646	·002639	·011	·019	·003968	·003229	·013	·020	·005291	·003726	·012	·021	·006614	·004164	·011	·021	·007937	·004558	·010	·021	·009259	·004920	·011	·021
758	·002642	·002635 +	·011	·019	·003963	·003225 +	·013	·020	·005284	·003722	·012	·021	·006605 +	·004158	·011	·021	·007926	·004552	·010	·021	·009247	·004913	·011	·021
759	·002639	·002632	·011	·019	·003958	·003221	·013	·020	·005277	·003717	·012	·021	·006596	·004153	·011	·021	·007916	·004546	·010	·021	·009235 −	·004907	·011	·021
760	·002635 +	·002628	·011	·019	·003953	·003217	·013	·020	·005270	·003712	·012	·021	·006588	·004147	·011	·021	·007905 +	·004540	·010	·021	·009223	·004900	·011	·021
761	·002632	·002625 −	·011	·019	·003947	·003212	·013	·020	·005263	·003707	·012	·021	·006579	·004142	·011	·021	·007895 −	·004534	·010	·021	·009211	·004894	·011	·021
762	·002628	·002621	·011	·019	·003942	·003208	·013	·020	·005256	·003702	·012	·021	·006570	·004136	·011	·021	·007884	·004528	·010	·021	·009198	·004888	·011	·021
763	·002625 −	·002618	·011	·019	·003937	·003204	·013	·020	·005249	·003697	·012	·021	·006562	·004131	·011	·021	·007874	·004522	·010	·021	·009186	·004881	·011	·021
764	·002621	·002614	·011	·019	·003932	·003200	·013	·020	·005242	·003692	·012	·021	·006553	·004126	·011	·021	·007864	·004516	·010	·021	·009174	·004875 −	·011	·021
765	·002618	·002611	·011	·019	·003927	·003196	·013	·020	·005236	·003688	·012	·021	·006545 −	·004120	·011	·021	·007853	·004510	·010	·021	·009162	·004869	·011	·021
766	·002614	·002608	·011	·019	·003922	·003191	·013	·020	·005229	·003683	·012	·021	·006536	·004115 −	·011	·021	·007843	·004505 −	·010	·021	·009150 +	·004862	·011	·021
767	·002611	·002604	·011	·019	·003916	·003187	·013	·020	·005222	·003678	·012	·021	·006527	·004109	·011	·021	·007833	·004499	·010	·021	·009138	·004856	·011	·021
768	·002608	·002601	·011	·019	·003911	·003183	·013	·020	·005215 +	·003673	·012	·021	·006519	·004104	·011	·021	·007823	·004493	·010	·021	·009126	·004850 −	·011	·021
769	·002604	·002597	·011	·019	·003906	·003179	·013	·020	·005208	·003668	·012	·021	·006510	·004099	·011	·021	·007813	·004487	·010	·021	·009115 −	·004843	·011	·021
770	·002601	·002594	·011	·019	·003901	·003175 −	·013	·020	·005202	·003664	·012	·021	·006502	·004093	·011	·021	·007802	·004481	·010	·021	·009103	·004837	·011	·021
771	·002597	·002591	·011	·019	·003896	·003171	·013	·020	·005195 −	·003659	·012	·021	·006494	·004088	·011	·021	·007792	·004475 +	·010	·021	·009091	·004831	·011	·021
772	·002594	·002587	·011	·019	·003891	·003167	·013	·020	·005188	·003654	·012	·021	·006485 +	·004083	·011	·021	·007782	·004470	·010	·021	·009079	·004825 −	·011	·021
773	·002591	·002584	·011	·019	·003886	·003163	·013	·020	·005181	·003650 −	·012	·021	·006477	·004078	·011	·021	·007772	·004464	·010	·021	·009067	·004818	·011	·021
774	·002587	·002581	·011	·019	·003881	·003159	·013	·020	·005175 −	·003645 −	·012	·021	·006468	·004072	·011	·021	·007762	·004458	·010	·021	·009056	·004812	·011	·021
775	·002584	·002577	·011	·019	·003876	·003155 −	·013	·020	·005168	·003640	·012	·021	·006460	·004067	·011	·021	·007752	·004452	·010	·021	·009044	·004806	·011	·021
776	·002581	·002574	·011	·019	·003871	·003150 +	·013	·020	·005161	·003635 +	·012	·021	·006452	·004062	·011	·021	·007742	·004447	·010	·021	·009032	·004800	·011	·021
777	·002577	·002571	·011	·019	·003866	·003146	·013	·020	·005155 −	·003631	·012	·021	·006443	·004057	·011	·021	·007732	·004441	·010	·021	·009021	·004794	·011	·021
778	·002574	·002567	·011	·019	·003861	·003142	·013	·020	·005148	·003626	·012	·021	·006435 +	·004052	·011	·021	·007722	·004435 +	·010	·021	·009009	·004788	·011	·021
779	·002571	·002564	·011	·019	·003856	·003138	·013	·020	·005141	·003622	·012	·021	·006427	·004046	·011	·021	·007712	·004430	·010	·021	·008997	·004782	·011	·021
780	·002567	·002561	·011	·019	·003851	·003134	·013	·020	·005135 −	·003617	·012	·021	·006418	·004041	·011	·021	·007702	·004424	·010	·021	·008986	·004775 +	·011	·021
781	·002564	·002558	·011	·019	·003846	·003130	·013	·020	·005128	·003612	·012	·021	·006410	·004036	·011	·021	·007692	·004418	·010	·021	·008974	·004769	·011	·021
782	·002561	·002554	·011	·019	·003841	·003126	·013	·020	·005122	·003608	·012	·021	·006402	·004031	·011	·021	·007682	·004413	·010	·021	·008963	·004763	·011	·021
783	·002558	·002551	·011	·019	·003836	·003122	·013	·020	·005115 +	·003603	·012	·021	·006394	·004026	·011	·021	·007673	·004407	·010	·021	·008951	·004757	·011	·021
784	·002554	·002548	·011	·019	·003831	·003118	·013	·020	·005109	·003598	·012	·021	·006386	·004021	·011	·021	·007663	·004402	·010	·021	·008940	·004751	·011	·021
785	·002551	·002545 −	·011	·019	·003827	·003114	·013	·020	·005102	·003594	·012	·021	·006378	·004016	·011	·021	·007653	·004396	·010	·021	·008929	·004745 +	·011	·021
786	·002548	·002541	·011	·019	·003822	·003110	·013	·020	·005096	·003589	·012	·021	·006369	·004010	·011	·021	·007643	·004390	·010	·021	·008917	·004739	·011	·021
787	·002545 −	·002538	·011	·019	·003817	·003106	·013	·020	·005089	·003585 −	·012	·021	·006361	·004005 +	·011	·021	·007634	·004385	·010	·021	·008906	·004733	·011	·021
788	·002541	·002535 −	·011	·019	·003812	·003103	·013	·020	·005083	·003580	·012	·021	·006353	·004000	·011	·021	·007624	·004379	·010	·021	·008895 −	·004727	·011	·021
789	·002538	·002532	·011	·019	·003807	·003099	·013	·020	·005076	·003576	·012	·021	·006345 −	·003995 +	·011	·021	·007614	·004374	·010	·021	·008883	·004721	·011	·021
790	·002535 −	·002528	·011	·019	·003802	·003095 −	·013	·020	·005070	·003571	·012	·021	·006337	·003990	·011	·021	·007605 −	·004368	·010	·021	·008872	·004715 +	·011	·021
791	·002532	·002525 +	·011	·019	·003797	·003091	·013	·020	·005063	·003567	·012	·021	·006329	·003985 +	·011	·021	·007595 −	·004363	·010	·021	·008861	·004709	·011	·021
792	·002528	·002522	·011	·019	·003793	·003087	·013	·020	·005057	·003562	·012	·021	·006321	·003980	·011	·021	·007585 +	·004357	·010	·021	·008850 −	·004703	·011	·021
793	·002525 +	·002519	·011	·019	·003788	·003083	·013	·020	·005051	·003558	·012	·021	·006313	·003975 +	·011	·021	·007576	·004352	·010	·021	·008838	·004697	·011	·021
794	·002522	·002516	·011	·019	·003783	·003079	·013	·020	·005044	·003553	·012	·021	·006305 +	·003970	·011	·021	·007566	·004346	·010	·021	·008827	·004692	·011	·021
795	·002519	·002513	·011	·019	·003778	·003075 +	·013	·020	·005038	·003549	·012	·021	·006297	·003965 +	·011	·021	·007557	·004341	·010	·021	·008816	·004686	·011	·021
796	·002516	·002509	·011	·019	·003774	·003071	·013	·020	·005031	·003544	·012	·021	·006289	·003960	·011	·021	·007547	·004335 +	·010	·021	·008805 +	·004680	·011	·021
797	·002513	·002506	·011	·019	·003769	·003068	·013	·020	·005025 +	·003540	·012	·021	·006281	·003955 +	·011	·021	·007538	·004330	·010	·021	·008794	·004674	·011	·021
798	·002509	·002503	·011	·019	·003764	·003064	·013	·020	·005019	·003535 +	·012	·021	·006274	·003950 +	·011	·021	·007528	·004325 −	·010	·021	·008783	·004668	·011	·021
799	·002506	·002500 −	·011	·019	·003759	·003060	·013	·020	·005013	·003531	·012	·021	·006266	·003945 +	·011	·021	·007519	·004319	·010	·021	·008772	·004662	·011	·021
800	·002503	·002497	·011	·019	·003755 −	·003056	·013	·020	·005006	·003527	·012	·021	·006258	·003940	·011	·021	·007509	·004314	·010	·021	·008761	·004657	·011	·021

N = size of sample	9				10				11			
	$\overline{\eta^2}$	$\sigma_{\overline{\eta^2}}$	P_1 λ_1 = 2·96	P_2 λ_2 = 2·50	$\overline{\eta^2}$	$\sigma_{\overline{\eta^2}}$	P_1 λ_1 = 2·92	P_2 λ_2 = 2·47	$\overline{\eta^2}$	$\sigma_{\overline{\eta^2}}$	P_1 λ_1 = 2·88	P_2 λ_2 = 2·44
751	·010667	·005298	·011	·021	·012000	·005615 +	·011	·021	·013333	·005915 +	·011	·021
752	·010652	·005291	·011	·021	·011984	·005608	·011	·021	·013316	·005907	·011	·021
753	·010638	·005284	·011	·021	·011968	·005601	·011	·021	·013298	·005899	·011	·021
754	·010624	·005277	·011	·021	·011952	·005593	·011	·021	·013280	·005892	·011	·021
755	·010610	·005270	·011	·021	·011936	·005586	·011	·021	·013263	·005884	·011	·021
756	·010596	·005263	·011	·021	·011921	·005578	·011	·021	·013245 +	·005876	·011	·021
757	·010582	·005256	·011	·021	·011905 −	·005571	·011	·021	·013228	·005869	·011	·021
758	·010568	·005249	·011	·021	·011889	·005564	·011	·021	·013210	·005861	·011	·021
759	·010554	·005242	·011	·021	·011873	·005556	·011	·021	·013193	·005853	·011	·021
760	·010540	·005235 +	·011	·021	·011858	·005549	·011	·021	·013175 +	·005846	·011	·021
761	·010526	·005229	·011	·021	·011842	·005542	·011	·021	·013158	·005838	·011	·021
762	·010512	·005222	·011	·021	·011827	·005535 −	·011	·021	·013141	·005830	·011	·021
763	·010499	·005215 −	·011	·021	·011811	·005528	·011	·021	·013123	·005823	·011	·021
764	·010485 −	·005208	·011	·021	·011796	·005520	·011	·021	·013106	·005815 +	·011	·021
765	·010471	·005201	·011	·021	·011780	·005513	·011	·021	·013089	·005808	·011	·021
766	·010458	·005195 −	·011	·021	·011765 −	·005506	·011	·021	·013072	·005800	·011	·021
767	·010444	·005188	·011	·021	·011749	·005499	·011	·021	·013055 −	·005793	·011	·021
768	·010430	·005181	·011	·021	·011734	·005492	·011	·021	·013038	·005785 +	·011	·021
769	·010417	·005174	·011	·021	·011719	·005485 −	·011	·021	·013021	·005778	·011	·021
770	·010403	·005168	·011	·021	·011704	·005478	·011	·021	·013004	·005770	·011	·021
771	·010390	·005161	·011	·021	·011688	·005471	·011	·021	·012987	·005763	·011	·021
772	·010376	·005154	·011	·021	·011673	·005463	·011	·021	·012970	·005755 +	·011	·021
773	·010363	·005148	·011	·021	·011658	·005456	·011	·021	·012953	·005748	·011	·021
774	·010349	·005141	·011	·021	·011643	·005449	·011	·021	·012937	·005740	·011	·021
775	·010336	·005135 −	·011	·021	·011628	·005442	·011	·021	·012920	·005733	·011	·021
776	·010323	·005128	·011	·021	·011613	·005435 +	·011	·021	·012903	·005726	·011	·021
777	·010309	·005121	·011	·021	·011598	·005429	·011	·021	·012887	·005718	·011	·021
778	·010296	·005115 −	·011	·021	·011583	·005422	·011	·021	·012870	·005711	·011	·021
779	·010283	·005108	·011	·021	·011568	·005415 −	·011	·021	·012853	·005704	·011	·021
780	·010270	·005102	·011	·021	·011553	·005408	·011	·021	·012837	·005697	·011	·021
781	·010256	·005095 +	·011	·021	·011538	·005401	·011	·021	·012821	·005689	·011	·021
782	·010243	·005089	·011	·021	·011524	·005394	·011	·021	·012804	·005682	·011	·021
783	·010230	·005082	·011	·021	·011509	·005387	·011	·021	·012788	·005675 −	·011	·021
784	·010217	·005076	·011	·021	·011494	·005380	·011	·021	·012771	·005668	·011	·021
785	·010204	·005069	·011	·021	·011480	·005374	·011	·021	·012755 +	·005661	·011	·021
786	·010191	·005063	·011	·021	·011465 −	·005367	·011	·021	·012739	·005653	·011	·021
787	·010178	·005057	·011	·021	·011450 +	·005360	·011	·021	·012723	·005646	·011	·021
788	·010165 +	·005050 +	·011	·021	·011436	·005353	·011	·021	·012706	·005639	·011	·021
789	·010152	·005044	·011	·021	·011421	·005346	·011	·021	·012690	·005632	·011	·021
790	·010139	·005038	·011	·021	·011407	·005340	·011	·021	·012674	·005625 −	·011	·021
791	·010127	·005031	·011	·021	·011392	·005333	·011	·021	·012658	·005618	·011	·021
792	·010114	·005025 −	·011	·021	·011378	·005326	·011	·021	·012642	·005611	·011	·021
793	·010101	·005019	·011	·021	·011364	·005320	·011	·021	·012626	·005604	·011	·021
794	·010088	·005012	·011	·021	·011349	·005313	·011	·021	·012610	·005597	·011	·021
795	·010076	·005006	·011	·021	·011335 +	·005306	·011	·021	·012594	·005590	·011	·021
796	·010063	·005000 −	·011	·021	·011321	·005300	·011	·021	·012579	·005583	·011	·021
797	·010050 +	·004994	·011	·021	·011307	·005293	·011	·021	·012563	·005576	·011	·021
798	·010038	·004987	·011	·021	·011292	·005286	·011	·021	·012547	·005569	·011	·021
799	·010025 +	·004981	·011	·021	·011278	·005280	·011	·021	·012531	·005562	·011	·021
800	·010013	·004975 −	·011	·021	·011264	·005273	·011	·021	·012516	·005555 +	·011	·021

N = size of sample	12				13				14			
	$\overline{\eta^2}$	$\sigma_{\overline{\eta^2}}$	P_1 λ_1 = 2·84	P_2 λ_2 = 2·42	$\overline{\eta^2}$	$\sigma_{\overline{\eta^2}}$	P_1 λ_1 = 2·80	P_2 λ_2 = 2·40	$\overline{\eta^2}$	$\sigma_{\overline{\eta^2}}$	P_1 λ_1 = 2·79	P_2 λ_2 = 2·39
751	·014667	·006200	·011	·022	·016000	·006471	·011	·022	·017333	·006731	·011	·021
752	·014647	·006191	·011	·022	·015979	·006462	·011	·022	·017310	·006722	·011	·021
753	·014628	·006183	·011	·022	·015957	·006454	·011	·022	·017287	·006713	·011	·021
754	·014608	·006175 +	·011	·022	·015936	·006445 +	·011	·022	·017264	·006704	·011	·021
755	·014589	·006167	·011	·022	·015915 +	·006437	·011	·022	·017241	·006695 +	·011	·021
756	·014570	·006159	·011	·022	·015894	·006428	·011	·022	·017219	·006686	·011	·021
757	·014550 +	·006151	·011	·022	·015873	·006420	·011	·022	·017196	·006678	·011	·021
758	·014531	·006143	·011	·022	·015852	·006412	·011	·022	·017173	·006669	·011	·021
759	·014512	·006135 −	·011	·022	·015831	·006403	·011	·022	·017150 +	·006660	·011	·021
760	·014493	·006127	·011	·022	·015810	·006395 −	·011	·022	·017128	·006652	·011	·021
761	·014474	·006119	·011	·022	·015789	·006387	·011	·022	·017105 +	·006643	·011	·021
762	·014455 −	·006111	·011	·022	·015769	·006378	·011	·022	·017083	·006634	·011	·021
763	·014436	·006103	·011	·022	·015748	·006370	·011	·022	·017060	·006626	·011	·021
764	·014417	·006095 −	·011	·022	·015727	·006362	·011	·022	·017038	·006617	·011	·021
765	·014398	·006087	·011	·022	·015707	·006353	·011	·022	·017016	·006608	·011	·021
766	·014379	·006079	·011	·022	·015686	·006345 +	·011	·022	·016993	·006600	·011	·021
767	·014360	·006071	·011	·022	·015666	·006337	·011	·022	·016971	·006591	·011	·021
768	·014342	·006063	·011	·022	·015645 +	·006329	·011	·022	·016949	·006583	·011	·021
769	·014323	·006056	·011	·022	·015625	·006321	·011	·022	·016927	·006574	·011	·021
770	·014304	·006048	·011	·022	·015605 −	·006312	·011	·022	·016905 +	·006566	·011	·021
771	·014286	·006040	·011	·022	·015584	·006304	·011	·022	·016883	·006557	·011	·021
772	·014267	·006032	·011	·022	·015564	·006296	·011	·022	·016861	·006549	·011	·021
773	·014249	·006024	·011	·022	·015544	·006288	·011	·022	·016839	·006541	·011	·021
774	·014230	·006017	·011	·022	·015524	·006280	·011	·022	·016818	·006532	·011	·021
775	·014212	·006009	·011	·022	·015504	·006272	·011	·022	·016796	·006524	·011	·021
776	·014194	·006001	·011	·022	·015484	·006264	·011	·022	·016774	·006516	·011	·021
777	·014175 +	·005994	·011	·022	·015464	·006256	·011	·022	·016753	·006507	·011	·021
778	·014157	·005986	·011	·022	·015444	·006248	·011	·022	·016731	·006499	·011	·021
779	·014139	·005978	·011	·022	·015424	·006240	·011	·022	·016710	·006491	·011	·021
780	·014121	·005971	·011	·022	·015404	·006232	·011	·022	·016688	·006482	·011	·021
781	·014103	·005963	·011	·022	·015385 −	·006224	·011	·022	·016667	·006474	·011	·021
782	·014085 −	·005956	·011	·022	·015365 −	·006216	·011	·022	·016645 +	·006466	·011	·021
783	·014066	·005948	·011	·022	·015345 +	·006208	·011	·022	·016624	·006458	·011	·021
784	·014049	·005941	·011	·022	·015326	·006201	·011	·022	·016603	·006450 −	·011	·021
785	·014031	·005933	·011	·022	·015306	·006193	·011	·022	·016582	·006441	·011	·021
786	·014013	·005925 +	·011	·022	·015287	·006185 −	·011	·022	·016561	·006433	·011	·021
787	·013995 −	·005918	·011	·022	·015267	·006177	·011	·022	·016539	·006425 +	·011	·021
788	·013977	·005911	·011	·022	·015248	·006169	·011	·022	·016518	·006417	·011	·021
789	·013959	·005903	·011	·022	·015228	·006162	·011	·022	·016497	·006409	·011	·021
790	·013942	·005896	·011	·022	·015209	·006154	·011	·022	·016477	·006401	·011	·021
791	·013924	·005888	·011	·022	·015190	·006146	·011	·022	·016456	·006393	·011	·021
792	·013906	·005881	·011	·022	·015171	·006138	·011	·022	·016435 −	·006385 +	·011	·021
793	·013889	·005874	·011	·022	·015152	·006131	·011	·022	·016414	·006377	·011	·021
794	·013871	·005866	·011	·022	·015132	·006123	·011	·022	·016393	·006369	·011	·021
795	·013854	·005859	·011	·022	·015113	·006115 +	·011	·022	·016373	·006361	·011	·021
796	·013836	·005852	·011	·022	·015094	·006108	·011	·022	·016352	·006353	·011	·021
797	·013819	·005844	·011	·022	·015075 +	·006100	·011	·022	·016332	·006345 +	·011	·021
798	·013802	·005837	·011	·022	·015056	·006093	·011	·022	·016311	·006337	·011	·021
799	·013784	·005830	·011	·022	·015038	·006085 +	·011	·022	·016291	·006330	·011	·021
800	·013767	·005823	·011	·022	·015019	·006078	·011	·022	·016270	·006322	·011	·021

n = number of arrays

N = size of sample	15				16				17				18				19				20			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}78$	P_2 $\lambda_2=2{\cdot}38$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}77$	P_2 $\lambda_2=2{\cdot}37$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}76$	P_2 $\lambda_2=2{\cdot}36$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}75$	P_2 $\lambda_2=2{\cdot}35$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}74$	P_2 $\lambda_2=2{\cdot}34$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}73$	P_2 $\lambda_2=2{\cdot}33$
751	·018667	·006980	·011	·021	·020000	·007220	·011	·021	·021333	·007452	·011	·021	·022667	·007676	·011	·021	·024000	·007893	·011	·021	·025333	·008104	·011	·021
752	·018642	·006971	·011	·021	·019973	·007210	·011	·021	·021305 –	·007442	·011	·021	·022636	·007666	·011	·021	·023968	·007883	·011	·021	·025300	·008093	·011	·021
753	·018617	·006962	·011	·021	·019947	·007201	·011	·021	·021277	·007432	·011	·021	·022606	·007656	·011	·021	·023936	·007872	·011	·021	·025266	·008082	·011	·021
754	·018592	·006952	·011	·021	·019920	·007192	·011	·021	·021248	·007422	·011	·021	·022576	·007646	·011	·021	·023904	·007862	·011	·021	·025232	·008072	·011	·021
755	·018568	·006943	·011	·021	·019894	·007182	·011	·021	·021220	·007413	·011	·021	·022546	·007636	·011	·021	·023873	·007852	·011	·021	·025199	·008061	·011	·021
756	·018543	·006934	·011	·021	·019868	·007173	·011	·021	·021192	·007403	·011	·021	·022517	·007626	·011	·021	·023841	·007841	·011	·021	·025166	·008051	·011	·021
757	·018519	·006925 +	·011	·021	·019841	·007163	·011	·021	·021164	·007393	·011	·021	·022487	·007616	·011	·021	·023810	·007831	·011	·021	·025132	·008040	·011	·021
758	·018494	·006916	·011	·021	·019815 +	·007154	·011	·021	·021136	·007384	·011	·021	·022457	·007606	·011	·021	·023778	·007821	·011	·021	·025099	·008030	·011	·021
759	·018470	·006907	·011	·021	·019789	·007145 –	·011	·021	·021108	·007374	·011	·021	·022427	·007596	·011	·021	·023747	·007811	·011	·021	·025066	·008019	·011	·021
760	·018445 +	·006898	·011	·021	·019763	·007135 +	·011	·021	·021080	·007364	·011	·021	·022398	·007586	·011	·021	·023715 +	·007801	·011	·021	·025033	·008009	·011	·021
761	·018421	·006889	·011	·021	·019737	·007126	·011	·021	·021053	·007355 –	·011	·021	·022368	·007576	·011	·021	·023684	·007790	·011	·021	·025000	·007999	·011	·021
762	·018397	·006880	·011	·021	·019711	·007117	·011	·021	·021025 –	·007345 +	·011	·021	·022339	·007566	·011	·021	·023653	·007780	·011	·021	·024967	·007988	·011	·021
763	·018373	·006871	·011	·021	·019685 +	·007108	·011	·021	·020997	·007336	·011	·021	·022310	·007556	·011	·021	·023622	·007770	·011	·021	·024934	·007978	·011	·021
764	·018349	·006862	·011	·021	·019659	·007098	·011	·021	·020970	·007326	·011	·021	·022280	·007547	·011	·021	·023591	·007760	·011	·021	·024902	·007968	·011	·021
765	·018325 –	·006853	·011	·021	·019634	·007089	·011	·021	·020942	·007317	·011	·021	·022251	·007537	·011	·021	·023560	·007750 +	·011	·021	·024869	·007957	·011	·021
766	·018301	·006844	·011	·021	·019608	·007080	·011	·021	·020915 +	·007307	·011	·021	·022222	·007527	·011	·021	·023529	·007740	·011	·021	·024837	·007947	·011	·021
767	·018277	·006836	·011	·021	·019582	·007071	·011	·021	·020888	·007298	·011	·021	·022193	·007517	·011	·021	·023499	·007730	·011	·021	·024804	·007937	·011	·021
768	·018253	·006827	·011	·021	·019557	·007062	·011	·021	·020860	·007288	·011	·021	·022164	·007508	·011	·021	·023468	·007720	·011	·021	·024772	·007927	·011	·021
769	·018229	·006818	·011	·021	·019531	·007053	·011	·021	·020833	·007279	·011	·021	·022135 +	·007498	·011	·021	·023437	·007710	·011	·021	·024740	·007916	·011	·021
770	·018205 +	·006809	·011	·021	·019506	·007044	·011	·021	·020806	·007270	·011	·021	·022107	·007489	·011	·021	·023407	·007700	·011	·021	·024707	·007906	·011	·021
771	·018182	·006800	·011	·021	·019481	·007035 –	·011	·021	·020779	·007260	·011	·021	·022078	·007479	·011	·021	·023377	·007691	·011	·021	·024675 +	·007896	·011	·021
772	·018158	·006792	·011	·021	·019455 +	·007026	·011	·021	·020752	·007251	·011	·021	·022049	·007469	·011	·021	·023346	·007681	·011	·021	·024643	·007886	·011	·021
773	·018135 –	·006783	·011	·021	·019430	·007017	·011	·021	·020725 +	·007242	·011	·021	·022021	·007460	·011	·021	·023316	·007671	·011	·021	·024611	·007876	·011	·021
774	·018111	·006774	·011	·021	·019405 –	·007008	·011	·021	·020699	·007233	·011	·021	·021992	·007450 +	·011	·021	·023286	·007661	·011	·021	·024580	·007866	·011	·021
775	·018088	·006766	·011	·021	·019380	·006999	·011	·021	·020672	·007223	·011	·021	·021964	·007441	·011	·021	·023256	·007651	·011	·021	·024548	·007856	·011	·021
776	·018065 –	·006757	·011	·021	·019355 –	·006990	·011	·021	·020645 +	·007214	·011	·021	·021935 +	·007431	·011	·021	·023226	·007642	·011	·021	·024516	·007846	·011	·021
777	·018041	·006748	·011	·021	·019330	·006981	·011	·021	·020619	·007205 –	·011	·021	·021907	·007422	·011	·021	·023196	·007632	·011	·021	·024485 –	·007836	·011	·021
778	·018018	·006740	·011	·021	·019305 +	·006972	·011	·021	·020592	·007196	·011	·021	·021879	·007412	·011	·021	·023166	·007622	·011	·021	·024453	·007826	·011	·021
779	·017995 –	·006731	·011	·021	·019280	·006963	·011	·021	·020566	·007187	·011	·021	·021851	·007403	·011	·021	·023136	·007613	·011	·021	·024422	·007816	·011	·021
780	·017972	·006723	·011	·021	·019255 +	·006954	·011	·021	·020539	·007178	·011	·021	·021823	·007394	·011	·021	·023107	·007603	·011	·021	·024390	·007806	·011	·021
781	·017949	·006714	·011	·021	·019231	·006945 +	·011	·021	·020513	·007168	·011	·021	·021795 –	·007384	·011	·021	·023077	·007593	·011	·021	·024359	·007796	·011	·021
782	·017926	·006706	·011	·021	·019206	·006937	·011	·021	·020487	·007159	·011	·021	·021767	·007375 –	·011	·021	·023047	·007584	·011	·021	·024328	·007786	·011	·021
783	·017903	·006697	·011	·021	·019182	·006928	·011	·021	·020460	·007150 +	·011	·021	·021739	·007366	·011	·021	·023018	·007574	·011	·021	·024297	·007777	·011	·021
784	·017880	·006689	·011	·021	·019157	·006919	·011	·021	·020434	·007141	·011	·021	·021711	·007356	·011	·021	·022989	·007565 –	·011	·021	·024266	·007767	·011	·021
785	·017857	·006680	·011	·021	·019133	·006910	·011	·021	·020408	·007132	·011	·021	·021684	·007347	·011	·021	·022959	·007555 +	·011	·021	·024235 –	·007757	·011	·021
786	·017834	·006672	·011	·021	·019108	·006902	·011	·021	·020382	·007123	·011	·021	·021656	·007338	·011	·021	·022930	·007546	·011	·021	·024204	·007747	·011	·021
787	·017812	·006663	·011	·021	·019084	·006893	·011	·021	·020356	·007114	·011	·021	·021628	·007329	·011	·021	·022901	·007536	·011	·021	·024173	·007738	·011	·021
788	·017789	·006655 +	·011	·021	·019060	·006884	·011	·021	·020330	·007105 +	·011	·021	·021601	·007319	·011	·021	·022872	·007527	·011	·021	·024142	·007728	·011	·021
789	·017766	·006647	·011	·021	·019036	·006876	·011	·021	·020305 –	·007096	·011	·021	·021574	·007310	·011	·021	·022843	·007517	·011	·021	·024112	·007718	·011	·021
790	·017744	·006638	·011	·021	·019011	·006867	·011	·021	·020279	·007088	·011	·021	·021546	·007301	·011	·021	·022814	·007508	·011	·021	·024081	·007709	·011	·021
791	·017722	·006630	·011	·021	·018987	·006858	·011	·021	·020253	·007079	·011	·021	·021519	·007292	·011	·021	·022785 –	·007498	·011	·021	·024051	·007699	·011	·021
792	·017699	·006622	·011	·021	·018963	·006850 –	·011	·021	·020228	·007070	·011	·021	·021492	·007283	·011	·021	·022756	·007489	·011	·021	·024020	·007689	·011	·021
793	·017677	·006614	·011	·021	·018939	·006841	·011	·021	·020202	·007061	·011	·021	·021465 –	·007274	·011	·021	·022727	·007480	·011	·021	·023990	·007680	·011	·021
794	·017654	·006605 +	·011	·021	·018916	·006833	·011	·021	·020177	·007052	·011	·021	·021438	·007265 –	·011	·021	·022699	·007470	·011	·021	·023960	·007670	·011	·021
795	·017632	·006597	·011	·021	·018892	·006824	·011	·021	·020151	·007043	·011	·021	·021411	·007256	·011	·021	·022670	·007461	·011	·021	·023929	·007661	·011	·021
796	·017610	·006589	·011	·021	·018868	·006816	·011	·021	·020126	·007035 –	·011	·021	·021384	·007247	·011	·021	·022642	·007452	·011	·021	·023899	·007651	·011	·021
797	·017588	·006581	·011	·021	·018844	·006807	·011	·021	·020101	·007026	·011	·021	·021357	·007238	·011	·021	·022613	·007443	·011	·021	·023869	·007642	·011	·021
798	·017566	·006572	·011	·021	·018821	·006799	·011	·021	·020075 +	·007017	·011	·021	·021330	·007229	·011	·021	·022585 –	·007433	·011	·021	·023839	·007632	·011	·021
799	·017544	·006564	·011	·021	·018797	·006790	·011	·021	·020050 +	·007009	·011	·021	·021303	·007220	·011	·021	·022556	·007424	·011	·021	·023810	·007623	·011	·021
800	·017522	·006556	·011	·021	·018773	·006782	·011	·021	·020025 +	·007000	·011	·021	·021277	·007211	·011	·021	·022528	·007415 +	·011	·021	·023780	·007613	·011	·021

N = size of sample	3				4				5			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 3·50	P_2 λ_2 = 2·94	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 3·20	P_2 λ_2 = 2·80	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 3·14	P_2 λ_2 = 2·68
801	·002500	·002494	·011	·019	·003750	·003052	·013	·020	·005000	·003522	·012	·021
802	·002497	·002491	·011	·019	·003745 +	·003049	·013	·020	·004994	·003518	·012	·021
803	·002494	·002488	·011	·019	·003741	·003045 −	·013	·020	·004988	·003514	·012	·021
804	·002491	·002484	·011	·019	·003736	·003041	·013	·020	·004981	·003509	·012	·021
805	·002488	·002481	·011	·019	·003731	·003037	·013	·020	·004975 +	·003505 −	·012	·021
806	·002484	·002478	·011	·019	·003727	·003033	·013	·020	·004969	·003500	·012	·021
807	·002481	·002475 +	·011	·019	·003722	·003030	·013	·020	·004963	·003496	·012	·021
808	·002478	·002472	·011	·019	·003717	·003026	·013	·020	·004957	·003492	·012	·021
809	·002475 +	·002469	·011	·019	·003713	·003022	·013	·020	·004950 +	·003488	·012	·021
810	·002472	·002466	·011	·019	·003708	·003018	·013	·020	·004944	·003483	·012	·021
811	·002469	·002463	·011	·019	·003704	·003015 −	·013	·020	·004938	·003479	·012	·021
812	·002466	·002460	·011	·019	·003699	·003011	·013	·020	·004932	·003475 −	·012	·021
813	·002463	·002457	·011	·019	·003695 −	·003007	·013	·020	·004926	·003470	·012	·021
814	·002460	·002454	·011	·019	·003690	·003004	·013	·020	·004920	·003466	·012	·021
815	·002457	·002451	·011	·019	·003686	·003000	·013	·020	·004914	·003462	·012	·021
816	·002454	·002448	·011	·019	·003681	·002996	·013	·020	·004908	·003458	·012	·021
817	·002451	·002445 −	·011	·019	·003676	·002993	·013	·020	·004902	·003453	·012	·021
818	·002448	·002442	·011	·019	·003672	·002989	·013	·020	·004896	·003449	·012	·021
819	·002445 −	·002439	·011	·019	·003667	·002985 +	·013	·020	·004890	·003445 +	·012	·021
820	·002442	·002436	·011	·019	·003663	·002982	·013	·020	·004884	·003441	·012	·021
821	·002439	·002433	·011	·019	·003659	·002978	·013	·020	·004878	·003437	·012	·021
822	·002436	·002430	·011	·019	·003654	·002974	·013	·020	·004872	·003433	·012	·021
823	·002433	·002427	·011	·019	·003650 −	·002971	·013	·020	·004866	·003428	·012	·021
824	·002430	·002424	·011	·019	·003645 +	·002967	·013	·020	·004860	·003424	·012	·021
825	·002427	·002421	·011	·019	·003641	·002964	·013	·020	·004854	·003420	·012	·021
826	·002424	·002418	·011	·019	·003636	·002960	·013	·020	·004848	·003416	·012	·021
827	·002421	·002415 +	·011	·019	·003632	·002957	·013	·020	·004843	·003412	·012	·021
828	·002418	·002413	·011	·019	·003628	·002953	·013	·020	·004837	·003408	·012	·021
829	·002415 +	·002410	·011	·019	·003623	·002949	·013	·020	·004831	·003404	·012	·021
830	·002413	·002407	·011	·019	·003619	·002946	·013	·020	·004825 +	·003400	·012	·021
831	·002410	·002404	·011	·019	·003614	·002942	·013	·020	·004819	·003395 +	·012	·021
832	·002407	·002401	·011	·019	·003610	·002939	·013	·020	·004813	·003391	·012	·021
833	·002404	·002398	·011	·019	·003606	·002935 +	·013	·020	·004808	·003387	·012	·021
834	·002401	·002395 +	·011	·019	·003601	·002932	·013	·020	·004802	·003383	·012	·021
835	·002398	·002392	·011	·019	·003597	·002928	·013	·020	·004796	·003379	·012	·021
836	·002395 +	·002389	·011	·019	·003593	·002925 −	·013	·020	·004790	·003375 +	·012	·021
837	·002392	·002387	·011	·019	·003589	·002921	·013	·020	·004785 −	·003371	·012	·021
838	·002389	·002384	·011	·019	·003584	·002918	·013	·020	·004779	·003367	·012	·021
839	·002387	·002381	·011	·019	·003580	·002914	·013	·020	·004773	·003363	·012	·021
840	·002384	·002378	·011	·019	·003576	·002911	·013	·020	·004768	·003359	·012	·021
841	·002381	·002375 +	·011	·019	·003571	·002907	·013	·020	·004762	·003355 +	·012	·021
842	·002378	·002372	·011	·019	·003567	·002904	·013	·020	·004756	·003351	·012	·021
843	·002375 +	·002370	·011	·019	·003563	·002901	·013	·020	·004751	·003347	·012	·021
844	·002372	·002367	·011	·019	·003559	·002897	·013	·020	·004745 −	·003343	·012	·021
845	·002370	·002364	·011	·019	·003555 −	·002894	·013	·020	·004739	·003339	·012	·021
846	·002367	·002361	·011	·019	·003550 +	·002890	·013	·020	·004734	·003335 +	·012	·021
847	·002364	·002358	·011	·019	·003546	·002887	·013	·020	·004728	·003331	·012	·021
848	·002361	·002356	·011	·019	·003542	·002883	·013	·020	·004723	·003328	·012	·021
849	·002358	·002353	·011	·019	·003538	·002880	·013	·020	·004717	·003324	·012	·021
850	·002356	·002350 +	·011	·019	·003534	·002877	·013	·020	·004711	·003320	·012	·021

N = size of sample	6				7				8			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 3·11	P_2 λ_2 = 2·63	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 3·08	P_2 λ_2 = 2·58	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 3·02	P_2 λ_2 = 2·54
801	·006250	·003936	·011	·021	·007500	·004308	·010	·021	·008750	·004651	·011	·021
802	·006242	·003931	·011	·021	·007491	·004303	·010	·021	·008739	·004645 −	·011	·021
803	·006234	·003926	·011	·021	·007481	·004298	·010	·021	·008728	·004639	·011	·021
804	·006227	·003921	·011	·021	·007472	·004292	·010	·021	·008717	·004633	·011	·021
805	·006219	·003916	·011	·021	·007463	·004287	·010	·021	·008706	·004628	·011	·021
806	·006211	·003911	·011	·021	·007453	·004282	·010	·021	·008696	·004622	·011	·021
807	·006203	·003906	·011	·021	·007444	·004277	·010	·021	·008685 −	·004616	·011	·021
808	·006196	·003902	·011	·021	·007435 −	·004271	·010	·021	·008674	·004611	·011	·021
809	·006188	·003897	·011	·021	·007426	·004266	·010	·021	·008663	·004605 −	·011	·021
810	·006180	·003892	·011	·021	·007417	·004261	·010	·021	·008653	·004599	·011	·021
811	·006173	·003887	·011	·021	·007407	·004256	·010	·021	·008642	·004594	·011	·021
812	·006165 +	·003882	·011	·021	·007398	·004250 +	·010	·021	·008631	·004588	·011	·021
813	·006158	·003878	·011	·021	·007389	·004245 +	·010	·021	·008621	·004582	·011	·021
814	·006150 +	·003873	·011	·021	·007380	·004240	·010	·021	·008610	·004577	·011	·021
815	·006143	·003868	·011	·021	·007371	·004235 −	·010	·021	·008600	·004571	·011	·021
816	·006135 −	·003863	·011	·021	·007362	·004230	·010	·021	·008589	·004566	·011	·021
817	·006127	·003859	·011	·021	·007353	·004224	·010	·021	·008578	·004560	·011	·021
818	·006120	·003854	·011	·021	·007344	·004219	·010	·021	·008568	·004555 −	·011	·021
819	·006112	·003849	·011	·021	·007335 −	·004214	·010	·021	·008557	·004549	·011	·021
820	·006105 +	·003845 −	·011	·021	·007326	·004209	·010	·021	·008547	·004543	·011	·021
821	·006098	·003840	·011	·021	·007317	·004204	·010	·021	·008537	·004538	·011	·021
822	·006090	·003835 +	·011	·021	·007308	·004199	·010	·021	·008526	·004532	·011	·021
823	·006083	·003831	·011	·021	·007299	·004194	·010	·021	·008516	·004527	·011	·021
824	·006075 +	·003826	·011	·021	·007290	·004189	·010	·021	·008505 +	·004521	·011	·021
825	·006068	·003821	·011	·021	·007282	·004184	·010	·021	·008495 +	·004516	·011	·021
826	·006061	·003817	·011	·021	·007273	·004179	·010	·021	·008485 −	·004511	·011	·021
827	·006053	·003812	·011	·021	·007264	·004174	·010	·021	·008475 −	·004505 +	·011	·021
828	·006046	·003808	·011	·021	·007255 +	·004168	·010	·021	·008464	·004500 −	·011	·021
829	·006039	·003803	·011	·021	·007246	·004163	·010	·021	·008454	·004494	·011	·021
830	·006031	·003798	·011	·021	·007238	·004158	·010	·021	·008444	·004489	·011	·021
831	·006024	·003794	·011	·021	·007229	·004154	·010	·021	·008434	·004484	·011	·021
832	·006017	·003789	·011	·021	·007220	·004149	·010	·021	·008424	·004478	·011	·021
833	·006010	·003785 −	·011	·021	·007212	·004144	·010	·021	·008413	·004473	·011	·021
834	·006002	·003780	·011	·021	·007203	·004139	·010	·021	·008403	·004468	·011	·021
835	·005995 +	·003776	·011	·021	·007194	·004134	·010	·021	·008393	·004462	·011	·021
836	·005988	·003771	·011	·021	·007186	·004129	·010	·021	·008383	·004457	·011	·021
837	·005981	·003767	·011	·021	·007177	·004124	·010	·021	·008373	·004452	·011	·021
838	·005974	·003762	·011	·021	·007168	·004119	·010	·021	·008363	·004446	·011	·021
839	·005967	·003758	·011	·021	·007160	·004114	·010	·021	·008353	·004441	·011	·021
840	·005959	·003753	·011	·021	·007151	·004109	·010	·021	·008343	·004436	·011	·021
841	·005952	·003749	·011	·021	·007143	·004104	·010	·021	·008333	·004430	·011	·021
842	·005945 +	·003744	·011	·021	·007134	·004099	·010	·021	·008323	·004425 +	·011	·021
843	·005938	·003740	·011	·021	·007126	·004095 −	·010	·021	·008314	·004420	·011	·021
844	·005931	·003736	·011	·021	·007117	·004090	·010	·021	·008304	·004415 −	·011	·021
845	·005924	·003731	·011	·021	·007109	·004085 −	·010	·021	·008294	·004410	·011	·021
846	·005917	·003727	·011	·021	·007101	·004080	·010	·021	·008284	·004404	·011	·021
847	·005910	·003722	·011	·021	·007092	·004075 +	·010	·021	·008274	·004399	·011	·021
848	·005903	·003718	·011	·021	·007084	·004071	·010	·021	·008264	·004394	·011	·021
849	·005896	·003714	·011	·021	·007075 +	·004066	·010	·021	·008255 −	·004389	·011	·021
850	·005889	·003709	·011	·021	·007067	·004061	·010	·021	·008245 −	·004384	·011	·021

n = number of arrays

N = size of sample	9				10				11			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2.96$	P_2 $\lambda_2 = 2.50$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2.92$	P_2 $\lambda_2 = 2.47$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2.88$	P_2 $\lambda_2 = 2.44$
801	·010000	·004969	·011	·021	·011250	·005267	·011	·021	·012500	·005548	·011	·021
802	·009988	·004963	·011	·021	·011236	·005260	·011	·021	·012484	·005541	·011	·021
803	·009975+	·004956	·011	·021	·011222	·005254	·011	·021	·012469	·005534	·011	·021
804	·009963	·004950+	·011	·021	·011208	·005247	·011	·021	·012453	·005528	·011	·021
805	·009950+	·004944	·011	·021	·011194	·005241	·011	·021	·012438	·005521	·011	·021
806	·009938	·004938	·011	·021	·011180	·005234	·011	·021	·012422	·005514	·011	·021
807	·009926	·004932	·011	·021	·011166	·005228	·011	·021	·012407	·005507	·011	·021
808	·009913	·004926	·011	·021	·011152	·005221	·011	·021	·012392	·005500+	·011	·021
809	·009901	·004920	·011	·021	·011139	·005215+	·011	·021	·012376	·005494	·011	·021
810	·009889	·004914	·011	·021	·011125−	·005209	·011	·021	·012361	·005487	·011	·021
811	·009877	·004908	·011	·021	·011111	·005202	·011	·021	·012346	·005480	·011	·021
812	·009864	·004902	·011	·021	·011097	·005196	·011	·021	·012330	·005474	·011	·021
813	·009852	·004896	·011	·021	·011084	·005190	·011	·021	·012315+	·005467	·011	·021
814	·009840	·004890	·011	·021	·011070	·005183	·011	·021	·012300	·005460	·011	·021
815	·009828	·004884	·011	·021	·011057	·005177	·011	·021	·012285+	·005453	·011	·021
816	·009816	·004878	·011	·021	·011043	·005171	·011	·021	·012270	·005447	·011	·021
817	·009804	·004872	·011	·021	·011029	·005164	·011	·021	·012255−	·005440	·011	·021
818	·009792	·004866	·011	·021	·011016	·005158	·011	·021	·012240	·005434	·011	·021
819	·009780	·004860	·011	·021	·011002	·005152	·011	·021	·012225−	·005427	·011	·021
820	·009768	·004854	·011	·021	·010989	·005145+	·011	·021	·012210	·005420	·011	·021
821	·009756	·004848	·011	·021	·010976	·005139	·011	·021	·012195+	·005414	·011	·021
822	·009744	·004842	·011	·021	·010962	·005133	·011	·021	·012180	·005407	·011	·021
823	·009732	·004837	·011	·021	·010949	·005127	·011	·021	·012165−	·005401	·011	·021
824	·009721	·004831	·011	·021	·010936	·005121	·011	·021	·012151	·005394	·011	·021
825	·009709	·004825−	·011	·021	·010922	·005114	·011	·021	·012136	·005388	·011	·021
826	·009697	·004819	·011	·021	·010909	·005108	·011	·021	·012121	·005381	·011	·021
827	·009685+	·004813	·011	·021	·010896	·005102	·011	·021	·012107	·005375−	·011	·021
828	·009674	·004807	·011	·021	·010883	·005096	·011	·021	·012092	·005368	·011	·021
829	·009662	·004802	·011	·021	·010870	·005090	·011	·021	·012077	·005362	·011	·021
830	·009650+	·004796	·011	·021	·010856	·005084	·011	·021	·012063	·005356	·011	·021
831	·009639	·004790	·011	·021	·010843	·005078	·011	·021	·012048	·005349	·011	·021
832	·009627	·004784	·011	·021	·010830	·005072	·011	·021	·012034	·005343	·011	·021
833	·009615+	·004779	·011	·021	·010817	·005066	·011	·021	·012019	·005336	·011	·021
834	·009604	·004773	·011	·021	·010804	·005060	·011	·021	·012005−	·005330	·011	·021
835	·009592	·004767	·011	·021	·010791	·005054	·011	·021	·011990	·005324	·011	·021
836	·009581	·004762	·011	·021	·010778	·005048	·011	·021	·011976	·005317	·011	·021
837	·009569	·004756	·011	·021	·010766	·005042	·011	·021	·011962	·005311	·011	·021
838	·009558	·004750+	·011	·021	·010753	·005036	·011	·021	·011947	·005305−	·011	·021
839	·009547	·004745−	·011	·021	·010740	·005030	·011	·021	·011933	·005298	·011	·021
840	·009535+	·004739	·011	·021	·010727	·005024	·011	·021	·011919	·005292	·011	·021
841	·009524	·004734	·011	·021	·010714	·005018	·011	·021	·011905−	·005286	·011	·021
842	·009512	·004728	·011	·021	·010702	·005012	·011	·021	·011891	·005280	·011	·021
843	·009501	·004722	·011	·021	·010689	·005006	·011	·021	·011876	·005273	·011	·021
844	·009490	·004717	·011	·021	·010676	·005000	·011	·021	·011862	·005267	·011	·021
845	·009479	·004711	·011	·021	·010664	·004994	·011	·021	·011848	·005261	·011	·021
846	·009467	·004706	·011	·021	·010651	·004988	·011	·021	·011834	·005255−	·011	·021
847	·009456	·004700	·011	·021	·010638	·004982	·011	·021	·011820	·005249	·011	·021
848	·009445+	·004695−	·011	·021	·010626	·004976	·011	·021	·011806	·005243	·011	·021
849	·009434	·004689	·011	·021	·010613	·004971	·011	·021	·011792	·005236	·011	·021
850	·009423	·004684	·011	·021	·010601	·004965−	·011	·021	·011779	·005230	·011	·021

N = size of sample	12				13				14			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2.84$	P_2 $\lambda_2 = 2.42$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2.80$	P_2 $\lambda_2 = 2.40$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1 = 2.79$	P_2 $\lambda_2 = 2.39$
801	·013750	·005815+	·011	·022	·015000	·006070	·011	·022	·016250	·006314	·011	·021
802	·013733	·005808	·011	·022	·014981	·006063	·011	·022	·016230	·006306	·011	·021
803	·013716	·005801	·011	·022	·014963	·006055−	·011	·022	·016209	·006298	·011	·021
804	·013699	·005794	·011	·022	·014944	·006048	·011	·022	·016189	·006291	·011	·021
805	·013682	·005787	·011	·022	·014925+	·006040	·011	·022	·016169	·006283	·011	·021
806	·013665−	·005779	·011	·022	·014907	·006033	·011	·022	·016149	·006275−	·011	·021
807	·013648	·005772	·011	·022	·014888	·006025+	·011	·022	·016129	·006267	·011	·021
808	·013631	·005765+	·011	·022	·014870	·006018	·011	·022	·016109	·006260	·011	·021
809	·013614	·005758	·011	·022	·014851	·006010	·011	·022	·016089	·006252	·011	·021
810	·013597	·005751	·011	·022	·014833	·006003	·011	·022	·016069	·006244	·011	·021
811	·013580	·005744	·011	·022	·014815−	·005996	·011	·022	·016049	·006237	·011	·021
812	·013564	·005737	·011	·022	·014797	·005988	·011	·022	·016030	·006229	·011	·021
813	·013547	·005730	·011	·022	·014778	·005981	·011	·022	·016010	·006221	·011	·021
814	·013530	·005723	·011	·022	·014760	·005974	·011	·022	·015990	·006214	·011	·021
815	·013514	·005716	·011	·022	·014742	·005967	·011	·022	·015971	·006206	·011	·021
816	·013497	·005709	·011	·022	·014724	·005959	·011	·022	·015951	·006199	·011	·021
817	·013480	·005702	·011	·022	·014706	·005952	·011	·022	·015931	·006191	·011	·021
818	·013464	·005695+	·011	·022	·014688	·005945−	·011	·022	·015912	·006184	·011	·021
819	·013447	·005688	·011	·022	·014670	·005938	·011	·022	·015892	·006176	·011	·021
820	·013431	·005681	·011	·022	·014652	·005930	·011	·022	·015873	·006169	·011	·021
821	·013415−	·005675−	·011	·022	·014634	·005923	·011	·022	·015854	·006161	·011	·021
822	·013398	·005668	·011	·022	·014616	·005916	·011	·022	·015834	·006154	·011	·021
823	·013382	·005661	·011	·022	·014599	·005909	·011	·022	·015815+	·006146	·011	·021
824	·013366	·005654	·011	·022	·014581	·005902	·011	·022	·015796	·006139	·011	·021
825	·013350−	·005647	·011	·022	·014563	·005895−	·011	·022	·015777	·006132	·011	·021
826	·013333	·005640	·011	·022	·014545+	·005888	·011	·022	·015758	·006124	·011	·021
827	·013317	·005634	·011	·022	·014528	·005881	·011	·022	·015738	·006117	·011	·021
828	·013301	·005627	·011	·022	·014510	·005874	·011	·022	·015719	·006110	·011	·021
829	·013285+	·005620	·011	·022	·014493	·005867	·011	·022	·015700	·006102	·011	·021
830	·013269	·005613	·011	·022	·014475+	·005860	·011	·022	·015682	·006095+	·011	·021
831	·013253	·005607	·011	·022	·014458	·005853	·011	·022	·015663	·006088	·011	·021
832	·013237	·005600	·011	·022	·014440	·005846	·011	·022	·015644	·006081	·011	·021
833	·013221	·005593	·011	·022	·014423	·005839	·011	·022	·015625	·006073	·011	·021
834	·013205+	·005587	·011	·022	·014406	·005832	·011	·022	·015606	·006066	·011	·021
835	·013189	·005580	·011	·022	·014388	·005825−	·011	·022	·015588	·006059	·011	·021
836	·013174	·005573	·011	·022	·014371	·005818	·011	·022	·015569	·006052	·011	·021
837	·013158	·005567	·011	·022	·014354	·005811	·011	·022	·015550+	·006044	·011	·022
838	·013142	·005560	·011	·022	·014337	·005804	·011	·022	·015532	·006037	·011	·022
839	·013126	·005554	·011	·022	·014320	·005797	·011	·022	·015513	·006030	·011	·022
840	·013111	·005547	·011	·022	·014303	·005790	·011	·022	·015495−	·006023	·011	·022
841	·013095+	·005541	·011	·022	·014286	·005783	·011	·022	·015476	·006016	·011	·022
842	·013080	·005534	·011	·022	·014269	·005777	·011	·022	·015458	·006009	·011	·022
843	·013064	·005527	·011	·022	·014252	·005770	·011	·022	·015439	·006002	·011	·022
844	·013049	·005521	·011	·022	·014235−	·005763	·011	·022	·015421	·005995−	·011	·022
845	·013033	·005515−	·011	·022	·014218	·005756	·011	·022	·015403	·005988	·011	·022
846	·013018	·005508	·011	·022	·014201	·005749	·011	·022	·015385−	·005981	·011	·022
847	·013002	·005502	·011	·022	·014184	·005743	·011	·022	·015366	·005974	·011	·022
848	·012987	·005495+	·011	·022	·014168	·005736	·011	·022	·015348	·005967	·011	·022
849	·012972	·005489	·011	·022	·014151	·005729	·011	·022	·015330	·005960	·011	·022
850	·012956	·005482	·011	·022	·014134	·005723	·011	·022	·015312	·005953	·011	·022

N = size of sample	15				16				17			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·78	P_2 $\lambda_2=$ 2·38	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·77	P_2 $\lambda_2=$ 2·37	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·76	P_2 $\lambda_2=$ 2·36
801	·017500	·006548	·011	·021	·018750	·006774	·011	·021	·020000	·006991	·011	·021
802	·017478	·006540	·011	·021	·018727	·006765 +	·011	·021	·019975 +	·006983	·011	·021
803	·017456	·006532	·011	·021	·018703	·006757	·011	·021	·019950 +	·006974	·011	·021
804	·017435 −	·006524	·011	·021	·018680	·006749	·011	·021	·019925 +	·006965 +	·011	·021
805	·017413	·006516	·011	·021	·018657	·006740	·011	·021	·019900	·006957	·011	·021
806	·017391	·006508	·011	·021	·018634	·006732	·011	·021	·019876	·006948	·011	·021
807	·017370	·006500 −	·011	·021	·018610	·006724	·011	·021	·019851	·006940	·011	·021
808	·017348	·006492	·011	·021	·018587	·006715 +	·011	·021	·019827	·006931	·011	·021
809	·017327	·006484	·011	·021	·018564	·006707	·011	·021	·019802	·006923	·011	·021
810	·017305 +	·006476	·011	·021	·018541	·006699	·011	·021	·019778	·006914	·011	·021
811	·017284	·006468	·011	·021	·018519	·006691	·011	·021	·019753	·006906	·011	·021
812	·017263	·006460	·011	·021	·018496	·006683	·011	·021	·019729	·006898	·011	·021
813	·017241	·006452	·011	·021	·018473	·006675 −	·011	·021	·019704	·006889	·011	·021
814	·017220	·006444	·011	·021	·018450 +	·006666	·011	·021	·019680	·006881	·011	·021
815	·017199	·006437	·011	·021	·018428	·006658	·011	·021	·019656	·006872	·011	·021
816	·017178	·006429	·011	·021	·018405 −	·006650 +	·011	·021	·019632	·006864	·011	·021
817	·017157	·006421	·011	·021	·018382	·006642	·011	·021	·019608	·006856	·011	·021
818	·017136	·006413	·011	·021	·018360	·006634	·011	·021	·019584	·006847	·011	·021
819	·017115 −	·006405 +	·011	·021	·018337	·006626	·011	·021	·019560	·006839	·011	·021
820	·017094	·006398	·011	·021	·018315 +	·006618	·011	·021	·019536	·006831	·011	·021
821	·017073	·006390	·011	·021	·018293	·006610	·011	·021	·019512	·006823	·011	·021
822	·017052	·006382	·011	·021	·018270	·006602	·011	·021	·019488	·006814	·011	·021
823	·017032	·006375 −	·011	·021	·018248	·006594	·011	·021	·019465 −	·006806	·011	·021
824	·017011	·006367	·011	·021	·018226	·006586	·011	·021	·019441	·006798	·011	·021
825	·016990	·006359	·011	·021	·018204	·006578	·011	·021	·019417	·006790	·011	·021
826	·016970	·006352	·011	·021	·018182	·006570	·011	·021	·019394	·006782	·011	·021
827	·016949	·006344	·011	·021	·018160	·006563	·011	·021	·019370	·006774	·011	·021
828	·016929	·006336	·011	·021	·018138	·006555 −	·011	·021	·019347	·006766	·011	·021
829	·016908	·006329	·011	·021	·018116	·006547	·011	·021	·019324	·006757	·011	·021
830	·016888	·006321	·011	·021	·018094	·006539	·011	·021	·019300	·006749	·011	·021
831	·016867	·006314	·011	·021	·018072	·006531	·011	·021	·019277	·006741	·011	·021
832	·016847	·006306	·011	·021	·018051	·006524	·011	·021	·019254	·006733	·011	·021
833	·016827	·006299	·011	·021	·018029	·006516	·011	·021	·019231	·006725 +	·011	·021
834	·016807	·006291	·011	·021	·018007	·006508	·011	·021	·019208	·006717	·011	·021
835	·016787	·006284	·011	·021	·017986	·006500 +	·011	·021	·019185 −	·006709	·011	·021
836	·016766	·006276	·011	·021	·017964	·006493	·011	·021	·019162	·006701	·011	·021
837	·016746	·006269	·011	·021	·017943	·006485 −	·011	·021	·019139	·006694	·011	·021
838	·016726	·006261	·011	·021	·017921	·006477	·011	·021	·019116	·006686	·011	·021
839	·016706	·006254	·011	·021	·017900	·006470	·011	·021	·019093	·006678	·011	·021
840	·016687	·006247	·011	·021	·017878	·006462	·011	·021	·019070	·006670	·011	·021
841	·016667	·006239	·011	·021	·017857	·006454	·011	·021	·019048	·006662	·011	·021
842	·016647	·006232	·011	·021	·017836	·006447	·011	·021	·019025 −	·006654	·011	·021
843	·016627	·006225 −	·011	·021	·017815 −	·006439	·011	·021	·019002	·006646	·011	·021
844	·016607	·006217	·011	·021	·017794	·006432	·011	·021	·018980	·006639	·011	·021
845	·016588	·006210	·011	·021	·017773	·006424	·011	·021	·018957	·006631	·011	·021
846	·016568	·006203	·011	·021	·017751	·006417	·011	·021	·018935 −	·006623	·011	·021
847	·016548	·006195 +	·011	·021	·017730	·006409	·011	·021	·018913	·006615 +	·011	·021
848	·016529	·006188	·011	·021	·017710	·006402	·011	·021	·018890	·006608	·011	·021
849	·016509	·006181	·011	·021	·017689	·006394	·011	·021	·018868	·006600	·011	·021
850	·016490	·006174	·011	·021	·017668	·006387	·011	·021	·018846	·006592	·011	·021

N = size of sample	18				19				20			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·75	P_2 $\lambda_2=$ 2·35	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·74	P_2 $\lambda_2=$ 2·34	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=$ 2·73	P_2 $\lambda_2=$ 2·33
801	·021250	·007202	·011	·021	·022500	·007406	·011	·021	·023750	·007604	·011	·021
802	·021223	·007193	·011	·021	·022472	·007397	·011	·021	·023720	·007595 −	·011	·021
803	·021197	·007184	·011	·021	·022444	·007388	·011	·021	·023691	·007585 +	·011	·021
804	·021171	·007175 +	·011	·021	·022416	·007379	·011	·021	·023661	·007576	·011	·021
805	·021144	·007166	·011	·021	·022388	·007370	·011	·021	·023632	·007567	·011	·021
806	·021118	·007158	·011	·021	·022360	·007360	·011	·021	·023602	·007557	·011	·021
807	·021092	·007149	·011	·021	·022333	·007351	·011	·021	·023573	·007548	·011	·021
808	·021066	·007140	·011	·021	·022305 −	·007342	·011	·021	·023544	·007539	·011	·021
809	·021040	·007131	·011	·021	·022277	·007333	·011	·021	·023515 −	·007530	·011	·021
810	·021014	·007123	·011	·021	·022250 −	·007325 −	·011	·021	·023486	·007520	·011	·021
811	·020988	·007114	·011	·021	·022222	·007316	·011	·021	·023457	·007511	·011	·021
812	·020962	·007105 +	·011	·021	·022195 −	·007307	·011	·021	·023428	·007502	·011	·021
813	·020936	·007097	·011	·021	·022167	·007298	·011	·021	·023399	·007493	·011	·021
814	·020910	·007088	·011	·021	·022140	·007289	·011	·021	·023370	·007484	·011	·021
815	·020885 −	·007079	·011	·021	·022113	·007280	·011	·021	·023342	·007475 −	·011	·021
816	·020859	·007071	·011	·021	·022086	·007271	·011	·021	·023313	·007466	·011	·021
817	·020833	·007062	·011	·021	·022059	·007262	·011	·021	·023284	·007457	·011	·021
818	·020808	·007054	·011	·021	·022032	·007254	·011	·021	·023256	·007448	·011	·021
819	·020782	·007045 +	·011	·021	·022005 −	·007245 −	·011	·021	·023227	·007439	·011	·021
820	·020757	·007037	·011	·021	·021978	·007236	·011	·021	·023199	·007430	·011	·021
821	·020732	·007028	·011	·021	·021951	·007228	·011	·021	·023171	·007421	·011	·021
822	·020706	·007020	·011	·021	·021924	·007219	·011	·021	·023143	·007412	·011	·021
823	·020681	·007011	·011	·021	·021898	·007210	·011	·021	·023114	·007403	·011	·021
824	·020656	·007003	·011	·021	·021871	·007201	·011	·021	·023086	·007394	·011	·021
825	·020631	·006995 −	·011	·021	·021845 −	·007193	·011	·021	·023058	·007385 +	·011	·021
826	·020606	·006986	·011	·021	·021818	·007184	·011	·021	·023030	·007377	·011	·021
827	·020581	·006978	·011	·021	·021792	·007176	·011	·021	·023002	·007368	·011	·021
828	·020556	·006969	·011	·021	·021765 +	·007167	·011	·021	·022975 −	·007359	·011	·021
829	·020531	·006961	·011	·021	·021739	·007159	·011	·021	·022947	·007350 +	·011	·021
830	·020507	·006953	·011	·021	·021713	·007150 +	·011	·021	·022919	·007341	·011	·021
831	·020482	·006945 −	·011	·021	·021687	·007142	·011	·021	·022892	·007333	·011	·021
832	·020457	·006936	·011	·021	·021661	·007133	·011	·021	·022864	·007324	·011	·021
833	·020433	·006928	·011	·021	·021635 −	·007125 −	·011	·021	·022837	·007315 +	·011	·021
834	·020408	·006920	·011	·021	·021609	·007116	·011	·021	·022809	·007307	·011	·021
835	·020384	·006912	·011	·021	·021583	·007108	·011	·021	·022782	·007298	·011	·021
836	·020359	·006903	·011	·021	·021557	·007099	·011	·021	·022754	·007289	·011	·021
837	·020335 −	·006895 +	·011	·021	·021531	·007091	·011	·021	·022727	·007281	·011	·021
838	·020311	·006887	·011	·021	·021505 +	·007083	·011	·021	·022700	·007272	·011	·021
839	·020286	·006879	·011	·021	·021480	·007074	·011	·021	·022673	·007264	·011	·021
840	·020262	·006871	·011	·021	·021454	·007066	·011	·021	·022646	·007255 +	·011	·021
841	·020238	·006863	·011	·021	·021429	·007058	·011	·021	·022619	·007246	·011	·021
842	·020214	·006855 −	·011	·021	·021403	·007049	·011	·021	·022592	·007238	·011	·021
843	·020190	·006847	·011	·021	·021378	·007041	·011	·021	·022565 +	·007230	·011	·021
844	·020166	·006839	·011	·021	·021352	·007033	·011	·021	·022539	·007221	·011	·021
845	·020142	·006831	·011	·021	·021327	·007024	·011	·021	·022512	·007213	·011	·021
846	·020118	·006823	·011	·021	·021302	·007016	·011	·021	·022485 +	·007204	·011	·021
847	·020095 −	·006815 −	·011	·021	·021277	·007008	·011	·021	·022459	·007196	·011	·021
848	·020071	·006807	·011	·021	·021251	·007000	·011	·021	·022432	·007187	·011	·021
849	·020047	·006799	·011	·021	·021226	·006992	·011	·021	·022406	·007179	·011	·021
850	·020024	·006791	·011	·021	·021201	·006984	·011	·021	·022379	·007171	·011	·021

n = number of arrays

N = size of sample	3				4				5				6				7				8			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}50$	P_2 $\lambda_2=2{\cdot}94$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}20$	P_2 $\lambda_2=2{\cdot}80$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}14$	P_2 $\lambda_2=2{\cdot}68$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}11$	P_2 $\lambda_2=2{\cdot}63$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}08$	P_2 $\lambda_2=2{\cdot}58$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}02$	P_2 $\lambda_2=2{\cdot}54$
851	·002353	·002347	·011	·019	·003529	·002873	·013	·020	·004706	·003316	·012	·021	·005882	·003705 +	·011	·021	·007059	·004056	·010	·021	·008235 +	·004379	·011	·021
852	·002350 +	·002345 −	·011	·019	·003525 +	·002870	·013	·020	·004700	·003312	·012	·021	·005875 +	·003701	·011	·021	·007051	·004051	·010	·021	·008226	·004374	·011	·021
853	·002347	·002342	·011	·019	·003521	·002867	·013	·020	·004695 −	·003308	·012	·021	·005869	·003696	·011	·021	·007042	·004047	·010	·021	·008216	·004368	·011	·021
854	·002345 −	·002339	·011	·019	·003517	·002863	·013	·020	·004689	·003304	·012	·021	·005862	·003692	·011	·021	·007034	·004042	·010	·021	·008206	·004363	·011	·021
855	·002342	·002336	·011	·019	·003513	·002860	·013	·020	·004684	·003300	·012	·021	·005855 −	·003688	·011	·021	·007026	·004037	·010	·021	·008197	·004358	·011	·021
856	·002339	·002334	·011	·019	·003509	·002857	·013	·020	·004678	·003297	·012	·021	·005848	·003683	·011	·021	·007018	·004033	·010	·021	·008187	·004353	·011	·021
857	·002336	·002331	·011	·019	·003505 −	·002853	·013	·020	·004673	·003293	·012	·021	·005841	·003679	·011	·021	·007009	·004028	·010	·021	·008178	·004348	·011	·021
858	·002334	·002328	·011	·019	·003501	·002850 −	·013	·020	·004667	·003289	·012	·021	·005834	·003675 −	·011	·021	·007001	·004023	·010	·021	·008168	·004343	·011	·021
859	·002331	·002326	·011	·019	·003497	·002847	·013	·020	·004662	·003285 +	·012	·021	·005828	·003671	·011	·021	·006993	·004019	·010	·021	·008159	·004338	·011	·021
860	·002328	·002323	·011	·019	·003492	·002843	·013	·020	·004657	·003281	·012	·021	·005821	·003666	·011	·021	·006985 −	·004014	·010	·021	·008149	·004333	·011	·021
861	·002326	·002320	·011	·019	·003488	·002840	·013	·020	·004651	·003277	·012	·021	·005814	·003662	·011	·021	·006977	·004009	·010	·021	·008140	·004328	·011	·021
862	·002323	·002317	·011	·019	·003484	·002837	·013	·020	·004646	·003274	·012	·021	·005807	·003658	·011	·021	·006969	·004005 −	·010	·021	·008130	·004323	·011	·021
863	·002320	·002315 −	·011	·019	·003480	·002833	·013	·020	·004640	·003270	·012	·021	·005800	·003654	·011	·021	·006961	·004000	·010	·021	·008121	·004318	·011	·021
864	·002317	·002312	·011	·019	·003476	·002830	·013	·020	·004635 −	·003266	·012	·021	·005794	·003649	·011	·021	·006952	·003995 +	·010	·021	·008111	·004313	·011	·021
865	·002315 −	·002309	·011	·019	·003472	·002827	·013	·020	·004630	·003262	·012	·021	·005787	·003645 +	·011	·021	·006944	·003991	·010	·021	·008102	·004308	·011	·021
866	·002312	·002307	·011	·019	·003468	·002824	·013	·020	·004624	·003259	·012	·021	·005780	·003641	·011	·021	·006936	·003986	·010	·021	·008092	·004303	·011	·021
867	·002309	·002304	·011	·019	·003464	·002820	·013	·020	·004619	·003255 −	·012	·021	·005774	·003637	·011	·021	·006928	·003982	·010	·021	·008083	·004298	·011	·021
868	·002307	·002301	·011	·019	·003460	·002817	·013	·020	·004614	·003251	·012	·021	·005767	·003633	·011	·021	·006920	·003977	·010	·021	·008074	·004293	·011	·021
869	·002304	·002299	·011	·019	·003456	·002814	·013	·020	·004608	·003247	·012	·021	·005760	·003628	·011	·021	·006912	·003973	·010	·021	·008065 −	·004288	·011	·021
870	·002301	·002296	·011	·019	·003452	·002811	·013	·020	·004603	·003244	·012	·021	·005754	·003624	·011	·021	·006904	·003968	·010	·021	·008055 +	·004283	·011	·021
871	·002299	·002294	·011	·019	·003448	·002807	·013	·020	·004598	·003240	·012	·021	·005747	·003620	·011	·021	·006897	·003963	·010	·021	·008046	·004279	·011	·021
872	·002296	·002291	·011	·019	·003444	·002804	·013	·020	·004592	·003236	·012	·021	·005741	·003616	·011	·021	·006889	·003959	·010	·021	·008037	·004274	·011	·021
873	·002294	·002288	·011	·019	·003440	·002801	·013	·020	·004587	·003232	·012	·021	·005734	·003612	·011	·021	·006881	·003954	·010	·021	·008028	·004269	·011	·021
874	·002291	·002286	·011	·019	·003436	·002798	·013	·020	·004582	·003229	·012	·021	·005727	·003608	·011	·021	·006873	·003950 −	·010	·021	·008018	·004264	·011	·021
875	·002288	·002283	·011	·019	·003432	·002795 −	·013	·020	·004577	·003225 +	·012	·021	·005721	·003604	·011	·021	·006865 −	·003945 +	·010	·021	·008009	·004259	·011	·021
876	·002286	·002280	·011	·019	·003429	·002791	·013	·020	·004571	·003221	·012	·021	·005714	·003600	·011	·021	·006857	·003941	·010	·021	·008000	·004254	·011	·021
877	·002283	·002278	·011	·019	·003425 −	·002788	·013	·020	·004566	·003218	·012	·021	·005708	·003595 +	·011	·021	·006849	·003936	·010	·021	·007991	·004249	·011	·021
878	·002281	·002275 +	·011	·019	·003421	·002785 +	·013	·020	·004561	·003214	·012	·021	·005701	·003591	·011	·021	·006842	·003932	·010	·021	·007982	·004245 −	·011	·021
879	·002278	·002273	·011	·019	·003417	·002782	·013	·020	·004556	·003210	·012	·021	·005695 −	·003587	·011	·021	·006834	·003927	·010	·021	·007973	·004240	·011	·021
880	·002275 +	·002270	·011	·019	·003413	·002779	·013	·020	·004551	·003207	·012	·021	·005688	·003583	·011	·021	·006826	·003923	·010	·021	·007964	·004235 −	·011	·021
881	·002273	·002268	·011	·019	·003409	·002776	·013	·020	·004545 +	·003203	·012	·021	·005682	·003579	·011	·021	·006818	·003919	·010	·021	·007955 −	·004230	·011	·021
882	·002270	·002265 +	·011	·019	·003405 +	·002772	·013	·020	·004540	·003200	·012	·021	·005675 +	·003575 +	·011	·021	·006810	·003914	·010	·021	·007946	·004225 +	·011	·021
883	·002268	·002262	·011	·019	·003401	·002769	·013	·020	·004535 +	·003196	·012	·021	·005669	·003571	·011	·021	·006803	·003910	·010	·021	·007937	·004221	·011	·021
884	·002265 +	·002260	·011	·019	·003398	·002766	·013	·020	·004530	·003192	·012	·021	·005663	·003567	·011	·021	·006795 +	·003905 +	·010	·021	·007928	·004216	·011	·021
885	·002262	·002257	·011	·019	·003394	·002763	·013	·020	·004525 −	·003189	·012	·021	·005656	·003563	·011	·021	·006787	·003901	·010	·021	·007919	·004211	·011	·021
886	·002260	·002255 −	·011	·019	·003390	·002760	·013	·020	·004520	·003185 +	·012	·021	·005650 −	·003559	·011	·021	·006780	·003897	·010	·021	·007910	·004206	·011	·021
887	·002257	·002252	·011	·019	·003386	·002757	·013	·020	·004515 −	·003182	·012	·021	·005643	·003555 +	·011	·021	·006772	·003892	·010	·021	·007901	·004202	·011	·021
888	·002255 −	·002250 −	·011	·019	·003382	·002754	·013	·020	·004510	·003178	·012	·021	·005637	·003551	·011	·021	·006764	·003888	·010	·021	·007892	·004197	·011	·021
889	·002252	·002247	·011	·019	·003378	·002751	·013	·020	·004505 −	·003174	·012	·021	·005631	·003547	·011	·021	·006757	·003883	·010	·021	·007883	·004192	·011	·021
890	·002250 −	·002245 −	·011	·019	·003375 −	·002748	·013	·020	·004499	·003171	·012	·021	·005624	·003543	·011	·021	·006749	·003879	·010	·021	·007874	·004188	·011	·021
891	·002247	·002242	·011	·019	·003371	·002745 −	·013	·020	·004494	·003167	·012	·021	·005618	·003539	·011	·021	·006742	·003875 −	·010	·021	·007865 +	·004183	·011	·021
892	·002245 −	·002240	·011	·019	·003367	·002741	·013	·020	·004489	·003164	·012	·021	·005612	·003535 +	·011	·021	·006734	·003870	·010	·021	·007856	·004178	·011	·021
893	·002242	·002237	·011	·019	·003363	·002738	·013	·020	·004484	·003160	·012	·021	·005605 +	·003531	·011	·021	·006726	·003866	·010	·021	·007848	·004174	·011	·021
894	·002240	·002235 −	·011	·019	·003359	·002735 +	·013	·020	·004479	·003157	·012	·021	·005599	·003527	·011	·021	·006719	·003862	·010	·021	·007839	·004169	·011	·021
895	·002237	·002232	·011	·019	·003356	·002732	·013	·020	·004474	·003153	·012	·021	·005593	·003523	·011	·021	·006711	·003857	·010	·021	·007830	·004164	·011	·021
896	·002235 −	·002230	·011	·019	·003352	·002729	·013	·020	·004469	·003150 −	·012	·021	·005587	·003519	·011	·021	·006704	·003853	·010	·021	·007821	·004160	·011	·021
897	·002232	·002227	·011	·019	·003348	·002726	·013	·020	·004464	·003146	·012	·021	·005580	·003516	·011	·021	·006696	·003849	·010	·021	·007813	·004155 −	·011	·021
898	·002230	·002225 −	·011	·019	·003344	·002723	·013	·020	·004459	·003143	·012	·021	·005574	·003512	·011	·021	·006689	·003845 −	·010	·021	·007804	·004150 +	·011	·021
899	·002227	·002222	·011	·019	·003341	·002720	·013	·020	·004454	·003139	·012	·021	·005568	·003508	·011	·021	·006682	·003840	·010	·021	·007795 +	·004146	·011	·021
900	·002225 −	·002220	·011	·019	·003337	·002717	·013	·020	·004449	·003136	·012	·021	·005562	·003504	·011	·021	·006674	·003836	·010	·021	·007786	·004141	·011	·021

N = size of sample	9				10				11			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}96$	P_2 $\lambda_2=2{\cdot}50$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}92$	P_2 $\lambda_2=2{\cdot}47$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}88$	P_2 $\lambda_2=2{\cdot}44$
851	·009412	·004678	·011	·021	·010588	·004959	·011	·021	·011765 −	·005224	·011	·021
852	·009401	·004673	·011	·021	·010576	·004953	·011	·021	·011751	·005218	·011	·021
853	·009390	·004667	·011	·021	·010563	·004947	·011	·021	·011737	·005212	·011	·021
854	·009379	·004662	·011	·021	·010551	·004942	·011	·021	·011723	·005206	·011	·021
855	·009368	·004656	·011	·021	·010539	·004936	·011	·021	·011710	·005200	·011	·021
856	·009357	·004651	·011	·021	·010526	·004930	·011	·021	·011696	·005194	·011	·021
857	·009346	·004646	·011	·021	·010514	·004924	·011	·021	·011682	·005188	·011	·021
858	·009335 −	·004640	·011	·021	·010502	·004919	·011	·021	·011669	·005182	·011	·021
859	·009324	·004635 −	·011	·021	·010490	·004913	·011	·021	·011655 +	·005176	·011	·021
860	·009313	·004629	·011	·021	·010477	·004907	·011	·021	·011641	·005170	·011	·021
861	·009302	·004624	·011	·021	·010465 +	·004902	·011	·021	·011628	·005164	·011	·021
862	·009292	·004619	·011	·021	·010453	·004896	·011	·021	·011614	·005158	·011	·021
863	·009281	·004613	·011	·021	·010441	·004890	·011	·021	·011601	·005152	·011	·021
864	·009270	·004608	·011	·021	·010429	·004885 −	·011	·021	·011587	·005146	·011	·021
865	·009259	·004603	·011	·021	·010417	·004879	·011	·021	·011574	·005140	·011	·021
866	·009249	·004598	·011	·021	·010405 −	·004874	·011	·021	·011561	·005134	·011	·021
867	·009238	·004592	·011	·021	·010393	·004868	·011	·021	·011547	·005128	·011	·021
868	·009227	·004587	·011	·021	·010381	·004862	·011	·021	·011534	·005122	·011	·021
869	·009217	·004582	·011	·021	·010369	·004857	·011	·021	·011521	·005117	·011	·021
870	·009206	·004576	·011	·021	·010357	·004851	·011	·021	·011507	·005111	·011	·021
871	·009195 +	·004571	·011	·021	·010345 −	·004846	·011	·021	·011494	·005105 −	·011	·021
872	·009185 −	·004566	·011	·021	·010333	·004840	·011	·021	·011481	·005099	·011	·021
873	·009174	·004561	·011	·021	·010321	·004835 −	·011	·021	·011468	·005093	·011	·021
874	·009164	·004556	·011	·021	·010309	·004829	·011	·021	·011455 −	·005087	·011	·021
875	·009153	·004550 +	·011	·021	·010297	·004824	·011	·021	·011442	·005082	·011	·021
876	·009143	·004545 +	·011	·021	·010286	·004818	·011	·021	·011429	·005076	·011	·021
877	·009132	·004540	·011	·021	·010274	·004813	·011	·021	·011416	·005070	·011	·021
878	·009122	·004535 −	·011	·021	·010262	·004807	·011	·021	·011403	·005064	·011	·021
879	·009112	·004530	·011	·021	·010251	·004802	·011	·021	·011390	·005059	·011	·021
880	·009101	·004525 −	·011	·021	·010239	·004796	·011	·021	·011377	·005053	·011	·021
881	·009091	·004520	·011	·021	·010227	·004791	·011	·021	·011364	·005047	·011	·021
882	·009081	·004515 −	·011	·021	·010216	·004786	·011	·021	·011351	·005042	·011	·021
883	·009070	·004509	·011	·021	·010204	·004780	·011	·021	·011338	·005036	·011	·021
884	·009060	·004504	·011	·021	·010193	·004775 −	·011	·021	·011325 +	·005030	·011	·021
885	·009050 −	·004499	·011	·021	·010181	·004769	·011	·021	·011312	·005025 −	·011	·021
886	·009040	·004494	·011	·021	·010169	·004764	·011	·021	·011299	·005019	·011	·021
887	·009029	·004489	·011	·021	·010158	·004759	·011	·021	·011287	·005013	·011	·021
888	·009019	·004484	·011	·021	·010147	·004753	·011	·021	·011274	·005008	·011	·021
889	·009009	·004479	·011	·021	·010135 +	·004748	·011	·021	·011261	·005002	·011	·021
890	·008999	·004474	·011	·021	·010124	·004743	·011	·021	·011249	·004997	·011	·021
891	·008989	·004469	·011	·021	·010112	·004738	·011	·021	·011236	·004991	·011	·021
892	·008979	·004464	·011	·021	·010101	·004732	·011	·021	·011223	·004985 +	·011	·021
893	·008969	·004459	·011	·021	·010090	·004727	·011	·021	·011211	·004980	·011	·021
894	·008959	·004454	·011	·021	·010078	·004722	·011	·021	·011198	·004974	·011	·021
895	·008949	·004449	·011	·021	·010067	·004716	·011	·021	·011186	·004969	·011	·021
896	·008939	·004444	·011	·021	·010056	·004711	·011	·021	·011173	·004963	·011	·021
897	·008929	·004439	·011	·021	·010045 −	·004706	·011	·021	·011161	·004958	·011	·021
898	·008919	·004434	·011	·021	·010033	·004701	·011	·021	·011148	·004952	·011	·021
899	·008909	·004430	·011	·021	·010022	·004696	·011	·021	·011136	·004947	·011	·021
900	·008899	·004425 −	·011	·021	·010011	·004690	·011	·021	·011123	·004941	·011	·021

N = size of sample	12				13				14			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}84$	P_2 $\lambda_2=2{\cdot}42$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}80$	P_2 $\lambda_2=2{\cdot}40$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}79$	P_2 $\lambda_2=2{\cdot}39$
851	·012941	·005476	·011	·022	·014118	·005716	·011	·022	·015294	·005946	·011	·022
852	·012926	·005469	·011	·022	·014101	·005709	·011	·022	·015276	·005939	·011	·022
853	·012911	·005463	·011	·022	·014085 −	·005703	·011	·022	·015258	·005932	·011	·022
854	·012896	·005457	·011	·022	·014068	·005696	·011	·022	·015240	·005925 +	·011	·022
855	·012881	·005450 +	·011	·022	·014052	·005689	$\cdot 011_5$	·022	·015222	·005918	·011	·022
856	·012865 +	·005444	·011	·022	·014035 +	·005683	·012	·022	·015205 −	·005911	·011	·022
857	·012850 +	·005438	·011	·022	·014019	·005676	·012	·022	·015187	·005904	·011	·022
858	·012835 −	·005431	·011	·022	·014002	·005670	·012	·022	·015169	·005898	·011	·022
859	·012821	·005425 +	·011	·022	·013986	·005663	·012	·022	·015152	·005891	·011	·022
860	·012806	·005419	·011	·022	·013970	·005657	·012	·022	·015134	·005884	·011	·022
861	·012791	·005413	·011	·022	·013953	·005650 +	·012	·022	·015116	·005877	·011	·022
862	·012776	·005406	·011	·022	·013937	·005644	·012	·022	·015099	·005871	·011	·022
863	·012761	·005400	·011	·022	·013921	·005637	·012	·022	·015081	·005864	·011	·022
864	·012746	·005394	·011	·022	·013905 −	·005631	·012	·022	·015064	·005857	·011	·022
865	·012731	·005388	·011	·022	·013889	·005624	·012	·022	·015046	·005850 +	·011	·022
866	·012717	·005382	·011	·022	·013873	·005618	·012	·022	·015029	·005844	·011	·022
867	·012702	·005375 +	·011	·022	·013857	·005611	·012	·022	·015012	·005837	·011	·022
868	·012687	·005369	·011	·022	·013841	·005605 −	·012	·022	·014994	·005830	·011	·022
869	·012673	·005363	·011	·022	·013825 −	·005598	·012	·022	·014977	·005824	·011	·022
870	·012658	·005357	·011	·022	·013809	·005592	·012	·022	·014960	·005817	·011	·022
871	·012644	·005351	·011	·022	·013793	·005586	·012	·022	·014943	·005810	·011	·022
872	·012629	·005345 −	·011	·022	·013777	·005579	·012	·022	·014925 +	·005804	·011	·022
873	·012615 −	·005339	·011	·022	·013761	·005573	·012	·022	·014908	·005797	·011	·022
874	·012600	·005333	·011	·022	·013746	·005567	·012	·022	·014891	·005791	·011	·022
875	·012586	·005327	·011	·022	·013730	·005560	·012	·022	·014874	·005784	·011	·022
876	·012571	·005321	·011	·022	·013714	·005554	·012	·022	·014857	·005777	·011	·022
877	·012557	·005315 −	·011	·022	·013699	·005548	·012	·022	·014840	·005771	·011	·022
878	·012543	·005309	·011	·022	·013683	·005541	·012	·022	·014823	·005764	·011	·022
879	·012528	·005303	·011	·022	·013667	·005535 +	·012	·022	·014806	·005758	·011	·022
880	·012514	·005297	·011	·022	·013652	·005529	·012	·022	·014790	·005751	·011	·022
881	·012500	·005291	·011	·022	·013636	·005523	·012	·022	·014773	·005745 −	·011	·022
882	·012486	·005285 −	·011	·022	·013621	·005516	·012	·022	·014756	·005738	·011	·022
883	·012472	·005279	·011	·022	·013605 +	·005510	·012	·022	·014739	·005732	·011	·022
884	·012458	·005273	·011	·022	·013590	·005504	·012	·022	·014723	·005726	·011	·022
885	·012443	·005267	·011	·022	·013575 −	·005498	·012	·022	·014706	·005719	·011	·022
886	·012429	·005261	·011	·022	·013559	·005492	·012	·022	·014689	·005713	·011	·022
887	·012415 +	·005255 +	·011	·022	·013544	·005486	·012	·022	·014673	·005706	·011	·022
888	·012401	·005249	·011	·022	·013529	·005479	·012	·022	·014656	·005700	·011	·022
889	·012387	·005243	·011	·022	·013514	·005473	·012	·022	·014640	·005694	·011	·022
890	·012373	·005237	·011	·022	·013498	·005467	·012	·022	·014623	·005687	·011	·022
891	·012360	·005232	·011	·022	·013483	·005461	·012	·022	·014607	·005681	·011	·022
892	·012346	·005226	·011	·022	·013468	·005455 +	·012	·022	·014590	·005675 −	·011	·022
893	·012332	·005220	·011	·022	·013453	·005449	·012	·022	·014574	·005668	·011	·022
894	·012318	·005214	·011	·022	·013438	·005443	·012	·022	·014558	·005662	·011	·022
895	·012304	·005208	·011	·022	·013423	·005437	·012	·022	·014541	·005656	·011	·022
896	·012291	·005203	·011	·022	·013408	·005431	·012	·022	·014525 +	·005649	·011	·022
897	·012277	·005197	·011	·022	·013393	·005425 −	·012	·022	·014509	·005643	·011	·022
898	·012263	·005191	·011	·022	·013378	·005419	·012	·022	·014493	·005637	·011	·022
899	·012249	·005185 +	·011	·022	·013363	·005413	·012	·022	·014477	·005631	·011	·022
900	·012236	·005180	·011	·022	·013348	·005407	·012	·022	·014461	·005624	·011	·022

n = number of arrays

N = size of sample	15: $\bar{\eta}^2$	15: $\sigma_{\bar{\eta}^2}$	15: P_1 $\lambda_1=2{\cdot}78$	15: P_2 $\lambda_2=2{\cdot}38$	16: $\bar{\eta}^2$	16: $\sigma_{\bar{\eta}^2}$	16: P_1 $\lambda_1=2{\cdot}77$	16: P_2 $\lambda_2=2{\cdot}37$
851	·016471	·006167	·011	·021	·017647	·006379	·011	·021
852	·016451	·006159	·011	·021	·017626	·006372	·011	·021
853	·016432	·006152	·011	·021	·017606	·006364	·011	·021
854	·016413	·006145 +	·011	·021	·017585 −	·006357	·011	·021
855	·016393	·006138	·011	·021	·017564	·006350 −	·011	·021
856	·016374	·006131	·011	·021	·017544	·006342	·011	·021
857	·016355 +	·006124	·011	·021	·017523	·006335 −	·011	·021
858	·016336	·006117	·011	·021	·017503	·006328	·011	·021
859	·016317	·006110	·011	·021	·017483	·006320	·011	·021
860	·016298	·006103	·011	·021	·017462	·006313	·011	·021
861	·016279	·006096	·011	·021	·017442	·006306	·011	·021
862	·016260	·006089	·011	·021	·017422	·006299	·011	·021
863	·016241	·006082	·011	·021	·017401	·006291	·011	·021
864	·016222	·006075 −	·011	·021	·017381	·006284	·011	·021
865	·016204	·006068	·011	·021	·017361	·006277	·011	·021
866	·016185 −	·006061	·011	·021	·017341	·006270	·011	·021
867	·016166	·006054	·011	·021	·017321	·006262	·011	·021
868	·016148	·006047	·011	·021	·017301	·006255 +	·011	·021
869	·016129	·006040	·011	·021	·017281	·006248	·011	·021
870	·016110	·006033	·011	·021	·017261	·006241	·011	·021
871	·016092	·006026	·011	·021	·017241	·006234	·011	·021
872	·016073	·006019	·011	·021	·017222	·006227	·011	·021
873	·016055 +	·006012	·011	·021	·017202	·006220	·011	·021
874	·016037	·006006	·011	·021	·017182	·006213	·011	·021
875	·016018	·005999	·011	·021	·017162	·006206	·011	·021
876	·016000	·005992	·011	·021	·017143	·006199	·011	·021
877	·015982	·005985 +	·011	·021	·017123	·006192	·011	·021
878	·015964	·005978	·011	·021	·017104	·006185 −	·011	·021
879	·015945 +	·005972	·011	·021	·017084	·006178	·011	·021
880	·015927	·005965 −	·011	·021	·017065 −	·006171	·011	·021
881	·015909	·005958	·011	·021	·017045 +	·006164	·011	·021
882	·015891	·005952	·011	·021	·017026	·006157	·011	·021
883	·015873	·005945 −	·011	·021	·017007	·006150 +	·011	·021
884	·015855 +	·005938	·011	·021	·016988	·006143	·011	·021
885	·015837	·005932	·011	·021	·016968	·006136	·011	·021
886	·015819	·005925 −	·011	·021	·016949	·006129	·011	·021
887	·015801	·005918	·011	·021	·016930	·006123	·011	·021
888	·015784	·005912	·011	·021	·016911	·006116	·011	·021
889	·015766	·005905 +	·011	·021	·016892	·006109	·011	·021
890	·015748	·005899	·011	·021	·016873	·006102	·011	·021
891	·015730	·005892	·011	·021	·016854	·006095 +	·011	·021
892	·015713	·005885 +	·011	·021	·016835 +	·006088	·011	·021
893	·015695 +	·005879	·011	·021	·016816	·006082	·011	·021
894	·015677	·005872	·011	·021	·016797	·006075 −	·011	·021
895	·015660	·005866	·011	·021	·016779	·006068	·011	·021
896	·015642	·005859	·011	·021	·016760	·006062	·011	·021
897	·015625	·005853	·011	·021	·016741	·006055 −	·011	·021
898	·015608	·005846	·011	·021	·016722	·006048	·011	·021
899	·015590	·005840	·011	·021	·016704	·006041	·011	·021
900	·015573	·005833	·011	·021	·016685 +	·006035 −	·011	·021

N = size of sample	17: $\bar{\eta}^2$	17: $\sigma_{\bar{\eta}^2}$	17: P_1 $\lambda_1=2{\cdot}76$	17: P_2 $\lambda_2=2{\cdot}36$	18: $\bar{\eta}^2$	18: $\sigma_{\bar{\eta}^2}$	18: P_1 $\lambda_1=2{\cdot}75$	18: P_2 $\lambda_2=2{\cdot}35$
851	·018824	·006584	·011	·021	·020000	·006783	·011	·021
852	·018801	·006577	·011	·021	·019976	·006775 +	·011	·021
853	·018779	·006569	·011	·021	·019953	·006767	·011	·021
854	·018757	·006562	·011	·021	·019930	·006759	·011	·021
855	·018735 +	·006554	·011	·021	·019906	·006752	·011	·021
856	·018713	·006546	·011	·021	·019883	·006744	·011	·021
857	·018692	·006539	·011	·021	·019860	·006736	·011	·021
858	·018670	·006531	·011	·021	·019837	·006728	·011	·021
859	·018648	·006524	·011	·021	·019814	·006720	·011	·021
860	·018626	·006516	·011	·021	·019790	·006713	·011	·021
861	·018605 −	·006509	·011	·021	·019767	·006705 +	·011	·021
862	·018583	·006501	·011	·021	·019744	·006697	·011	·021
863	·018561	·006494	·011	·021	·019722	·006690	·011	·021
864	·018540	·006486	·011	·021	·019699	·006682	·011	·021
865	·018519	·006479	·011	·021	·019676	·006674	·011	·021
866	·018497	·006471	·011	·021	·019653	·006667	·011	·021
867	·018476	·006464	·011	·021	·019630	·006659	·011	·021
868	·018454	·006457	·011	·021	·019608	·006652	·011	·021
869	·018433	·006449	·011	·021	·019585 −	·006644	·011	·021
870	·018412	·006442	·011	·021	·019563	·006636	·011	·021
871	·018391	·006435 −	·011	·021	·019540	·006629	·011	·021
872	·018370	·006427	·011	·021	·019518	·006621	·011	·021
873	·018349	·006420	·011	·021	·019495 +	·006614	·011	·021
874	·018328	·006413	·011	·021	·019473	·006606	·011	·021
875	·018307	·006406	·011	·021	·019451	·006599	·011	·021
876	·018286	·006398	·011	·021	·019429	·006591	·011	·021
877	·018265 −	·006391	·011	·021	·019406	·006584	·011	·021
878	·018244	·006384	·011	·021	·019384	·006577	·011	·021
879	·018223	·006377	·011	·021	·019362	·006569	·011	·021
880	·018203	·006369	·011	·021	·019340	·006562	·011	·021
881	·018182	·006362	·011	·021	·019318	·006554	·011	·021
882	·018161	·006355 +	·011	·021	·019296	·006547	·011	·021
883	·018141	·006348	·011	·021	·019274	·006540	·011	·021
884	·018120	·006341	·011	·021	·019253	·006532	·011	·021
885	·018100	·006334	·011	·021	·019231	·006525 −	·011	·021
886	·018079	·006327	·011	·021	·019209	·006518	·011	·021
887	·018059	·006320	·011	·021	·019187	·006510	·011	·021
888	·018038	·006313	·011	·021	·019166	·006503	·011	·021
889	·018018	·006306	·011	·021	·019144	·006496	·011	·021
890	·017998	·006299	·011	·021	·019123	·006489	·011	·021
891	·017978	·006292	·011	·021	·019101	·006481	·011	·021
892	·017957	·006285 −	·011	·021	·019080	·006474	·011	·021
893	·017937	·006278	·011	·021	·019058	·006467	·011	·021
894	·017917	·006271	·011	·021	·019037	·006460	·011	·021
895	·017897	·006264	·011	·021	·019016	·006453	·011	·021
896	·017877	·006257	·011	·021	·018994	·006446	·011	·021
897	·017857	·006250 −	·011	·021	·018973	·006439	·011	·021
898	·017837	·006243	·011	·021	·018952	·006431	·011	·021
899	·017817	·006236	·011	·021	·018931	·006424	·011	·021
900	·017798	·006229	·011	·021	·018910	·006417	·011	·021

N = size of sample	19: $\bar{\eta}^2$	19: $\sigma_{\bar{\eta}^2}$	19: P_1 $\lambda_1=2{\cdot}74$	19: P_2 $\lambda_2=2{\cdot}34$	20: $\bar{\eta}^2$	20: $\sigma_{\bar{\eta}^2}$	20: P_1 $\lambda_1=2{\cdot}73$	20: P_2 $\lambda_2=2{\cdot}33$
851	·021176	·006975 +	·011	·021	·022353	·007162	·011	·021
852	·021152	·006967	·011	·021	·022327	·007154	·011	·021
853	·021127	·006959	·011	·021	·022300	·007146	·011	·021
854	·021102	·006951	·011	·021	·022274	·007137	·011	·021
855	·021077	·006943	·011	·021	·022248	·007129	·011	·021
856	·021053	·006935 +	·011	·021	·022222	·007121	·011	·021
857	·021028	·006927	·011	·021	·022196	·007113	·011	·021
858	·021004	·006919	·011	·021	·022170	·007105 −	·011	·021
859	·020979	·006911	·011	·021	·022145 −	·007096	·011	·021
860	·020955 −	·006903	·011	·021	·022119	·007088	·011	·021
861	·020930	·006895 +	·011	·021	·022093	·007080	·011	·021
862	·020906	·006887	·011	·021	·022067	·007072	·011	·021
863	·020882	·006880	·011	·021	·022042	·007064	·011	·021
864	·020857	·006872	·011	·021	·022016	·007056	·011	·021
865	·020833	·006864	·011	·021	·021991	·007048	·011	·021
866	·020809	·006856	·011	·021	·021965 +	·007040	·011	·021
867	·020785 +	·006848	·011	·021	·021940	·007032	·011	·021
868	·020761	·006840	·011	·021	·021915 −	·007024	·011	·021
869	·020737	·006833	·011	·021	·021889	·007016	·011	·021
870	·020713	·006825 −	·011	·021	·021864	·007008	·011	·021
871	·020690	·006817	·011	·021	·021839	·007000	·011	·021
872	·020666	·006809	·011	·021	·021814	·006992	·011	·021
873	·020642	·006802	·011	·021	·021789	·006984	·011	·021
874	·020619	·006794	·011	·021	·021764	·006976	·011	·021
875	·020595 −	·006786	·011	·021	·021739	·006968	·011	·021
876	·020571	·006779	·011	·021	·021714	·006960	·011	·021
877	·020548	·006771	·011	·021	·021689	·006952	·011	·021
878	·020525 −	·006763	·011	·021	·021665 −	·006945 −	·011	·021
879	·020501	·006756	·011	·021	·021640	·006937	·011	·021
880	·020478	·006748	·011	·021	·021615 +	·006929	·011	·021
881	·020455 −	·006740	·011	·021	·021591	·006921	·011	·021
882	·020431	·006733	·011	·021	·021566	·006913	·011	·021
883	·020408	·006725 +	·011	·021	·021542	·006906	·011	·021
884	·020385 +	·006718	·011	·021	·021518	·006898	·011	·021
885	·020362	·006710	·011	·021	·021493	·006890	·011	·021
886	·020339	·006703	·011	·021	·021469	·006882	·011	·021
887	·020316	·006695 +	·011	·021	·021445 −	·006875 −	·011	·021
888	·020293	·006688	·011	·021	·021421	·006867	·011	·021
889	·020270	·006680	·011	·021	·021396	·006860	·011	·021
890	·020247	·006673	·011	·021	·021372	·006852	·011	·021
891	·020225 −	·006666	·011	·021	·021348	·006844	·011	·021
892	·020202	·006658	·011	·021	·021324	·006837	·011	·021
893	·020179	·006651	·011	·021	·021300	·006829	·011	·021
894	·020157	·006643	·011	·021	·021277	·006822	·011	·021
895	·020134	·006636	·011	·021	·021253	·006814	·011	·021
896	·020112	·006629	·011	·021	·021229	·006807	·011	·021
897	·020089	·006621	·011	·021	·021205 +	·006799	·011	·021
898	·020067	·006614	·011	·021	·021182	·006792	·011	·021
899	·020045 −	·006607	·011	·021	·021158	·006784	·011	·021
900	·020022	·006600	·011	·021	·021135 −	·006777	·011	·021

N = size of sample	3				4				5			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}50$	P_2 $\lambda_2=2{\cdot}94$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}20$	P_2 $\lambda_2=2{\cdot}80$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}14$	P_2 $\lambda_2=2{\cdot}68$
901	·002222	·002217	·011	·019	·003333	·002714	·013	·020	·004444	·003132	·012	·021
902	·002220	·002215 −	·011	·019	·003330	·002711	·013	·020	·004440	·003129	·012	·021
903	·002217	·002212	·011	·019	·003326	·002708	·013	·020	·004435 −	·003125 +	·012	·021
904	·002215 −	·002210	·011	·019	·003322	·002705 +	·013	·020	·004430	·003122	·012	·021
905	·002212	·002208	·011	·019	·003319	·002702	·013	·020	·004425 −	·003118	·012	·021
906	·002210	·002205 +	·011	·019	·003315 −	·002699	·013	·020	·004420	·003115 −	·012	·021
907	·002208	·002203	·011	·019	·003311	·002696	·013	·020	·004415 +	·003112	·012	·021
908	·002205 +	·002200	·011	·019	·003308	·002693	·013	·020	·004410	·003108	·012	·021
909	·002203	·002198	·011	·019	·003304	·002690	·013	·020	·004405 +	·003105 −	·012	·021
910	·002200	·002195 +	·011	·019	·003300	·002687	·013	·020	·004400	·003101	·012	·021
911	·002198	·002193	·011	·019	·003297	·002684	·013	·020	·004396	·003098	·012	·021
912	·002195 +	·002191	·011	·019	·003293	·002681	·013	·020	·004391	·003095 −	·012	·021
913	·002193	·002188	·011	·019	·003289	·002678	·013	·020	·004386	·003091	·012	·021
914	·002191	002186	·011	·019	·003286	·002676	·013	·020	·004381	·003088	·012	·021
915	·002188	·002183	·011	·019	·003282	·002673	·013	·020	·004376	·003084	·012	·021
916	·002186	·002181	·011	·019	·003279	·002670	·013	·020	·004372	·003081	·012	·021
917	·002183	·002179	·011	·019	·003275 +	·002667	·013	·020	·004367	·003078	·012	·021
918	·002181	·002176	·011	·019	·003272	·002664	·013	·020	·004362	·003074	·012	·021
919	·002179	·002174	·011	·019	·003268	·002661	·013	·020	·004357	·003071	·012	·021
920	·002176	·002172	·011	·019	·003264	·002658	·013	·020	·004353	·003068	·012	·021
921	·002174	·002169	·011	·019	·003261	·002655 +	·013	·020	·004348	·003064	·012	·021
922	·002172	·002167	·011	·019	·003257	·002652	·013	·020	·004343	·003061	·012	·021
923	·002169	·002164	·011	·019	·003254	·002650 −	·013	·020	·004338	·003058	·012	·021
924	·002167	·002162	·011	·019	·003250 +	·002647	·013	·020	·004334	·003054	·012	·021
925	·002165 −	·002160	·011	·019	·003247	·002644	·013	·020	·004329	·003051	·012	·021
926	·002162	·002157	·011	·019	·003243	·002641	·013	·020	·004324	·003048	·012	·021
927	·002160	·002155 +	·011	·019	·003240	·002638	·013	·020	·004320	·003045 −	·012	·021
928	·002157	·002153	·011	·019	·003236	·002635 +	·013	·020	·004315 −	·003041	·012	·021
929	·002155 +	·002151	·011	·019	·003233	·002632	·013	·020	·004310	·003038	·012	·021
930	·002153	·002148	·011	·019	·003229	·002630	·013	·020	·004306	·003035 −	·012	·021
931	·002151	·002146	·011	·019	·003226	·002627	·013	·020	·004301	·003032	·012	·021
932	·002148	·002144	·011	·019	·003222	·002624	·013	·020	·004296	·003028	·012	·021
933	·002146	·002141	·011	·019	·003219	·002621	·013	·020	·004292	·003025 +	·012	·021
934	·002144	·002139	·011	·019	·003215 +	·002618	·013	·020	·004287	·003022	·012	·021
935	·002141	·002137	·011	·019	·003212	·002616	·013	·020	·004283	·003019	·012	·021
936	·002139	·002134	·011	·019	·003209	·002613	·013	·020	·004278	·003015 +	·012	·021
937	·002137	·002132	·011	·019	·003205 +	·002610	·013	·020	·004274	·003012	·012	·021
938	·002134	·002130	·011	·019	·003202	·002607	·013	·020	·004269	·003009	·012	·021
939	·002132	·002128	·011	·019	·003198	·002604	·013	·020	·004264	·003006	·012	·021
940	·002130	·002125 +	·011	·019	·003195 −	·002602	·013	·020	·004260	·003003	·012	·021
941	·002128	·002123	·011	·019	·003191	·002599	·013	·020	·004255 +	·002999	·012	·021
942	·002125 +	·002121	·011	·019	·003188	·002596	·013	·020	·004251	·002996	·012	·021
943	·002123	·002119	·011	·019	·003185 −	·002593	·013	·020	·004246	·002993	·012	·021
944	·002121	·002116	·011	·019	·003181	·002591	·013	·020	·004242	·002990	·012	·021
945	·002119	·002114	·011	·019	·003178	·002588	·013	·020	·004237	·002987	·012	·021
946	·002116	·002112	·011	·019	·003175 −	·002585 +	·013	·020	·004233	·002984	·012	·021
947	·002114	·002110	·011	·019	·003171	·002582	·013	·020	·004228	·002980	·012	·021
948	·002112	·002107	·011	·019	·003168	·002580	·013	·020	·004224	·002977	·012	·021
949	·002110	·002105 +	·011	·019	·003165 −	·002577	·013	·020	·004219	·002974	·012	·021
950	·002107	·002103	·011	·019	·003161	·002574	·013	·020	·004215 −	·002971	·012	·021

N = size of sample	6				7				8			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}11$	P_2 $\lambda_2=2{\cdot}63$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}08$	P_2 $\lambda_2=2{\cdot}58$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}02$	P_2 $\lambda_2=2{\cdot}54$
901	·005556	·003500 −	·011	·021	·006667	·003832	·010	·021	·007778	·004137	·011	·021
902	·005549	·003496	·011	·021	·006659	·003828	·010	·021	·007769	·004132	·011	·021
903	·005543	·003492	·011	·021	·006652	·003823	·010	·021	·007761	·004127	·011	·021
904	·005537	·003488	·011	·021	·006645 −	·003819	·010	·021	·007752	·004123	·011	·021
905	·005531	·003485 −	·011	·021	·006637	·003815 +	·010	·021	·007743	·004118	·011	·021
906	·005525 −	·003481	·011	·021	·006630	·003811	·010	·021	·007735 −	·004114	·011	·021
907	·005519	·003477	·011	·021	·006623	·003807	·010	·021	·007726	·004109	·011	·021
908	·005513	·003473	·011	·021	·006615 +	·003802	·010	·021	·007718	·004105 −	·011	·021
909	·005507	·003469	·011	·021	·006608	·003798	·010	·021	·007709	·004100	·011	·021
910	·005501	·003465 +	·011	·021	·006601	·003794	·010	·021	·007701	·004096	·011	·021
911	·005495 −	·003462	·011	·021	·006593	·003790	·010	·021	·007692	·004091	·011	·021
912	·005488	·003458	·011	·021	·006586	·003786	·010	·021	·007684	·004087	·011	·021
913	·005482	·003454	·011	·021	·006579	·003782	·010	·021	·007675 +	·004082	·011	·021
914	·005476	·003450 +	·011	·021	·006572	·003778	·010	·021	·007667	·004078	·011	·021
915	·005470	·003447	·011	·021	·006565 −	·003773	·010	·021	·007659	·004074	·011	·021
916	·005464	·003443	·011	·021	·006557	·003769	·010	·021	·007650 +	·004069	·011	·021
917	·005459	·003439	·011	·021	·006550 +	·003765 +	·010	·021	·007642	·004065 −	·011	·021
918	·005453	·003435 +	·011	·021	·006543	·003761	·010	·021	·007634	·004060	·011	·021
919	·005447	·003432	·011	·021	·006536	·003757	·010	·021	·007625 +	·004056	·011	·021
920	·005441	·003428	·011	·021	·006529	·003753	·010	·021	·007617	·004052	·011	·021
921	·005435 −	·003424	·011	·021	·006522	·003749	·010	·021	·007609	·004047	·011	·021
922	·005429	·003420	·011	·021	·006515 −	·003745 −	·010	·021	·007600	·004043	·011	·021
923	·005423	·003417	·011	·021	·006508	·003741	·010	·021	·007592	·004038	·011	·021
924	·005417	·003413	·011	·021	·006501	·003737	·010	·021	·007584	·004034	·011	·021
925	·005411	·003409	·011	·021	·006494	·003733	·010	·021	·007576	·004030	·011	·021
926	·005405 +	·003406	·011	·021	·006486	·003729	·010	·021	·007568	·004025 +	·011	·021
927	·005400	·003402	·011	·021	·006479	·003725 −	·010	·021	·007559	·004021	·011	·021
928	·005394	·003398	·011	·021	·006472	·003721	·010	·021	·007551	·004017	·011	·021
929	·005388	·003395 −	·011	·021	·006466	·003717	·010	·021	·007543	·004012	·011	·021
930	·005382	·003391	·011	·021	·006459	·003713	·010	·021	·007535 −	·004008	·011	·021
931	·005376	·003388	·011	·021	·006452	·003709	·010	·021	·007527	·004004	·011	·021
932	·005371	·003384	·011	·021	·006445 −	·003705 −	·010	·021	·007519	·004000	·011	·021
933	·005365 −	·003380	·011	·021	·006438	·003701	·010	·021	·007511	·003995 +	·011	·021
934	·005359	·003377	·011	·021	·006431	·003697	·010	·021	·007503	·003991	·011	·021
935	·005353	·003373	·011	·021	·006424	·003693	·010	·021	·007495 −	·003987	·011	·021
936	·005348	·003369	·011	·021	·006417	·003689	·010	·021	·007487	·003983	·011	·021
937	·005342	·003366	·011	·021	·006410	·003685 +	·010	·021	·007479	·003978	·011	·021
938	·005336	·003362	·011	·021	·006403	·003681	·010	·021	·007471	·003974	·011	·021
939	·005330	·003359	·011	·021	·006397	·003677	·010	·021	·007463	·003970	·011	·021
940	·005325 −	·003355 +	·011	·021	·006390	·003673	·010	·021	·007455 −	·003966	·011	·021
941	·005319	·003352	·011	·021	·006383	·003670	·010	·021	·007447	·003961	·011	·021
942	·005313	·003348	·011	·021	·006376	·003666	·010	·021	·007439	·003957	·011	·021
943	·005308	·003345 −	·011	·021	·006369	·003662	·010	·021	·007431	·003953	·011	·021
944	·005302	·003341	·011	·021	·006363	·003658	·010	·021	·007423	·003949	·011	·021
945	·005297	·003337	·011	·021	·006356	·003654	·010	·021	·007415 +	·003945 −	·011	·021
946	·005291	·003334	·011	·021	·006349	·003650 +	·010	·021	·007407	·003941	·011	·021
947	·005285 +	·003330	·011	·021	·006342	·003646	·010	·021	·007400	·003936	·011	·021
948	·005280	·003327	·011	·021	·006336	·003643	·010	·021	·007392	·003932	·011	·021
949	·005274	·003323	·011	·021	·006329	·003639	·010	·021	·007384	·003928	·011	·021
950	·005269	·003320	·011	·021	·006322	·003635 −	·010	·021	·007376	·003924	·011	·021

n = number of arrays

N = size of sample	9				10				11				12				13				14			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}96$	P_2 $\lambda_2=2{\cdot}50$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}92$	P_2 $\lambda_2=2{\cdot}47$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}88$	P_2 $\lambda_2=2{\cdot}44$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}84$	P_2 $\lambda_2=2{\cdot}42$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}80$	P_2 $\lambda_2=2{\cdot}40$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2{\cdot}79$	P_2 $\lambda_2=2{\cdot}39$
901	·008889	·004420	·011	·021	·010000	·004685 +	·011	·021	·011111	·004936	·011	·021	·012222	·005174	·011	·022	·013333	·005401	·012	·022	·014444	·005618	·011	·022
902	·008879	·004415 –	·011	·021	·009989	·004680	·011	·021	·011099	·004930	·011	·021	·012209	·005168	·011	·022	·013319	·005395 –	·012	·022	·014428	·005612	·011	·022
903	·008869	·004410	·011	·021	·009978	·004675 –	·011	·021	·011086	·004925 +	·011	·021	·012195 +	·005162	·011	·022	·013304	·005389	·012	·022	·014412	·005606	·011	·022
904	·008859	·004405 +	·011	·021	·009967	·004670	·011	·021	·011074	·004920	·011	·021	·012182	·005157	·011	·022	·013289	·005383	·012	·022	·014396	·005600	·011	·022
905	·008850 –	·004400	·011	·021	·009956	·004665 –	·011	·021	·011062	·004914	·011	·021	·012168	·005151	·011	·022	·013274	·005377	·012	·022	·014381	·005594	·011	·022
906	·008840	·004396	·011	·021	·009945 –	·004659	·011	·021	·011050 –	·004909	·011	·021	·012155 –	·005146	·011	·022	·013260	·005371	·012	·022	·014365 –	·005587	·011	·022
907	·008830	·004391	·011	·021	·009934	·004654	·011	·021	·011038	·004903	·011	·021	·012141	·005140	·011	·022	·013245 +	·005365 +	·012	·022	·014349	·005581	·011	·022
908	·008820	·004386	·011	·021	·009923	·004649	·011	·021	·011025 +	·004898	·011	·021	·012128	·005134	·011	·022	·013230	·005360	·012	·022	·014333	·005575 +	·011	·022
909	·008811	·004381	·011	·021	·009912	·004644	·011	·021	·011013	·004893	·011	·021	·012115 –	·005129	·011	·022	·013216	·005354	·012	·022	·014317	·005569	·011	·022
910	·008801	·004376	·011	·021	·009901	·004639	·011	·021	·011001	·004887	·011	·021	·012101	·005123	·011	·022	·013201	·005348	·012	·022	·014301	·005563	·011	·022
911	·008791	·004371	·011	·021	·009890	·004634	·011	·021	·010989	·004882	·011	·021	·012088	·005117	·011	·022	·013187	·005342	·012	·022	·014286	·005557	·011	·022
912	·008782	·004367	·011	·021	·009879	·004629	·011	·021	·010977	·004877	·011	·021	·012075 –	·005112	·011	·022	·013172	·005336	·012	·022	·014270	·005551	·011	·022
913	·008772	·004362	·011	·021	·009868	·004624	·011	·021	·010965 –	·004871	·011	·021	·012061	·005106	·011	·022	·013158	·005330	·012	·022	·014254	·005545 –	·011	·022
914	·008762	·004357	·011	·021	·009858	·004619	·011	·021	·010953	·004866	·011	·021	·012048	·005101	·011	·022	·013143	·005325 –	·012	·022	·014239	·005539	·011	·022
915	·008753	·004352	·011	·021	·009847	·004614	·011	·021	·010941	·004861	·011	·021	·012035 +	·005095 +	·011	·022	·013129	·005319	·012	·022	·014223	·005533	·011	·022
916	·008743	·004348	·011	·021	·009836	·004609	·011	·021	·010929	·004855 +	·011	·021	·012022	·005090	·011	·022	·013115 –	·005313	·012	·022	·014208	·005527	·011	·022
917	·008734	·004343	·011	·021	·009825 +	·004604	·011	·021	·010917	·004850 +	·011	·021	·012009	·005084	·011	·022	·013100	·005307	·012	·022	·014192	·005521	·011	·022
918	·008724	·004338	·011	·021	·009815 –	·004599	·011	·021	·010905 +	·004845 –	·011	·021	·011996	·005079	·011	·022	·013086	·005302	·012	·022	·014177	·005515 –	·011	·022
919	·008715 –	·004334	·011	·021	·009804	·004594	·011	·021	·010893	·004840	·011	·021	·011983	·005073	·011	·022	·013072	·005296	·012	·022	·014161	·005509	·011	·022
920	·008705 +	·004329	·011	·021	·009793	·004589	·011	·021	·010881	·004834	·011	·021	·011970	·005068	·011	·022	·013058	·005290	·012	·022	·014146	·005503	·011	·022
921	·008696	·004324	·011	·021	·009783	·004584	·011	·021	·010870	·004829	·011	·021	·011957	·005062	·011	·022	·013043	·005284	·012	·022	·014130	·005497	·011	·022
922	·008686	·004320	·011	·021	·009772	·004579	·011	·021	·010858	·004824	·011	·021	·011944	·005057	·011	·022	·013029	·005279	·012	·022	·014115 +	·005491	·011	·022
923	·008677	·004315 –	·011	·021	·009761	·004574	·011	·021	·010846	·004819	·011	·021	·011931	·005051	·011	·022	·013015 +	·005273	·012	·022	·014100	·005485 +	·011	·022
924	·008667	·004310	·011	·021	·009751	·004569	·011	·021	·010834	·004814	·011	·021	·011918	·005046	·011	·022	·013001	·005267	·012	·022	·014085 –	·005479	·011	·022
925	·008658	·004306	·011	·021	·009740	·004564	·011	·021	·010823	·004809	·011	·021	·011905 –	·005040	·011	·022	·012987	·005262	·012	·022	·014069	·005474	·011	·022
926	·008649	·004301	·011	·021	·009730	·004559	·011	·021	·010811	·004803	·011	·021	·011892	·005035 +	·011	·022	·012973	·005256	·012	·022	·014054	·005468	·011	·022
927	·008639	·004296	·011	·021	·009719	·004554	·011	·021	·010799	·004798	·011	·021	·011879	·005030	·011	·022	·012959	·005250 +	·012	·022	·014039	·005462	·011	·022
928	·008630	·004292	·011	·021	·009709	·004550 –	·011	·021	·010787	·004793	·011	·021	·011866	·005024	·011	·022	·012945 –	·005245 –	·012	·022	·014024	·005456	·011	·022
929	·008621	·004287	·011	·021	·009698	·004545 –	·011	·021	·010776	·004788	·011	·021	·011853	·005019	·011	·022	·012931	·005239	·012	·022	·014009	·005450 –	·011	·022
930	·008611	·004283	·011	·021	·009688	·004540	·011	·021	·010764	·004783	·011	·021	·011841	·005014	·011	·022	·012917	·005234	·012	·022	·013994	·005444	·011	·022
931	·008602	·004278	·011	·021	·009677	·004535 –	·011	·021	·010753	·004778	·011	·021	·011828	·005008	·011	·022	·012903	·005228	·012	·022	·013978	·005439	·011	·022
932	·008593	·004273	·011	·021	·009667	·004530	·011	·021	·010741	·004773	·011	·021	·011815 +	·005003	·011	·022	·012889	·005222	·012	·022	·013963	·005433	·011	·022
933	·008584	·004269	·011	·021	·009657	·004525 +	·011	·021	·010730	·004768	·011	·021	·011803	·004997	·011	·022	·012876	·005217	·012	·022	·013948	·005427	·011	·022
934	·008574	·004264	·011	·021	·009646	·004520	·011	·021	·010718	·004762	·011	·021	·011790	·004992	·011	·022	·012862	·005211	·012	·022	·013934	·005421	·011	·022
935	·008565 +	·004260	·011	·021	·009636	·004516	·011	·021	·010707	·004757	·011	·021	·011777	·004987	·011	·022	·012848	·005206	·012	·022	·013919	·005415 +	·011	·022
936	·008556	·004255 +	·011	·021	·009626	·004511	·011	·021	·010695 +	·004752	·011	·021	·011765 –	·004982	·011	·022	·012834	·005200	·012	·022	·013904	·005410	·011	·022
937	·008547	·004251	·011	·021	·009615 +	·004506	·011	·021	·010684	·004747	·011	·021	·011752	·004976	·011	·022	·012821	·005195 –	·012	·022	·013889	·005404	·011	·022
938	·008538	·004246	·011	·021	·009605 +	·004501	·011	·021	·010672	·004742	·011	·021	·011740	·004971	·011	·022	·012807	·005189	·012	·022	·013874	·005398	·011	·022
939	·008529	·004242	·011	·021	·009595 –	·004497	·011	·021	·010661	·004737	·011	·021	·011727	·004966	·011	·022	·012793	·005184	·012	·022	·013859	·005393	·011	·022
940	·008520	·004237	·011	·021	·009585 –	·004492	·011	·021	·010650 –	·004732	·011	·021	·011715 –	·004960	·011	·022	·012780	·005178	·012	·022	·013845 –	·005387	·011	·022
941	·008511	·004233	·011	·021	·009574	·004487	·011	·021	·010638	·004727	·011	·022	·011702	·004955 +	·011	·022	·012766	·005173	·012	·022	·013830	·005381	·011	·022
942	·008502	·004228	·011	·021	·009564	·004482	·011	·021	·010627	·004722	·011	·022	·011690	·004950 +	·011	·022	·012752	·005167	·012	·022	·013815 +	·005375 +	·011	·022
943	·008493	·004224	·011	·021	·009554	·004478	·011	·021	·010616	·004717	·011	·022	·011677	·004945 –	·011	·022	·012739	·005162	·012	·022	·013800	·005370	·011	·022
944	·008484	·004219	·011	·021	·009544	·004473	·011	·021	·010604	·004712	·011	·022	·011665 –	·004940	·011	·022	·012725 +	·005156	·012	·022	·013786	·005364	·011	·022
945	·008475 –	·004215 –	·011	·021	·009534	·004468	·011	·021	·010593	·004707	·011	·022	·011653	·004934	·011	·022	·012712	·005151	·012	·022	·013771	·005359	·011	·022
946	·008466	·004210	·011	·021	·009524	·004463	·011	·021	·010582	·004702	·011	·022	·011640	·004929	·011	·022	·012698	·005146	·012	·022	·013757	·005353	·011	·022
947	·008457	·004206	·011	·021	·009514	·004459	·011	·021	·010571	·004697	·011	·022	·011628	·004924	·011	·022	·012685 –	·005140	·012	·022	·013742	·005347	·011	·022
948	·008448	·004202	·011	·021	·009504	·004454	·011	·021	·010560	·004692	·011	·022	·011616	·004919	·011	·022	·012672	·005135 –	·012	·022	·013728	·005342	·011	·022
949	·008439	·004197	·011	·021	·009494	·004449	·011	·021	·010549	·004688	·011	·022	·011603	·004914	·011	·022	·012658	·005129	·012	·022	·013713	·005336	·011	·022
950	·008430	·004193	·011	·021	·009484	·004445 –	·011	·021	·010537	·004683	·011	·022	·011591	·004909	·011	·022	·012645 –	·005124	·012	·022	·013699	·005331	·011	·022

N = size of sample	15				16				17			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·78$	P_2 $\lambda_2=2·38$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·77$	P_2 $\lambda_2=2·37$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·76$	P_2 $\lambda_2=2·36$
901	·015556	·005827	·011	·021	·016667	·006028	·011	·021	·017778	·006222	·011	·021
902	·015538	·005821	·011	·021	·016648	·006022	·011	·021	·017758	·006216	·011	·021
903	·015521	·005814	·011	·021	·016630	·006015 −	·011	·021	·017738	·006209	·011	·021
904	·015504	·005808	·011	·021	·016611	·006008	·011	·021	·017719	·006202	·011	·021
905	·015487	·005802	·011	·021	·016593	·006002	·011	·021	·017699	·006195 +	·011	·021
906	·015470	·005795 +	·011	·021	·016575 −	·005995 +	·011	·021	·017680	·006188	·011	·021
907	·015453	·005789	·011	·021	·016556	·005989	·011	·021	·017660	·006182	·011	·021
908	·015436	·005782	·011	·021	·016538	·005982	·011	·021	·017641	·006175 −	·011	·021
909	·015419	·005776	·011	·021	·016520	·005976	·011	·021	·017621	·006168	·011	·021
910	·015402	·005770	·011	·021	·016502	·005969	·011	·021	·017602	·006161	·011	·021
911	·015385 −	·005764	·011	·021	·016484	·005963	·011	·021	·017582	·006155 −	·011	·021
912	·015368	·005757	·011	·021	·016465 +	·005956	·011	·021	·017563	·006148	·011	·021
913	·015351	·005751	·011	·021	·016447	·005950 −	·011	·021	·017544	·006141	·011	·021
914	·015334	·005745 −	·011	·021	·016429	·005943	·011	·021	·017525 −	·006135 −	·011	·021
915	·015317	·005739	·011	·021	·016411	·005937	·011	·021	·017505 +	·006128	·011	·021
916	·015301	·005732	·011	·021	·016393	·005930	·011	·021	·017486	·006121	·011	·021
917	·015284	·005726	·011	·021	·016376	·005924	·011	·021	·017467	·006115 −	·011	·021
918	·015267	·005720	·011	·021	·016358	·005917	·011	·021	·017448	·006108	·011	·021
919	·015251	·005714	·011	·021	·016340	·005911	·011	·021	·017429	·006102	·011	·021
920	·015234	·005708	·011	·021	·016322	·005905 −	·011	·021	·017410	·006095 −	·011	·021
921	·015217	·005702	·011	·021	·016304	·005898	·011	·021	·017391	·006088	·011	·021
922	·015201	·005695 +	·011	·021	·016287	·005892	·011	·021	·017372	·006082	·011	·021
923	·015184	·005689	·011	·021	·016269	·005886	·011	·021	·017354	·006075 +	·011	·021
924	·015168	·005683	·011	·021	·016251	·005879	·011	·021	·017335 −	·006069	·011	·021
925	·015152	·005677	·011	·021	·016234	·005873	·011	·021	·017316	·006062	·011	·021
926	·015135 +	·005671	·011	·021	·016216	·005867	·011	·021	·017297	·006056	·011	·021
927	·015119	·005665 −	·011	·021	·016199	·005861	·011	·021	·017279	·006049	·011	·021
928	·015102	·005659	·011	·021	·016181	·005854	·011	·021	·017260	·006043	·011	·021
929	·015086	·005653	·011	·021	·016164	·005848	·011	·021	·017241	·006036	·011	·021
930	·015070	·005647	·011	·021	·016146	·005842	·011	·021	·017223	·006030	·011	·021
931	·015054	·005641	·011	·021	·016129	·005836	·011	·021	·017204	·006024	·011	·021
932	·015038	·005635 −	·011	·021	·016112	·005829	·011	·021	·017186	·006017	·011	·021
933	·015021	·005629	·011	·021	·016094	·005823	·011	·021	·017167	·006011	·011	·021
934	·015005 +	·005623	·011	·021	·016077	·005817	·011	·021	·017149	·006004	·011	·021
935	·014989	·005617	·011	·021	·016060	·005811	·011	·021	·017131	·005998	·011	·021
936	·014973	·005611	·011	·021	·016043	·005805 −	·011	·021	·017112	·005992	·011	·021
937	·014957	·005605 −	·011	·021	·016026	·005798	·011	·021	·017094	·005985 +	·011	·021
938	·014941	·005599	·011	·021	·016009	·005792	·011	·021	·017076	·005979	·011	·021
939	·014925 +	·005593	·011	·021	·015991	·005786	·011	·021	·017058	·005973	·011	·021
940	·014909	·005587	·011	·021	·015974	·005780	·011	·021	·017039	·005966	·011	·021
941	·014894	·005581	·011	·021	·015957	·005774	·011	·021	·017021	·005960	·011	·021
942	·014878	·005575 +	·011	·021	·015940	·005768	·011	·021	·017003	·005954	·011	·021
943	·014862	·005569	·011	·021	·015924	·005762	·011	·021	·016985 +	·005948	·011	·021
944	·014846	·005564	·011	·021	·015907	·005756	·011	·021	·016967	·005941	·011	·021
945	·014831	·005558	·011	·021	·015890	·005750 −	·011	·021	·016949	·005935 +	·011	·021
946	·014815 −	·005552	·011	·021	·015873	·005744	·011	·021	·016931	·005929	·011	·021
947	·014799	·005546	·011	·021	·015856	·005738	·011	·021	·016913	·005923	·011	·021
948	·014784	·005540	·011	·021	·015839	·005732	·011	·021	·016895 +	·005917	·011	·021
949	·014768	·005535 −	·011	·021	·015823	·005726	·011	·021	·016878	·005910	·011	·021
950	·014752	·005529	·011	·021	·015806	·005720	·011	·021	·016860	·005904	·011	·021

N = size of sample	18				19				20			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·75$	P_2 $\lambda_2=2·35$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·74$	P_2 $\lambda_2=2·34$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=2·73$	P_2 $\lambda_2=2·33$
901	·018889	·006410	·011	·021	·020000	·006592	·011	·021	·021111	·006769	·011	·021
902	·018868	·006403	·011	·021	·019978	·006585 +	·011	·021	·021088	·006762	·011	·021
903	·018847	·006396	·011	·021	·019956	·006578	·011	·021	·021064	·006754	·011	·021
904	·018826	·006389	·011	·021	·019934	·006571	·011	·021	·021041	·006747	·011	·021
905	·018805 +	·006382	·011	·021	·019912	·006564	·011	·021	·021018	·006740	·011	·021
906	·018785 −	·006375 +	·011	·021	·019890	·006556	·011	·021	·020994	·006732	·011	·021
907	·018764	·006368	·011	·021	·019868	·006549	·011	·021	·020971	·006725 −	·011	·021
908	·018743	·006361	·011	·021	·019846	·006542	·011	·021	·020948	·006718	·011	·021
909	·018722	·006354	·011	·021	·019824	·006535 −	·011	·021	·020925 +	·006710	·011	·021
910	·018702	·006347	·011	·021	·019802	·006528	·011	·021	·020902	·006703	·011	·021
911	·018681	·006341	·011	·021	·019780	·006521	·011	·021	·020879	·006696	·011	·021
912	·018661	·006334	·011	·021	·019759	·006514	·011	·021	·020856	·006688	·011	·021
913	·018640	·006327	·011	·021	·019737	·006507	·011	·021	·020833	·006681	·011	·021
914	·018620	·006320	·011	·021	·019715 +	·006500 −	·011	·021	·020811	·006674	·011	·021
915	·018600	·006313	·011	·021	·019694	·006492	·011	·021	·020788	·006667	·011	·021
916	·018579	·006306	·011	·021	·019672	·006485 +	·011	·021	·020765 +	·006659	·011	·021
917	·018559	·006299	·011	·021	·019651	·006478	·011	·021	·020742	·006652	·011	·021
918	·018539	·006293	·011	·021	·019629	·006471	·011	·021	·020720	·006645 +	·011	·021
919	·018519	·006286	·011	·021	·019608	·006465 −	·011	·021	·020697	·006638	·011	·021
920	·018498	·006279	·011	·021	·019587	·006458	·011	·021	·020675 −	·006631	·011	·021
921	·018478	·006272	·011	·021	·019565 +	·006451	·011	·021	·020652	·006624	·011	·021
922	·018458	·006266	·011	·021	·019544	·006444	·011	·021	·020630	·006617	·011	·021
923	·018438	·006259	·011	·021	·019523	·006437	·011	·021	·020607	·006610	·011	·021
924	·018418	·006252	·011	·021	·019502	·006430	·011	·021	·020585 +	·006602	·011	·021
925	·018398	·006245 +	·011	·021	·019481	·006423	·011	·021	·020563	·006595 +	·011	·021
926	·018378	·006239	·011	·021	·019459	·006416	·011	·021	·020541	·006588	·011	·021
927	·018359	·006232	·011	·021	·019438	·006409	·011	·021	·020518	·006581	·011	·021
928	·018339	·006225 +	·011	·021	·019417	·006402	·011	·021	·020496	·006574	·011	·021
929	·018319	·006219	·011	·021	·019397	·006396	·011	·021	·020474	·006567	·011	·021
930	·018299	·006212	·011	·021	·019376	·006389	·011	·021	·020452	·006560	·011	·021
931	·018280	·006206	·011	·021	·019355 −	·006382	·011	·021	·020430	·006553	·011	·021
932	·018260	·006199	·011	·021	·019334	·006375 +	·011	·021	·020408	·006546	·011	·021
933	·018240	·006192	·011	·021	·019313	·006368	·011	·021	·020386	·006539	·011	·021
934	·018221	·006186	·011	·021	·019293	·006362	·011	·021	·020364	·006532	·011	·021
935	·018201	·006179	·011	·021	·019272	·006355 −	·011	·021	·020343	·006526	·011	·021
936	·018182	·006173	·011	·021	·019251	·006348	·011	·021	·020321	·006519	·011	·021
937	·018162	·006166	·011	·021	·019231	·006342	·011	·021	·020299	·006512	·011	·021
938	·018143	·006160	·011	·021	·019210	·006335 −	·011	·021	·020277	·006505 −	·011	·021
939	·018124	·006153	·011	·021	·019190	·006328	·011	·021	·020256	·006498	·011	·021
940	·018104	·006147	·011	·021	·019169	·006322	·011	·021	·020234	·006491	·011	·021
941	·018085 +	·006140	·011	·021	·019149	·006315 −	·011	·021	·020213	·006484	·011	·021
942	·018066	·006134	·011	·021	·019129	·006308	·011	·021	·020191	·006478	·011	·021
943	·018047	·006127	·011	·021	·019108	·006302	·011	·021	·020170	·006471	·011	·021
944	·018028	·006121	·011	·021	·019088	·006295 −	·011	·021	·020148	·006464	·011	·021
945	·018008	·006115 −	·011	·021	·019068	·006288	·011	·021	·020127	·006457	·011	·021
946	·017989	·006108	·011	·021	·019048	·006282	·011	·021	·020106	·006450 +	·011	·021
947	·017970	·006102	·011	·021	·019027	·006275 +	·011	·021	·020085 −	·006444	·011	·021
948	·017951	·006095 +	·011	·021	·019007	·006269	·011	·021	·020063	·006437	·011	·021
949	·017932	·006089	·011	·021	·018987	·006262	·011	·021	·020042	·006430	·011	·021
950	·017914	·006083	·011	·021	·018967	·006256	·011	·021	·020021	·006424	·011	·021

n = number of arrays

N = size of sample	3				4				5				6				7				8			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}50$	P_2 $\lambda_2=2{\cdot}94$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}20$	P_2 $\lambda_2=2{\cdot}80$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}14$	P_2 $\lambda_2=2{\cdot}68$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}11$	P_2 $\lambda_2=2{\cdot}63$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}08$	P_2 $\lambda_2=2{\cdot}58$	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 $\lambda_1=3{\cdot}02$	P_2 $\lambda_2=2{\cdot}54$
951	·002105+	·002101	·011	·019	·003158	·002572	·013	·020	·004211	·002968	·012	·021	·005263	·003316	·011	·021	·006316	·003631	·010	·021	·007368	·003920	·011	·021
952	·002103	·002099	·011	·019	·003155−	·002569	·013	·020	·004206	·002965−	·012	·021	·005258	·003313	·011	·021	·006309	·003627	·010	·021	·007361	·003916	·011	·021
953	·002101	·002096	·011	·019	·003151	·002566	·013	·020	·004202	·002962	·012	·021	·005252	·003310	·011	·021	·006303	·003623	·010	·021	·007353	·003912	·011	·021
954	·002099	·002094	·011	·019	·003148	·002564	·013	·020	·004197	·002959	·012	·021	·005247	·003306	·011	·021	·006296	·003620	·010	·021	·007345+	·003908	·011	·021
955	·002096	·002092	·011	·019	·003145−	·002561	·013	·020	·004193	·002955+	·012	·021	·005241	·003303	·011	·021	·006289	·003616	·010	·021	·007338	·003904	·011	·021
956	·002094	·002090	·011	·019	·003141	·002558	·013	·020	·004188	·002952	·012	·021	·005236	·003299	·011	·021	·006283	·003612	·010	·021	·007330	·003899	·011	·021
957	·002092	·002088	·011	·019	·003138	·002556	·013	·020	·004184	·002949	·012	·021	·005230	·003296	·011	·021	·006276	·003608	·010	·021	·007322	·003895+	·011	·021
958	·002090	·002086	·011	·019	·003135−	·002553	·013	·020	·004180	·002946	·012	·021	·005225−	·003292	·011	·021	·006270	·003605−	·010	·021	·007315−	·003891	·011	·021
959	·002088	·002083	·011	·019	·003132	·002550+	·013	·020	·004175+	·002943	·012	·021	·005219	·003289	·011	·021	·006263	·003601	·010	·021	·007307	·003887	·011	·021
960	·002086	·002081	·011	·019	·003128	·002548	·013	·020	·004171	·002940	·012	·021	·005214	·003285+	·011	·021	·006257	·003597	·010	·021	·007299	·003883	·011	·021
961	·002083	·002079	·011	·019	·003125	·002545−	·013	·020	·004167	·002937	·012	·021	·005208	·003282	·011	·021	·006250	·003593	·010	·021	·007292	·003879	·011	·021
962	·002081	·002077	·011	·019	·003122	·002542	·013	·020	·004162	·002934	·012	·021	·005203	·003279	·011	·021	·006243	·003590	·010	·021	·007284	·003875+	·011	·021
963	·002079	·002075−	·011	·019	·003119	·002540	·013	·020	·004158	·002931	·012	·021	·005198	·003275+	·011	·021	·006237	·003586	·010	·021	·007277	·003871	·011	·021
964	·002077	·002073	·011	·019	·003115+	·002537	·013	·020	·004154	·002928	·012	·021	·005192	·003272	·011	·021	·006231	·003582	·010	·021	·007269	·003867	·011	·021
965	·002075−	·002070	·011	·019	·003112	·002534	·013	·020	·004149	·002925−	·012	·021	·005187	·003268	·011	·021	·006224	·003579	·010	·021	·007261	·003863	·011	·021
966	·002073	·002068	·011	·019	·003109	·002532	·013	·020	·004145+	·002922	·012	·021	·005181	·003265+	·011	·021	·006218	·003575−	·010	·021	·007254	·003859	·011	·021
967	·002070	·002066	·011	·019	·003106	·002529	·013	·020	·004141	·002919	·012	·021	·005176	·003262	·011	·021	·006211	·003571	·010	·021	·007246	·003855+	·011	·021
968	·002068	·002064	·011	·019	·003102	·002527	·013	·020	·004137	·002916	·012	·021	·005171	·003258	·011	·021	·006205−	·003567	·010	·021	·007239	·003851	·011	·021
969	·002066	·002062	·011	·019	·003099	·002524	·013	·020	·004132	·002913	·012	·021	·005165+	·003255+	·011	·021	·006198	·003564	·010	·021	·007231	·003847	·011	·021
970	·002064	·002060	·011	·019	·003096	·002521	·013	·020	·004128	·002910	·012	·021	·005160	·003252	·011	·021	·006192	·003560	·010	·021	·007224	·003843	·011	·021
971	·002062	·002058	·011	·019	·003093	·002519	·013	·020	·004124	·002907	·012	·021	·005155−	·003248	·011	·021	·006186	·003557	·010	·021	·007216	·003839	·011	·021
972	·002060	·002055+	·011	·019	·003090	·002516	·013	·020	·004119	·002904	·012	·021	·005149	·003245−	·011	·021	·006179	·003553	·010	·021	·007209	·003836	·011	·021
973	·002058	·002053	·011	·019	·003086	·002514	·013	·020	·004115+	·002901	·012	·021	·005144	·003242	·011	·021	·006173	·003549	·010	·021	·007202	·003832	·011	·021
974	·002055+	·002051	·011	·019	·003083	·002511	·013	·020	·004111	·002898	·012	·021	·005139	·003238	·011	·021	·006166	·003546	·010	·021	·007194	·003828	·011	·021
975	·002053	·002049	·011	·019	·003080	·002508	·013	·020	·004107	·002895−	·012	·021	·005133	·003235+	·011	·021	·006160	·003542	·010	·021	·007187	·003824	·011	·021
976	·002051	·002047	·011	·019	·003077	·002506	·013	·020	·004103	·002892	·012	·021	·005128	·003232	·011	·021	·006154	·003538	·010	·021	·007179	·003820	·011	·021
977	·002049	·002045−	·011	·019	·003074	·002503	·013	·020	·004098	·002889	·012	·021	·005123	·003228	·011	·021	·006148	·003535−	·010	·021	·007172	·003816	·011	·021
978	·002047	·002043	·011	·019	·003071	·002501	·013	·020	·004094	·002886	·012	·021	·005118	·003225+	·011	·021	·006141	·003531	·010	·021	·007165−	·003812	·011	·021
979	·002045−	·002041	·011	·019	·003067	·002498	·013	·020	·004090	·002883	·012	·021	·005112	·003222	·011	·021	·006135−	·003528	·010	·021	·007157	·003808	·011	·021
980	·002043	·002039	·011	·019	·003064	·002496	·013	·020	·004086	·002880	·012	·021	·005107	·003219	·011	·021	·006129	·003524	·010	·021	·007150+	·003804	·011	·021
981	·002041	·002037	·011	·019	·003061	·002493	·013	·020	·004082	·002877	·012	·021	·005102	·003215−	·011	·021	·006122	·003520	·010	·021	·007143	·003800	·011	·021
982	·002039	·002035−	·011	·019	·003058	·002491	·013	·020	·004077	·002874	·012	·021	·005097	·003212	·011	·021	·006116	·003517	·010	·021	·007136	·003797	·011	·021
983	·002037	·002033	·011	·019	·003055−	·002488	·013	·020	·004073	·002871	·012	·021	·005092	·003209	·011	·021	·006110	·003513	·010	·021	·007128	·003793	·011	·021
984	·002035−	·002030	·011	·019	·003052	·002486	·013	·020	·004069	·002869	·012	·021	·005086	·003206	·011	·021	·006104	·003510	·010	·021	·007121	·003789	·011	·021
985	·002033	·002028	·011	·019	·003049	·002483	·013	·020	·004065+	·002866	·012	·021	·005081	·003202	·011	·021	·006098	·003506	·010	·021	·007114	·003785+	·011	·021
986	·002030	·002026	·011	·019	·003046	·002480	·013	·020	·004061	·002863	·012	·021	·005076	·003199	·011	·021	·006091	·003503	·010	·021	·007107	·003781	·011	·021
987	·002028	·002024	·011	·019	·003043	·002478	·013	·020	·004057	·002860	·012	·021	·005071	·003196	·011	·021	·006085+	·003499	·010	·021	·007099	·003777	·011	·021
988	·002026	·002022	·011	·019	·003040	·002475+	·013	·020	·004053	·002857	·012	·021	·005066	·003193	·011	·021	·006079	·003496	·010	·021	·007092	·003774	·011	·021
989	·002024	·002020	·011	·019	·003036	·002473	·013	·020	·004049	·002854	·012	·021	·005061	·003189	·011	·021	·006073	·003492	·010	·021	·007085+	·003770	·011	·021
990	·002022	·002018	·011	·019	·003033	·002470	·013	·020	·004044	·002851	·012	·021	·005056	·003186	·011	·021	·006067	·003488	·010	·021	·007078	·003766	·011	·021
991	·002020	·002016	·011	·019	·003030	·002468	·013	·020	·004040	·002848	·012	·021	·005051	·003183	·011	·021	·006061	·003485−	·010	·021	·007071	·003762	·011	·021
992	·002018	·002014	·011	·019	·003027	·002466	·013	·020	·004036	·002845+	·012	·021	·005045−	·003180	·011	·021	·006054	·003481	·010	·021	·007064	·003758	·011	·021
993	·002016	·002012	·011	·019	·003024	·002463	·013	·020	·004032	·002843	·012	·021	·005040	·003177	·011	·021	·006048	·003478	·010	·021	·007056	·003755−	·011	·021
994	·002014	·002010	·011	·019	·003021	·002461	·013	·020	·004028	·002840	·012	·021	·005035−	·003173	·011	·021	·006042	·003474	·010	·021	·007049	·003751	·011	·021
995	·002012	·002008	·011	·019	·003018	·002458	·013	·020	·004024	·002837	·012	·021	·005030	·003170	·011	·021	·006036	·003471	·010	·021	·007042	·003747	·011	·021
996	·002010	·002006	·011	·019	·003015+	·002456	·013	·020	·004020	·002834	·012	·021	·005025+	·003167	·011	·021	·006030	·003468	·010	·021	·007035+	·003743	·011	·021
997	·002008	·002004	·011	·019	·003012	·002453	·013	·020	·004016	·002831	·012	·021	·005020	·003164	·011	·021	·006024	·003464	·010	·021	·007028	·003740	·011	·021
998	·002006	·002002	·011	·019	·003009	·002451	·013	·020	·004012	·002828	·012	·021	·005015+	·003161	·011	·021	·006018	·003461	·010	·021	·007021	·003736	·011	·021
999	·002004	·002000	·011	·019	·003006	·002448	·013	·020	·004008	·002826	·012	·021	·005010	·003158	·011	·021	·006012	·003457	·010	·021	·007014	·003732	·011	·021
1000	·002002	·001998	·011	·019	·003003	·002446	·013	·020	·004004	·002823	·012	·021	·005005+	·003154	·011	·021	·006006	·003454	·010	·021	·007007	·003729	·011	·021

N = size of sample	9: $\bar{\eta}^2$	9: $\sigma_{\bar{\eta}^2}$	9: P_1 $\lambda_1=2·96$	9: P_2 $\lambda_2=2·50$	10: $\bar{\eta}^2$	10: $\sigma_{\bar{\eta}^2}$	10: P_1 $\lambda_1=2·92$	10: P_2 $\lambda_2=2·47$
951	·008421	·004188	·011	·021	·009474	·004440	·011	·021
952	·008412	·004184	·011	·021	·009464	·004435 +	·011	·021
953	·008403	·004180	·011	·021	·009454	·004431	·011	·021
954	·008395 −	·004175 −	·011	·021	·009444	·004426	·011	·021
955	·008386	·004171	·011	·021	·009434	·004422	·011	·021
956	·008377	·004167	·011	·021	·009424	·004417	·011	·021
957	·008368	·004162	·011	·021	·009414	·004412	·011	·021
958	·008359	·004158	·011	·021	·009404	·004408	·011	·021
959	·008351	·004154	·011	·021	·009395 −	·004403	·011	·021
960	·008342	·004149	·011	·021	·009385 −	·004399	·011	·021
961	·008333	·004145 −	·011	·021	·009375	·004394	·011	·021
962	·008325 −	·004141	·011	·021	·009365 +	·004390	·011	·021
963	·008316	·004136	·011	·021	·009356	·004385 −	·011	·021
964	·008307	·004132	·011	·021	·009346	·004380	·011	·021
965	·008299	·004128	·011	·021	·009336	·004376	·011	·021
966	·008290	·004124	·011	·021	·009326	·004371	·011	·021
967	·008282	·004119	·011	·021	·009317	·004367	·011	·021
968	·008273	·004115 +	·011	·021	·009307	·004362	·011	·021
969	·008264	·004111	·011	·021	·009298	·004358	·011	·021
970	·008256	·004107	·011	·021	·009288	·004353	·011	·021
971	·008247	·004102	·011	·021	·009278	·004349	·011	·021
972	·008239	·004098	·011	·021	·009269	·004345 −	·011	·021
973	·008230	·004094	·011	·021	·009259	·004340	·011	·021
974	·008222	·004090	·011	·021	·009250 −	·004336	·011	·021
975	·008214	·004086	·011	·021	·009240	·004331	·011	·021
976	·008205 +	·004082	·011	·021	·009231	·004327	·011	·021
977	·008197	·004077	·011	·021	·009221	·004322	·011	·021
978	·008188	·004073	·011	·021	·009212	·004318	·011	·021
979	·008180	·004069	·011	·021	·009202	·004314	·011	·021
980	·008172	·004065 −	·011	·021	·009193	·004309	·011	·021
981	·008163	·004061	·011	·021	·009184	·004305 −	·011	·021
982	·008155 −	·004057	·011	·021	·009174	·004301	·011	·021
983	·008147	·004053	·011	·021	·009165 −	·004296	·011	·021
984	·008138	·004048	·011	·021	·009156	·004292	·011	·021
985	·008130	·004044	·011	·021	·009146	·004288	·011	·021
986	·008122	·004040	·011	·021	·009137	·004283	·011	·021
987	·008114	·004036	·011	·021	·009128	·004279	·011	·021
988	·008105 +	·004032	·011	·021	·009119	·004275 −	·011	·021
989	·008097	·004028	·011	·021	·009109	·004270	·011	·021
990	·008089	·004024	·011	·021	·009100	·004266	·011	·021
991	·008081	·004020	·011	·021	·009091	·004262	·011	·021
992	·008073	·004016	·011	·021	·009082	·004257	·011	·021
993	·008065 −	·004012	·011	·021	·009073	·004253	·011	·021
994	·008056	·004008	·011	·021	·009063	·004249	·011	·021
995	·008048	·004004	·011	·021	·009054	·004245 −	·011	·021
996	·008040	·004000	·011	·021	·009045 +	·004240	·011	·021
997	·008032	·003996	·011	·021	·009036	·004236	·011	·021
998	·008024	·003992	·011	·021	·009027	·004232	·011	·021
999	·008016	·003988	·011	·021	·009018	·004228	·011	·021
1000	·008008	·003984	·011	·021	·009009	·004223	·011	·021

N = size of sample	11: $\bar{\eta}^2$	11: $\sigma_{\bar{\eta}^2}$	11: P_1 $\lambda_1=2·88$	11: P_2 $\lambda_2=2·44$	12: $\bar{\eta}^2$	12: $\sigma_{\bar{\eta}^2}$	12: P_1 $\lambda_1=2·84$	12: P_2 $\lambda_2=2·42$
951	·010526	·004678	·011	·022	·011579	·004903	·011	·022
952	·010515 +	·004673	·011	·022	·011567	·004898	·011	·022
953	·010504	·004668	·011	·022	·011555 −	·004893	·011	·022
954	·010493	·004663	·011	·022	·011542	·004888	·011	·022
955	·010482	·004658	·011	·022	·011530	·004883	·011	·022
956	·010471	·004653	·011	·022	·011518	·004878	·011	·022
957	·010460	·004649	·011	·022	·011506	·004873	·011	·022
958	·010449	·004644	·011	·022	·011494	·004868	·011	·022
959	·010438	·004639	·011	·022	·011482	·004863	·011	·022
960	·010428	·004634	·011	·022	·011470	·004858	·011	·022
961	·010417	·004629	·011	·022	·011458	·004853	·011	·022
962	·010406	·004625 −	·011	·022	·011446	·004848	·011	·022
963	·010395 +	·004620	·011	·022	·011435 −	·004843	·011	·022
964	·010384	·004615 −	·011	·022	·011423	·004838	·011	·022
965	·010373	·004610	·011	·022	·011411	·004833	·011	·022
966	·010363	·004605 +	·011	·022	·011399	·004828	·011	·022
967	·010352	·004601	·011	·022	·011387	·004823	·011	·022
968	·010341	·004596	·011	·022	·011375 +	·004818	·011	·022
969	·010331	·004591	·011	·022	·011364	·004813	·011	·022
970	·010320	·004587	·011	·022	·011352	·004808	·011	·022
971	·010309	·004582	·011	·022	·011340	·004803	·011	·022
972	·010299	·004577	·011	·022	·011329	·004798	·011	·022
973	·010288	·004573	·011	·022	·011317	·004793	·011	·022
974	·010277	·004568	·011	·022	·011305 +	·004788	·011	·022
975	·010267	·004563	·011	·022	·011294	·004783	·011	·022
976	·010256	·004559	·011	·022	·011282	·004779	·011	·022
977	·010246	·004554	·011	·022	·011270	·004774	·011	·022
978	·010235 +	·004549	·011	·022	·011259	·004769	·011	·022
979	·010225 −	·004545 −	·011	·022	·011247	·004764	·011	·022
980	·010215 −	·004540	·011	·022	·011236	·004759	·011	·022
981	·010204	·004535 +	·011	·022	·011224	·004754	·011	·022
982	·010194	·004531	·011	·022	·011213	·004750 −	·011	·022
983	·010183	·004526	·011	·022	·011202	·004745 −	·011	·022
984	·010173	·004522	·011	·022	·011190	·004740	·011	·022
985	·010163	·004517	·011	·022	·011179	·004735 +	·011	·022
986	·010152	·004513	·011	·022	·011168	·004730	·011	·022
987	·010142	·004508	·011	·022	·011156	·004726	·011	·022
988	·010132	·004503	·011	·022	·011145 −	·004721	·011	·022
989	·010121	·004499	·011	·022	·011134	·004716	·011	·022
990	·010111	·004494	·011	·022	·011122	·004711	·011	·022
991	·010101	·004490	·011	·022	·011111	·004707	·011	·022
992	·010091	·004485 +	·011	·022	·011100	·004702	·011	·022
993	·010081	·004481	·011	·022	·011089	·004697	·011	·022
994	·010070	·004476	·011	·022	·011078	·004693	·011	·022
995	·010060	·004472	·011	·022	·011066	·004688	·011	·022
996	·010050 +	·004467	·011	·022	·011055 +	·004683	·011	·022
997	·010040	·004463	·011	·022	·011044	·004678	·011	·022
998	·010030	·004459	·011	·022	·011033	·004674	·011	·022
999	·010020	·004454	·011	·022	·011022	·004669	·011	·022
1000	·010010	·004450 −	·011	·022	·011011	·004665 −	·011	·022

N = size of sample	13: $\bar{\eta}^2$	13: $\sigma_{\bar{\eta}^2}$	13: P_1 $\lambda_1=2·80$	13: P_2 $\lambda_2=2·40$	14: $\bar{\eta}^2$	14: $\sigma_{\bar{\eta}^2}$	14: P_1 $\lambda_1=2·79$	14: P_2 $\lambda_2=2·39$
951	·012632	·005119	·012	·022	·013684	·005325 −	·011	·022
952	·012618	·005113	·012	·022	·013670	·005319	·011	·022
953	·012605 +	·005108	·012	·022	·013655 +	·005314	·011	·022
954	·012592	·005103	·012	·022	·013641	·005308	·011	·022
955	·012579	·005097	·012	·022	·013627	·005303	·011	·022
956	·012565 +	·005092	·012	·022	·013613	·005297	·011	·022
957	·012552	·005087	·012	·022	·013598	·005292	·011	·022
958	·012539	·005082	·012	·022	·013584	·005286	·011	·022
959	·012526	·005076	·012	·022	·013570	·005281	·011	·022
960	·012513	·005071	·012	·022	·013556	·005275 +	·011	·022
961	·012500	·005066	·012	·022	·013542	·005270	·011	·022
962	·012487	·005061	·012	·022	·013528	·005264	·011	·022
963	·012474	·005055 +	·012	·022	·013514	·005259	·011	·022
964	·012461	·005050 +	·012	·022	·013499	·005254	·011	·022
965	·012448	·005045 −	·012	·022	·013485 +	·005248	·011	·022
966	·012435 +	·005040	·012	·022	·013472	·005243	·011	·022
967	·012422	·005035 −	·012	·022	·013458	·005237	·011	·022
968	·012410	·005029	·012	·022	·013444	·005232	·011	·022
969	·012397	·005024	·012	·022	·013430	·005227	·011	·022
970	·012384	·005019	·012	·022	·013416	·005221	·011	·022
971	·012371	·005014	·012	·022	·013402	·005216	·011	·022
972	·012358	·005009	·012	·022	·013388	·005211	·011	·022
973	·012346	·005004	·012	·022	·013374	·005205 +	·011	·022
974	·012333	·004999	·012	·022	·013361	·005200	·011	·022
975	·012320	·004994	·012	·022	·013347	·005195 −	·011	·022
976	·012308	·004988	·012	·022	·013333	·005189	·011	·022
977	·012295 +	·004983	·012	·022	·013320	·005184	·011	·022
978	·012282	·004978	·012	·022	·013306	·005179	·011	·022
979	·012270	·004973	·012	·022	·013292	·005174	·011	·022
980	·012257	·004968	·012	·022	·013279	·005168	·011	·022
981	·012245 −	·004963	·012	·022	·013265 +	·005163	·011	·022
982	·012232	·004958	·012	·022	·013252	·005158	·011	·022
983	·012220	·004953	·012	·022	·013238	·005153	·011	·022
984	·012208	·004948	·012	·022	·013225 −	·005148	·011	·022
985	·012195 +	·004943	·012	·022	·013211	·005142	·011	·022
986	·012183	·004938	·012	·022	·013198	·005137	·011	·022
987	·012170	·004933	·012	·022	·013185 −	·005132	·011	·022
988	·012158	·004928	·012	·022	·013171	·005127	·011	·022
989	·012146	·004923	·012	·022	·013158	·005122	·011	·022
990	·012133	·004918	·012	·022	·013145 −	·005117	·011	·022
991	·012121	·004913	·012	·022	·013131	·005111	·011	·022
992	·012109	·004908	·012	·022	·013118	·005106	·011	·022
993	·012097	·004904	·012	·022	·013105 −	·005101	·011	·022
994	·012085 −	·004899	·012	·022	·013092	·005096	·011	·022
995	·012072	·004894	·012	·022	·013078	·005091	·011	·022
996	·012060	·004889	·012	·022	·013065 +	·005086	·011	·022
997	·012048	·004884	·012	·022	·013052	·005081	·011	·022
998	·012036	·004879	·012	·022	·013039	·005076	·011	·022
999	·012024	·004874	·012	·022	·013026	·005071	·011	·022
1000	·012012	·004869	·012	·022	·013013	·005066	·011	·022

n = number of arrays

N = size of sample	15				16			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·78	P_2 λ_2 = 2·38	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·77	P_2 λ_2 = 2·37
951	·014737	·005523	·011	·021	·015789	·005714	·011	·021
952	·014721	·005517	·011	·021	·015773	·005708	·011	·021
953	·014706	·005511	·011	·021	·015756	·005702	·011	·021
954	·014690	·005506	·011	·021	·015740	·005696	·011	·021
955	·014675 +	·005500 +	·011	·021	·015723	·005690	·011	·021
956	·014660	·005494	·011	·021	·015707	·005684	·011	·021
957	·014644	·005489	·011	·021	·015690	·005678	·011	·021
958	·014629	·005483	·011	·021	·015674	·005672	·011	·021
959	·014614	·005477	·011	·021	·015658	·005667	·011	·021
960	·014599	·005472	·011	·021	·015641	·005661	·011	·021
961	·014583	·005466	·011	·021	·015625	·005655 −	·011	·021
962	·014568	·005460	·011	·021	·015609	·005649	·011	·021
963	·014553	·005455 −	·011	·021	·015593	·005643	·011	·021
964	·014538	·005449	·011	·021	·015576	·005637	·011	·021
965	·014523	·005443	·011	·021	·015560	·005632	·011	·021
966	·014508	·005438	·011	·021	·015544	·005626	·011	·021
967	·014493	·005432	·011	·021	·015528	·005620	·011	·021
968	·014478	·005427	·011	·021	·015512	·005614	·011	·021
969	·014463	·005421	·011	·021	·015496	·005608	·011	·021
970	·014448	·005416	·011	·021	·015480	·005603	·011	·021
971	·014433	·005410	·011	·021	·015464	·005597	·011	·021
972	·014418	·005405 −	·011	·021	·015448	·005591	·011	·021
973	·014403	·005399	·011	·021	·015432	·005586	·011	·021
974	·014388	·005394	·011	·021	·015416	·005580	·011	·021
975	·014374	·005388	·011	·021	·015400	·005574	·011	·021
976	·014359	·005383	·011	·021	·015385 −	·005569	·011	·021
977	·014344	·005377	·011	·021	·015369	·005563	·011	·021
978	·014330	·005372	·011	·021	·015353	·005557	·011	·021
979	·014315 −	·005366	·011	·021	·015337	·005552	·011	·021
980	·014300	·005361	·011	·021	·015322	·005546	·011	·021
981	·014286	·005355 +	·011	·021	·015306	·005540	·011	·021
982	·014271	·005350 −	·011	·021	·015291	·005535 −	·011	·021
983	·014257	·005345 −	·011	·021	·015275 −	·005529	·011	·021
984	·014242	·005339	·011	·021	·015259	·005524	·011	·021
985	·014228	·005334	·011	·021	·015244	·005518	·011	·021
986	·014213	·005328	·011	·021	·015228	·005513	·011	·021
987	·014199	·005323	·011	·021	·015213	·005507	·011	·021
988	·014184	·005318	·011	·021	·015198	·005501	·011	·021
989	·014170	·005312	·011	·021	·015182	·005496	·011	·021
990	·014156	·005307	·011	·021	·015167	·005490	·011	·021
991	·014141	·005302	·011	·021	·015152	·005485 −	·011	·021
992	·014127	·005296	·011	·021	·015136	·005479	·011	·021
993	·014113	·005291	·011	·021	·015121	·005474	·011	·021
994	·014099	·005286	·011	·021	·015106	·005469	·011	·021
995	·014085 −	·005281	·011	·021	·015091	·005463	·011	·021
996	·014070	·005275 +	·011	·021	·015075 +	·005458	·011	·021
997	·014056	·005270	·011	·021	·015060	·005452	·011	·021
998	·014042	·005265 −	·011	·021	·015045 +	·005447	·011	·021
999	·014028	·005260	·011	·021	·015030	·005441	·011	·021
1000	·014014	·005254	·011	·021	·015015 +	·005436	·011	·021

N = size of sample	17				18			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·76	P_2 λ_2 = 2·36	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·75	P_2 λ_2 = 2·35
951	·016842	·005898	·011	·021	·017895 −	·006076	·011	·021
952	·016824	·005892	·011	·021	·017876	·006070	·011	·021
953	·016807	·005886	·011	·021	·017857	·006064	·011	·021
954	·016789	·005880	·011	·021	·017838	·006057	·011	·021
955	·016771	·005874	·011	·021	·017820	·006051	·011	·021
956	·016754	·005867	·011	·021	·017801	·006045 −	·011	·021
957	·016736	·005861	·011	·021	·017782	·006039	·011	·021
958	·016719	·005855 +	·011	·021	·017764	·006032	·011	·021
959	·016701	·005849	·011	·021	·017745 +	·006026	·011	·021
960	·016684	·005843	·011	·021	·017727	·006020	·011	·021
961	·016667	·005837	·011	·021	·017708	·006014	·011	·021
962	·016649	·005831	·011	·021	·017690	·006007	·011	·021
963	·016632	·005825 +	·011	·021	·017672	·006001	·011	·021
964	·016615 −	·005819	·011	·021	·017653	·005995 +	·011	·021
965	·016598	·005813	·011	·021	·017635 −	·005989	·011	·021
966	·016580	·005807	·011	·021	·017617	·005983	·011	·021
967	·016563	·005801	·011	·021	·017598	·005977	·011	·021
968	·016546	·005795 +	·011	·021	·017580	·005971	·011	·021
969	·016529	·005789	·011	·021	·017562	·005964	·011	·021
970	·016512	·005783	·011	·021	·017544	·005958	·011	·021
971	·016495 −	·005778	·011	·021	·017526	·005952	·011	·021
972	·016478	·005772	·011	·021	·017508	·005946	·011	·021
973	·016461	·005766	·011	·021	·017490	·005940	·011	·021
974	·016444	·005760	·011	·021	·017472	·005934	·011	·021
975	·016427	·005754	·011	·021	·017454	·005928	·011	·021
976	·016410	·005748	·011	·021	·017436	·005922	·011	·021
977	·016393	·005742	·011	·021	·017418	·005916	·011	·021
978	·016377	·005737	·011	·021	·017400	·005910	·011	·021
979	·016360	·005731	·011	·021	·017382	·005904	·011	·021
980	·016343	·005725 −	·011	·021	·017365 −	·005898	·011	·021
981	·016327	·005719	·011	·021	·017347	·005892	·011	·021
982	·016310	·005713	·011	·021	·017329	·005886	·011	·021
983	·016293	·005708	·011	·021	·017312	·005880	·011	·021
984	·016277	·005702	·011	·021	·017294	·005874	·011	·021
985	·016260	·005696	·011	·021	·017276	·005868	·011	·021
986	·016244	·005690	·011	·021	·017259	·005862	·011	·021
987	·016227	·005685 −	·011	·021	·017241	·005857	·011	·021
988	·016211	·005679	·011	·021	·017224	·005851	·011	·021
989	·016194	·005673	·011	·021	·017206	·005845 −	·011	·021
990	·016178	·005668	·011	·021	·017189	·005839	·011	·021
991	·016162	·005662	·011	·021	·017172	·005833	·011	·021
992	·016145 +	·005656	·011	·021	·017154	·005827	·011	·021
993	·016129	·005651	·011	·021	·017137	·005822	·011	·021
994	·016113	·005645 −	·011	·021	·017120	·005816	·011	·021
995	·016097	·005639	·011	·021	·017103	·005810	·011	·021
996	·016080	·005634	·011	·021	·017085 +	·005804	·011	·021
997	·016064	·005628	·011	·021	·017068	·005798	·011	·021
998	·016048	·005623	·011	·021	·017051	·005793	·011	·021
999	·016032	·005617	·011	·021	·017034	·005787	·011	·021
1000	·016016	·005611	·011	·021	·017017	·005781	·011	·021

N = size of sample	19				20			
	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·74	P_2 λ_2 = 2·34	$\bar{\eta}^2$	$\sigma_{\bar{\eta}^2}$	P_1 λ_1 = 2·73	P_2 λ_2 = 2·33
951	·018947	·006249	·011	·021	·020000	·006417	·011	·021
952	·018927	·006243	·011	·021	·019979	·006410	·011	·021
953	·018908	·006236	·011	·021	·019958	·006404	·011	·021
954	·018888	·006230	·011	·021	·019937	·006397	·011	·021
955	·018868	·006223	·011	·021	·019916	·006390	·011	·021
956	·018848	·006217	·011	·021	·019895 +	·006384	·011	·021
957	·018828	·006210	·011	·021	·019874	·006377	·011	·021
958	·018809	·006204	·011	·021	·019854	·006370	·011	·021
959	·018789	·006197	·011	·021	·019833	·006364	·011	·021
960	·018770	·006191	·011	·021	·019812	·006357	·011	·021
961	·018750	·006185 −	·011	·021	·019792	·006351	·011	·021
962	·018730	·006178	·011	·021	·019771	·006344	·011	·021
963	·018711	·006172	·011	·021	·019751	·006338	·011	·021
964	·018692	·006166	·011	·021	·019730	·006331	·011	·021
965	·018672	·006159	·011	·021	·019710	·006325 −	·011	·021
966	·018653	·006153	·011	·021	·019689	·006318	·011	·021
967	·018634	·006147	·011	·021	·019669	·006312	·011	·021
968	·018614	·006140	·011	·021	·019648	·006305 +	·011	·021
969	·018595 +	·006134	·011	·021	·019628	·006299	·011	·021
970	·018576	·006128	·011	·021	·019608	·006292	·011	·021
971	·018557	·006122	·011	·021	·019588	·006286	·011	·021
972	·018538	·006115 +	·011	·021	·019567	·006280	·011	·021
973	·018519	·006109	·011	·021	·019547	·006273	·011	·021
974	·018499	·006103	·011	·021	·019527	·006267	·011	·021
975	·018480	·006097	·011	·021	·019507	·006261	·011	·021
976	·018462	·006091	·011	·021	·019487	·006254	·011	·021
977	·018443	·006084	·011	·021	·019467	·006248	·011	·021
978	·018424	·006078	·011	·021	·019447	·006241	·011	·021
979	·018405 −	·006072	·011	·021	·019427	·006235 +	·011	·021
980	·018386	·006066	·011	·021	·019408	·006229	·011	·021
981	·018367	·006060	·011	·021	·019388	·006223	·011	·021
982	·018349	·006054	·011	·021	·019368	·006216	·011	·021
983	·018330	·006048	·011	·021	·019348	·006210	·011	·021
984	·018311	·006041	·011	·021	·019329	·006204	·011	·021
985	·018293	·006035 +	·011	·021	·019309	·006198	·011	·021
986	·018274	·006029	·011	·021	·019289	·006191	·011	·021
987	·018256	·006023	·011	·021	·019270	·006185 +	·011	·021
988	·018237	·006017	·011	·021	·019250 +	·006179	·011	·021
989	·018219	·006011	·011	·021	·019231	·006173	·011	·021
990	·018200	·006005 +	·011	·021	·019211	·006167	·011	·021
991	·018182	·005999	·011	·021	·019192	·006160	·011	·021
992	·018163	·005993	·011	·021	·019173	·006154	·011	·021
993	·018145 +	·005987	·011	·021	·019153	·006148	·011	·021
994	·018127	·005981	·011	·021	·019134	·006142	·011	·021
995	·018109	·005975 +	·011	·021	·019115 −	·006136	·011	·021
996	·018090	·005969	·011	·021	·019095 +	·006130	·011	·021
997	·018072	·005963	·011	·021	·019076	·006124	·011	·021
998	·018054	·005958	·011	·021	·019057	·006118	·011	·021
999	·018036	·005952	·011	·021	·019038	·006112	·011	·021
1000	·018018	·005946	·011	·021	·019019	·006106	·011	·021

A CONTRIBUTION TO BASQUE CRANIOMETRY.

By G. M. MORANT, D.Sc.

For a number of reasons the ethnology of the Basques of the south-west of France and the north of Spain is of particular interest. Cultural peculiarities, and the fact that the Basque language is structurally different from all others of Western Europe, suggest that the people have been isolated for a long period, but their origins are not disclosed by any historical records. The earlier attempts to discover the relationships of the race were based almost entirely on philological evidence and the most fantastic and often diametrically opposed theories have been brought forward by different writers*. At one time or another the Basques have been supposed akin to the ancient Egyptians, Guanches, Berbers, Etruscans, Phoenicians, Lapps, Finns, Bulgarians, or to Asiatic races; others have seen in them the unique descendants of a prehistoric population of Europe, or the sole survivors of Atlantis! The first anthropological contribution to the subject which is of any importance was made by Paul Broca in 1862†. He describes a series of 60 skulls from Zaraus which had been presented to the Paris Anthropological Society by Gonzales Velasco. "Ces crânes," Broca writes, "ont été extraits sans aucun choix, et dans l'ordre où le hasard les présentait, d'un cimetière de la province de Guipuzcoa (Espagne), dans une petite localité où les Basques, depuis les temps historiques, n'ont subi aucun mélange de race." The mean cephalic index was found to be 77·7. The only other measurement considered in this paper is the cranial capacity given as 1486·9‡. The inionic protuberance and the imprint of the muscles of the neck were observed to be particularly feeble. In the following year Broca published another paper on the Zaraus skulls controverting the opinion expressed by Pruner-Bey that they were racially heterogeneous§. A number of additional measurements of the unsexed series are given and it is suggested that the Basques are most closely related to the peoples of the north of Africa. It is Broca again who makes the next important contribution to the craniology of this race‖. His paper deals principally with a series of 58 Basque skulls from the French town of Saint Jean-de-Luz, which is

* An interesting account of a number of these theories is given by W. Z. Ripley in Chapter VIII of *The Races of Europe*.

† "Sur les caractères du crâne des Basques." *Bulletins de la Société d'Anthropologie de Paris*, t. III. 1862, pp. 579—597.

‡ This value is probably too high. A criticism of Broca's method of determining cranial capacities is given in Alice Lee and Karl Pearson: "Data for the Problem of Evolution in Man. VI. A First Study of the Correlation of the Human Skull." *Phil. Trans. Royal Society, London.* Series A. Vol. 196, 1901, pp. 225—264.

§ "Sur les crânes basques." *Bulletins de la Société d'Anthropologie de Paris*, t. IV. 1863, pp. 38—72.

‖ "Sur les crânes basques de Saint Jean-de-Luz." *Ibid.* 2e série, t. III. 1868, pp. 43—101.

less than 30 miles from Zaraus, though separated from it by the Pyrenees. The cemetery in this case had been disused by A.D. 1532. The mean cephalic index of the 57 French specimens is given as 80·25 which is significantly greater than the value for the Spanish collection. Numerous other measurements given for the two unsexed series suggest that other characters also differ significantly, though, in general, the means agree closely. The collection from Zaraus had been augmented in the meantime by the addition of 19 skulls*. The mean cephalic index of these is given as 76 0, which is not significantly less than the value of 77·7 found for the original 60.

In 1925 the present writer was able to study the Spanish Basque crania preserved in the Musée Broca† and he re-measured and drew contours of the 39 specimens which had been supposed male by Broca. Individual measurements and means are given in Appendix II below. This partial duplication of a somewhat laborious task was desirable for a number of reasons; individual measurements of the skulls have not previously been published, the unsexed means given by Broca do not serve modern requirements and there is no indication in his published papers of the number of specimens on which each is based; also, a number of additional measurements and type contours are now given‡.

Since Broca's day a considerable number of living Basques have been measured, chiefly by Aranzadi in Spain and Collignon in France. Their studies have shown that there is a real difference between the cephalic indices of the populations on the two sides of the Pyrenees, as had been suggested by the cranial series, but nearly all other characters are closely similar and there is every reason to believe that the Spanish and French Basques are varieties of the same race§. There appears to be a fairly general agreement in relating it most closely to ancient Egyptian and

* Paul Broca: "Crânes basques de Zaraus." *Bulletins de la Société d'Anthropologie de Paris*, 2e série, t. I. 1866, pp. 470—473.

† He was indebted to Professor Léonce Manouvrier, the late Sécretaire général of the Société d'Anthropologie de Paris, for permission to undertake this study.

‡ It must be admitted, too, that there is sufficient reason to question the accuracy of some of the means given in Broca's published works on this subject. The mean basio-bregmatic height of 128·75 given for the 60 unsexed Zaraus skulls on p. 58 of the 1863 memoir and on p. 64 of the 1868 memoir is an almost impossibly low value. It gives a height-length index of 67·4, which appears to be smaller than any other mean recorded for a cranial series. For the 39 male specimens from Zaraus I find a mean height of 130·8 and the height-length index is 70·5: these values were checked by measurements of the sagittal type contour. Again, on p. 80 of the 1868 memoir the mean nasal height is given as 42·35, which eads to a nasal index of 52·6. This is higher than any other nasal index recorded for a European race, whereas it is clear that, in reality, the Basque nasal index is extremely low. Corrections of these obvious inaccuracies would give a better correspondence between Broca's unsexed means of the Zaraus and St Jean-de-Luz series.

§ In an article in *L'Anthropologie* (t. v. 1894, pp. 276—287), in which Collignon summarises the results of his important memoir published in the *Mémoires de la Société d'Anthropologie de Paris* (3e série, t. I. 1895), this writer remarks (p. 286): "...Broca, ne jugeant la population basque française que d'après des crânes provenant de la plus déplorable localité qu'il fût possible de choisir à ce point de vue (St Jean-de-Luz) d'une ville cosmopolite par excellence depuis plusieurs siècles, appliquait à tort l'impression, exacte d'ailleurs, qu'il ressentait au reste du pays...." The variabilities of the male St Jean-de-Luz series could be calculated from Broca's manuscript records, and it would be interesting to compare them with the values given for Spanish Basque crania in Table III below.

modern North African races. Apart from the measurements of very small numbers of skulls, no further advance in Basque craniology appears to have been made until 1892*. A few measurements of 489 Spanish crania in the Museum of Madrid were published then: they comprise the sexed distributions and means of the cephalic and nasal indices for different provinces, and the means only (in whole millimetres) of the calvarial length and breadth. There are 46 male skulls from Guipuzcoa and 5 from Navarre and the means agree excellently with those of the series from Zaraus which is preserved at Paris†. A considerable number of sexed mean measurements of the skulls from Guipuzcoa at Madrid were published in 1913‡ and in the following year the individual measurements of the specimens in the same collection were given for the first time§. There are 14 male and 15 female skulls from Zaraus and 23 male and 18 female skulls from neighbouring localities. Sexed means of this Guipuzcoan series are given in Table I below. A study of the facial triangle of Basque skulls was published by Aranzadi in 1917||. Individual measurements of the sides and angles of the triangle are given for 53 male and 40 female specimens from the provinces of Guipuzcoa, Visoaya, Navarre and Alava. There is a good agreement with the corresponding measurements of the Basque series at Paris¶. An inter-racial comparison made by Aranzadi in this paper shows that the Basque skull is peculiarly orthognathous: the chord from basion to alveolar point, the nasal angle and the gnathic index shown are almost, if not quite, extreme values for all races of man. Finally, in 1922, Aranzadi published a synthesis of his measurements of Basque skulls preserved in Spanish museums**. He deals with 82 male and 83 female specimens, but unfortunately the individual measurements are not provided and many of the means are given to the nearest millimetre only and without any indication of the number of individuals on which each is based.

* L. de Hoyos Sáinz and T. de Aranzadi: "Un avance á la antropología de España." *Anales de la Sociedad (Española) de Historia Natural*, t. XXI. 1892, pp. 1—71.

† The male means are:

	Cephalic Index (100 B/L)	Nasal Index (100 NB/NH')
Basques from Guipuzcoa and Navarre (Hoyos Sáinz and Aranzadi)	76·5 (46)	44·6 (45)
Basques from Zaraus (Morant)	77·2 (39)	44·8 (35)

‡ T. de Aranzadi: "Cráneos de Guipúzcoa." *Asociación Española para el Progreso de las Ciencias. Congreso de Madrid*, 1913.

§ T. de Aranzadi: "Sur quelques corrélations du trou occipital des crânes basques." *Bulletins et Mémoires de la Société d'Anthropologie de Paris*, 6e série, t. V. 1914, pp. 325—382.

|| "El triángulo facial de los cráneos vascos." *Memorias de la Real Sociedad Española de Historia Natural*, t. X. 1917, No. 8a.

¶ The male means are:

	$G'H$	GL	LB	$N\angle$	$A\angle$	$B\angle$
Spanish Basques (Aranzadi)...	70·8 (53)	92·9 (53)	100·3 (53)	62°·9 (53)	74°·3 (53)	42°·8 (53)
Basques from Zaraus (Morant)	70·7 (31)	90·7 (37)	99·6 (39)	61°·9 (31)	75°·0 (31)	43°·1 (31)

** "Síntesis métrica de cráneos vascos." *Revue internationale des études basques*, t. XIII. 1922, pp. 1—32.

TABLE I.

Mean Measurements of Series of Basque Crania from Guipuzcoa.

Character*	Measured by			
	Aranzadi		Morant	Aranzadi and Morant
	Female	Male	Male	Male
L	179·0 (33)	186·2 (37)	185·8 (39)	186·0 (76)
B	138·9 (33)	142·8 (37)	143·5 (39)	143·2 (76)
B''	116·9 (33)	120·2 (37)	119·7 (39)	119·9 (76)
B'	95·0 (33)	96·6 (37)	97·1 (39)	96·9 (76)
Bi-asterionic B	108·9 (33)	112·8 (37)	113·2 (39)	113·0 (76)
Bi-auricular B	120·4 (33)	125·7 (36)	[123·3 (37)]†	124·5 (73)
H'	125·0 (33)	131·8 (37)	130·8 (39)	131·3 (76)
LB	95·8 (33)	101·5 (36)	99·6 (39)	100·5 (75)
S	363·5 (33)	374·4 (37)	375·2 (39)	374·8 (76)
S_1	125·3 (33)	128·8 (37)	129·5 (39)	129·2 (76)
S_2	122·9 (33)	127·6 (37)	126·0 (39)	126·8 (76)
S_3	115·3 (33)	118·3 (37)	119·8 (39)	119·1 (76)
Broca's Q'	299·1 (31)	307·1 (37)	—	307·1 (37)
Glabella U	510·1 (33)	527·3 (37)	-	527·3 (37)
fml	34·6 (33)	35·5 (37)	36·4 (39)	36·0 (76)
fmb	29·2 (33)	29·7 (37)	30·9 (39)	30·3 (76)
$G'H$	66·7 (19)	70·3 (30)	70·7 (31)	70·5 (61)
GL	90·4 (19)	93·3 (30)	90·7 (37)	91·9 (67)
NH'	49·2 (33)	51·1 (37)	51·9 (37)	51·5 (74)
NB	22·9 (31)	22·7 (37)	23·2 (35)	22·9 (72)
DC	20·2 (33)	20·9 (37)	21·1 (37)	21·0 (74)
O_1'	38·3 (32)	38·3 (37)	39·1 (35) R	38·7 (72)
O_2	33·5 (33)	32·6 (37)	33·6 (35) R	33·1 (72)
J	122·4 (32)	129·4 (31)	128·8 (34)	129·1 (65)
GB	86·8 (33)	89·1 (34)	90·1 (31)	89·6 (65)
External bi-orb. B	98·3 (33)	101·5 (37)	—	101·5 (37)
Bi-jugal B	104·7 (29)	109·3 (34)	-	109·3 (34)
Basio-palatal L	42·6 (31)	44·0 (37)	[41·4 (36)]†	42·7 (73)
$100\ B/L$	77·6 (33)	76·7 (37)	77·2 (39)	77·0 (76)
$100\ H'/L$	69·9 (33)	70·9 (37)	70·5 (39)	70·7 (76)
$100\ B/H'$	111·3 (33)	108·4 (37)	109·9 (39)	109·2 (76)
$100\,(B-H')/L$	7·7 (33)	5·9 (37)	6·8 (39)	6·4 (76)
$100\,fmb/fml$	84·3 (33)	86·6 (37)	85·1 (39)	85·8 (76)
$100\ G'H/GB$	76·9 (19)	78·9 (30)	78·9 (26)	78·9 (56)
$100\ NB/NH'$	46·6 (31)	44·3 (37)	44·8 (35)	44·5 (72)
$100\ O_2/O_1'$	87·3 (32)	85·3 (37)	86·3 (34) R	85·8 (71)
$100\ G'H/J$	55·0 (19)	54·5 (25)	55·2 (28)	54·9 (53)
$N\angle$	64°·9 (19)	63°·0 (30)	61°·9 (31)	62°·4 (61)
$A\angle$	73°·3 (19)	74°·9 (30)	75°·0 (31)	75°·0 (61)
$B\angle$	41°·8 (19)	42°·1 (30)	43°·1 (31)	42°·6 (61)
Daubenton's $\angle$	−4°·6 (33)	−1°·3 (37)	[+0°·2 (36)]†	−0°·6 (73)

* The measurements are defined in Appendix I.
† From type contour.

The mean measurements of the male Basque series preserved at Paris and Madrid are given in Table I and it will be seen that all the differences between

corresponding characters are remarkably small. Actually no difference is significant, and there is complete justification for considering that the two samples were drawn from the same homogeneous population. The pooled means given in the same table are based on numbers of individuals which are adequate enough to provide a reliable description of the type. They indicate that the Basque skull is typically European in all respects: there is nothing to suggest that it is more closely related to any non-European types than any other Western European forms are. The characters of the calvaria are in no way peculiar. It has been suggested that the occipital index (defined in Appendix I) has lower values for European races than for any others in the world. The means available range from 58·0 to 68·3*. In this respect the Basques are precisely similar to neighbouring races. All means shown below are based on more than 30 male crania. It is interesting to find that this character makes a fairly definite distinction between the races of Western Europe (whether dolicho- or brachycephalic) on the one hand, and the Egyptian and modern races of Eastern Europe on the other.

Oc. I.	Cranial Series
58—59	Seventeenth century English (Farringdon Street)[1] 58·0, Walser (Vorarlberg)[2] 58·1, Anglo-Saxons 58·2[3], Basques 58·3, Bavarians (Alps)[4] 59·0.
59—60	Reihengraber[5] 59·5.
60—61	Predynastic Egyptians (Naqada *A* and *Q*) 60·2[6], Swiss (Daniser)[7] 60·3, Czechs[8] 60·5, Guanche[9] 60 6, Egyptians (XVIIIth—XXth Dynasties)[10] 60·7.
61—62	Egyptians (XXVIth—XXXth Dynasties)[11] 61·5, Abyssinians (Tigre)[12] 62·0.
62—68	Rumanians[13] 62·7, Serbo-Croats[14] 62·8, Greeks[15] 62·9, Turks[16] 63·3.

[1] *Biometrika*, Vol. XVIII. p. 29.
[2] *Zeitschrift für Ethnologie*, Bd. XLIV. S. 509.
[3] *Biometrika*, Vol. XVIII. p. 82.
[4] *Beiträge z. Anthrop. u. Urgesch. Bayerns*, Bd. XVIII. S. 1.
[5] Pooled mean. *Biometrika*, Vol. XX B. p. 313.
[6] *Biometrika*, Vol. XVII. p. 15.
[7] *Zeitschrift f. Morph. u. Anthrop.* Bd. XV. Tafeln, S. 544.
[8] *Archiv f. Anthropologie*, Bd. XXXIX. S. 282.
[9] Detloff v. Behr: *Metrische Studien an 152 Guanchenschädeln.* Stuttgart, 1908.
[10] Hermann Stahr: *Die Rassenfrage im antiken Ägypten.* Leipzig, 1907.
[11] *Biometrika*, Vol. XVI. p. 337.
[12] Sergio Sergi: *Crania Habessinica.* Rome, 1912.
[13] *Densch. d. k. Akad. d. Wissensch. Wien, Math. Nat. Kl.* Bd. XXX. S. 107—136.
[14] Weisbach: Supp. 1884. *Zeitschrift f. Ethnologie*, S. 70—2.
[15] *Mitth. d. Anthrop. Gesellsch. in Wien*, Bd. XI. S. 72.
[16] *Ibid.* Bd. III. S. 206—213.

Some of the facial characters of the Basque skull are more peculiar than its calvarial ones. The narrowness of the face is one of the most salient features of the living head for this race and the same narrowness marks the cranium. The breadth between the lowest points on the malar-maxillary sutures (GB) is 89·6: the reliable male means nearest to this value available for other European types are 90·9 for the Whitechapel English† and 91·4 for the Farringdon Street English‡, while 22 other series have values greater than 92. The Basque nasal breadth of 22·9 is also extreme: for 46 other European races the lowest male means are 23·1 for both French Soldiers§

* *Biometrika*, Vol. XVI. 1924, pp. 334 and 335.
† *Ibid.* Vol. III. p. 208.
‡ *Ibid.* Vol. XVIII. p. 28.
§ See references appended to Table II.

and Lowland Scottish*. The Basque bi-zygomatic breadth of 129·1 is less characteristic than GB and NB, since all Egyptian types have this measurement below 129·0, and European races with smaller values are Sardinians† 127·7, Portuguese‡ 127·7 and Great Russians§ 128·8. Several Western European types have male means for this character between 130 and 131. The facial heights of the Basque type are not outstanding, so that some of the indices expressing the ratios of these heights to the unusual breadths may be expected to be unusual also. The upper facial index ($100\, G'H/GB$) has the extreme value of 78·9. The closest approach is shown by the Würtemberger 78·1‖, Austrians (Vienna) 77·5¶ and Farringdon Street English 77·1**, while 22 other European series have lower values. The Basque nasal index ($100\, NB/NH'$) of 44·5 is low, but not extreme, as still lower values are recorded for Belgian Franks 43·9*, Breton Gallots 44·3†† and Portuguese‡ 44·4, and the Lowland Scottish* have the same mean of 44·5. Forty-three other European series are found to have higher nasal indices than these. The marked orthognathism of the Basque skull has been insisted upon by Aranzadi and, like the narrowness of the face, this is a striking characteristic of the living head. The basal length (from nasion to basion) and the upper facial height (from nasion to alveolar point) are not unusual, but the length from basion to alveolar point (GL) again appears to be extreme. The male mean is 91·9, and the lowest means found among 26 other European series are 92·3 for Czechs‡‡ and 93·3 for Modern Cretans§§. The Basque nasal angle of 62°·4 is just equalled by one recorded for Serbo-Croats‖‖, while the next smallest mean is 63°·3 for the Modern Cretans. The Basque alveolar angle of 75°·0 is large but not extreme, and the basal angle is not peculiar. The alveolar profile angle of 88°·7 appears to be the largest given for any race. It is particularly interesting to find this association in the type of an extremely narrow face and marked orthognathism. For 48 racial series from all parts of the world an inter-racial correlation of $+\cdot 747 \pm \cdot 040$ has been found between the nasal index and nasal angle¶¶. For some of these facial characters the Basque type appears to have values which are extreme for all modern races of man; such are the bi-maxillary and nasal breadths, the chord from basion to alveolar point and the upper facial index. Other characters which are almost extreme are the occipital and nasal indices and the nasal and alveolar angles. These facts do not dissociate the Basques from other European races. The types noted above which have characters approaching most closely to the extremes for the Spanish race need not be presumed to be the ones most nearly related to it; relationship can only be estimated with safety by considering all the more important features of the skull.

* See references appended to Table II.

† *Zeitschrift f. Morph. und Anthrop.* Bd. XIII. S. 444.

‡ Ferraz de Macedo: *Crime et Criminel.* Lisbon, 1892, p. 52.

§ *Mémoires de l'Académie Impériale des Sciences de St Pétersbourg*, VIIe série, t. XXXII. pp. 1—81.

‖ *Die anthropologischen Sammlungen Deutschlands.* Tübingen Catalogue.

¶ *Zeitschrift der k. k. Gesellschaft der Ärzte in Wien, Medizinische Jahrbücher*, XX Jahrgang. H. i—iv.

** *Biometrika*, Vol. XVIII. p. 28.

†† *Revue d'Anthropologie*, t. II. p. 627.

‡‡ *Archiv f. Anthropologie*, Bd. XXXIX. S. 281, *et seq.*

§§ *Zeitschrift für Ethnologie*, Bd. XLIII. S. 322—325.

‖‖ Weisbach: Supp. 1884. *Zeitschrift f. Ethnologie*, S. 66—77. Means given, *Biometrika*, Vol. XX B. pp. 366—367.

¶¶ *Annals of Eugenics*, Vol. II. p. 386.

TABLE II.

Coefficients of Racial Likeness between Basque and other Series.

		Crude C.R.L.'s						C.R.L.'s reduced to $\frac{\bar{n}_1 \bar{n}_2}{\bar{n}_1 + \bar{n}_2} = 50$		
		Basques (Morant) (34·7)*		Basques (Aranzadi) (35·6)		Basques (Pooled) (61·5)		Basques (Morant)	Basques (Aranzadi)	Basques (Pooled)
		C.R.L.	No. of Characters	C.R.L.	No. of Characters	C.R.L.	No. of Characters			
Basques (Morant)	(34·7)*	—	—	0·20 ± ·19	23	—	—	—	0·57 ± ·19	—
Basques (Aranzadi)	(35·6)	0·20 ± ·19	23	—	—	—	—	0·57 ± ·19	—	—
British Iron Age[1]	(52·3)	1·98 ± ·20	21	1·26 ± ·20	21	2·58 ± ·20	22	4·74 ± ·20	2·97 ± ·20	4·56 ± ·20
Lowland Scottish[2]	(34·7)	1·88 ± ·20	21	1·59 ± ·20	21	2·22 ± ·20	22	5·42 ± ·20	4·52 ± ·20	5·00 ± ·20
Pompeians[3]	(52·3)	2·07 ± ·20	22	2·29 ± ·20	21	3·72 ± ·20	22	4·95 ± ·20	5·40 ± ·20	6·57 ± ·20
Etruscans[4]	(79·6)	3·57 ± ·18	26	3·49 ± ·20	21	4·68 ± ·18	26	7·38 ± ·18	7·09 ± ·20	6·74 ± ·18
Belgian Franks[5]	(35·8)	2·46 ± ·23	16	2·82 ± ·23	16	3·48 ± ·22	17	6·99 ± ·23	7·92 ± ·23	7·70 ± ·22
Parisians: Cité[6]	(54·6)	3·46 ± ·21	19	3·38 ± ·21	19	4·47 ± ·21	20	8·16 ± ·21	7·86 ± ·21	7·73 ± ·21
English: Farringdon Street[7]	(95·4)	2·90 ± ·17	30	5·38 ± ·19	23	5·87 ± ·17	30	5·71 ± ·17	10·39 ± ·19	7·85 ± ·17
Parisians: l'Ouest[8]	(76·7)	3·58 ± ·22	17	4·86 ± ·22	17	5·90 ± ·22	18	7·49 ± ·22	10·00 ± ·22	8·65 ± ·22
Guanche[9]	(240·2)	3·57 ± ·19	23	6·43 ± ·19	23	8·81 ± ·19	24	5·89 ± ·19	10·37 ± ·19	8·99 ± ·19
Mediaeval Austrians[10]	(41·5)	4·66 ± ·29	10	2·98 ± ·32	8	4·80 ± ·29	10	12·33 ± ·29	7·76 ± ·32	9·68 ± ·29
French Soldiers[11]	(56·0)	6·70 ± ·20	21	3·09 ± ·23	16	7·61 ± ·20	21	15·65 ± ·20	7·09 ± ·23	12·99 ± ·20

* The numbers in brackets are the mean numbers of skulls ($\bar{n}$) for the characters used in computing the coefficients.

[1] G. M. Morant: "A First Study of the Craniology of England and Scotland from Neolithic to Early Historic Times, with Special Reference to the Anglo-Saxon Skulls in London Museums." *Biometrika*, Vol. XVIII. (1926), pp. 56—98. The skulls in the "British Iron Age" series are chiefly of Romano-British date, and the greater number of them came from the south of England. Revised means are given in *Biometrika*, Vol. XX B. pp. 372—373.

[2] W. Turner: "A Contribution to the Craniology of the People of Scotland. Part I—Anatomical." *Transactions of the Royal Society of Edinburgh*, Vol. XL. Part III (1903), pp. 547—613. Measurements of six of Turner's groups were pooled (see *Biometrika*, Vol. XVIII. (1926), p. 22) to give the "Lowland Scottish" means (*Ibid.* p. 38: the "*Q*" in this table should be "Broca's *Q'*," and it is not the same as Macdonell's "Bregmatic *Q'*").

[3] G. Nicolucci: *Crania Pompeiana.* Naples (1882). Means are given in *Biometrika*, Vol. XX B. pp. 370—371.

[4] E. Schmidt: *Die anthropologischen Sammlungen Deutschlands.* Leipzig Catalogue (1887). Means are given in *Biometrika*, Vol. XX B. pp. 370—371.

[5] E. Houzé: "Les Francs de la Nécropole de Cipley, Hainaut." *Bulletins et Mémoires de la Société d'Anthropologie de Bruxelles*, t. XXXII. (1913), pp. cix—cxli. The skulls are of 6th and 7th century date.

[6] The means were abstracted from Broca's manuscript catalogue with the kind permission of M. Papillault. The series from the "Cimetière de la Cité" is earlier than the 13th century.

[7] B. G. E. Hooke: "A Third Study of the English Skull, with Special Reference to the Farringdon Street Crania." *Biometrika*, Vol. XVIII. (1926), pp. 1—55. This London cemetery was used in the 17th century.

[8] Means from Broca's manuscript catalogue. The series from the "Cimetière de l'Ouest" can be dated between 1788 and 1824.

[9] E. A. Hooton: "The Ancient Inhabitants of the Canary Islands." *Harvard African Studies*, Vol. VII. (1925).

[10] C. Toldt: "Die Schädelformen in den österreichischen Wohngebieten der Altslawen—einst und jetzt." *Mitteilungen der anthropologischen Gesellschaft in Wien*, Bd. XLII. (1912), S. 247—280. The skulls came from 11 cemeteries in Lower Austria and one in Moravia. All can be dated between A.D. 500 and 1200. Means are given in *Biometrika*, Vol. XX B. pp. 374—375.

[11] Büdinger: *Die anthropologischen Sammlungen Deutschlands.* Munich Catalogue (1892). The skulls are of French soldiers who died in Munich during the Franco-Prussian War. Means are given in *Biometrika*, Vol. XX B. pp. 370—371.

Comparison of the several Basque series with one another and with other racial types was made by Professor Karl Pearson's method of the coefficient of racial likeness*. Using the standard deviations given for the Farringdon Street series of seventeenth century London skulls† a coefficient is found between the male means of the two Basque series of 0·20 ± ·19 for 23 characters. There is thus complete statistical justification for considering that the two samples were drawn from identically the same population. No single character shows a significant difference, so it is probable that in cases where the two workers followed the same definitions of measurements they interpreted them in identically the same way. The standard deviations of the pooled Basque series are given in Table III below and most of them are rather smaller than the corresponding English values. Using the Basque standard deviations, the coefficient between the series measured by Aranzadi and Morant respectively is increased to 0·35 ± ·19, but it is still insignificant.

Comparison was made with a considerable number of other European cranial series‡ and all the coefficients found which denote close resemblance are given in Table II. The collections at Paris and Madrid are almost exactly equal in size and their corresponding coefficients with the other series are of the same order. But the coefficient between any series and the pooled Basques is in every case greater than that between the same series and either of the two smaller Basque samples. These differences are evidently due to the differences in the sizes of the samples and not to differences in degrees of relationship. To obviate this effect, each coefficient was reduced to the value it would have if each series in the comparison contained 100 individuals, and these adjusted values may be compared with one another directly. In 8 of the 11 cases the pooled Basque series has a coefficient lying between the two found with its component halves. The orders in which the 3 Basque series arrange the 11 other types are similar, but by no means identical. Such divergences must be expected when small samples are being dealt with and it would evidently be fallacious to attribute them in this case to differences in racial constitution. The comparison with the pooled series will be most reliable. It is remarkably similar to the British Iron Age and Lowland Scottish types, the bonds in these cases being quite as close as those usually found between neighbouring and contemporary European races. The divergence from the Pompeians and Etruscans is a little greater. These connections are appreciably closer than any which can be found with French series. Comparison is made with two Parisian populations of different dates, and with a series of soldiers who probably came from the north of France. The coefficients with these are not less than the one with seventeenth century Londoners, though

* "On the Coefficient of Racial Likeness." *Biometrika*, Vol. XVIII. 1926, pp. 105—117. The form of the coefficient used was:

$$\text{C.R.L.} = S\frac{1}{M}\frac{n_s n_{s'}}{n_s + n_{s'}} \times \frac{(m_s - m_{s'})^2}{\sigma_s^2} - 1 + \frac{1}{M} \pm \cdot 67449\sqrt{\frac{2}{M}\left(1 - \frac{1}{M}\right)}.$$

† B G. E. Hooke: "A Third Study of the English Skull, with Special Reference to the Farringdon Street Crania." *Ibid.* pp. 1—55.

‡ Coefficients of racial likeness between the Basque series measured by the present writer and 40 other series are given in *Ibid.* Vol. XX B. 1928, pp. 317—327.

all are of a decidedly lower order than any which can be found between the Basques and any central or southern French series. Comparison could not be made with any modern Spanish types. Among the various races, all of Asiatic origin, which have at one time or another been supposed closely allied to the Basques, on the ground of physical or other evidence, the Lapps, Finns and Egyptians may be rejected entirely. No close relationships have been noted with adjacent types, but all the most closely allied ones, apart from the Guanche, belong to Western Europe. The connections in these cases are quite as close as any that can be found for a number of other European races. Although the Basque skull possesses some unusual features, it is not by any means an isolated type, and there is no reason to believe that its origin was essentially different from that of any other racial form of skull. Its geographical isolation has been explained by the wildest hypotheses, but the ethnic history of Western Europe in recent times has been so unsettled that that isolation, though unusual, is not surprising.

The standard deviations and coefficients of variation of the characters available for the pooled series of male Basque skulls are given in Table III (p. 76). Comparison is there made with the coefficients of variation of the absolute measurements, and with the standard deviations of indices and angles, obtained for a seventeenth century London series from a single graveyard* and for an Egyptian series of the XXVIth—XXXth Dynasties also from a single cemetery†. The variability of the former series may be supposed typical of a random sample taken from a racially homogeneous population coming from the less isolated parts of Europe in modern times. For every character in the table the English constants are in excess of the Egyptian, although few of the differences are definitely significant. Twenty-two measurements provide these data for all three series. In the case of 9 of them the Basque variabilities are between the English and Egyptian values; for 5 the Basque variabilities exceed the English and for 7 they are less than the Egyptian. For the remaining character—the cephalic index—the Basque and Egyptian standard deviations are exactly equal. Judging from all the measurements, the Basque series may be supposed to exceed the Egyptian in variability by as much as it falls short of the English. There is a surprisingly small difference in this respect between the isolated Spanish people from a single province and the Londoners. The futility of the assumption often made that the variabilities for any single measurement can provide a reliable measure of the relative degrees of homogeneity of a number of series is evident from the above comparison.

The three type contours were constructed in the usual way‡ for the 37 male Basque skulls preserved at Paris. They are reproduced in Figs. 1—3 and the mean measurements used in their construction are given in Tables IV—VI. The only

* B. G. E. Hooke: "A Third Study of the English Skull, with Special Reference to the Farringdon Street Crania." *Biometrika*, Vol. XVIII. 1926, pp. 1—55. The variabilities quoted are based on numbers of skulls varying between 48 and 153.

† Karl Pearson and Adelaide G. Davin: "On the Biometric Constants of the Human Skull." *Ibid.* Vol. XVI. 1924, pp. 328—363. The variabilities quoted are all based on more than 780 skulls.

‡ See *Ibid.* Vol. XIV. pp. 227—244.

closely related series for which these contours have been published is that of the seventeenth century Londoners from the Farringdon Street graveyard and com-

TABLE III.

Constants of Variation for the Basque and other Series. Male Skulls.

Characters	Basques No.	Basques Standard Deviations	Basques Coefficients of Variation	English (Farringdon Street)	Egyptians (XXVIth–XXXth Dynasties)
				Coefficients of Variation	
L	76	$6{\cdot}88 \pm {\cdot}38$	$3{\cdot}70 \pm {\cdot}20$	$3{\cdot}42 \pm {\cdot}12$	$3{\cdot}09 \pm {\cdot}05$
B	76	$5{\cdot}02 \pm {\cdot}27$	$3{\cdot}51 \pm {\cdot}19$	$4{\cdot}14 \pm {\cdot}17$	$3{\cdot}43 \pm {\cdot}05$
B'	76	$3{\cdot}96 \pm {\cdot}22$	$4{\cdot}09 \pm {\cdot}22$	$4{\cdot}73 \pm {\cdot}18$	$4{\cdot}28 \pm {\cdot}07$
B''	76	$5{\cdot}52 \pm {\cdot}30$	$4{\cdot}60 \pm {\cdot}25$	—	—
Bi-asterionic B	76	$4{\cdot}75 \pm {\cdot}26$	$4{\cdot}20 \pm {\cdot}23$	—	—
H'	76	$4{\cdot}95 \pm {\cdot}27$	$3{\cdot}77 \pm {\cdot}21$	$3{\cdot}90 \pm {\cdot}17$	—
LB	75	$4{\cdot}28 \pm {\cdot}24$	$4{\cdot}26 \pm {\cdot}24$	$4{\cdot}47 \pm {\cdot}20$	$3{\cdot}90 \pm {\cdot}06$
S	76	$14{\cdot}20 \pm {\cdot}78$	$3{\cdot}79 \pm {\cdot}21$	$3{\cdot}75 \pm {\cdot}16$	$3{\cdot}36 \pm {\cdot}05$
S_1	76	$6{\cdot}63 \pm {\cdot}36$	$5{\cdot}13 \pm {\cdot}28$	$5{\cdot}00 \pm {\cdot}19$	$4{\cdot}48 \pm {\cdot}08$
S_2	76	$7{\cdot}62 \pm {\cdot}42$	$6{\cdot}03 \pm {\cdot}33$	$6{\cdot}24 \pm {\cdot}25$	$5{\cdot}77 \pm {\cdot}09$
S_3	76	$6{\cdot}70 \pm {\cdot}37$	$5{\cdot}63 \pm {\cdot}31$	$6{\cdot}50 \pm {\cdot}27$	$5{\cdot}91 \pm {\cdot}10$
$G'H$	61	$4{\cdot}04 \pm {\cdot}25$	$5{\cdot}73 \pm {\cdot}35$	$6{\cdot}31 \pm {\cdot}33$	$5{\cdot}90 \pm {\cdot}10$
GL	67	$4{\cdot}61 \pm {\cdot}27$	$5{\cdot}02 \pm {\cdot}29$	$5{\cdot}66 \pm {\cdot}34$	$5{\cdot}10 \pm {\cdot}08$
J	65	$5{\cdot}21 \pm {\cdot}31$	$4{\cdot}04 \pm {\cdot}24$	$3{\cdot}69 \pm {\cdot}27$	$3{\cdot}55 \pm {\cdot}06$
GB	65	$4{\cdot}83 \pm {\cdot}29$	$5{\cdot}39 \pm {\cdot}32$	$6{\cdot}74 \pm {\cdot}38$	$4{\cdot}90 \pm {\cdot}08$
NH'	74	$3{\cdot}01 \pm {\cdot}17$	$5{\cdot}85 \pm {\cdot}33$	$6{\cdot}75 \pm {\cdot}45$	—
NB	72	$1{\cdot}74 \pm {\cdot}10$	$7{\cdot}60 \pm {\cdot}43$	$8{\cdot}17 \pm {\cdot}43$	$7{\cdot}27 \pm {\cdot}12$
O_1'	73	$1{\cdot}61 \pm {\cdot}09$	$4{\cdot}16 \pm {\cdot}23$	$(R)\ 4{\cdot}20 \pm {\cdot}22$	—
O_2	74	$2{\cdot}00 \pm {\cdot}11$	$6{\cdot}04 \pm {\cdot}34$	$(R)\ 6{\cdot}88 \pm {\cdot}36$	$(R)\ 5{\cdot}67 \pm {\cdot}09$
DC	74	$2{\cdot}10 \pm {\cdot}12$	$10{\cdot}00 \pm {\cdot}56$	$9{\cdot}73 \pm {\cdot}51$	—
fml	76	$2{\cdot}39 \pm {\cdot}13$	$6{\cdot}64 \pm {\cdot}36$	$8{\cdot}15 \pm {\cdot}36$	$6{\cdot}95 \pm {\cdot}11$
fmb	76	$2{\cdot}32 \pm {\cdot}13$	$7{\cdot}66 \pm {\cdot}42$	$7{\cdot}19 \pm {\cdot}33$	$7{\cdot}18 \pm {\cdot}11$
				Standard Deviations	
$100\ B/L$	76	$2{\cdot}68 \pm {\cdot}15$	—	$3{\cdot}48 \pm {\cdot}14$	$2{\cdot}68 \pm {\cdot}06$
$100\ H'/L$	76	$2{\cdot}96 \pm {\cdot}16$	—	$3{\cdot}24 \pm {\cdot}14$	—
$100\ B/H'$	76	$5{\cdot}25 \pm {\cdot}29$	—	$5{\cdot}27 \pm {\cdot}23$	—
$100\ (B-H')/L$	76	$3{\cdot}48 \pm {\cdot}19$	—	$3{\cdot}59 \pm {\cdot}16$	—
$100\ fmb/fml$	76	$5{\cdot}23 \pm {\cdot}29$	—	$5{\cdot}90 \pm {\cdot}27$	$5{\cdot}79 \pm {\cdot}09$
$100\ G'H/GB$	56	$4{\cdot}88 \pm {\cdot}31$	—	$6{\cdot}24 \pm {\cdot}37$	$4{\cdot}96 \pm {\cdot}08$
$100\ NB/NH'$	72	$3{\cdot}32 \pm {\cdot}19$	—	—	—
$100\ O_2/O_1'$	74	$4{\cdot}38 \pm {\cdot}24$	—	$(R)\ 6{\cdot}40 \pm {\cdot}34$	—
$N\angle$	61	$3^\circ{\cdot}55 \pm {\cdot}22$	—	$3^\circ{\cdot}68 \pm {\cdot}22$	$3^\circ{\cdot}31 \pm {\cdot}06$
$A\angle$	61	$3^\circ{\cdot}56 \pm {\cdot}22$	—	$3^\circ{\cdot}63 \pm {\cdot}22$	$3^\circ{\cdot}46 \pm {\cdot}06$
$B\angle$	61	$2^\circ{\cdot}83 \pm {\cdot}17$	—	$3^\circ{\cdot}65 \pm {\cdot}22$	$2^\circ{\cdot}66 \pm {\cdot}04$

parison will be restricted to them. The two transverse types are almost identical. The English figure has a height (MA) 1·3 mm. less than the Basque, but that difference, and the maximum found between the parallels, are not significant. It may be noted

that the Basque section is almost perfectly symmetrical, the maximum difference between the right and left sides of the same parallel being 0·9 mm. in favour of the left side. For the English section the maximum difference is 2·7 mm. in favour of the right side. The horizontal type contour of the Basque skull is far more asymmetrical. All the parallels on the left side exceed the corresponding ones on the right and the differences increase as the lines approach the occiput. The left side of the 7th parallel exceeds the right by 1·4 mm., for the 8th the difference is 2·5, for the 9th 3·8, for the 10th 4·5 and for $O\frac{1}{2}$ it reaches a maximum of 5·6 mm. It is evident from the appearance of the figure that the occipital region is quite markedly asymmetrical. Such a condition is more accentuated on this figure than on any other type contour that has yet been constructed, and usually there is no suggestion of it. For the English section the maximum difference between the two sides of the same parallel is 1·5 mm. and in this case it is the right side which is in excess. Some confirmation of the asymmetry noted will be needed before it will be safe to assert that it is a racial characteristic of the Basque skull. When the outlines for the two races are superposed with the point F and the axes (FO) coincident there is a close correspondence anterior to the 7th parallels. The maximum breadths are exactly equal. But the English figure exceeds the Basque in length by 3·7 mm. If the symmetrical English contour is then rotated about F until there is an angle of 0°·5 between the axes the correspondence becomes appreciably better. The outlines

TABLE IV.

Mean Measurements of 37 Male Basque Transverse Contours.

	MA	*1*	*M½*	*2*	*3*	*4*	*5*	*6*	*7*
R ...	111·3	58·6	61·4	63·7	66·8	68·7*	68·9	68·1	66·1
L ...		58·6	61·0	64·2	67·7	69·4	69·7*	68·1	65·5

	8	*9*	*10*	*A½*	*ZRy*	*ZRx*
R ...	61·3	52·2	37·7	19·5	61·4	2·4
L ...	60·7	52·1	37·5	18·8	61·7	3·5

* Mean of 36 contours.

are now almost coincident as far back as the 8th parallel of the English type. This suggests that it is merely the position of the point O—below the lambda—which is asymmetrically placed on the Basque type †.

† The writer is willing to admit that there may have been a constant error in marking the point O on the individual contours, but there has been no suggestion of this in the case of the other series with which he has dealt.

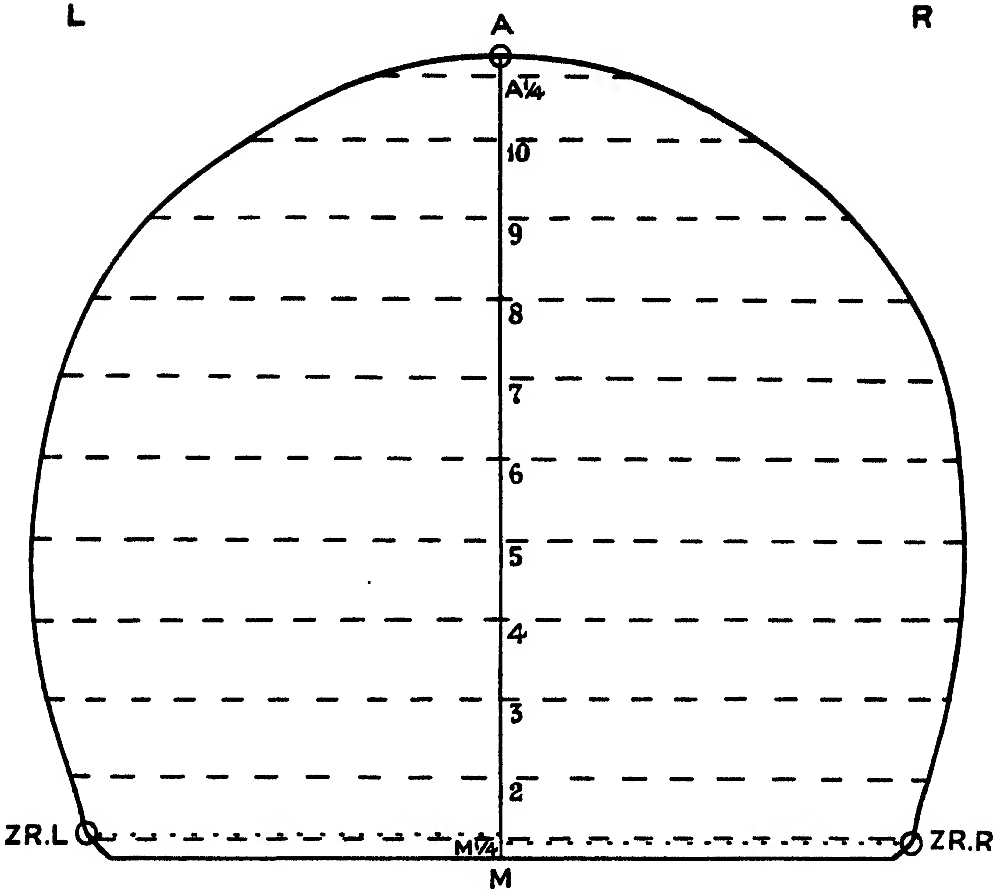


FIG. I. Transverse Type Contour, based on 37 ♂ Basque Skulls.

TABLE V.

Mean Measurements of 37 Male Basque Horizontal Contours.

	FO	$F\frac{1}{4}$	$F\frac{1}{2}$	*2*	$2\frac{1}{2}$	*3*	*4*	*5*	*6*
R ...	182·1	21·4	33·1	45·2	47·5	50·0	56·6*	64·2*	68·3
L ...		22·1	34·6	46·8	49·4	51·0	57·6	64·5	69·1

	7	*8*	*9*	*10*	$O\frac{1}{2}$	*Ty*	*Tx*
R ...	69·2	66·2	58·3	43·5	20·9	47·6	20·9
L ...	70·6*	68·7	62·1	48·0	26·5	49·5	21·3

* Mean of 36 contours.

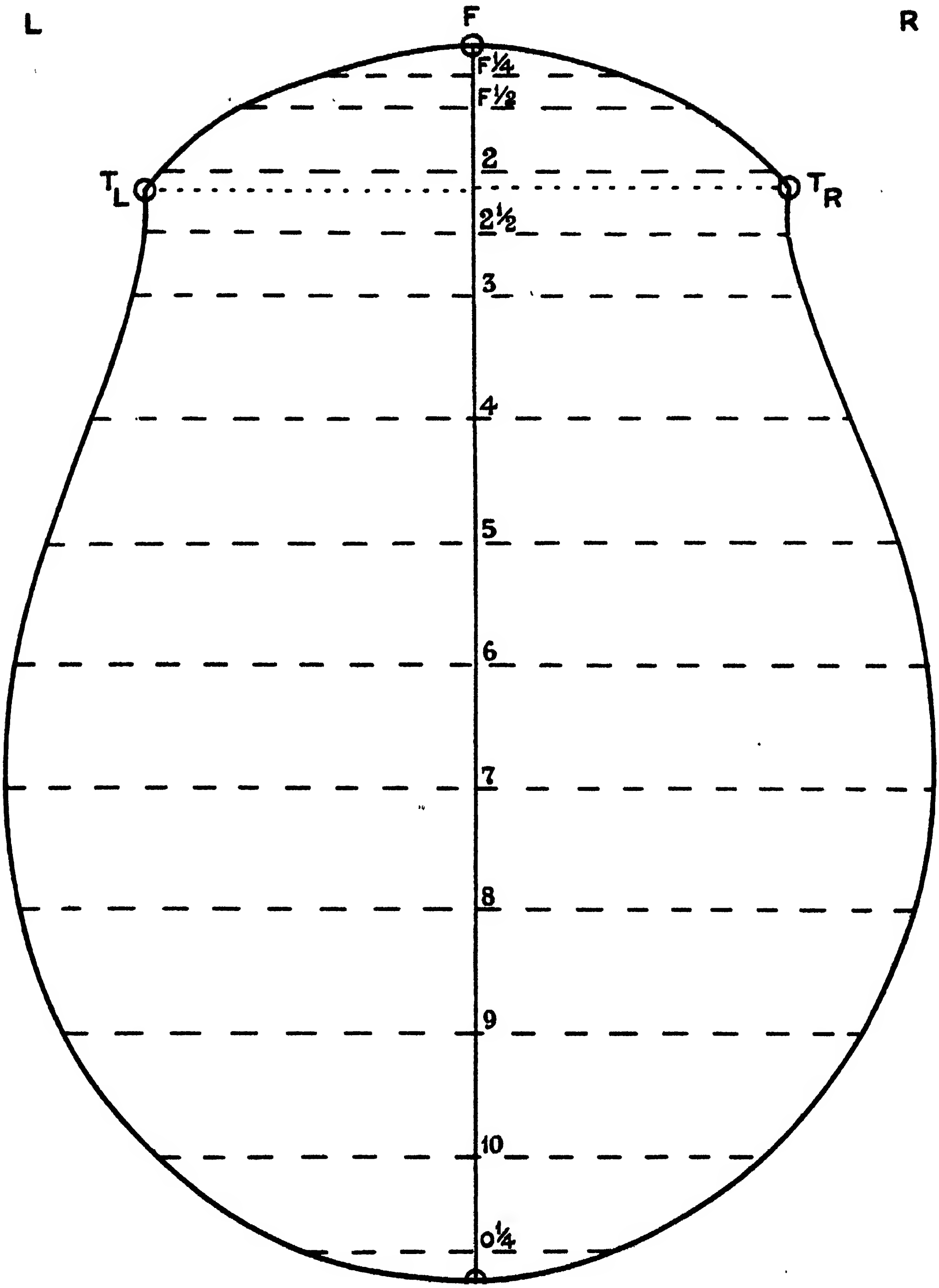


FIG. II. Horizontal Type Contour, based on 37 ♂ Basque Skulls.

TABLE VI.

Mean Measurements of 37 Male Basque Median Sagittal Contours.

$N\gamma$	Ordinates above $N\gamma$								
	$0=N$	$N\frac{1}{4}$	*1*	*2*	*3*	*4*	*5*	*6*	*7*
180·8 (37)	19·5 (37)	36·8 (36)	56·7 (37)	69·2 (37)	76·2 (37)	79·9 (37)	80·7 (37)	79·8 (37)	75·5 (37)

Ordinates above $N\gamma$				Ordinates below $N\gamma$					
8	*9*	$\gamma\frac{1}{4}$	$\gamma\frac{1}{8}$	$N\frac{1}{4}$	*1*	*2*	*8*	*9*	$\gamma\frac{1}{8}$
65·6 (37)	46·3 (37)	20·1 (37)	14·5 (37)	64·7 (31)	57·1 (37)	54·2 (37)	52·0 (37)	44·4 (37)	34·2 (37)

Ordinates below $N\gamma$	Vertex		Bregma		Glabella		Occipital Point	
$\gamma\frac{1}{4}$	x from N	y	x from N	y	x from N	y	x from γ	y
26·0 (37)	91·0 (37)	81·4 (37)	77·1 (37)	80·3 (37)	2·5 (37)	8·7 (37)	0·5 (37)	−3·7 (37)*

Lambda		Sub.-Orb. Point		Auricular Point		Opisthion		Inion	
x from γ	y	x from N	y	x from γ	y	x from γ	y	x from γ	y
5·9 (37)	21·3 (37)	10·1 (37)	31·1 (37)	95·2 (37)	31·1 (37)	58·8 (37)	57·2 (37)	15·2 (37)	43·3 (37)

Basion		Alveolar Point		Nose				
from γ	from N	from N	from Bas.	(i)	(ii)	(iii)	$\angle LN\gamma$	NL
106·6 (36)	99·1 (36)	72·4 (19)	90·1 (19)	1·5 (37)	3·8 (37)	7·6 (31)	125°·2 (12)	23·0 (12)

Frontal		Occipital		Max. Sub. to $N\lambda$		Max. Sub. to GI		Sp.	
Max. Sub. to $N\beta$		Max. Sub. to λ Op.							
x from N	y	x from λ	y	x from N	y	x from G	y	x from N	y
50·2 (37)	26·6 (37)	52·7 (37)	29·3 (37)	81·2 (37)	71·2 (37)	100·7 (37)	102·2 (37)	65·3 (36)	34·0 (36)

Sub. from ½ Bas.-Sp. Chord		Palate				N. S.		Crossing of Alv.-N.S. Chord
		P'		P				
x from Bas.	y	x from N	y	x from Alv.	y	x from N	y	from Alv.
12·9 (36)	0·5 (36)	44·9 (37)	53·4 (37)	32·7 (19)	17·5 (19)	5·5 (37)	52·0 (37)	8·9 (19)

* The occipital point is below the $N\gamma$ line.

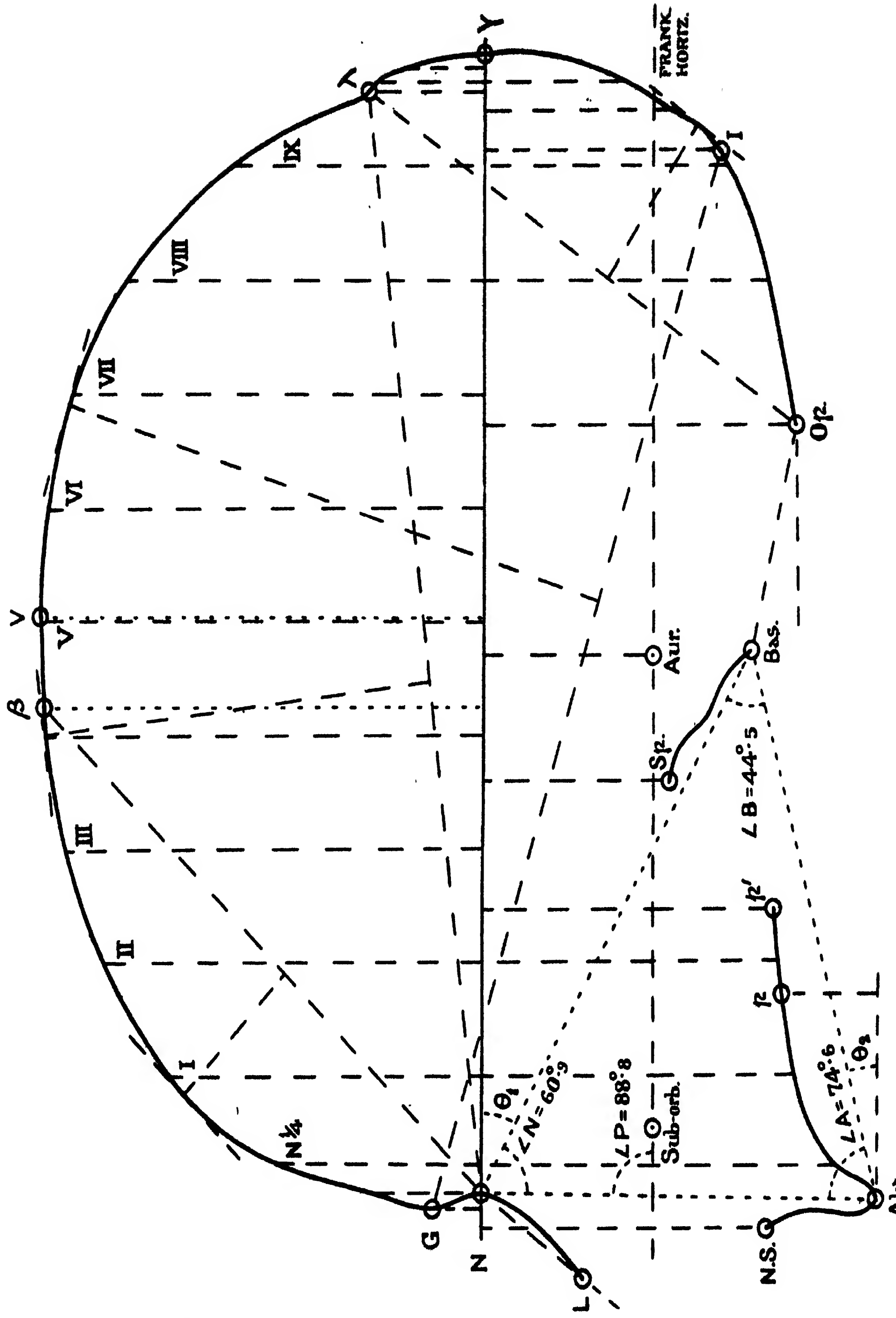


FIG. III. Sagittal Type Contour, based on 37 ♂ Basque Skulls.

Greater differences are found between the sagittal type contours of the two series compared. When they are superposed with $N\gamma$ lines and the nasions coincident the outlines of the nasal bones and the glabella regions also coincide. The English skull has an extremely retreating frontal bone, but the Basque does not differ from it greatly in this respect. The angle $\beta N\gamma$ is $45^\circ\cdot 1$ for the former and $46^\circ\cdot 2$ for the latter. The contours cross near their vertices and the outline of the obelion and occipital regions is significantly more protruding for the English than for the Basque figure. The contours cross again near the inions, which are almost coincident, and from the inion to the opisthion the Basque outline recedes further from the $N\gamma$ line. The lines indicating the basi-occipitals are practically coincident. The areas of the two calvarial sections in this plane are almost equal, the English being slightly the greater. The roofs of the palates are exactly equidistant from the horizontal base line and the same is almost true for the anterior nasal spines and alveolar points, but there is a sensible difference between the prognathism of the English and Basque types. The resemblances of the three contours compared are so close that it can only be inferred that the two racial types are intimately related. The comparison of direct measurements has shown, however, that there are some other Western European types which appear to be more closely related to the Basques than the seventeenth century Londoners are.

Conclusions. The principal purpose of the present paper has been to provide individual and mean measurements of a series of 39 male Basque skulls preserved in the Musée Broca. They came from Zaraus in the Spanish province of Guipuzcoa. Comparison with another series measured by Aranzadi of 37 male specimens from the same province shows that there is full justification for considering that the two samples were drawn randomly from the same homogeneous population. The variability of that population, as measured by the pooled samples, is rather less than for Londoners interred in a single seventeenth century graveyard, but greater than for a dynastic Egyptian series. The Basque skull is characterised by a peculiarly narrow facial skeleton—so that its nasal and upper facial indices are almost extreme for all races of man—and it is markedly orthognathous. These features do not dissociate the type from neighbouring ones. A close resemblance is found to several other Western European races. In spite of their present-day isolation, it is extremely probable that the Basques are more closely related to some existing or extinct races of Western Europe than to any others.

APPENDIX I. *Definitions of Measurements.*

The greater number of the following index letters denoting measurements are those used normally by workers in the Biometric Laboratory: F = Flower's ophryo-occipital length. L = maximum glabella-occipital length in median sagittal plane. B = maximum calvarial breadth. B' = minimum frontal breadth. B'' = maximum frontal breadth. Bi-asterionic B = chord asterion R. to asterion L. H = Frankfurt vertical height from basion. H' = basio-bregmatic height. LB = nasion to basion.

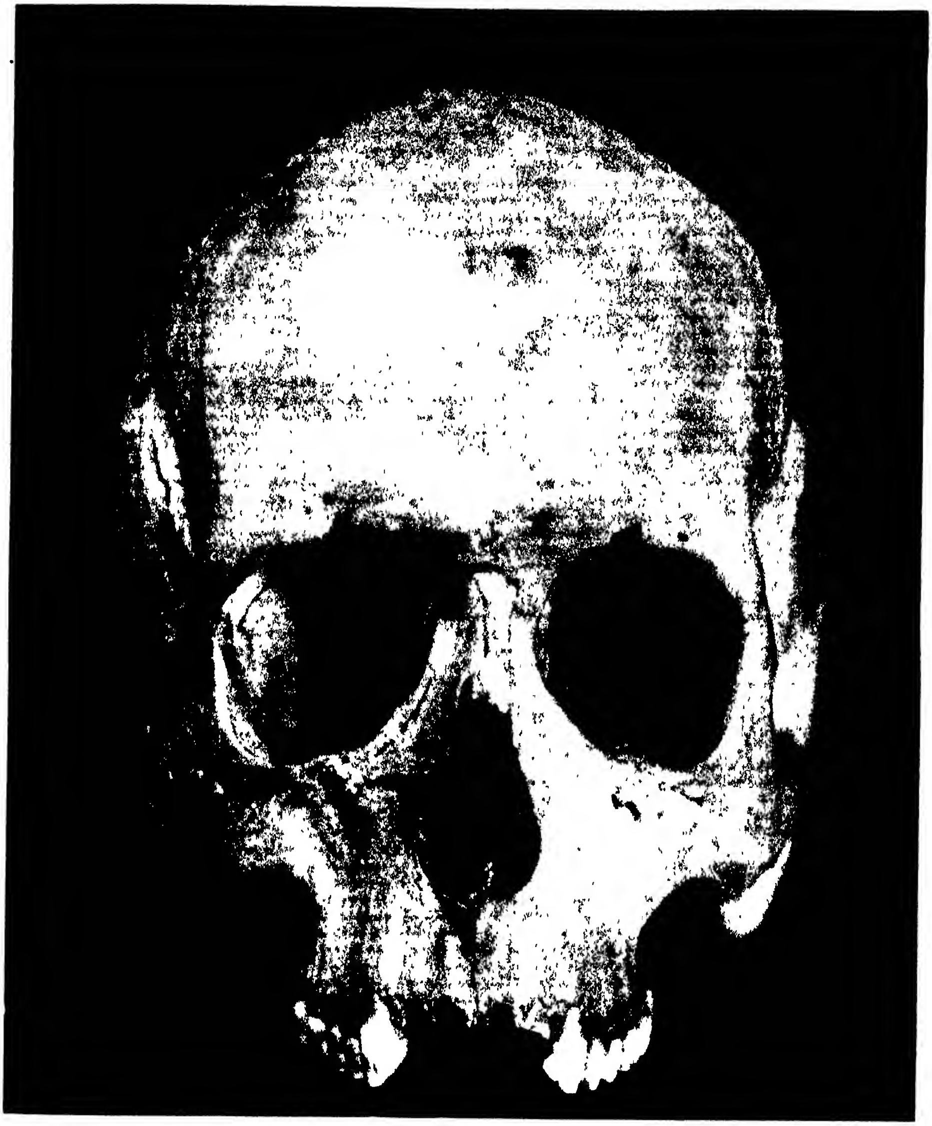
Q' = Frankfurt vertical transverse arc from auricular point R. to auricular point L.* Bregmatic Q' = transverse arc similar to Q' through bregma. Broca's transverse arc = arc terminating at *points sus-auriculaires* R. and L. and passing through bregma. Bi-auricular B = breadth between *points sus-auriculaires* R. and L. S = median sagittal arc from nasion to opisthion. S_1 = arc nasion to bregma. S_2 = arc bregma to lambda. S_3 = arc lambda to opisthion. S_1' = chord nasion to bregma. S_2' = chord bregma to lambda. S_3' = chord lambda to opisthion. U = maximum horizontal circumference above superciliary ridge and through ophryon. Glabella U = maximum horizontal circumference through glabella. fml = basion to opisthion. fmb = maximum breadth of *foramen magnum*. PH = tip of anterior nasal spine to alveolar point. $G'H$ = nasion to alveolar point. GL = basion to alveolar point. GB = chord between lowest points on malar-maxillary sutures R. and L. J = maximum bi-zygomatic breadth. External bi-orbital B = maximum breadth between external surfaces of orbital processes of frontal bone. Bi-jugal B = breadth between Broca's *points jugals* R. and L. NH' = nasal height from nasion to base of anterior nasal spine. NH, R. and L. = Frankfurt nasal height from nasion to lowest point on edge of pyriform aperture R. and L. NB = maximum breadth of pyriform aperture. DC = chord dacryon R. to dacryon L. DA = arc dacryon R. to dacryon L. DS = minimum subtense from bridge of nose to dacryal chord. SC = minimum chord between naso-maxillary sutures. SS = subtense from bridge of nose to simotic chord. O_1, R. and L. = maximum breadth of orbit R. and L. using curvature method (see *Biometrika*, Vol. I. p. 130 and Vol. VIII. pp. 311 and 312). O_1', R. and L. = orbital breadth from dacryon R. and L. O_2, R. and L. = orbital height R. and L. whether taken perpendicular to O_1 or O_1'. G_1 = palate length from tip of posterior nasal spine to median point on an imaginary line tangential to inner alveolar borders of the central incisors. G_1' = palate length from base of posterior nasal spine to same anterior terminal. G_2 = palate breadth between inner alveolar walls at second molars. EH = palate depth from G_2 chord taken with Pearson's uraniscometer. Basio-palatal L = basion to tip of posterior nasal spine.

Various indices are calculated from the above absolute measurements. The Occipital Index (*Oc. I.*), defined to be $100\frac{S_3}{S_3'}\sqrt{\frac{S_3}{24(S_3-S_3')}}$, was found with the aid of Tildesley's table of this function (*Biometrika*, Vol. XIII. pp. 261—262). $P\angle$ is the angle between the line joining nasion to alveolar point—not the prosthion—and the Frankfurt horizontal plane. $N\angle$, $A\angle$ and $B\angle$ are the angles of the triangle of which the nasion, alveolar point and basion are the apices. They were found from the chords $G'H$, GL and LB in the manner described by Fawcett (*Biometrika*, Vol. I. p. 418) with the aid of Pearson's trigonometer. θ_1 is the angle between the line joining basion to nasion and the Frankfurt horizontal, i.e. $180° - P\angle - N\angle$. θ_2 is the angle between the line joining basion to alveolar point and the horizontal, i.e. $P\angle - A\angle$. Daubenton's $\angle$ is, by Broca's definition, the angle between the sagittal axis of the *foramen magnum* (i.e. the chord joining

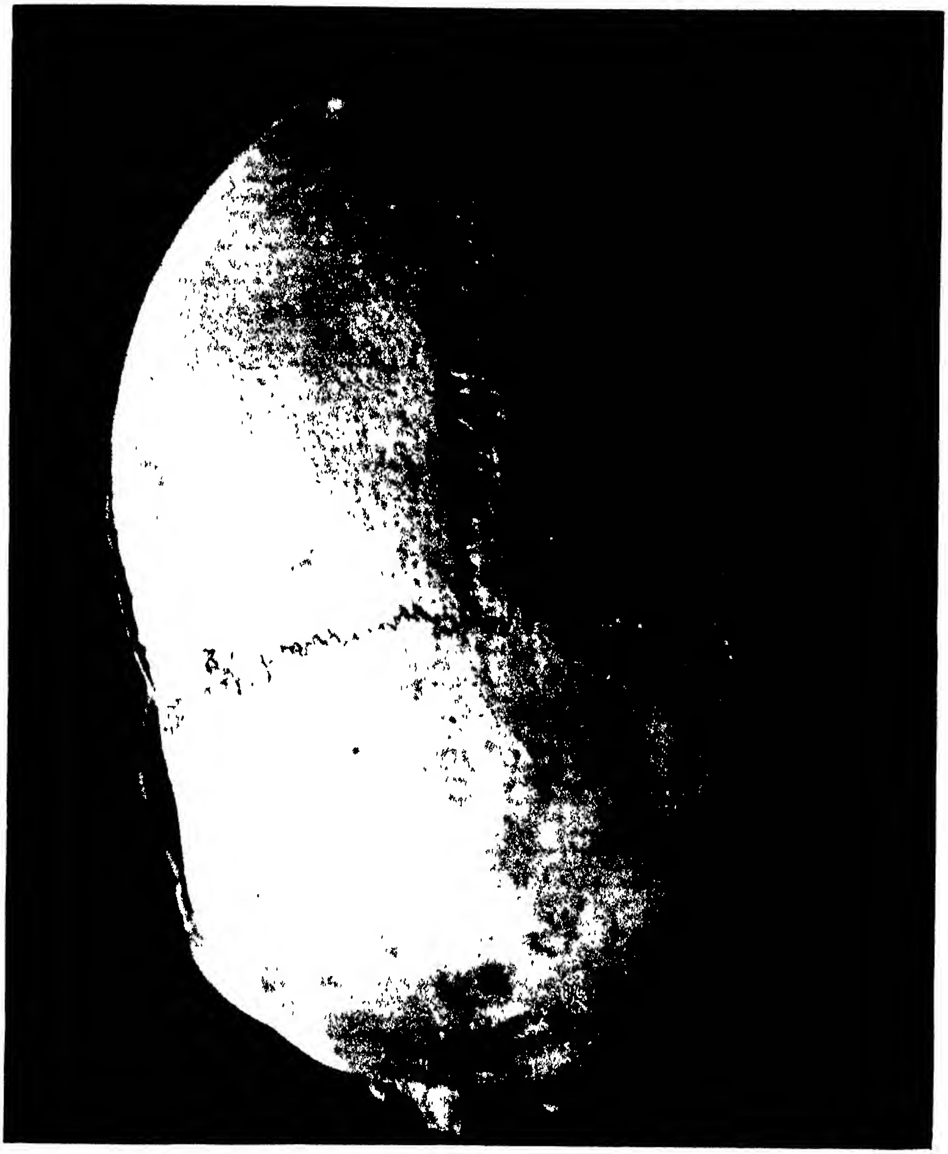
* These auricular points correspond to Martin's "porions."

basion and opisthion) and the line joining the opisthion to the meet of the *ligne sous orbitaire* with the median sagittal plane. It is positive if the basion falls below that line.

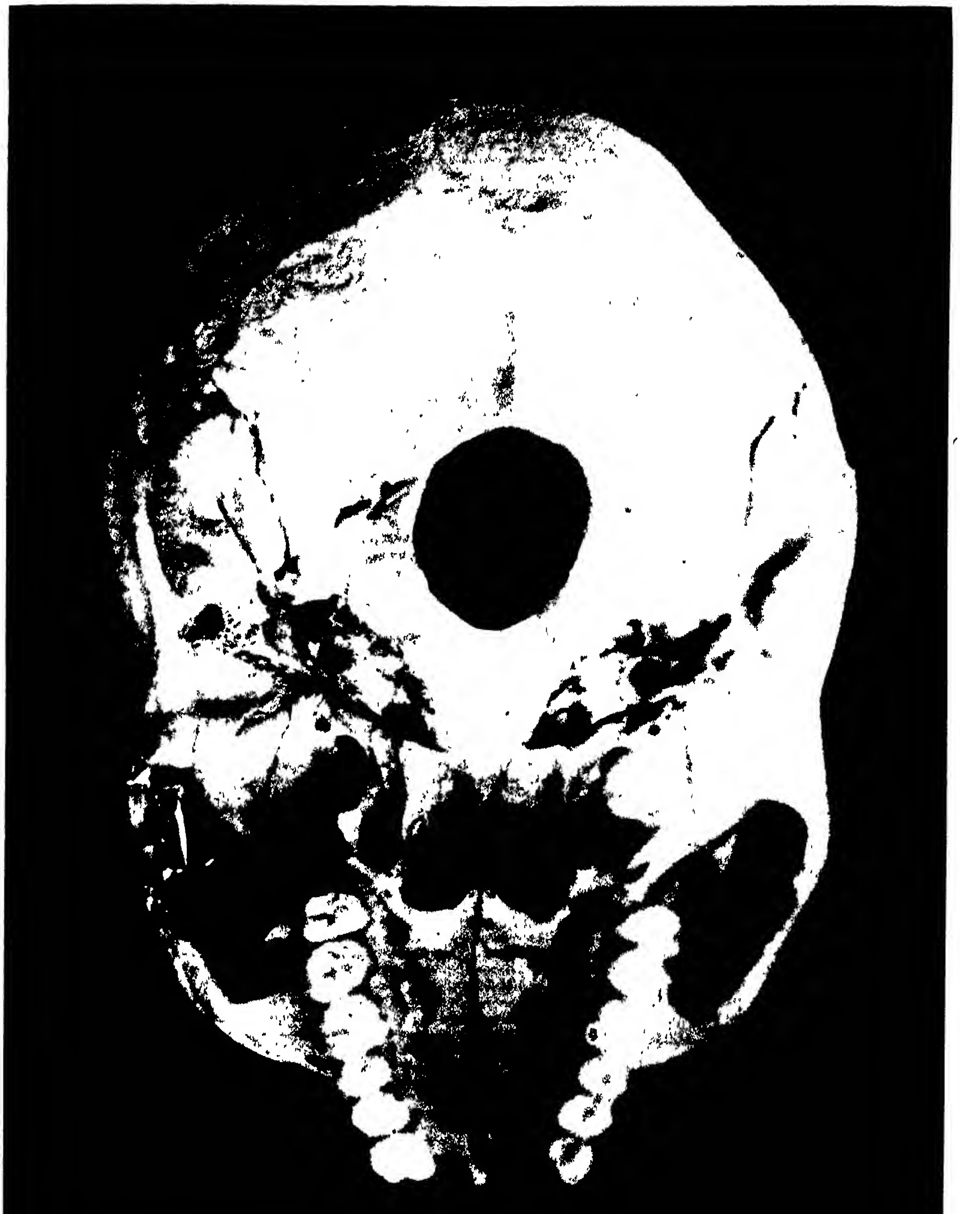
Plates. Plates I—V show five aspects of a normal male Basque skull taken with the focal plane of the camera parallel or perpendicular to the Frankfurt horizontal plane. The specimen is No. 58 of the first (1862) series in the Musée Broca, and its measurements are given in Appendix II. The reduction is approximately to eight-tenths natural size.



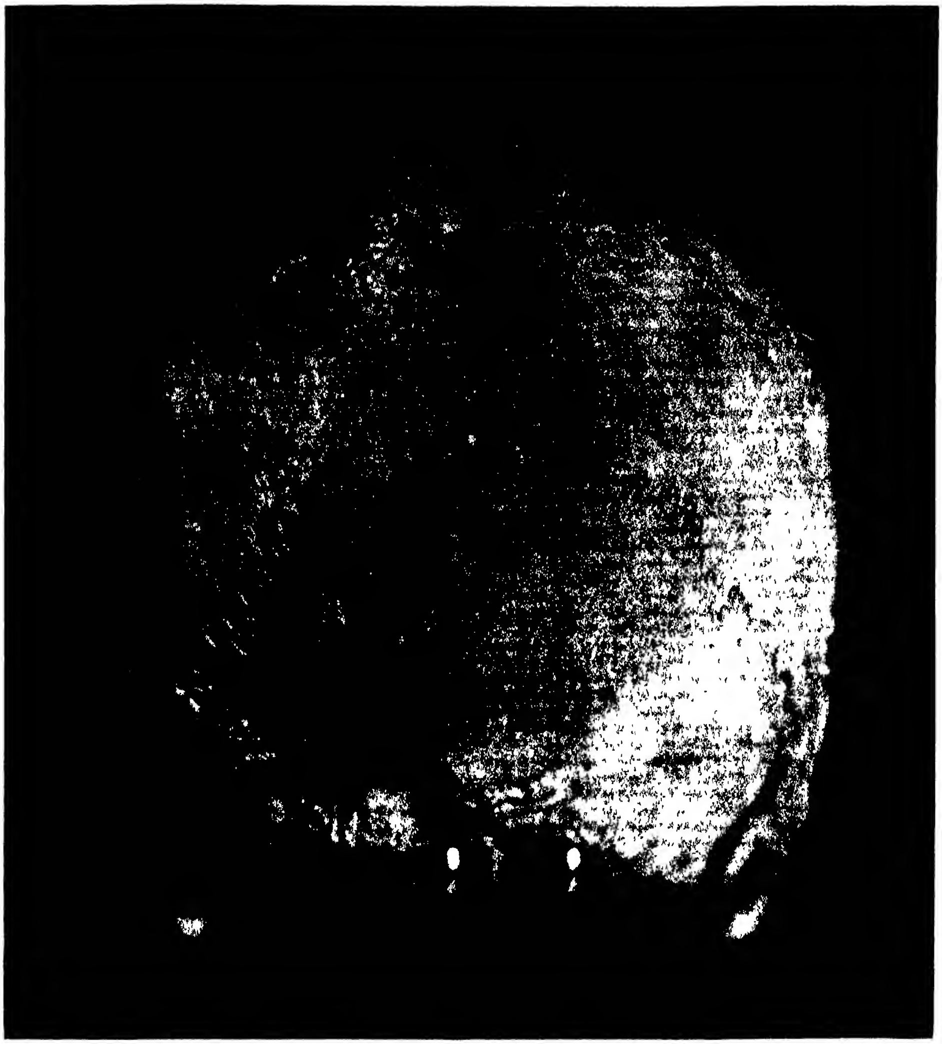
Skull of Adult Male. *Norma facialis.*



Skull of Adult Male. *Norma verticalis.*



Skull of Adult Male. *Norma basalis.*



Skull of Adult Male. *Norma occipitalis.* (The bright elliptical spots below inion belong to craniophor.)

ON MEASUREMENT OF THE INTERNAL DIAMETERS OF THE SKULL IN RELATION:
(I) TO THE PREDICTION OF ITS CAPACITY,
(II) TO THE "PRE-EMINENCE" OF THE LEFT HEMISPHERE.

By M. F. HOADLEY (with some assistance from K. PEARSON).

(I) *On the Prediction of Cranial Capacity from Internal and from External Measurements.*

(i) *Introductory.* It is well known, at any rate to readers of *Biometrika*, that prediction formulae for the capacity of the skull can be based on the measurement of external diameters or external arcs, and that within a definite race they give fairly satisfactory results. But the application of formulae found from the data for one race to the determination of the capacity of a second race is not so satisfactory and caution must be used in drawing inferences from even mean capacities so determined. The need for such formulae is considerable; it may arise: (i) from the condition of a series of crania being such that either from their broken or from their fragile condition it is not possible to obtain the cranial capacity directly; (ii) from a desire to ascertain cranial capacity from measurements on the living. The latter need can of course not be met by any proposal to replace external by internal measurements. The present paper arises from the consideration of whether in the former case we shall much improve our prediction formulae by using internal instead of external measurements. This involves a consideration of the further question of whether the gain is so considerable that it more than compensates for the increased labour of taking the additional measurements.

After a consideration of various suggested instruments for measuring internal lengths of the cranium by insertion through the foramen magnum *, we saw in the *Anthropologischer Anzeiger*, Jahrg. II, S. 129—31, an article by Dr Hans Weinert entitled "Ein neuer Messzirkel zur Ermittelung von Innenmassen," and we concluded that it was easier to experiment with an already constructed instrument than to experiment in constructing one. Accordingly we procured from Messrs Alig and Baumgärtel, of Aschaffenburg, one of Dr Weinert's instruments. No directions for the use of the instrument are provided with it or in the above paper, only in the latter we find a diagram illustrating what is apparently the median sagittal plane of the skull with the instrument set for measuring the length. In this diagram the maximum foraminal length is almost as great

* Including an ingenious model by Sir W. Flinders Petrie.

as the distance from the basion to the tip of the dorsum sellae; with such a magnitude of the foramen it is possible to place the hinge of the measuring circle (Weinert's callipers) entirely within the foramen as in the diagram. With crania with more moderate foramina we have not found this possible, and greater difficulty may then arise in taking the internal measurements; it is less easy to grope for the maximum diameter. It is ungracious to criticise an instrument which has been designed and constructed with considerable thought and care, but if it be feasible to reduce the size of the hinge and of the arms in the immediate neighbourhood of the hinge, we believe it would be a distinct advantage. For taking the internal height of the skull this instrument was not used. The skull in norma basalis was adjusted to the Frankfurt plane and the height (H_i) from basion to inner table then measured by a vertical rod. The maximum internal breadth (B_i) was ascertained by aid of Weinert's callipers. We found it required some experience for the same person to repeat closely the same measurement, but after practise this became easy. Several measurements were taken, and if these were in close agreement the *largest* was adopted as the maximum internal breadth; if the measurements were not in close agreement the measurer started afresh. The greatest care was taken to maintain the line between the measuring points of the callipers horizontal and perpendicular to the median sagittal plane; the skull rested during the measurement with foramen upwards in the standard position. After considerable practise the taking of breadths internally seemed to have reached a satisfactory degree of accuracy, but no craniometrician could, we believe, purchase Weinert's callipers, and straight off, without a large amount of experimenting, hope to obtain exact results even for the internal breadth. The handling of the callipers requires to be patiently learnt.

Now the object of measuring the internal diameters of the brain box is to obtain a system of measurements more nearly representing the cavity itself than can be found when we include (by using external diameters) the thickness of the bone and that of the frontal sinus. Dr Weinert has applied his callipers to determine the internal diameters, in particular the length of the cranium, in numerous specimens of mammals and in particular of the primates including man*, but we cannot find that he has given any very detailed account of how the length is to be taken. On pp. 354—356 we have much the same account of his callipers as in the *Anthropologischer Anzeiger*. The diagrams, p. 344, seem to suggest that he takes it in the sagittal plane. Now it seems to us that the objections to this are twofold, one arising from the purpose to be obtained by the measurement, and the other from the difficulty of manipulating the instrument. The reader who will examine a sectioned skull will find that the median sagittal section is ridged to a greater or less extent both anteriorly and posteriorly; we have the crest for the attachment of the falx cerebri, the crista galli of the ethmoid, and the internal occipital protuberance and the sagittal ridge associated with it. These ridges are difficult to feel with the tip of the callipers, and if one tip has been adjusted to the anterior ridge, it will

* See his memoir "Die Ausbildung der Stirnhöhlen als stammesgeschichtliches Merkmal," *Zeitschrift für Morphologie und Anthropologie*, Bd. xxv. S. 243—357, 365—418.

be almost sure to slip off while the other tip is being adjusted to the posterior ridge; we found it practically impossible to determine satisfactorily, by groping about in the unseen cavity of the skull, the maximum distance between two ridges of this kind. We do not believe that the maximum internal diameter in the sagittal plane can be satisfactorily determined, at least with callipers pointed as in Weinert's instrument; to fix the reading face of the callipers on the ridge a flat surface to the tip is needful. But we hold that if the ridges could be obtained by successful groping and the maximum distance between them ascertained, we should be reducing the true cranial maximum distance by bony excrescences in precisely the same manner as the external length is fictitiously magnified by the thickness of bone and the frontal sinus. We therefore rejected after trial the attempt to obtain an internal diameter in the sagittal plane. A slight examination of the brain box shows that the maximum longitudinal diameters of the cavity lie right and left of the anterior and posterior median ridges; these are relatively easy to ascertain and accordingly we took as our maximum internal length (L_i) the mean of the two internal maximum lengths measured right and left of the median ridges and parallel to the median sagittal plane. The success of this approach to the problem will be appreciated when we say that L_i thus found had the high correlation of ·8193 with the cranial capacity as found by seed in the usual manner of the Biometric Laboratory.

(ii) *Material Selected.* The material chosen for this investigation consisted of 729 adult male skulls of the long 26th—30th Dynasties Egyptian series in the Biometric Laboratory. The necessary external measurements, viz. L = glabellar-occipital length, B = maximum parietal breadth, H = basion to point vertically above it with skull adjusted on craniophor to Frankfurt horizontal, and also C = capacity obtained by packing skull tightly with mustard seed and then weighing, were of course already available. The three internal measurements L_i, B_i and H_i were taken as described in the above introductory note.

Twenty correlation tables were formed between the various measurements. It was then possible to obtain all the other twenty-seven correlations by aid of formulae involving the correlations already found and the known standard deviations. This simplification arises, because we are dealing with various differences in the thickness of the skull found by subtracting an internal from an external measurement. The correlations r_{L,L_i} and $r_{B,B-B_i}$ were found both by forming tables and by formulae, in order to estimate the degree of divergence when such formulae are used instead of forming tables. We have

$$\begin{aligned} r_{L,L_i} &= \cdot 78042 \text{ by table} \\ &= \cdot 78169 \text{ by formula.} \\ r_{B,B-B_i} &= -\cdot 12761 \text{ by table} \\ &= -\cdot 12831 \text{ by formula.} \end{aligned}$$

These differences are due to the grouping in the tables and are of no importance for our present purpose.

The formulae used for calculating these indirectly deduced correlations were the following:

(1) $$r_{x,x_i} = \frac{\sigma^2_x + \sigma^2_{x_i} - \sigma^2_{x-x_i}}{2\sigma_x\sigma_{x_i}}.$$

From this were found r_{L,L_i}, r_{B,B_i} and r_{H,H_i}.

(2) $$r_{x,x-x_i} = \frac{\sigma_x - r_{x,x_i}\cdot\sigma_{x_i}}{\sigma_{x-x_i}}.$$

From this were found $r_{L,L-L_i}$, $r_{L_i,L-L_i}$ and the four corresponding correlations for breadth and height.

(3) $$r_{x,y-y_i} = \frac{\sigma_y r_{x,y} - \sigma_{y_i} r_{x,y_i}}{\sigma_{y-y_i}}.$$

This gave $r_{C,L-L_i}$ and two corresponding ones $r_{C,B-B_i}$, $r_{C,H-H_i}$, and further $r_{L,B-B_i}$, $r_{L_i,B-B_i}$, $r_{L,H-H_i}$, $r_{L_i,H-H_i}$, $r_{x-x_i,y-y_i}$ and the eight corresponding correlations

(4) $$r_{x_i,y} = \frac{\sigma_{x_i}\sigma_{y_i}r_{x_i,y_i} + \sigma_x\sigma_y r_{xy} - \sigma_x\sigma_{y_i}r_{x,y_i} - \sigma_{x-x_i}\sigma_{y-y_i}r_{x-x_i,y-y_i}}{\sigma_{x_i}\cdot\sigma_y},$$

This provides $r_{H_i,B}$, $r_{L_i,H}$ and $r_{L_i,B}$.

Table I gives the means, standard deviations and coefficients of variation. The internal length is considerably more variable than the external length while in the

TABLE I.

	Mean	S.D.	V
C	1440·30 ± 2·84	113·67 ± 2·01	7·89 ± ·14
L	185·53 ± ·14	5·63 ± ·10	3·04 ± ·05
L_i	170·44 ± ·14	5·77 ± ·10	3·39 ± ·06
B	139·16 ± ·11	4·65 ± ·08	3·34 ± ·06
B_i	132·14 ± ·11	4·41 ± ·08	3·33 ± ·06
H	134·31 ± ·13	5·02 ± ·09	3·74 ± ·07
H_i	128·85 ± ·12	4·86 ± ·07	3·77 ± ·07
$L-L_i$	15·10 ± ·09	3·77 ± ·07	24·95 ± ·47
$B-B_i$	7·03 ± ·05	2·15 ± ·04	30·58 ± ·59
$H-H_i$	5·46 ± ·04	1·75 ± ·03	32·23 ± ·63
P	3472·91 ± 5·67	266·98 ± 4·71	7·69 ± ·14
P_i	2906·52 ± 5·82	232·96 ± 4·11	8·02 ± ·14
L_R*	171·04 ± ·15	5·96 ± ·11	3·48 ± ·06
L_L*	170·05 ± ·14	5·69 ± ·10	3·35 ± ·06

case of the breadths and heights the difference in the coefficients is negligible, being well within the probable error. Consequently the internal product is slightly more variable than the external one.

The correlations are given in Table II and are classified in Table III (p. 91). There is no significant difference in the correlations $r_{C,H}$ and r_{C,H_i} but r_{C,L_i} shows a considerable increase on $r_{C,L}$ while r_{C,B_i} is slightly greater than $r_{C,B}$. The total result of these values is to produce a correlation between capacity and internal product

* From second series of measurements. $L_i = \frac{1}{2}(L_R + L_L)$ in this table is from the first series.

TABLE II.

Correlations.

	L	L_i	B	B_i	H	H_i	$L-L_i$	$B-B_i$	$H-H_i$	P	P_i
C	·67246 ±·0137	·81933 ±·0082	·67563 ±·0136	·72414 ±·0119	·54730 ±·0175	·53434 ±·0178	−·25085 ±·0234	−·02296 ±·0250	+·08525 ±·0248	·82081 ±·0082	·89586 ±·0049
L	—	·78042 ±·0098	·41280 ±·0207	·38576 ±·0213	·32110 ±·0224	·26979 ±·0232	·29586 ±·0228	·10195 ±·0247	·17102 ±·0243	—	—
L_i	·78042 ±·0098	—	·42861 ±·0204	·46530 ±·0196	·46709 ±·0195	·45800 ±·0197	−·36448 ±·0217	−·02667 ±·0250	·06730 ±·0249	—	—
B	·41280 ±·0207	·42861 ±·0204	—	·88827 ±·0053	·33392 ±·0222	·28140 ±·0230	−·04011 ±·0249	·34156 ±·0221	·17553 ±·0242	—	—
B_i	·38576 ±·0213	·46530 ±·0196	·88827 ±·0053	—	·27035 ±·0232	·24149 ±·0235	−·13668 ±·0245	−·12761 ±·0246	·10429 ±·0247	—	—
H	·32110 ±·0224	·46709 ±·0195	·33392 ±·0222	·27035 ±·0232	—	·93734 ±·0030	−·23597 ±·0236	·16779 ±·0243	·26368 ±·0232	—	—
H_i	·26979 ±·0232	·45800 ±·0197	·28140 ±·0230	·24149 ±·0235	·93734 ±·0030	—	−·29866 ±·0228	·11324 ±·0247	−·08892 ±·0248	—	—
$L-L_i$	·29586 ±·0228	−·36448 ±·0217	−·04011 ±·0249	−·13668 ±·0245	−·23597 ±·0236	−·29866 ±·0228	—	·19306 ±·0241	·15225 ±·0244	—	—
$B-B_i$	·10195 ±·0247	−·02667 ±·0250	·34156 ±·0221	−·12761 ±·0246	·16779 ±·0243	·11324 ±·0247	·19306 ±·0241	—	·16618 ±·0243	—	—
$H-H_i$	·17102 ±·0243	·06730 ±·0249	·17553 ±·0242	·10429 ±·0247	·26368 ±·0232	−·08892 ±·0248	·15225 ±·0244	·16618 ±·0243	—	—	—

of ·89586 as compared with ·82081 for external product and capacity. The correlations r_{L,L_i}, r_{B,B_i}, r_{H,H_i} are all high, H and H_i being the most closely correlated and L and L_i the least.

It must be remembered that whereas $H-H_i$ is a measure of the thickness of the bone at the top of the skull, $B-B_i$ and $L-L_i$ are respectively twice the thickness of the parietal bone and the thickness of the frontal plus the occipital bones. The mean thickness on the parietal bone where the skull has greatest horizontal breadth is thus 3·52 approximately.

On the few divided male crania in the Laboratory these values for the bone thickness were by no means unreasonable, especially considering the light build of

the Egyptian skull. Of the 15 mm. thickness of bone in the cranial length about $\frac{3}{5}$ may be taken to be frontal and $\frac{2}{5}$ occipital.

The correlations $r_{L, L-L_i}$, $r_{B, B-B_i}$, $r_{H, H-H_i}$ are all positive and lie between ·25 and ·35 indicating that the thickness increases significantly but not rapidly with increase of the external measurement. On the other hand, it is interesting to note that $r_{L_i, L-L_i}$, $r_{B_i, B-B_i}$, $r_{H_i, H-H_i}$ are all negative, but the only one of importance is the first which is − ·36.

We conclude from the first of these results that a large skull has on the average a large thickness of bone, and that a large brain cavity has on the average a small bone thickness. It will be seen that these conclusions are what we should expect as an outcome of "spurious correlation," if the internal and external measurements were really uncorrelated.

Next turning to the three thicknesses $L - L_i$, $B - B_i$ and $H - H_i$ we see that while only very moderately correlated with each other, the correlations are all positive, or a thick brain case tends to be thick in all directions. Thickness of bone in the length is more highly correlated with that of the breadth, i.e. on the parietals, than with that of the height, i.e. at the apex of the skull; and the thickness at the apex is slightly but not significantly more correlated with that on the parietals than with the sum of the thicknesses on the frontal and occipital bones. As a matter of fact none of these bone thickness correlations are really significantly different, and it would suffice to say that the thicknesses of the skull in those places tested have with one another the very moderate correlation of about ·17. From this it is clear that there is a great deal of independence about the thickness of cranial bone at different points. Turning to the influence of bone thickness on cranial capacity, we see that the thickness on the parietals has no sensible influence on the capacity; that at the apex has a very slight *positive* effect, or if a skull has a thick crown the capacity might be expected to be very minutely *larger*.

It will be seen that the highest correlations are between corresponding external and internal lengths, these correlations being in the descending order heights, breadths, lengths*. The correlations of the length, breadth and height thicknesses with the external diameters are on the whole so small that there is little hope of obtaining any close estimate of the thicknesses from these external measurements. To this point we shall return later. As a rule an external measurement is more highly correlated with another external measurement than with the corresponding internal measurement, but the external breadth and height are more closely associated with the internal than with the external length; this is markedly the case for the external height. In the case of the internal measurements they also are not always more highly correlated with each other than with the corresponding external measurements. Thus L_i is more highly correlated with B_i than with B, but is slightly more correlated with H than B_i. B_i again is more highly correlated with L_i than with L, but slightly more correlated with H than H_i. Finally H_i is far more highly correlated with L_i than with L, but somewhat less correlated with B_i than B.

* This order is precisely that of the thickness of bone in these directions.

TABLE III.

Classified Table of Correlations.

−0·40 to −0·30	*−0·30 to −0·10*	*−0·10 to +0·10*	*+0·10 to +0·20*	*+0·20 to +0·40*	*+0·40 to +0·60*	*+·0·60 to +0·80*	*+0·80 to +1·00*
−0·40 to −0·35 Int. Length and Diff. of Ext. and Int. Lengths	−0·30 to −0·25 Int. Height and Diff. of Ext. and Int. Lengths Capacity and Diff. of Ext. and Int. Lengths −0·25 to −0·20 Ext. Height and Diff. of Ext. and Int. Lengths −0·20 to −0·15 — −0·15 to −0·10 Int. Breadth with Diff. of Ext. and Int. Lengths Int. Breadth with Diff. of Ext. and Int. Breadths	−0·10 to −0·05 Int. Height with Diff. of Ext. and Int. Heights −0·05 to 0·00 Ext. Breadth with Diff. of Ext. and Int. Lengths Int. Length with Diff. of Ext. and Int. Breadths Capacity with Diff. of Ext. and Int. Breadths 0·00 to +0·05 — +0·05 to +0·10 Int. Length and Diff. of Ext. and Int. Heights Capacity and Diff. of Ext. and Int. Heights	+0·10 to +0·15 Ext. Length and Diff. of Ext. and Int. Breadths Int. Breadth and Diff. of Ext. and Int. Heights Int. Height and Diff. of Ext. and Int. Breadths +0·15 to +0·20 Diff. of Ext. and Int. Lengths and Diff. of Ext. and Int. Heights Diff. of Ext. and Int. Breadths and Diff. of Ext. and Int. Heights Ext. Height and Diff. of Ext. and Int. Breadths Ext. Breadth and Diff. of Ext. and Int. Heights Ext. Length and Diff. of Ext. and Int. Heights Diff. of Ext. and Int. Lengths and Diff. of Ext. and Int. Breadths	+0·20 to +0·25 Int. Breadth and Int. Height +0·25 to +0·30 Ext. Height and Diff. of Ext. and Int. Heights Ext. Length and Int. Height Ext. Height and Int. Breadth Ext. Breadth and Int. Height Ext. Length and Diff. of Ext. and Int. Lengths +0·30 to +0·35 Ext.. Length and Ext. Height Ext. Breadth and Ext. Height Ext. Breadth and Diff. of Ext. and Int. Breadths +0·35 to +0·40 Ext. Length and Int. Breadth	+0·40 to +0·45 Ext. Length and Ext. Breadth Int. Length and Ext. Breadth +0·45 to +0·50 Int. Height and Int. Length Ext. Height and Int. Length Int. Length and Int. Breadth +0·50 to +0·55 Capacity and Int. Height Capacity and Ext. Height +0·55 to +0·60 —	+0·60 to +0·65 — +0·65 to +0·70 Capacity and Ext. Length Capacity and Ext. Breadth +0·70 to +0·75 Capacity and Int. Breadth +0·75 to +0·80 Ext. Length and Int. Length	+0·80 to +0·85 Capacity and Int. Length Capacity and Ext. Product +0·85 to +0·90 Ext. Breadth and Int. Breadth Capacity and Int. Product +0·90 to +0·95 Ext. Height and Int. Height +0·95 to +1·00 —

The first line of Table II indicates the advantages to be obtained by using internal rather than external measurements. We note that the correlations of the internal and external heights with capacity are nearly equal; what advantage there is being in favour of the external height. But the internal breadth with capacity exceeds by 7·2 °/₀ the external breadth with capacity correlation, while that of capacity with internal length is 21·8 °/₀ better than that with external length. It thus appears clear that the internal measurements, difficult as they are to make, will give in the case of the length considerably and in case of the breadth slightly better results than the corresponding external measurements. Accordingly the continuous products P for the external and P_i for the internal diameters were formed and the regression equations of the capacity C on these determined, with the following results for probable capacity $\tilde{C}$:

$$\tilde{C} = \cdot 0003495\, P + 226\cdot 52 \pm 44/\sqrt{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a),$$

$$\tilde{C} = \cdot 0004372\, P_i + 169\cdot 57 \pm 34/\sqrt{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (b).$$

It will be seen that the probable error of the estimate is reduced from $44/\sqrt{n}$ to $34/\sqrt{n}$ or 22·7 °/₀ by using internal instead of external diameters. The correlation between capacity and continuous product was raised from ·82081 to ·89586, the chief factor here being undoubtedly the change from L to L_i.

Now this increased accuracy of prediction is of great importance, but it involves serious consequences, for undoubtedly the determination of the internal measurements is a difficult task and not to be lightly undertaken. Accordingly attempts were made to determine L_i, B_i and H_i from L, B and H, and then to use (b).

The regression formulae are:

$$\left.\begin{aligned} L_i &= \cdot 69527\, L + \cdot 09155\, B + \cdot 25867\, H - 6\cdot 04011 \pm \frac{2\cdot 25113}{\sqrt{n}} \\ B_i &= \cdot 02423\, L + \cdot 84103\, B - \cdot 03156\, H + 14\cdot 81886 \pm \frac{1\cdot 35994}{\sqrt{n}} \\ H_i &= -\cdot 02217\, L - \cdot 02797\, B + \cdot 92439\, H + 12\cdot 70355 \pm \frac{1\cdot 13385}{\sqrt{n}} \end{aligned}\right\} \quad ..(c).$$

A first sample of 20 skulls was taken, but not at random, being distributed with rough uniformity over the whole range of capacity, rather than at random over the distribution of frequency. The results were as follows:

Table of ·67449 × *Square Root Mean Square Residual.*
1*st Sample*, 20 *crania.*

(i) Internal Product from (b)	39·69 cm.³, expected 34 cm.³
(ii) Internal Diameters obtained by subtracting mean thicknesses from external diameters and using (b)	42·15 „
(iii) L_i found from (c), and B_i and H_i by subtracting mean thicknesses	45·75 „
(iv) External Product from (a)	48·33 „ expected 44 cm.³
(v) L_i, B_i, H_i all found from (c) and then (b) used	52·02 „

(iii) was undertaken with a view of reducing the labour of (v), as H_i is not better than H, and B_i is only slightly better than B. The result is somewhat anomalous. After actual internal measurement of the three diameters, we obtain the closest estimate by merely subtracting the average thicknesses from the corresponding external measurements. The next best result is when we obtain L_i only from the regression formulae (c), and B_i and H_i by subtracting mean thicknesses. The worst result—worse than using the external diametral product with (a)—arises from finding L_i, B_i and H_i from the regression formulae (c). This is of course paradoxical, but the source of the paradox is to be sought, we think, in the sample, which had emphasised too much the skulls with very small and very large capacities, and it is usually towards the tails that the linear regression equations prove less satisfactory. It will be noted that the values obtained by both (a) and (b) are considerably in excess of the expected.

It seemed advisable accordingly to select a second sample, this time of 24 skulls, choosing them at random from the frequency, actually by taking out every thirtieth card from the pack of data cards.

Table of ·67449 × *Square Root Mean Square Residual.*
2nd Sample, 24 *crania.*

(i)	From Internal Diametral Product	33·27 cm.^{3}, expected 34 cm.^{3}
(ii)	From values of L_i, B_i, H_i from (c)	42·62 cm.^{3}
(iii)	From External Diametral Product	43·32 cm.^{3}, expected 44 cm.^{3}
(iv)	By subtracting mean thicknesses from external diameters	44·97 cm.^{3}
(v)	By subtracting mean thicknesses to obtain B_i and H_i and determining L_i from L and H*	45·84 cm.^{3}
(vi)	By subtracting mean thicknesses to obtain B_i and H_i and determining L_i from L, B and H	45·93 cm.^{3}

The expected result from external measurements is 44 cm.^{3}; the sample gives 43. The best result obtained by endeavouring to estimate the internal from the external diameters is provided by (ii), practically 43 also. All the other values are in excess of 44, and none in any way approaches the internal diametral product value of 33. Accordingly we do not seem able to predict with any serviceable degree of accuracy L_i, B_i and H_i from L, B and H with a view to using (b).

The general conclusions of this section of our paper are: (i) that internal measurements have an accuracy about 23 °/$_o$ greater than external measurements; (ii) that the thicknesses of the bone are so slightly correlated with the external measurements that it is not possible to obtain from the latter good approximations to their values; and (iii) that we must either be contented with the degree of accuracy provided by the external diametral product, or if we wish to improve on that accuracy we must practise the rather difficult technique of internal measurement.

* The requisite regression formula is $L_i = \cdot722492\,L + \cdot277208\,H - \cdot84808$.

If the process of internal measurement has only to be applied occasionally to isolated skulls, we are inclined to doubt whether the technique is worth learning and facility in it maintained for the case of such increased accuracy, and this conclusion may be emphasised by the fact that such isolated skulls may not belong to the same race or even to a race allied to the race (or possibly one or two races) for which the prediction formula for internal measurements may have been ascertained.

(II) *On the Pre-eminence of one or other Cerebral Hemisphere.*

(i) *Introductory.* Aretaeus, a Greek physician of the first century of our era, was probably the first observer who has recorded the decussation of the pyramidal tracts, those from the left side passing to the right hemisphere and those from the right side to the left hemisphere*. But he did not apparently note that injuries to the left side of the brain were accompanied not only by paralysis of the right side, but often by loss of speech. Though there may have been isolated cases previously noted, Dax in 1865 appears to have been the first who emphasised the view that injury to the left side of the brain was often accompanied by aphasia. The association of aphasia with paralysis of the right side as a result of injury to the left side of the brain has accordingly been spoken of as Dax's Law†. On the basis of this law the faculty of speech was associated with the left hemisphere and taken in conjunction with right-handedness it became customary to speak of a "pre-eminence" of the left hemisphere. There has, as far as we are aware, been no exact definition of this "pre-eminence" as to whether it is to be sought for in sensation, perception, volition, etc., but it was supposed that if it existed it would certainly manifest itself in physical characters, which could be measured or appreciated. And search was made for somewhat gross physical differences between the right and left sides of the brain, in particular between the right and left cerebral hemispheres, at first in weight or volume and later in the depth of furrows and complexity of folds. The investigation was rendered exceedingly difficult owing to the task of dividing the hemispheres from one another, to the fact that some of the brains may have belonged to sinistral not dextral individuals, and that even if the ante-mortem laterality of the individual had been ascertained the factor of lateral educability could be called

* In his Book v, Chapter vii "On Palsy," he writes:

"Should any part below the head begin to be affected such as the membrane enclosing the spiral marrow, then parts which are synonymous and connected suffer from the resolution, viz. those on the right from an affection of the right side.... But if the head is first affected on the right side, the nerves on the left suffer, and again the nerves on the right from a resolution taking place on the left, which is owing to a change in the course of the nerves, for those that begin from the right do not run in a straight line on the same side to their extremities, but immediately after their origin or rise pass to the opposite side, interchanging one with another like the letter X." *Aretaeus, consisting of eight Books on the Causes, Symptoms and Cure of acute and chronic Diseases; translated from the original Greek.* By John Moffat, M.D., London.

† "Lésions de la moitié gauche de l'encéphale coïncidant avec l'oubli des signes de la Pensée," *Gazette hebdomadaire de médecine et de chirurgie*, T. II. Série 2, pp. 259—262, Paris, 1865. Dax collected 371 observations, and in 87 cases a lesion of the left hemisphere coincided with a lesion of the faculty of speech; 53 cases of lesion of the right hemisphere were accompanied by conservation of this faculty. Six cases appeared to contradict the law and 225 cases provided no information one way or the other.

into play to account for apparently exceptional cases. Added to all this there is the question of errors of observation, and in none of the investigations we have come across, although the errors of measurement were admitted to be large, was there any attempt to distinguish them from the errors of random sampling. Various writers decided not to pay attention to differences in weight, for example, under 3 grs. or under 10 grs., as they imagined the order of their errors of measurement to be lower or higher. Considering the amount of fluid in the brain, and how it may drain off when the hemispheres are separated, it is quite conceivable that if an anatomist had the habit of measuring the left hemisphere before the right, he would find the former the heavier*. The difficulties of hemisphere weighing are so great that it is little wonder that the "pre-eminence of the left hemisphere," if it be associated with greater weight, has remained unproven.

The principle of the "pre-eminence" of the left hemisphere was carried far in England by Boyd and Ogle, and in France by Broca, and seems to have become almost dogmatic until quite recent times†.

R. Boyd published in the *Phil. Trans.* a long series of brain weights in 1861‡. Unfortunately he did not provide the individual values, but only the means in certain age groups. His Table II (pp. 254—262) contains the results of measuring the weights of 295 male and 233 female patients of the Somerset County Lunatic Asylum. In the case of 290 of the former and 229 of the latter the weights of the right and left cerebal hemispheres are given separately. Unfortunately no details are provided of the methods of measurement nor are we told how the hemispheres were divided.

The results in ozs. are as follows:

Mean Weights of Hemispheres.

Ages in years	Men			Women		
	No.	Right	Left	No.	Right	Left
Under *30*	44	20·89	21·05	30	19·21	19·51
30—40	61	19·82	19·94	46	18·63	18·84
40—50	76	19·49	19·67	48	18·05	18·24
50—60	42	20·44	20·73	39	18·66	18·75
60—70	39	20·66	20·86	41	18·37	18·53
70—80	20	20·25	20·47	20	17·97	18·09
Over *80*	8	18·97	18·62	5	17·20	17·39

* 100 grs. of fluid may pass out of an extracted brain and it can lose 5 grs. by evaporation per two hours.

† See for example Byron Bramwell in the *Lancet*, 1899, Vol. I. pp. 1473—9. "In perfectly healthy right-handed persons who do not inherit a tendency to left-handedness, the driving or leading speech centres are (with perhaps rare exceptions, but I know of no recorded cases which definitely prove this) situated in the left hemisphere of the brain; and vice versa in left-handed persons the leading or driving speech centres are so far as we know usually but probably less constantly situated in the right hemisphere."

‡ Vol. CLI. pp. 241—262.

In all fourteen classes, except that of men over 80 years of age, the mean weight of the left hemisphere exceeds that of the right. If we exclude individuals over 80 years of age, we have

	Mean for Men			Mean for Women		
	No.	Right	Left	No.	Right	Left
ozs. ...	282	20·137	20·322	224	18·482	18·662
grs. ...		570·55	575·79		523·66	528·76

These numbers might be considered by some as fairly conclusive, although no probable errors are given. But when we find directly opposed results given by later writers, we wish that some details of his procedure had been provided by Boyd. Could he possibly have measured his left hemisphere usually before the right?

Wagner, in a work* published in the year following Boyd's, found in the case, however, of only 18 brains: Right hemisphere 427 grs., Left hemisphere 426 grs., the latter having a slightly less mean weight; but to judge by his average weights his method must have been wholly different from Boyd's. Thurnam in 1866 measured the brains of 257 males and 213 females and in all but two cases the cerebral hemispheres apart†. He found for the mean male right hemisphere 570·63 grs.—a value extraordinarily close to Boyd's 570·55 grs.—but his value for the male left hemisphere was only 569·78 grs., slightly less than that for the right, but really insignificantly different; the excess of right, however, was present in six out of eight age groups. For the females the mean right hemisphere weighed 511·13 grs. and the left again less—510·85 grs. In the age groups two means were equal, three in excess for the right and three in excess for the left. Thus Thurnam's results by no means confirmed Boyd's, but seemed to indicate if anything equality in the hemispheres as far as weight was concerned.

Broca appears very early to have cast in his lot with the supporters of the pre-eminence of the left hemisphere. Broca's actual measurements have we believe never been published and we owe the account of them to his pupil Topinard‡. Broca is said to have weighed 264 male and 139 female brains, and to have found the mean of the *right* hemisphere greater than that of the left by 1·93 grs. in men and 0·03 gr. in women. Thus Broca's results seem to accord better with those of Thurnam than with those of Boyd. The fact which Broca states—that on 19 out of 20 occasions the lesions which produce aphasia are on the left—led him to insist that the left side of the brain is that which works by preference. Broca added that manual dextrality in virtue of

l'entre-croisement des faisceaux médullaires dans la moelle allongée est une autre preuve du fonctionnement plus ordinaire du cerveau gauche.

* *Vorstudien des menschlichen Gehirns*, 1862, Bd. II. §§89—92.

† *Journal of Mental Science*, April, 1866.

‡ *Éléments d'Anthropologie générale*, Paris, 1885, p. 581.

Not daunted by the failure to show the left hemisphere the heavier, Broca proceeded to the harder task of separating the lobes of the hemispheres and weighing each separately. He divided into frontal, temporo-parietal and occipital lobes, and found the following results for mean values:

Weights of	*Men* (258)	*Women* (135)
Right – Left Frontal Lobes	– 2·50 grs.	– 1·50 grs.
" " Temporo-parietal Lobes	+ 1·92 grs.	+ ·80 gr.
" " Occipital Lobes	+ 1·57 grs.	+ ·03 gr.

Thus according to Broca the male frontal lobe is heavier on the *left* by 2·50 grs. and the remainder of the hemisphere heavier on the right by 3·49 grs. We cannot test the significance of these results, because no probable errors can be found without the individual measurements. Topinard says that Broca's register indicates that the weights of the frontal lobes were equal in 12 cases, the left in excess in 136 and the right in 94, the total of which, 242, does not accord with the 258 male cases in which we are told the frontal lobe was weighed. Topinard (p. 584) tells us that with these extracts from Broca's register he has confirmed the results drawn from the two series, one from Bicêtre consisting of 19 subjects and the other from Saint-Antoine of 18 subjects actually published by Broca, for therein Broca found the right hemisphere somewhat heavier than the left as a whole, but the left frontal lobe in excess by "une quantité très notable" Topinard gives no figures and the numbers were far from adequate to base any sound conclusions on*. Topinard, however,

* The data appear in the *Bulletins de la Société d'Anthropologie*, 2 série, 1875, pp. 534—6, in a paper under Broca's name entitled: "Sur les poids relatifs des deux hémisphères cérébraux et de leurs lobes frontaux." The following results are given:

	Hemisphere R.	Hemisphere L.	Frontal Lobe R.	Frontal Lobe L.
Hospice de Bicêtre (mean age 62 years 6 months) 19 males.	531·31	530·84	227·57	232·10
Hôpital Saint-Antoine (mean age 50 years 1 month) 18 males:	575·83	574·89	245·05	248 50

In this paper Broca says that since 1861, when he recognised that the faculty of language is localised in the third frontal convolution of the left hemisphere, he had weighed separately the two hemispheres and their principal components in all the autopsies he had made at the hospitals. He had made, he said, 440 detailed observations, which filled three great registers and awaited reduction. Meanwhile, he gives an abstract of the cases (37!) cited above. In the discussion Broca was asked if he thought the greater weight of the left frontal lobe was due to the part played by the third convolution. He replied that the frontal lobe contained more than this convolution, but that the third convolution was situated at the level and behind the small cranial region, which is called the pterion; he considered that the size of the pterion depends in part on the volume of the third convolution, and he asserted that the mean value of the left pterion is a little larger than the right. Broca gave no figures, and the problem would be an interesting one if a satisfactory measure of the pterion could be derived. Pressed further as to whether he thought the influence of the third convolution could affect the development of the frontal lobe, Broca replied that he was "tout disposé à croire à cette influence" (p. 536). Bertillon remarked that there were probably cerebral dextralists and sinistralists as there were manual sinistralists and dextralists.

Dr Morant kindly measured for us the arc from the krotaphion to the sphenion on the left and right sides of 65 Egyptian male crania of the 26th to 30th Dynasties. This arc seems a reasonable measure of the size of the pterion. The right arc was greater in 28 cases, the left in 35 and there was equality in 2 cases. The mean value of the right arc was 11·677 ± ·3741, and of the left arc 12·195 ± ·3699. The difference of the means, left minus right, was ·518 ± ·519, or the difference was, on the number of skulls measured, insignificant. On these crania accordingly it was not possible to confirm Broca's opinion that the pterionic area was greater on the left side.

clearly holds that excess of weight is a measure of excess of functioning, and believes that the brain is left-handed for certain functions and right-handed for others, a point of view adopted later by Riese as we shall note below.

It will be seen that Broca did not base a pre-eminence of the left hemisphere on greater general weight. Topinard would apparently account for the discrepancy between Boyd and Thurnam on the ground of the latter having measured the brains of the insane; this in fact Broca was also doing in his 1875 paper, and it must be borne in mind when considering later Braune's data. Meanwhile mention must be made of two English investigators, Bastian and Ogle. The characteristic of the first is cautious statement accompanied by hesitation to push undemonstrated theories to extremes; the tendency of the latter was to seize an hypothesis which, if true, would much simplify our ideas and press it forward before its foundations were well established. It is not our purpose to deal with the whole of the wide literature discussing the "pre-eminence of the left hemisphere," still less with that of the still more extensive topic of the localisation of function. But as some of the doubts expressed by Bastian are again coming to the fore, it may be well to recall them to the reader. Bastian wrote two years before Ogle's paper*: "I am therefore strongly inclined still to believe in the similarity of function and practical equality of education of the two cerebral hemispheres, notwithstanding all that has been said of late in opposition to this doctrine," and later on in the same paper he continues:

> In short, if anything like localisation of function is possible in the cerebral hemispheres, then I believe it would occur, and could be accounted for, rather in this way: that insomuch as we have certain distinct avenues of knowledge (through the Sense Organs and their proximate nerve ganglia), and that the cerebral hemispheres are the parts concerned in the elaboration of impressions so derived†, we can well understand that the impressions entering through one gate or sense-avenue, may pass through the substance and towards the periphery of these Cerebral hemispheres in certain definite directions, and according to accustomed routes. Then, the impressions entering through another gate of knowledge, or avenue of sense, may, and probably do, pursue a different direction through its substance, so that at the periphery the fibres and cells concerned in the condition and elaboration of these impressions may exist in maximum quantity in different portions of the surface of the hemispheres—though in part they may occupy jointly the same area, and be intertwined with the fibres and cells concerned in the elaboration of the previously mentioned set of impressions. And so on with the various sense organs and their ultimate expansions in the form of what I would call "Perceptive centres" in the cerebral hemispheres. Thus, though there may be much overlapping of areas, and though the area pertaining to the impressions of any particular sense in the cerebral hemispheres may be a very extended one (not to speak of the still further complication brought about by the communication established between the nerve cells of one sense area with those of others in the same hemisphere, and of the probable union by means of commissural fibres between analogous parts of the two hemispheres), still it may well be that certain portions of the surface of the cerebral hemispheres might correspond more especially to the maximum amount of nerve cells and fibres pertaining to some one or other of the various senses. I should expect, therefore, that the parts concerned in the production of the emotional feelings related to any particular sense or senses, as well as in the

* *Journal of Mental Science*, January, 1869, "Note on the Localisation of Function in the 'Cerebral Hemispheres.'"

† "Converting them in fact into what I call Perceptions—using this term in its ordinary psychological acceptation." (Footnote by Bastian.)

production of volitional stimuli to which these might give rise, would be those parts of the Convolutional grey matter that represented, as it were, the Perceptive Centres of the senses in question.

In his lectures of six years later* Bastian takes a somewhat modified view, but is still unprepared to give absolute predominance to the left hemisphere. Here he admits that in the great majority of cases in which a right-sided paralysis accompanied by aphasia presents itself it is produced by a lesion of the left hemisphere. "In fact a long series of observations has compelled us to recognize the greatly superior activity of the left hemisphere, as compared with the right, in initiating motor acts subservient to intellectual expression. Just as the left hemisphere has undoubtedly to initiate the muscular acts by which writing is effected in right-handed individuals so it would appear that from this same half of the brain the incitations habitually pass over which are destined to excite the motor acts of speech—even though the muscles concerned are bilaterally disposed, and always act in concert on two sides of the larynx, fauces, tongue and lips" (pp. 205—6).

Bastian suggests that this initiatory action of the left hemisphere in relation to speech-movements may be connected with a slight precedence in its development, and this itself be a more or less remote consequence of an inherited tendency to right-handedness (p. 206). He considers that the situation in the left hemisphere affected in aphasic individuals may be (i) in or around the 3rd frontal convolution, (ii) in the white substance between this convolution and the left corpus striatum, or (iii) in the latter itself. On the whole, he prefers the third or Broca's convolution† as the source of the volitional stimuli which incite motor acts of speech; it is a portion of the brain having intimate functional relations with many other parts of the hemisphere.

Bastian then turns to certain difficulties attached to a theory of absolute dominance of one hemisphere; he recalls the fact that there seem to be definite cases where aphasia occurring with left-handed persons goes with left rather than right-sided hemiplegia. Further, he considers that while the left hemisphere is more especially concerned in the performance of voluntary motor acts of speech it would appear that there are also certain peculiarities of the function pertaining more especially to the right hemisphere. Lesions on that side are more frequently and rapidly fatal than those of the left and hemiplegic symptoms more severe and lasting. Lesions of the right hemisphere lead more frequently than those of the opposite side to disorders of nutrition (p. 210), and further, hysterical paralysis occurs more frequently in left than right limbs‡. Another noteworthy point is that

* H. Charlton Bastian: *On Paralysis from Brain Disease in its Common Forms*, London, 1875. See especially Lecture IV.

† The inferior frontal convolution on the left has been given Broca's name.

‡ Brown-Séquard recorded left side paralysed in 97 as against right side in only 24 instances. N. Saveliew ("Gehirnembolie," *Virchows Archiv*, Bd. 135, p. 121) states that the *left* hemisphere is the chosen locality for embolisms. But this does not follow from his own data of 104 cases in which 29 were on right side, 39 bilateral, and 36 left sided. What we have to compare are 29 R. and 36 L. occurring in 65 cases; the mean for indifference would be 32·5, and the difference 3·5 does not seem significant

lesions of the right hemisphere are more apt to give rise to paralysis or convulsions on the same side of limbs or face than in the case of lesions of the left hemisphere, and such cases Bastian holds cannot all be due to error of record though perhaps some are. "The occurrence of paralysis or convulsion on the same side as the brain lesion is with our present state of knowledge quite inexplicable; still more mysterious when we find one hemisphere apparently more apt than the other to produce such an anomaly" (p. 212).

If Bastian's views tend, as we think, to a differentiation of function rather than a dominance of the left hemisphere, there appears little reason *à priori* to anticipate a greater weight or size in the left hemisphere.

We now pass to Dr William Ogle, whose paper, entitled "On Dextral Pre-eminence," was published in 1871*. It is by no means clear whether Ogle in speaking of pre-eminence of the left hemisphere intends to attribute to it the same kind of dominance in mental functioning that the right hand usually has in manual, but his statements suggest this.

He begins by speaking of dextrality as a characteristic peculiar to many forms of life. In man he found 57 men out of 1000 or 5·7 %, and 28 women out of 1000 or 2·8 % manual sinistralists, a conclusion pointing in the same direction, but not as sweeping as Hippocrates' γυνὴ οὐδεμία ἀμφιδέξιος. Ogle says that out of 23 monkeys in the Zoological Gardens, 20 were right and only 3 were left-handed, but he admits the experiment requires great caution. He states that the leg on which a parrot stands when it is given a nut is always the same, and that of 86 parrots, 63 stood on the right and 23 on the left leg—a fair Mendelian quarter! He meets the objection that 63 out of the 86 parrots took the nut in their *left* claw and so were sinistralists by the statement that they must select a leg to stand on before they can take the nut. But the mental process of the parrot might be: "Here is a nut, I must take it with my more efficient left claw, and so I will stand on my right leg." Ogle following Dax asserts that the mental faculties concerned in speech are lodged in the left cerebral hemisphere (p. 292). In 1867† Ogle had suggested that in left-handed men they would be lodged on the right. In 100 cases with palsy, 97 had palsy on the right and were right-handed, 3 had palsy on the left and were left-handed. On the basis of this "there can remain no fair doubt that right-handedness depends on some predominance of the left brain, and left-handedness, when it occurs, on a transposition of this structural peculiarity whatever it may be" (p. 292).

Ogle next cites Boyd's measurements on weights of the two hemispheres, and quotes Bastian's results as to the specific gravity of grey matter on the left side being

on 65 cases. The data of Strauss (22 cases) and Butin (38 cases) do not seem comparable with Saveliew's, for they contain not a single instance of bilateral embolism which occurs in 34 % of Saveliew's cases. For the same reason Meissner's 32 cases showing only 10 % of bilateral attack are hardly comparable.

* *Medico-Chirurgical Transactions (Roy. Med. and Chir. Soc.)*, Vol. xxxv. 2nd Series, 1871, pp. 279—301.

† *St George's Hospital Reports*, Vol. II. p. 122, 1867.

higher than on the right* in order to pass from a pre-eminence in functioning to a predominance in physical characters. There is no reason really, if there be a pre-eminence in functioning, which might solely connote a pre-eminence of commissures, why this should involve a preponderance in size or weight, or even in the complexity of convolutions, which, however, Ogle asserts that Dr Broadbent and he have both found for the *frontal* convolutions on the left side. The left hemisphere is not only, Ogle tells us, heavier, but more highly developed than its fellow, and this is the explanation of dextral pre-eminence. To prove his point Ogle obtained the brains of *two* left-handed women and submitted them to Dr Broadbent; the latter reported on them with drawings (not reproduced), and the conclusion is stated thus: "The ordinary conditions of the two hemispheres were in each of these brains reversed, the greater complexity of convolution occurring in both on the right side and not on the left" as—Ogle adds—"I had anticipated" (pp. 294—5).

It will be seen that the ordinary condition, i.e. greater convolutional complexity on left, is here assumed to be proven. Broca himself—without providing actual statistics—had limited his assertion to the frontal lobe: "The convolutions are more numerous in the left frontal lobe than in the right, and the converse condition exists in the occipital lobes, where the right is richer in convolutions than the left."

It is clear that if we may suppose, as indeed we know, that man is bilaterally asymmetrical, then the odds are only three to one that a pair of left-handed women will exhibit pre-eminent hemispheres both on the right side. The data are far too slender to carry complete conviction. To meet the objection that the greater development of the left brain may be the consequence not the cause of the greater use of the right side, Ogle cites Gratiolet's remarks, that the convolutions of the

* Bastian's results are given in a paper, "On the Specific Gravity of different parts of the Human Brain," *Journal of Mental Science*, No. 56, January 1866, p. 29. They seem to have been averaged for 27 brains, and the results are as follows:

	Averages of Grey Matter	
Convolutions	Left Hemisphere	Right Hemisphere
Frontal	1·0291	1·0276
Parietal	1·0300	1·0296
Occipital	1·0320	1·0316

How far these results are of real significance it is not possible to say as no statement is given as to the variation due either to errors of measurement or to random sampling.

For the sane Bastian found for averages:

Specific gravity of grey matter: Left side 1·0300, Right side 1·0296.

For the insane, without regard to side, 1·0325.

The specific gravity of white matter was always higher than that of grey matter, and without distinction of side was 1·0404 for sane and 1·0405 for insane. Our author did not find the heaviest or lightest brains with the highest or lowest specific gravities.

Danilewsky (*Medicinisches Centralblatt*, 1880, No. 14, April 3), taking *three* brains only, considered the grey and white substances of the brain separately, also dealing with their specific gravity. He found the distribution of grey and white substances in the two hemispheres nearly alike ("nahezu gleich"). The differences between the specific gravities obtained by different observers are all of the order of the differences recorded by Bastian as existing for right and left hemispheres or for sane and insane.

left frontal lobe appear earlier in the foetus than the corresponding convolutions on the right. As Gratiolet has been frequently quoted as in some way confirming the pre-eminence of the left hemisphere, we searched his book* and could only discover a single very modest paragraph on this point, where he is discussing the development of the brain:

Il m'a semblé, par suite d'une série d'observations consciencieusement étudiées, que les deux hémisphères ne se développaient pas d'une manière absolument symétrique. Ainsi le développement des plis frontaux paraît se faire plus vite à gauche qu'à droite, tandis que l'inverse a lieu pour les plis du lobe occipito-sphénoidal. Du moins, dans tous les cas que j'ai observés, ai-je vu la scissure parallèle qui distingue le pli marginal inférieur se dessiner à droite avant de se montrer à gauche. (p. 241.)

Here is no dogmatic assertion as to pre-eminence of the left hemisphere; in certain districts Gratiolet thinks he has observed the left, in others the right develop *earlier*; he makes no statement that *earlier* development is accompanied by greater complexity. Ecker, a first-class anatomist, dealing with the development of the furrows and convolutions of the brain in the human foetus†, states that he had found in foetal twins, in the one both frontal furrows present, in the other the first frontal furrow wholly absent on the *left*. That the left side always precedes the right in the development of furrows and convolutions, "as Gratiolet has asserted," Ecker could in no way confirm. But Gratiolet really made no sweeping assertion of this kind.

We now return to Ogle and cite the following words, in which we are responsible for those italicised:

Seeing, however, that we *know*, if the arguments I have used in the earlier part of this paper be valid, that some or other anatomical difference between the two sides *must* precede the right-handedness, and moreover that this difference *must* be somewhere in the brain (for how otherwise can the facts I have brought forward concerning aphasia be explained?) it appears to me only rational to *suppose*, when one finds such an anatomical difference between the two hemispheres as that *now revealed*, that this anatomical difference is the antecedent for which one was searching. (pp. 295—6.)

This paragraph begs two questions: (i) whether the difference of function must necessarily involve macroscopic differences, and (ii) whether the evidence of such differences existing has really been provided.

Ogle now proceeds to settle what has given rise to this "pre-eminence of the left side of the brain." He examined the cervical vessels and says that he found in 12 out of 17 cases of right-handed men the common or internal carotid was larger on the left than the right. He admits that the interpretation of the larger left carotid is rather dubious‡, but tries to strengthen his result by saying that the left carotid is also less tortuous and so blood will flow more abundantly to the

* F. Leuret et P. Gratiolet: *Anatomie comparée du Système, considéré dans ses Rapports avec l'Intelligence*, T. II (Gratiolet), pp. 241—2, 1857.

† *Archiv für Anthropologie*, Bd. III. S. 215, 1868.

‡ If there be no essential differentiation, then the anticipated number would be 8·5, while the observed number is 12, a deviation of 3·5, which is only 2·5 times its probable error of 1·39. The result is therefore dubious statistically as well as anatomically.

left hemisphere and to this he would attribute its [supposed] greater development*. Naturally persons with the viscera inverted should be left-handed. This is not universally the case; no statistics are, however, provided. Cases of inverted viscera and right-handedness Ogle attributes to education in dextrality. He cites G. St Hilaire's statement that inversion of viscera is more common in men than women, and holds that this is in accordance with manual sinistrality being more common with men than women. Finally he says that carotids in parrots confirm his views (see our p. 100).

Such is the memoir of Ogle, which created much interest in its day, and spread almost as a dogma the view that the left hemisphere is pre-eminent, without defining in which of many characters the pre-eminence is supposed to exist†!

We are not able in this slight study to consider all the papers that have been published on this very obscure problem, but before we turn to quite recent work we may refer to a paper by Wilhelm Braune, entitled: "Das Gewichtsverhältniss der rechten zur linken Hirnhälfte beim Menschen" (*Archiv für Anatomie und Entwickelungsgeschichte*, Jhg. 1891, S. 253—270). This is a good paper not only for its references and criticism of earlier works, but to a certain extent for its new data. Braune considers that up to 1891 no definite dominance had been proved for either hemisphere, and holds that the observed differences were within the error-limits. We do not feel clear that he distinguished between the errors of random sampling and of measurement; or between the error of the mean and that of an individual measurement. Neither he nor any of his predecessors makes the slightest investigation of what these error-limits may be. He considers that if the result of the "Nervenfaserkreuzung" produces for dextralists a greater development of the left hemisphere, then manual dextrality should always to some degree cause excess of the left hemisphere or at least some part of it, and there should only be excess of the right in cases of left-handedness. This he states is in nowise true (S. 259).

Braune's 100 weighings were made on hospital cadavers, partly by himself and partly by other anatomists, and he says they were made in uniform or standard manner, and included no insane or criminals. The brain as a whole was measured and also its parts. We are compelled regretfully to doubt his statement as to the absolute standardisation of methods of measurement, because in the cases of some

* Moutier (cited by Bouvicini, *Wiener medicinische Wochenschrift*, No. 23, 1926) argues in favour of asymmetry, but did not find the left side vascular system pre-eminent.

† A good illustration of this occurs in a paper by Hasse, "Ueber Gesichtsasymmetrie" (*Archiv für Anatomie*, 1887, S. 124). Hasse is very properly defending the Greek sculptors against an anatomist's charge that they were not true to nature, because their creations are not symmetrical. No human being is symmetrical, he says. But he then goes on to attribute this asymmetry to the dominance of one hemisphere, and this the left one; this dominance is a sequence to its *greater volume*, which is again the result of greater muscular development of the right side due to a greater use of the right upper part of the body (!). In the same way Lombroso speaks of the left hemisphere in normal persons as being more pre-eminent than the right in both weight and complexity of convolutions. He then proceeds to state that criminals reverse this ordinary rule of normal persons, and says that among criminals 41 % were asymmetrical on right, 20 % on left and in 38 % the heads were equal, that is we suppose symmetrical. It is not easy to understand how if the head be asymmetrical it can be described as asymmetrical on one side rather than the other.

of his contributors the sum of the weights of the parts nearly approaches the weight of the whole, while in other cases that sum may differ by as much as 100 grs.* It looks therefore suspiciously as if some of the anatomists had weighed the brain as a whole before and others after separation into parts. Further as nine of the hundred cases were suicides, one died of chronic alcoholism, and two in the "Zuchthaus" (besides the failure to state that the remainder had never been in asylum or jail), we cannot convince ourselves that Braune's statement as to the absence of the insane and of criminals is wholly correct, or that his material is thereby raised above that of Boyd, Thurnam, or Broca.

Braune has not given the means of the several series of measurements, but these we have taken with the following results:

Encephalon		Cerebrum		Cerebellum	
Right Half	Left Half	Right Half	Left Half	Right Half	Left Half
630·62 grs.	629·99 grs.	551·23 grs.	549·66 grs.	78·93 grs.	79·76 grs.

Or, the right encephalic hemisphere is 0·63 gr. in excess, the right cerebral hemisphere is 1·57 grs. in excess and the right cerebellar hemisphere 0·83 gr. in defect. Even should any of these differences prove to be significant† it is very hard to believe that they are sufficient to indicate a higher development or pre-eminence on either side of the brain.

Braune gives the above results in the form:

Encephalon:	R. excess 47.	Total excess 267·98 grs.	Average 5·70 grs.
	L. " 52.	" " 213·2 grs.	" 4·10 grs.
	One case of equality.		
Cerebrum:	R. excess 54.	Total excess 273·4 grs.	Average 5·06 grs.
	L. " 37.	" " 129·0 grs.	" 3·49 grs.
	One case of equality and eight not measured.		
Cerebellum:	R. excess 33.	Total excess 85·75 grs.	Average 2·60 grs.
	L. " 54.	" " 168·55 grs.	" 3·12 grs.
	Five cases of equality, and eight not measured.		

Braune emphasises this excess of the left cerebellar hemisphere, and says that the predominance in weight of the left cerebellar hemisphere corresponds with a 53·8 °/。 of bulging which he found in 91 skulls of various races he examined. The case is more in favour of left bulging than he credits it with being, for he has counted those with no bulge as if they bulged to the right. He says that 12 cases occurred in which the right hemisphere weighed 10 grs., or more, in excess of the left, but none of those was associated with left-handedness.

On the whole Braune's paper is good, when we consider how little appreciation most anatomists have of the need of statistical reduction. Braune concludes by

* Let the reader examine the relative equality of the sum of the parts and the whole after Brain No. 86, and note how little equality there is before No. 86. Note especially No. 25.

† We have determined the probable from the mean errors based on the excess differences of R. and L. values published by Braune. We reach the results 0·63 ± ·41, 1·57 ± ·35, 0·83 ± ·22. Thus in the cases of the cerebrum and the cerebellum significant differences can possibly be said to exist in weight between R. and L. sides, for these differences are 4·5 and 3·8 times their probable errors.

stating that if size or weight be causally associated with the unequal muscular division on the two sides of the body then this asymmetry ought constantly to follow the muscle and bone distribution, but this is not the fact. Clearly we are justified in saying that the work of Thurnam and Braune is at least as weighty as that of Boyd and Ogle.

Braune's data (if not his statements) confirm Thurnam's results that the right hemisphere is slightly heavier than the left, and they support the view of Wilde, although less emphatically, that the left cerebellum is heavier than the right (see our p. 96 and p. 109).

We will now pass to the more recent literature and consider to what extent it throws further light on the nature of this "pre-eminence of the left hemisphere." In the first place we have an inaugural dissertation by Eugen Rübel entitled: *Ueber das Gewicht der rechten u. linken Grosshirn-Hemisphäre im gesunden und kranken Zustand*, Würzburg, 1908. Our author begins with an account of other investigators' work and enlarges on the difficulty of the problem, the dividing of the brain, the drainage of the fluid and so forth. He considers they have come to opposed conclusions owing to imperfections and want of standardisation in their methods of separating the hemispheres. But he gives no evidence that other anatomists have used less care than himself, nor does he explain in detail his own method of procedure. He says that, up to his own investigations, no author had in the problem of the relative weight of the hemispheres taken into consideration the skull capacity. He remarks:

> Wenn die Differenz in Prozenten zwischen Schädelkapazität und Hirngewicht eine pathologische ist, so muss man nach derselben annehmen, dass das Gehirn selbst in seiner Materie, durch eine akute oder chronische Gehirnkrankheit verändert war. Eine auffällige Gewichtsdifferenz der Hemisphären wäre dann natürlich in erster Linie als Folge der Gehirnkrankheit anzusehen. (S. 8.)

Rübel now proceeds to table his cases according to the percentage difference between "Schädelkapazität und Hirngewicht," which reads as a difference between a volume and a weight! He neither explains how he reduces volume to weight, nor how he has determined the skull capacities of the subjects from whom the brains have been extracted. We have no statement or numbers supplied for the reduction, and as the specific gravities of the grey and white matters differ, and their relative proportions vary from brain to brain, and further the lateral ventricles differ in size, surely some detailed explanation is essential for the proper understanding of tables which speak merely of the percentage difference between skull capacity and weight. The author concludes that when there is no one-sided influence of disease both hemispheres are of equal weight. He thus appears to have reached the same point as Braune, but we have no idea of how he gets there and feel desperately inclined to echo the Schlagwort: "Weg mit Dissertationen!"

The next writer we have to notice is far more suggestive. M. Inglessis proceeded to take "frontal sections" of hardened brains*. These sections were taken 8 to

* "Untersuchungen über Symmetrie und Asymmetrie der menschlichen Groshirnhemisphären." *Zeitschrift für die gesamte Neurologie und Psychiatrie*, Bd. xcv. S. 464—472. 1924.

10 mm. apart parallel to the plane through the auricular axis perpendicular to "Rieger's horizontal plane*." Photographs were then taken of the faces of these slabs and the areas right and left measured on the photographs by aid of a planimeter. No attempt seems to have been made to reconstruct the volume of the hemispheres from the known thickness of the slabs and the areas. It may be remarked that asymmetry in the positions of the auricular passages would produce artificial asymmetry of the sections. Taking 3 °/₀ difference as negligible our author proceeds to measure the percentage difference of his hemispheres; he does not as far as we can see state on what his percentage difference is measured. We take it that symmetry means in his case equality of areas, and that he terms that side "asymmetrical" which has the greater area. He divides the brain into a "forward" part, i.e. that in front of the standard plane through the auricular axis, and a rearward part, namely that behind this plane. On the basis of measurements for 200 brains Inglessis found:

Forward	part	symmetrical,	rearward	part	symmetrical, 13 : 6 °/₀.
,,	,,	,,	,,	,,	asymmetrical, 168 : 84 °/₀.
,,	,,	asymmetrical,	,,	,,	symmetrical, 0 : 0 °/₀.
,,	,,	,,	,,	,,	asymmetrical, 19 : 5 °/₀.

Thus of the forward parts 181 were symmetrical and only 19 asymmetrical, but of the rearward parts 187 were asymmetrical and only 13 symmetrical. The 19 brains with asymmetrical forward parts were chiefly those of persons with organic brain disease, in particular, paralysis. Age and sex seemed to have little influence on the asymmetry, and the influence of disease was not definitely significant for this number of brains.

It will be seen that if we can trust Inglessis' results the asymmetry was practically confined to the occipital lobes, and we must answer the question put to Broca as to whether the enlarged frontal portion of the left hemisphere was due to the organ of speech being on the left side, by saying that the frontal portion of the left hemisphere is not greater than that of the right, and thus it is idle to question whether it is due to the pre-eminence of the left third frontal convolution behind the pterion. Inglessis thus directly contradicts Broca's view. Turning now to right and left asymmetries, i.e. to which is larger, Inglessis finds that of the 19 forward asymmetries 11 were left and 8 right, i.e. the difference is not significant, but of the 187 cases of rearward asymmetries 161 were left and only 16 right. Thus according to our author the brain is asymmetrical, but this asymmetry is confined chiefly to the occipital part of the hemispheres and here the left hemisphere predominates. If we combine the conclusions of Braune and Inglessis we should accept a predominance of the left side of the encephalon in the occipital portion of the cerebrum and in the cerebellum. But if this be true, is it needful to associate it with differentiation of psychical function in the two hemispheres? May it not be that a majority of persons having their hearts on the left side find it easier to sleep for the major portion of the night on the right side? While the brain is growing this

* For a definition of this plane the author refers to an inaugural dissertation of Gertrud Wolf: *Ueber die Lage der Ohrachse, in Beziehung zum Schädel und Gehirn.* Würzburg, 1918.

must give greater pressure on the occipital than on the frontal portion of the right hemisphere, while the left hemisphere would be free of such pressure. Hence might arise the asymmetry observed by Braune and Inglessis. We do not press the point, but it seems as worthy of consideration as any theory of left pre-eminence. It is also consistent with the view of H. Reichardt*, who holds that there is no sensible difference between the weights of right and left hemispheres even when the skull is asymmetrical.

We now turn to a second memoir by Inglessis entitled: "Ueber Kapazitätsunterschiede der linken und rechten Hälfte am Schädel bei Menschen (insbesondere Geisteskranken) und über Hirnasymmetrien †." The object of this paper is to show that there is an extensive agreement between the skull and the brain sides which predominate ("überwiegen"). Inglessis does not appear to have had the whole skulls from which the brains were extracted, but only the skull-caps ("Kalotten") which were removed for the purpose of extracting the brains. These caps were taken off by saw-cut; and he says that they were sawn off at the plane going through the upper borders of the orbits and the uppermost points of the auricular passages. Now our own experience shows us that the upper borders of the orbits and the two auricular points in the vast majority of crania do not lie in one plane, and to take off a skull-cap with any exactitude in this way would be a task of the greatest difficulty demanding an accuracy hardly attainable in the post-mortem room. Inglessis considers the skull-cap thus obtained gives the most important portion of the cerebrum. He then divides the skull-cap into the right and left halves by a sheet of lead. This is taken through the crista galli, in the concavity through the sulcus longitudinalis superior, and posteriorly through the "middle" of the protuberantia occipitalis. He does not say how he cut the lead sheet to fit the individual skull-cap, or how, having done so, the right and left capacities were determined by water up to a border which could not be uniplanar. The whole process seems an extraordinarily difficult and necessarily rough one, but Inglessis concludes that the capacities thus determined agreed with the frontal section method in showing the predominance of the left hemisphere; thus the predominance of the left hemisphere in the case of the skull-cap must have been in the capacity of the occipital region. Our author attributes in chief although not entirely the difference in capacities to the deviations of the falx cerebri to right or left ‡.

Deviation of Falx to right	Falx straight	Deviation of Falx to left
59·4 %	19·8 %	20·8 %
Asymmetry to left	Symmetrical	Asymmetry to right
63·5 %	6·3 %	30·2 %
Predominance of left in skull-cap	Equality	Predominance of right in skull-cap
63·5 %	5·2 %	31·3 %
Lateral Ventricle greater on right	Ventricles equal	Lateral Ventricle greater on left
66·8 %	17·8 %	15·4 %

* *Arbeit aus der psychiatr. Klinik zu Würzburg*, Heft I. S. 44, und Heft VI. S. 341, 611.

† *Zeitschrift für die gesamte Neurologie und Psychiatrie*, Bd. XCVII. S. 354—378, 1925.

‡ As determined by the sulcus longitudinalis superior deviating to right or left, but this does not appear to measure the full possibility of deviation in the falx cerebri.

The measurements were made on 96 skull-caps only. Our author terms this an "ausgedehnte Uebereinstimmung im Ueberwiegen der linken Seite." In women Inglessis found that the predominance of the left hemisphere was *less*; this should be correlated with more left-handedness in women than men, which does not appear to be the case.

If we might trust Inglessis' method and results, they would give us confidence in the belief that internal measurements on the skull will more or less accurately describe what is the position with regard to the brain.

A paper by Rasdolsky, entitled "The Asymmetry of the Hemispheres of the Brain in Man and the Animals," appeared in 1925*. The author starts with the statement that the functions of speech, writing, reading, customary and expressive actions, the visual and auditory gnosis are only perfectly developed in man in one predominant hemisphere; and that except in man the two hemispheres are quite symmetrically organised. Presumably therefore manual laterality does not occur in apes, monkeys or parrots (see our p. 100). He attributes right-handedness to the position of the heart and the effect that the expenditure of much energy on the left side would have on the heart (described as the theory of Astuazaturoff and Weber). There are no experimental data given and the paper is a somewhat dogmatic attempt to account on a developmental basis for the pre-eminence of the left hemisphere which Rasdolsky considers proven. If apes and monkeys, having a left-sided situation of the heart, be considered as animals, is it correct to assert that their hemispheres are symmetrical and they have no manual laterality?

In 1926 we have a paper by C. U. A. Kappers on "The relative Weight of the Brain Cortex in Human Races and in some Animals, and the Asymmetry of the Hemispheres†." Here we are concerned with the weights of the grey matter in right and left hemispheres taken in relation to the total weight of the hemisphere and of the brain. Kappers' procedure is clearly a long and laborious one, and it could not be easily applied to determine in adequate numbers either the mean of the difference of these relative weights or the probable error of this difference. He applies it only to three Dutch and three Chinese brains which are far too few to really determine (i) average differences, (ii) any excess or defect in right or left hemisphere, or (iii) any differentiation of European and Chinese brains.

J. Wilde in the same year, in a paper entitled "Ueber das Gewichtsverhältniss der Hirnhälften beim Menschen‡," returns to the old problem of weighing the two sides of the brain. He dealt with 200 brains of individuals from 17 to 77 years of age of which 125 were male and 75 female, and says that his greatest error as shown by control measurings was 0·25 gr. The brains were those of Letts, Germans and Russians. We have only seen Wallenberg's account of the paper in the *Zentralblatt f. d. gesammte Neurologie u. Psychiatrie* (Bd. LV. S. 393—4), but the divisions and measurements seem to have been carefully made. The total weight

* *The Journal of Nervous and Mental Disease*, Vol. LXII. pp. 119—132, N.Y. 1925.

† *Ibid.* Vol. LXIV. pp. 118—124, N.Y. 1928.

‡ Paper from the Neurologic Institute of Riga. *Latvijas Univ. raksti*, Vol. XIV. pp. 271—288, 1926.

of the right side was somewhat the heavier, and this applied also to the cerebrum. On the other hand the (cerebellum + pons Varolii + medulla oblongata) was heavier on the left side than the right. Thus this last brain weight paper appears to confirm the results of Thurnam, Broca and Braune, who found the right hemisphere somewhat heavier (see our pp. 96, 97, 104), and of Braune again who found the cerebellum heavier on the left (see our p. 104), and to confute those of Boyd (see our p. 95), who found the left-hand hemisphere heavier. Our author concludes that:

"The weight of the brain is accordingly no witness to the functional superiority or inferiority of the brain (exception being made of that of the microcephalic idiot which falls below the normal limits)."

Walther Riese in the following year (1927) published a paper* on: "Die Überwertigkeit der einen Hemisphäre auf Grund hirnmorphologischer und hirnpathologischer Untersuchungen." He criticises strongly the view that superiority belongs to one hemisphere or to the other. He asserts that we must investigate in each locality which hemisphere has the greatest area of functioning substance, furrows and convolutions both being examined. It will be found in this way that one hemisphere may dominate for one tract and the other for another. Riese considers that all grades of superiority may occur and these may vary from one tract to a second. A manual dextralist may inherit only the laterality of one tract, and this give rise to a very complicated laterality in the brain. Our author cites with approval Kleist, who considered that for any brain tract there might be sinistrality, equality or dextrality. Thus a genuine manual dextralist might on his supposed inferior right hemisphere actually have tracts of superior brain dextrality (S. 227—8). According to Riese (who does not cite Ogle, but appeals to a single case examined by Flechsig) there may be a superiority of blood vessels on the left side, but it is not yet proven †.

Direct investigations of whether the left hemisphere is wholly directive have recently been made. We may refer first to R. A. Pfeifer, who has considered the matter in his "Bemerkungen zur Links- und Rechtshändigkeit ‡." He starts by saying that if we accept the "Aproxielehre" of Liepmann we are forced to the conclusion that the superiority of the left hemisphere almost entirely excludes any participation of the right in action. But observations on left and right-handedness teach us that both hemispheres influence each other in alternating manner in action; this Pfeifer says is very obvious in the use of bilateral innervatory identical musculature, as when the two hands make the same or opposed movements. In the use of bilateral innervatory identical musculature the anatomical certainty of the course of the motion is most remarkable, and we attribute it to the linkage of homologous districts in the brain by the nerve fibres of the corpus callosum ("Balkenfasern") and thus the transit of innervatory impulses from one side to the identical innervatory

* *Monatsschrift für Psychiatrie und Neurologie*, Bd. LXIV. S. 195—228.

† Riese's main material consisted of three brains of manual dextralists with disturbance of speech and right-handed lesions. In these cases he was able from convolutions and sulci to show morphological right side superiority.

‡ *Münchener medizinische Wochenschrift*, Bd. LXXIV. i. S. 846, 1927.

musculature of the other. Thus in writing with the right hand arises by way of latent co-practice the mirror writing of the left hand; bad writing of the left hand flows from this acquired tendency to mirror writing. From these and other instances and illustrations, Pfeifer draws the conclusion that the so-called dominance of the left is not independent of the corresponding tract on the right hemisphere.

Some attempt to measure this reciprocal influence of the cerebral hemispheres has been recently made by J. Wysocki* and by Wysocki in conjunction with L. Zbyszewski*. Their process was associated with the phenomena termed by Brown-Séquard "dynamogénie."

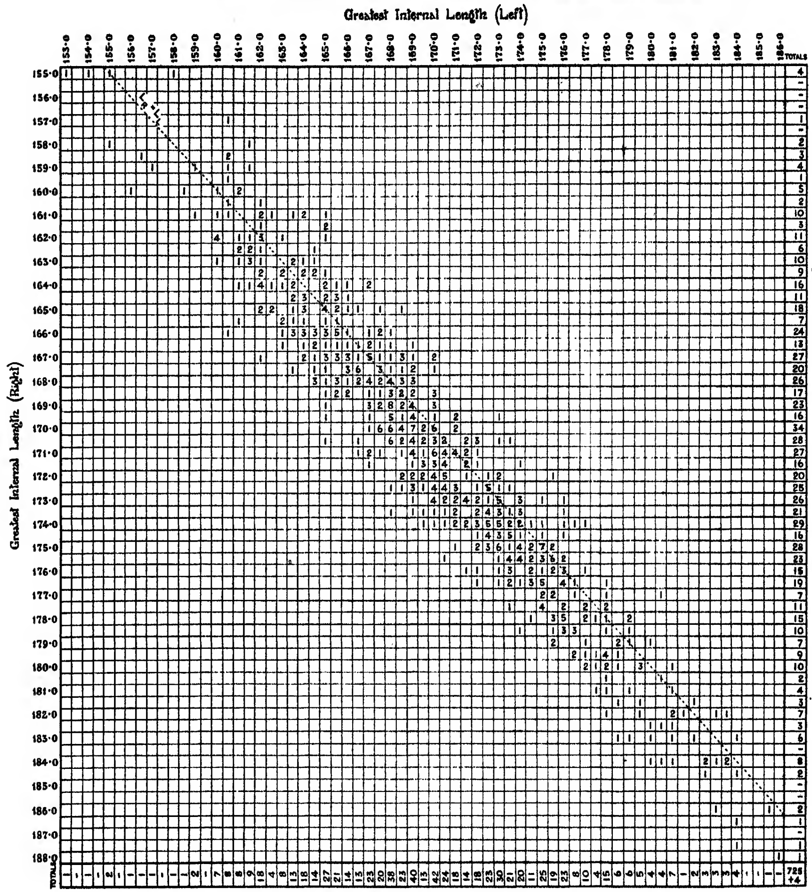
L'entre-croisement de deux excitations dans un groupe de cellules nerveuses, à l'intersection de deux neurones, ne détermine pas toujours des actions d'arrêt, mais peut donner lieu, au contraire, à un renforcement de l'excitation. (*loc. cit.* p. 1009.)

The same class of phenomena has been termed by Exner "Bahnung." Choosing homologous motor tracts of the surface of the two hemispheres of the brain, our authors excited them in diverse ways, and found that these excitations had a reciprocal influence. It would thus seem possible by exciting one hemisphere in a definite region to strengthen the effects which flow from exciting the other hemisphere in the same region, or there is some evidence to indicate that the two hemispheres to a greater or less extent cooperate.

(ii) *Our own Material.* It will be seen from the previous analysis of papers dealing with the pre-eminence of one or other hemisphere that the earlier and somewhat dogmatic views have been largely called into question, and that the pre-eminence claimed for the left hemisphere has not been shown hitherto definitely to be associated with corresponding pre-eminence in such gross characters as weight or size. The measurements referred to in the first section of this paper permit a definite answer to be given to the relative pre-eminence of the hemispheres in one measurement of size, namely their length. The maximum lengths of the right and left sides of the cranial cavity were taken in 729 male Egyptian skulls. These lengths were taken on either side of the crista galli to points above the occipital protuberance, the instrument being held parallel to the median sagittal plane, and endeavours made to pass from the most anterior point of the frontal to the most posterior point of the occipital bone. Three trials were made to obtain the maximum on each side, and the greatest of these was selected as the maximum for each side. If these lengths be L_R and L_L, then $\frac{1}{2}(L_R + L_L)$ was the quantity used for correlation purposes in connection with the capacity of the skull. This quantity $\frac{1}{2}(L_R + L_L)$ was entered on the card of the skull, and no record preserved of which component length was L_R and which L_L. The same system of measurements was again repeated for the whole 729 crania, record being kept of L_R and L_L separately. The maximum values right and left were read to the nearest half millimetre and Fig. 1 provides a complete statement of all the corresponding values and a picture of the results.

* *Comptes rendus de la Société de Biologie*, Tom. XCVI. pp. 572—575, March, 1927, and Tom. XCII. p. 1009 (1925), Tom. XCIII. p. 1629 (1925).

Lengths of Right & Left Hemispheres to the unit 0·5mm of measurement



In addition: 1 at Right 146·0 & Left 147·0   1 at Right 151·0 & Left 147·0   1 at Right 149·5 & Left 154·0   1 at Right 190·0 & Left 190·0

Fig. 1.

The broken line represents equality in the lengths of the two hemispheres.

To face p. 110

We will investigate first how we can determine the error due to the measurer. Let z_1 be the mean of the two lengths in the first measurement and z_2 be the corresponding quantity for the second measurement. Let $\tilde{L}_R$ and $\tilde{L}_L$ be the true lengths of the hemispheres, and e_1, e_2, e_1', e_2' the errors made in the measurements taken of the four lengths, then

$$z_1 = \tfrac{1}{2}(\tilde{L}_R + \tilde{L}_L) + \tfrac{1}{2}e_1 + \tfrac{1}{2}e_2, \quad z_2 = \tfrac{1}{2}(\tilde{L}_R + \tilde{L}_L) + \tfrac{1}{2}e_1' + \tfrac{1}{2}e_2',$$

and accordingly

$$z_1 - z_2 = \tfrac{1}{2}(e_1 + e_2 - e_1' - e_2'),$$

and if σ denotes a standard deviation

$$\sigma^2_{z_1-z_2} = \tfrac{1}{4}(\sigma^2_{e_1} + \sigma^2_{e_2} + \sigma^2_{e_1'} + \sigma^2_{e_2'}),$$

approximately, for there is no reason to suppose high correlation of errors.

Now $\sigma^2_{e_1} = \sigma^2_{e_1'}$ and $\sigma^2_{e_2} = \sigma^2_{e_2'}$, for they represent the variation in measuring the same quantity at a short interval. Further we have no strong reason to assert that any sensible difference was made in the manner of measuring left and right hemisphere lengths. Thus it seems fair to take $\sigma^2_{e_1} = \sigma^2_{e_2}$, and accordingly we have $\sigma_{z_1-z_2} = \sigma_e$, where e stands for e_1, e_2, e_1' or e_2'. In the next place the frequency distribution was formed of $z_1 - z_2$, i.e. the difference of $\tfrac{1}{2}(L_R + L_L)$ at the first and second measurements, and the mean and standard deviation obtained of this difference. They are in mm.:

$$\text{Mean (1st measurement} - \text{2nd measurement)} = -\cdot 1104,$$
$$\text{Standard Deviation } \sigma_{z_1-z_2} = 1\cdot 7856.$$

Hence: Mean/probable error of mean $-\cdot 1104/\cdot 0446 = 2\cdot 47$,

and accordingly as the ratio is less than 2·5, it is not at all improbable that the difference in the means of $\tfrac{1}{2}(L_R + L_L)$ at first and second measurements may be due to errors of measurement. Now let us consider the means of L_R and L_L; the probable error of measurement of both is ·0446, and accordingly the probable error of their difference, if due to errors of measurement, is ·0631, but the means of L_R and L_L differ by ·9945 mm. or by 15·75 times the probable error of the difference. Thus although the probable error of a single measurement $= \cdot 67449 \times 1\cdot 7856 = 1\cdot 2043$ mm. is considerable—as it must be when we remember the difficulties of taking an internal measurement by way of the foramen magnum—yet the process is sufficiently accurate when we take the means on 729 crania to prove that a mean length of right hemisphere of 171·0446 is significantly greater than a mean length of left hemisphere of 170·0501 mm. In other words, if we may take our sample of over 700 male Egyptian skulls as representative, the right hemisphere is in the main of greater length, and therefore probably of greater capacity than the left.

The actual constants obtained from the measurements are:

Mean $L_R = \bar{L}_R = 171\cdot 0446$ mm., Standard Deviation $= 5\cdot 9551$ mm.

Mean $L_L = \bar{L}_L = 170\cdot 0501$ mm., Standard Deviation $= 5\cdot 6908$ mm.

Correlation coefficient $= \cdot 9603$*.

* From the grouped Table IV on p. 112. Worked from the formula

$$r_{L_R L_L} = (\sigma^2_{L_R} + \sigma^2_{L_L} - \sigma^2_{L_R - L_L}) / (2\sigma_{L_R}\sigma_{L_L}),$$

we have $r_{L_R L_L} = \cdot 9602$.

TABLE IV.

Internal Lengths of Right and Left Hemispheres (Condensed Table).

Internal Length. Left Hemisphere in mm.

Internal Length. Right Hemisphere in mm.	146·5—148·5	—150·5	—152·5	—154·5	—156·5	—158·5	—160·5	—162·5	—164·5	—166·5	—168·5	—170·5	—172·5	—174·5	—176·5	—178·5	—180·5	—182·5	—184·5	—186·5	—188·5	—190·5	Totals
144·5—146·5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—148·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—150·5	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—152·5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—154·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—156·5	—	—	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
—158·5	—	—	—	—	1·25	·25	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4·5
—160·5	—	—	—	—	1·25	1·75	4·25	5·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	12·5
—162·5	—	—	—	—	—	—	6·75	12·25	4·75	4·25	—	—	—	—	—	—	—	—	—	—	—	—	28
—164·5	—	—	—	—	—	—	1	16	14·75	9·75	2	—	—	—	—	—	—	—	—	—	—	—	43·5
—166·5	—	—	—	—	—	—	·5	4·5	21·5	25·25	8·25	1	—	—	—	—	—	—	—	—	—	—	61
—168·5	—	—	—	—	—	—	—	1	7	29·75	34·25	16	—	—	—	—	—	—	—	—	—	—	88
—170·5	—	—	—	—	—	—	—	—	—	5·25	42·25	39	7	2	—	—	—	—	—	—	—	—	95·5
—172·5	—	—	—	—	—	—	—	—	—	1·25	10·5	44·75	25·25	6·75	1	—	—	—	—	—	—	—	89·5
—174·5	—	—	—	—	—	—	—	—	—	—	1·75	17·25	31·75	37·25	7	1·5	—	—	—	—	—	—	96·5
—176·5	—	—	—	—	—	—	—	—	—	—	—	·5	9	36·75	35·5	1·75	—	—	—	—	—	—	83·5
—178·5	—	—	—	—	—	—	—	—	—	—	—	—	·5	4·75	27·25	11·5	3	·5	—	—	—	—	47·5
—180·5	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	5·75	16·75	7·75	1·25	—	—	—	—	32
—182·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4	4·5	6	2	—	—	—	16·5
—184·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	4·75	5·5	5·75	—	—	—	16·5
—186·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	2·25	1	—	—	3·5
—188·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5	1	—	—	2·5
—190·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	2	—	—	3	3·5	3	13·5	41	48	75·5	99	118·5	73·5	88	76·5	36	20	13·5	11·5	2	—	1	729

If we work from the formula:

$$\text{Correlation coefficient} = \frac{\sigma_R^2+\sigma_L^2}{2\sigma_R\sigma_L} - \frac{\pi}{\sigma_R\sigma_L}\left\{\Sigma\,\frac{(L_R-\bar{L}_R-(L_L-\bar{L}_L))}{N}\right\}^2,$$

where Σ is a summation for all values of $L_R-\bar{L}_R$ greater than $L_L-\bar{L}_L$, we find

$$\text{Correlation coefficient} = \cdot 9614.$$

These values are, of course, uncorrected for errors of measurement, and the last two obtained by very different methods are in good accordance.

Since $L_R=\tilde{L}_R+\epsilon_1$ and we may suppose ϵ_1 independent of the length measured, we have:

$$\sigma^2_{L_R}=\sigma^2_{\tilde{L}_R}+\sigma^2_{\epsilon_1},$$

and this leads to

$$\sigma_{\tilde{L}_R}=5{\cdot}681053, \text{ and } \sigma_{\tilde{L}_L}=5{\cdot}403434,$$

for the true variabilities of L_R and L_L. Thus the true variabilities of L_R and L_L are 3·18 and 3·03 times the variability due to error of measurement, i.e. 1·785568.

We are not able, however, to correct $r_{L_R L_L}$ for the errors of measurement; for if we multiply the crude value ·9603 by $\dfrac{\sigma_{L_R}\sigma_{L_L}}{\sigma_{\tilde{L}_R}\sigma_{\tilde{L}_L}}$, we find the corrected value somewhat exceeds unity or our σ_ϵ is *too large*. This we believe arises from the hypotheses (*a*) that measuremental errors in determining L_R and L_L are uncorrelated, for correlation may arise from the approximate symmetry of the skull which creates the same type of difficulty on either side, or again from the nature of the individual foramen magnum; and (*b*) the same conditions may influence the errors ϵ_1, ϵ_1' (or ϵ_2, ϵ_2') when measuring the same length twice. Such correlations will influence not only the size of σ_ϵ and so the sizes of $\sigma_{\tilde{L}_R}$ and $\sigma_{\tilde{L}_L}$, but also the value of the product-moment $S(L_R L_L)$. Actually

$$r_{\tilde{L}_R\tilde{L}_L}=\frac{\frac{1}{N}S(L_R-\bar{L}_R)(L_L-\bar{L}_L)-\sigma_\epsilon^2 r_{\epsilon_1\epsilon_2}}{\sqrt{(\sigma^2_{L_R}-\sigma_\epsilon^2)(\sigma^2_{L_L}-\sigma_\epsilon^2)}},$$

but as our data are inadequate to find $r_{\epsilon_1\epsilon_2}$ (= probably $r_{\epsilon_1'\epsilon_2'}$), and we do not know $r_{\epsilon_1\epsilon_1'}$ (= probably $r_{\epsilon_2\epsilon_2'}$), or $r_{\epsilon_1\epsilon_2'}$ (= probably $r_{\epsilon_1'\epsilon_2}$) we cannot determine σ_ϵ^2. Our results, however, do indicate that these correlations differ from zero, even if the associations be only slight.

We have seen that the errors of measurement are unlikely to account for the difference between mean L_R and mean L_L, we may now consider whether their difference ·9945 could easily arise from random sampling*. As we do not know

* $L_R=\tilde{L}_R+\epsilon_1$, hence: mean L_R = mean $\tilde{L}_R$ + mean ϵ_1. In taking the mean L_R − mean L_L = mean $\tilde{L}_R$ − mean $\tilde{L}_L$ we merely suppose mean ϵ_1 = mean ϵ_2 and not necessarily mean ϵ_1 = zero. It might be supposed that mean ϵ_1 could not be zero as we are aiming at $\tilde{L}_R$ = true maximum length, but it must be remembered that this length is to be measured parallel to the median plane, and if there be error in this adjustment we may exceed the true maximum, further there may be errors in excess in reading the instrument or in resetting it after extraction, so that we cannot by any means assert that *all* ϵ's are positive.

the true value of the correlation of $\tilde{L}_R$ and $\tilde{L}_L$ we are compelled to use that of L_R and L_L, i.e. ·9603, which cannot differ much from it. We have then

$$\sigma^2_{\tilde{L}_R-\tilde{L}} = \sigma^2_{\tilde{L}_R} + \sigma^2_{\tilde{L}_L} - 2\sigma_{\tilde{L}_R}\sigma_{\tilde{L}_L} \times \cdot 9603$$
$$= 2{\cdot}514{,}4343,$$

or,
$$\sigma_{\tilde{L}_R-\tilde{L}_L} = 1{\cdot}585{,}697.$$

Accordingly the probable error of $\tilde{L}_R - \tilde{L}_L$, or ·9945, is ·0396; thus the difference is over 25 times its probable error.

Lastly if we consider the observed difference of mean L_R – mean L_L, its observed standard deviation is 1·664,3155 and therefore the probable error of the difference is ·0416, or the difference is 23·9 times the probable error, this last proceeding neglecting the distinction between variation due to random sampling and that due to measuremental errors. Whichever way we approach the problem we find the significance of the difference between the lengths of right and left hemispheres emphasised.

We may attempt to compare our results for greater right hemispherical length with those of Braune and others for the relative weights. Thus let us consider two skulls both of which are *symmetrical*, but one of which has the mean internal length 171·0446 and the other 170·0501—i.e. the mean right and left hemisphere lengths—further, we will suppose the internal height and breadth to have their mean values, the capacities of these would be, by the formula on p. 92, 1435·39 and 1442·80 cubic centimetres*. Now Braune gives 1260·61 grs. for the mean weight of the German encephalon; assuming his material was chiefly male, the cranial capacity of Germans would be about 1500 cm.^{3} or 1 cm.^{3} of capacity corresponds to ·8406 gr. of brain weight. Our two symmetrical brains would therefore correspond to weights of 1206·59 and 1212·82 grs. respectively for the entire brain. But Braune found the ratio of weight of encephalon to cerebrum to be 1260·61 to 1100·89, or the factor is ·87329, accordingly the weights of the cerebrum in these two symmetrical brains would be 1053·70 and 1059·14 or their hemispheres 526·85 and 529·57. If these two hemispheres be put together to form a single asymmetrical brain with the larger weight on the right side, they would correspond to a mean skull with the difference in length of right hemisphere over the left equal to ·9945 mm. This would give a preponderance in weight of right hemisphere of brain = 2·72 grs. Braune found a difference of 1·57 grs., which we have shown to be significant, and Broca of 1·97 grs. The difference between our result and Braune's is about three times the probable error of the difference. Such a difference might very well be anticipated for we are dealing with two very different races at very

* The reader must bear in mind that the mean value $\bar{P}_i$ of the internal diametral product is not the product of the three mean internal diameters, i.e. $\bar{L}_i\bar{B}_i\bar{H}_i$, which is 2901·94 cm.^{3} corresponding to a capacity of 1438·30 cm.^{3}, and not 1440·30 cm.^{3}, the mean value. The true mean $\bar{P}_i$ to a first approximation is

$$\bar{P}_i = \bar{L}_i\bar{B}_i\bar{H}_i(1 + v_{B_i}v_{H_i}r_{B_iH_i} + v_{H_i}v_{L_i}r_{H_iL_i} + v_{L_i}v_{B_i}r_{L_iB_i}),$$

where the v's are coefficients of variation. This gives $\bar{P}_i = 2906{\cdot}03$ cm.^{3}— a reasonable approach to the true value 2906·58 cm.^{3}. The slight difference 0·5 cm.^{3} is due chiefly to the effect of grouping in determining the means of P_i, L_i, B_i and H_i, and not to the need for a further term in the approximate value.

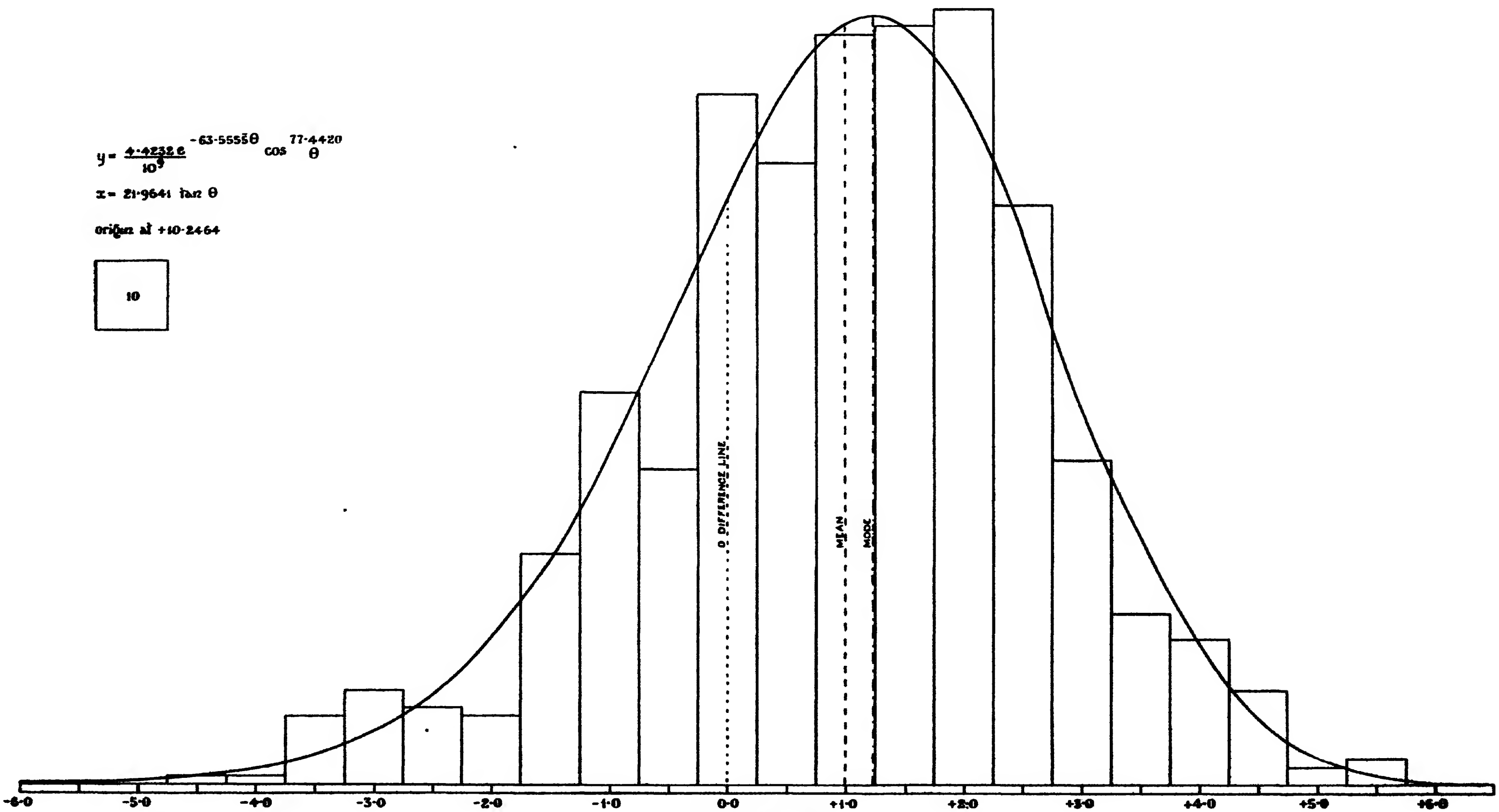
different periods of history, and we have further neglected all differences in size of the hemispheres except their length. It may well be that there exists some compensatory factor at work, so that a long cerebral hemisphere may be less high and broad. Further our method of approximation is very rough, it being improbable that the weights of encephalon and cerebrum are really proportional to their cubic capacities. Still the fact remains that the best weighings and the most complete series of measurements yet made lead to the same conclusion, a pre-eminence of the right, not the left side of the cerebrum. If it were not that so many anatomists have sought to associate a psychical pre-eminence of the left with size or weight, we should have been inclined to say *ab initio* that such pre-eminence, if it exists, would have little relation to such crude physical characters as these. Indeed we should doubt whether it would be found markedly exhibited in anything short of microscopic differences in corresponding tracts of the grey substance of the two hemispheres.

Before we conclude this matter we may draw some further inferences from our Fig. 1, on p. 110 above, where the readings are taken to the nearest half millimetre. The dotted diagonal line marks the cases in which to within the unit of measurement the two hemispheres were of equal length. There are 81 cases or 11·11 °/ₒ. Adding up the cases in lines parallel to the diagonal line we get the accompanying frequency distribution for $L_R - L_L$, the length of right hemisphere minus length of left. This shows us that in 68·59 °/ₒ of cases the right hemisphere was larger than the left, and that it was only less than the left in 20·30 °/ₒ cases. If we divide up the 11·11 °/ₒ, in which to our unit of measurement the lengths are equal, in the ratio of the excess of left over right to excess of right over left, we find on adding these 2·54 °/ₒ and 8·57 °/ₒ to the 20·30 °/ₒ and 68·59 °/ₒ respectively, 22·84 °/ₒ of cases of the left hemisphere in excess and 77·16 °/ₒ cases of the right. As the process is a rough one these percentages are close enough to 25 °/ₒ and 75 °/ₒ for a very ardent Mendelian to suggest that lesser size of left hemisphere is inherited as a Mendelian recessive, and corresponds with the asserted 25 °/ₒ of left-handedness. But if size has anything to do with dominance of control this relationship will scarcely work, as it is the left hemisphere which is supposed to control the right-handed person's movements.

TABLE V.

$L_R - L_L$ in mms.

−4·5	−4·0	−3·5	−3·0	−2·5	−2·0	−1·5	−1·0	−0·5	0	+0·5	+1·0	+1·5	+2·0	+2·5	+3·0	+3·5	+4·0	+4·5	+5·0	+5·5	Totals
1	1	8	11	9	8	27	46	37	81	73	88	89	91	68	38	20	17	11	2	3	729
Left greater 148									81	500 Right greater											729
20·30 °/ₒ									11·11 °/ₒ	68·59 °/ₒ											100 °/ₒ
22·84 °/ₒ										77·16 °/ₒ											100 °/ₒ



Right-Left Length

Fig. 2.

The equation to the Type IV curve is:

$$y = \frac{\cdot 44232}{10^8} e^{-63\cdot 565539 \tan^{-1}(x/21\cdot 964093)} \Big/ \left(1 + \frac{x}{(21\cdot 964093)^2}\right)^{38\cdot 720973},$$

the distance from origin of curve to mean being $-18\cdot 503496$, and from mean to mode $+\cdot 477868$.

The unit of x is 0·5 mm. The standard deviation $= 3\cdot 328631 = 1\cdot 6643155$ in mm.

To test the percentages still further the frequency was fitted with a Pearson Type IV curve (see Fig. 2) and the areas on either side of zero read off with a planimeter; the percentages then obtained were:

Length of R. Hemisphere > Length of L. Hemisphere 75·48 %,
" " " < " " " 24·52 %.

In other words the number of cases in which the left hemisphere is shorter than the right is exceedingly close to the Mendelian quarter.

We may judge from these results that the right hemisphere of the cerebrum, so far from being inferior in size or weight to the left hemisphere, is slightly superior to it, and the proportion of cases in which it is in excess of the left corresponds closely to the number in which the right side of the skeleton exceeds the left, or, as some have asserted, the proportion of right-handed to left-handed individuals, which has been claimed to be a Mendelian three-quarters. We do not consider that this is in any way opposed to the view that—at any rate for some functions—the left hemisphere may predominate, because we do not believe that a volitional predominance is of necessity associated with such gross characters as size or weight. Consequently we do not think that the present result must be taken in any way to strengthen the assertion recently made on the basis of monumental art that the ancient Egyptians were a left-handed race*.

There is a slight—a very slight—predominance of gross physical characters, possibly in the ratio of 75 to 25 for the right side of the body, and this extends to the right side of the brain notwithstanding the decussation of nerves, which it has been held must connote a gross physical pre-eminence of the left hemisphere in manual dextralists.

Appendix of Tables.

I.	Capacity and Internal Breadth.
II.	" " External "
III.	" " Internal Height.
IV.	" " External "
V.	" " Internal Length.
VI.	" " External "
VII.	Length Thickness and Breadth Thickness.
VIII.	" " " Height "
IX.	Breadth " " " "
X.	External Length and External Breadth.
XI.	" Breadth " " Height.
XII.	" Length " " "

* See *Man*, Vol. XXVIII. p. 187, but also Vol. XXIX. p. 24.

XIII.	Internal	Length	and	Internal	Breadth.
XIV.	„	Breadth	„	„	Height.
XV.	„	Length	„	„	„
XVI.	External	Length	and	Internal	Length.
XVII.	„	„	„	„	Breadth.
XVIII.	„	„	„	„	Height.
XIX.	Internal	Breadth	and	External	Height.
XX.	„	„	„	Breadth	Thickness.
XXI.	External	Diametral	Product	and	Capacity.
XXII.	Internal	„	„	„	„

APPENDIX OF TABLES

TABLE I. *Capacity and Internal Breadt*

Internal Breadth in mm. (Central Values

Capacity in cm.³ (Central Values).

	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	Totals
1025	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
1075	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1125	1	—	1	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
1175	—	·5	1·5	·5	1·5	1	1	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
1225	—	1	—	·5	·5	5·5	2	1	2·5	2	1·5	2	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19
1275	—	—	—	3·5	1·5	1·5	3	6	6·25	3·75	10·5	3	·5	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40·5
1325	—	—	—	—	1·5	1·5	5·5	9	3·75	7·25	12	10·5	9·5	5·5	3·5	4	3·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	77·5
1375	—	—	1	1	—	2	2	5·5	7	9·5	14	11	17·5	13	9·25	5·75	5	3	—	2	—	—	—	—	—	—	—	—	—	—	108·5
1425	—	—	—	—	1	1·5	·5	2	5	7·5	17	10·5	20	21·5	19·25	16·25	7	4	1·5	1·5	1·5	1	—	—	—	—	—	—	—	—	138·5
1475	—	—	—	—	—	—	1·5	1	1	2·5	7	5·25	12·75	14·5	15·25	16·25	10	11	6·5	8·5	2·5	2	—	—	—	—	—	—	—	—	117·5
1525	—	—	—	—	—	—	1	·5	·5	·5	1·5	3·75	8·75	11·5	12·75	10·75	14	4	5·75	5·75	7	3	·5	—	·5	·5	—	—	—	—	92·5
1575	—	—	—	—	—	—	—	—	—	—	—	1	1·5	5·5	5·5	4	10·5	7·5	4·75	6·25	9	4·5	1	·5	2·5	—	1	—	·5	·5	66
1625	—	—	—	—	—	—	—	—	—	—	—	—	1·5	1·5	·5	3	3	4	2·5	5·5	1·5	1	1·5	2	·5	—	—	—	—	—	28
1675	—	—	—	—	—	—	—	—	—	—	—	—	—	3	—	1	1	3	2·5	1·5	3·5	3·5	·5	1·5	·5	—	1	—	—	—	22·5
1725	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	1	—	—	—	—	—	—	—	1·5
1775	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	·5	·5	—	—	2
1825	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1875	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1925	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1975	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2025	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	2	1·5	3·5	5·5	6	13	16·5	26	28·5	34·5	63·5	47	72·5	76	67	61	54	37	24	31·5	25·5	15	4·5	4	5	·5	2·5	·5	·5	·5	729

TABLE II. *Capacity and External Breadth.*

External Breadth in mm. (Central Values).

Capacity in cm.^{3} (Central Values)	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	...	161	Totals
1025	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	1
1075	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	—
1125	—	—	·5	1·5	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	3
1175	—	—	—	·5	1·5	1	3	1	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	10
1225	—	—	1	4	2	1	—	1	1·5	3·5	1	2	—	·5	·5	—	1	—	—	—	—	—	—	—	—	—	—	—	—	...	—	19
1275	1	—	1·5	1·5	1	1·5	4	7	5·5	5·5	4·5	3·5	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	40·5
1325	—	—	—	—	2	1·5	3	8·5	6	5	14	8·5	10·5	5	6	4	2·5	1	—	—	—	—	—	—	—	—	—	—	—	...	—	77·5
1375	1	—	1	—	2	2	3	7·5	8	4	13·5	13·5	18·5	9·5	8·5	5·5	6	2	2	1	—	—	—	—	—	—	—	—	—	...	—	108·5
1425	—	—	1	1	—	—	—	2	4·5	8	17.5	12·5	15	24	13	15	11·5	5·5	3	2	1	·5	1·5	—	—	—	—	—	—	...	—	138·5
1475	—	—	—	—	1	1	—	1·5	1·5	2·5	11·5	10·5	12·5	8·5	12	8	15	7	11·5	3·5	4·5	2·5	1·5	1·5	—	—	—	—	—	...	—	117·5
1525	—	—	—	1	—	—	—	1	—	2	6·5	5·5	8·5	4	12	14	5·5	7	6	8	5·5	3	·5	—	2·5	—	—	—	—	...	—	92·5
1575	—	—	—	—	—	—	—	—	—	—	—	3	3·5	2·5	5	3·5	8	4·5	6	8·5	5·5	7·5	2	2·5	3	—	—	—	1	...	—	66
1625	—	—	—	—	—	—	—	—	—	—	—	—	1	3	1	6	3	2	3	2	2	—	1·5	2	1·5	—	—	—	—	...	—	28
1675	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1·5	1	3·75	2·25	5	—	3	2	—	1	·5	·5	—	—	...	—	22·5
1725	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	1	—	—	...	—	1·5
1775	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	...	1	2
1825	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	—
1875	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	—
1925	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	—
1975	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	...	—	—
2025	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	...	—	1
Totals	2	—	5	10	10	8	13	29·5	27	33·5	69·5	59	71·5	60	59	57·5	53·5	33	34	30	18·5	16·5	10	6	9	·5	1·5	—	1	...	1	729

TABLE III. *Capacity and Internal Height.*

Internal Height in mm. (Central Values).

Capacity in cm.³ (Central Values).

	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	Totals
1025	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
1075	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1125	—	—	—	—	1	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
1175	·5	·5	—	—	1	—	1	2	2	—	—	—	—	1·5	1	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	10
1225	—	—	—	—	1	2	1·5	1	2	·5	4	3	2·5	·5	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	19
1275	·25	·25	—	—	1	2·5	3·75	1·75	2	3·5	4	5·75	5·25	4	1·5	—	2	1	1·5	·5	—	—	—	—	—	—	—	—	—	40·5
1325	·25	·25	—	—	1·5	2·5	3·25	8·25	7	6·5	6·5	8·75	7·25	5·5	7·5	6·5	1·5	2·5	2	—	—	—	—	—	—	—	—	—	—	77·5
1375	1	1	1·5	1·5	—	·5	2·5	5·5	4·5	5	9·5	9·75	13·25	15	9	4	6·75	6·75	5·5	3·5	1·5	—	—	—	—	—	·5	·5	—	108·5
1425	—	—	·5	1·5	3	2·5	5·25	1·25	2·5	4·5	7	13·25	6·25	15	18·5	10·5	11·25	11·75	5·25	5·75	3·5	4·5	3	1·5	·5	—	—	—	—	138·5
1475	—	·5	·5	·5	1·5	·5	·25	·75	3·5	4	5	1·5	9	13·5	13·75	9·75	11·25	8·75	8·75	10·25	2·5	3·5	2	3·5	2	·5	—	—	—	117·5
1525	—	—	—	—	·5	1	·5	1·5	2·5	1	5	5·5	4	8·5	9·5	10·5	9·25	7·25	9	4·5	1·5	4·5	3·5	1·5	·5	1	—	—	—	92·5
1575	—	—	—	—	—	·5	·5	1	·5	1	3·5	2·5	2	2·5	·75	4·75	6	7	5·5	9·25	3·75	5	5	2	2·5	·5	—	—	—	66
1625	—	—	—	—	—	—	—	—	·5	·5	—	—	·5	1·5	1	3	3	1·5	3·5	2·75	5·75	2	·5	1	·5	·5	—	—	—	28
1675	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	1·5	·5	2·5	1·5	·5	1·75	1·25	3	1	4	1	1	1	—	1	22·5
1725	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	1	—	—	—	—	—	—	—	1·5
1775	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	1	—	—	—	—	—	—	—	2
1825	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1875	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1925	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1975	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2025	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	3	2·5	2·5	3·5	10·5	12	18·5	23	28	26·5	44·5	50·5	50·5	67·5	65	50	53·5	49	42	39	20	24·5	15	13·5	8	3·5	1·5	·5	1	729

TABLE IV. *Capacity and External Height.*

External Height in mm. (Central Values).

Capacity in cm.³ (Central Values).	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	Totals
1025	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
1075	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1125	—	—	—	—	—	1	—	—	—	·5	·5	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
1175	—	·5	·5	1	—	—	1	1	2	—	1·5	·5	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	10
1225	—	—	—	—	—	·5	2	3·5	2	3	·5	4	·5	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	19
1275	—	—	·5	·5	1	1·5	2·5	1	2	8	1·5	5·5	3·5	5	1	1·5	1·5	2	1	—	1	—	—	—	—	—	—	—	—	—	40·5
1325	—	—	—	—	—	2	3	6	10·5	10	5	7	5	7	7	6	3·5	1·5	1	·5	2·5	—	—	—	—	—	—	—	—	—	77·5
1375	2	—	—	1	1	1·5	2·5	3·5	3·5	7·5	11	10	7	12	9	8	9	9	3	2	4	1	—	—	1	—	—	—	—	—	108·5
1425	—	—	1	2	4	1·25	1·25	2	2	7	7	13	7·5	15·5	14·5	11	11	11	6·5	8·5	2·5	3	2·5	3	·5	—	—	1	—	—	138·5
1475	—	1	—	·5	·5	·25	1·75	1	4	2·5	2	5·5	8·5	8·5	13	14	10	9·5	6	7·5	4	7	3·5	1	3	2	—	1	—	—	117·5
1525	—	—	—	—	—	—	·5	·5	2	·5	5·5	4	4	8	9	10·5	10	9·5	8·5	5·5	4	5	1·5	2	—	1	1	—	—	—	92·5
1575	—	—	—	—	—	—	—	2	—	2	—	3·5	1·5	3	1·5	2	4·5	5·5	9·75	4·25	9	6·5	3·5	1·5	1	1·5	3·5	—	—	—	66
1625	—	—	—	—	—	—	—	—	—	—	1	—	—	2·5	·5	2	1	3·5	2·75	2·75	5·5	1	1	1·5	2	—	—	1	—	—	28
1675	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	—	1	2	·5	1·5	1·5	2	3	1·5	2·5	1	2	—	—	1	22·5
1725	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	1	—	—	—	—	1·5
1775	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	2
1825	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1875	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1925	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1975	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2025	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	2	2·5	2	5	6·5	8	14·5	20·5	28	41·	35·5	53	37·5	64·5	56·5	57	54·5	53·5	39	33·5	34·5	25·5	15	10·5	11	6·5	6·5	4	—	1	729

TABLE V. *Capacity and Internal Length*, $\frac{1}{2}(L_R + L_L)$.

Internal Length in mm. (Central Values).

Capacity in cm.³ (Central Values).	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	Totals
1025	·75	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
1075	—	—	—	—		—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1125	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
1175	—	—		—	—	·5	1·5	·5	·5	2	1	2	·25	·75	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
1225	—	—	—	—	—	—	·5	1·25	·25	1	—	·5	2·25	2·75	4·25	2	·25	1·75	·25	1	—	—	—	—	—	—	—	—	—	·75	—	—	—	—	—	—	—	—	—	—	—	—	—	19
1275	—	—	—	—	—	—	—	—	—	—	—	·75	·25	3	3·5	5·75	3·875	3·125	8	2·25	3·25	2·75	1·75	1·25	—	—	·25	·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40·5
1325	—	—	—		—	—	—	—	—	—	—	—	1·25	1	2·25	4·5	7·625	11·875	11·25	7·25	6·75	11	4	3·25	2·5	2	—	—	—	·75	·25	—	—	—	—	—	—	—	—	—	—	—	—	77·5
1375	—	—	—	—	—	—	—	—	—	—	—	—	1	1·25	2·25	1·5	6·25	4·75	5·25	12	12	16·25	13·375	9·875	7·75	6·25	4	·75	2	1	·75	·25	—	—	—	—	—	—	—	—	—	—	—	108·5
1425	—	—	—	—	—	—	—	—	—	—	—	—	·25	·75	—	1	2	2·75	2·5	6·5	8·25	12·25	15·875	21·25	21	17·375	12·5	5·5	4·75	3	·75	·25	—	—	—	—	—	—	—	—	—	—	—	138·5
1475	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	2·75	4	9·25	15	15·5	11·625	9·625	17·125	9·625	7·75	5	2·75	3	·75	2·25	—	—	1	—	—	—	—	—	—	117·5
1525	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·75	·25	—	—	1	1·75	2·75	1·5	6·875	7·625	8·25	6·625	6·875	14·75	12	8·625	5·625	4·25	·5	·75	1·75	—	—	—	—	—	—	—	92·5
1575	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2·75	2·375	3·875	8·5	7·5	8·25	4·25	9·625	3·375	5·5	2·75	2·25	1·5	2·25	·25	—	—	—	—	—	66
1625	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	2·625	2·75	2·5	1·5	5	2·75	2·75	1·75	2	2·25	·25	1	·75	—	—	—	—	—	28
1675	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·75	1	·75	·75	1·25	·5	2·75	3·75	3	2·25	2·25	·75	1·5	·75	—	—	22·5
1725	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	·5	—	—	—	—	—	—	—	—	—	1·5
1775	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·75	·25	—	·75	·25	—	—	—	—	2
1825	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1875	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1925	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1975	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2025	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	1
Totals	·75	·25	1	1	—	·5	2	1·75	·75	3	1	3·25	5·25	9·5	12·75	16	20·25	25·25	27·75	32·75	36	55·25	51·5	60·75	53	50	52·25	34·25	40	32·5	26·5	16·5	13·75	10·75	9·75	6·75	6·5	4	1	1·5	·75	·5	·5	729

TABLE VI. *Capacity and External Length.*

External Length (Central Values).

Capacity in cm.³ (Central Values).	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	Totals
1025	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
1075	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1125	1	—	1	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
1175	—	—	1	—	—	—	2	3	—	2	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
1225	—	—	1	—	—	3	1·5	1	·5	1	3	1	—	3	2	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19
1275	—	—	1	—	—	1	2	1·5	4	1·5	2	1·75	3	4·25	5·5	2·5	2·5	3	3	·5	·5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40·5
1325	—	—	—	—	—	—	·5	1	2	·5	4·5	7·25	9·5	7·75	11·5	5·5	8·5	7	3	2	1·5	3·5	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	77·5
1375	—	—	—	—	—	1	—	3	1	3	5·5	6	11·5	9·5	13	11·75	6·75	8·5	5	6	5·5	3	2·5	2·5	1·5	·5	1·5	—	—	—	—	—	—	—	—	—	—	108·5
1425	—	—	—	—	—	—	—	1	—	1	2	8·5	6·5	7	7	9·25	12·25	12·5	15·75	11·75	7·5	10·5	6	9·5	4·5	4	1	—	·5	·5	—	—	—	—	—	—	—	138·5
1475	—	—	—	—	—	—	—	·5	·5	2·5	2	1	4·5	5·5	9·5	6	8·5	10	18·25	9·25	6·5	7·5	6·5	3·75	5·75	4·5	3	—	1	—	·5	·5	—	—	—	—	—	117·5
1525	—	—	—	—	—	—	—	—	—	—	1	1	—	·5	5	2·5	10	4·5	5·5	15	4·5	6	5·5	6·25	6·75	7	1·5	4·5	2	2·5	—	1	—	—	—	—	—	92·5
1575	—	—	—	—	—	—	—	—	—	—	—	—	—	1	·5	2	3·5	2	5·5	4	3·5	9·5	5·5	5·5	5	3·5	4	4	2	—	4	1	—	—	—	—	—	66
1625	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	·5	1	·5	1·5	·5	3	5·5	3·5	3	·5	·5	1	2	2	—	2	—	—	—	—	—	—	28
1675	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	3·5	2·5	2	2	2·25	2·75	1·5	—	—	·5	·5	1	1	·5	·5	22·5
1725	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	1	—	—	—	1·5
1775	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	1	—	—	2
1825	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1875	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1925	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1975	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2025	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	1·5	·5	4	—	—	5	6	11	8	11·5	21	26·5	36	38·5	56·5	41·5	53	48	58·5	49	32·5	47·5	34	34	26	22	14·5	13·5	9	3·5	7	3	1·5	2	2	·5	·5	729

To face p. 122

TABLE VII. *Length Thickness and Breadth Thickness.*

Difference of External and Internal Lengths in mm.

Rows: Difference of External and Internal Breadths in mm.

	6	6·5	7	7·5	8	8·5	9	9·5	10	10·5	11	11·5	12	12·5	13	13·5	14	14·5	15	15·5	16	16·5	17	17·5	18	18·5	19	19·5	20	20·5	21	21·5	22	22·5	23	23·5	24	24·5	25	25·5	26	26·5	27	27·5	28	28·5	Totals
2·5	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
3	—	—	—	1	—	·5	1	·5	1	—	—	—	·5	·5	—	—	—	—	1	2	2·5	·5	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	12
3·5	—	—	—	—	—	—	—	1	—	1	1	1	1	1	2	1	1·5	1	2	·5	1·5	·5	—	1	—	—	—	1	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	20
4	—	—	—	—	—	—	1	—	1	—	·5	4·5	3	2	2	2	·5	1	2·5	1	4	1	3	1	1	—	1	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	34
4·5	—	—	—	—	1	·5	·5	—	1	1	·5	·5	2	4	—	1	3	2	1	1	—	1·5	1·5	·5	1	1	1	1	1	1·5	·5	·5	—	—	1	—	—	—	—	—	—	—	—	—	—	—	31
5	—	1·5	·5	1	1·5	3·5	2·5	1·5	1	2·5	4·5	2	3	2	1·5	3	2	4	3	1·5	1·5	1·5	3	4·5	2·5	4·5	1	1·5	2·5	1	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	66
5·5	—	·5	·5	1	1	1·5	·5	2·5	1·5	·5	·5	1·5	2·5	2	2·5	3·5	2	1	1·5	1·5	2	2·5	1	3	2·5	3	1	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	45
6	—	1	1	1·5	·5	1	1	—	·5	3·5	3	3·5	4·5	3·5	2·5	6·5	8·5	3	5	2	2·5	2·5	6	3	1	1	·5	5	3·5	1	—	—	—	1	·5	·5	—	—	—	—	—	—	—	—	—	—	80
6·5	—	1	—	—	—	·5	·5	2·5	·5	1	2·5	4	2	3·5	2·5	4	2·5	4·5	1	4·5	3	4	1·5	2	3	·5	·5	2·5	2	·5	1·5	—	1	—	—	—	1	—	—	—	—	—	—	—	—	1	61
7	1	—	·5	1	1·5	—	·5	3·5	1	·5	2	6·5	1	1·5	5·5	5	5	2·5	4	6·5	3	2·5	1·5	3	5	3	·5	·5	·5	1·5	·5	·5	—	1·5	·5	1	1	—	—	—	—	—	—	—	—	—	75
7·5	—	—	—	—	·5	2·5	3·5	·5	1·5	1·5	1	—	·5	·5	1	1·5	3·5	4	1	2	1·5	2·5	1	3	1·5	2·5	3	1·5	·5	2	·5	·5	2	1	—	—	—	—	—	—	—	—	—	—	—	—	48
8	—	—	—	—	—	—	—	1	2	·5	4	1·5	3	3·5	5	·5	3	4·5	1·5	1·5	3·5	5	4	4	5·5	1·5	2·5	2·5	2·5	1·5	—	·5	·5	1	1	1	·5	·5	—	—	—	—	—	—	—	—	69
8·5	1	—	—	—	—	1	1	—	—	—	·5	2	·5	1·5	2	2	3	3·5	1	2·5	2·5	1	2	3	1·5	·5	—	·5	1	1	·5	—	3	·5	·5	·5	·5	—	—	—	—	—	—	—	—	—	40
9	—	—	·5	·5	—	—	—	—	—	—	1·5	·5	1·5	1·5	2	·5	3	2	2	4·5	1	1·5	3·5	3	1	2	1	—	1·5	1·5	—	1	—	—	·5	·5	1	—	·5	·5	—	—	—	—	—	—	40
9·5	—	—	—	·5	·5	—	—	1·5	1	·5	1·5	·5	1	·5	1	·5	—	—	·5	·5	1	—	—	1·5	1·5	1·5	·5	·5	2·5	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	20
10	—	—	—	—	—	—	1	—	—	—	—	1	—	·5	·5	4	3	—	2	2	1	—	·5	1	1	2	·5	—	2	1	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	25
10·5	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	1	—	2	—	1	—	1·5	2·5	—	—	·5	·5	1	—	—	·5	1	1·5	—	—	—	—	1	—	—	—	—	—	—	—	—	—	15
11	—	—	—	—	—	—	·5	·5	·5	1	1	·5	—	1	—	·5	1	1·5	1	2·5	·5	—	—	2	—	1·5	·5	—	—	—	—	1	—	1	—	—	—	1	—	—	—	—	—	—	—	—	19
11·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	1	1	—	—	1	1	—	·5	·5	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	8
12	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	·5	1·5	—	—	1	—	1·5	·5	·5	2·5	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11
12·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	·5	2	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
13·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
15·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	4	3	6·5	6·5	11	13·5	15	12·5	13·5	24	29·5	26·5	31·5	33	36·5	43·5	36·5	34·5	37·5	34	31·5	28·5	39	31	25·5	17·5	18	21·5	13	8	7	9	6	4	3·5	6	2·5	1·5	·5	—	—	—	—	—	1	729

TABLE VIII. *Length Thickness and Height Thickness.*

Difference of External and Internal Lengths in mm.

Rows: Difference of External and Internal Heights in mm.

	6	6·5	7	7·5	8	8·5	9	9·5	10	10·5	11	11·5	12	12·5	13	13·5	14	14·5	15	15·5	16	16·5	17	17·5	18	18·5	19	19·5	20	20·5	21	21·5	22	22·5	23	23·5	24	24·5	25	25·5	26	26·5	27	27·5	28	28·5	Totals
1·5	—	—	—	·5	·5	·5	·5	1	—	·5	·5	1	—	—	—	1	—	·5	·5	1	1	—	—	·5	·5	—	—	1	—	1	—	1	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	14
2	—	—	—	1	—	—	1	—	—	—	—	—	1	1	3·5	·5	1	—	1·5	1·5	—	—	1	1	—	·5	·5	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	17
2·5	—	—	—	1	—	—	—	—	1	—	—	—	—	·5	1	1·5	2	2	—	1·5	·5	2	·5	1·5	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	18
3	1	—	·5	·5	—	—	—	—	—	—	1	2	3	—	2	1	3·5	2·5	2·5	1	1·5	1	1	1	—	2·5	·5	·5	·5	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	30
3·5	—	—	—	—	1	1·5	1	2·5	·5	2	3·5	2	1·5	1·5	1·5	4	3·5	2·5	·5	—	2	2	1	2	1	—	1	—	·5	·5	·5	·5	1	1	—	—	—	—	—	—	—	—	—	—	—	—	42
4	—	·5	·5	—	—	1	—	2·5	2	3	3	2·5	1·5	2·5	3	4	1	3·5	4	4·5	4	3·5	·5	—	1·5	2	1	2·5	4·5	1·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	60
4·5	—	1	—	·5	1·5	1	2·5	1·5	1·5	2·5	4	5	2	·5	4	1·5	2·5	4	3·5	4	2·5	4	4	3	3	·5	1	1	1	—	1	—	1	·5	1	·5	—	—	—	—	—	—	—	—	—	—	67
5	—	1·5	·5	1	·5	1·5	1·5	2·5	1·5	1·5	1	2·5	5·5	9	1	3·5	5·5	4	6·5	5	4	3·5	5·5	2·5	3·5	5	2	·5	—	1	—	1	—	—	1	1	—	—	—	—	—	—	—	—	—	1	87
5·5	—	—	—	1·5	·5	1·5	1·5	—	—	2	3	5	4·5	4	5	4	4	2·5	1·5	4·5	4·5	3	3·5	3	4·5	2	1·5	3	1	—	·5	1	3	·5	—	—	1	—	—	—	—	—	—	—	—	—	77
6	—	—	—	—	—	—	1	—	2·5	·5	2·5	1·5	4·5	2·5	·5	4·5	8·5	5	4·5	1·5	1·5	1	3·5	6·5	3	5	1·5	2·5	1·5	3	3	·5	1	—	—	—	·5	1·5	—	—	—	—	—	—	—	—	75
6·5	1	·5	1	·5	—	·5	1	1·5	·5	·5	1·5	2	1	1	3	3·5	5·5	2	5	4	3·5	3	3	4	4	2	1	2·5	1·5	·5	1·5	1·5	1	·5	·5	1·5	1	—	1	—	—	—	—	—	—	—	69
7	—	—	—	—	1·5	3·5	·5	2·5	—	·5	2·5	2	·5	3	4	4·5	3·5	3	1	2	3	1	1·5	5	4	1·5	2	1·5	3	1·5	·5	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	60
7·5	—	·5	·5	—	—	—	2	1	2	·5	·5	2	—	3	2	—	1	3	·5	3	2·5	4	1·5	3·5	1	·5	3	·5	2	—	·5	·5	1	1	—	—	3	—	—	—	—	—	—	—	—	—	46
8	—	—	—	—	—	—	1	—	1	—	·5	1·5	·5	2·5	—	2	·5	·5	1	1·5	1	1·5	1	1·5	1	1	·5	—	1·5	1·5	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	24
8·5	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	·5	1	·5	1·5	2	1·5	2	—	·5	1·5	1	1	·5	2·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	18
9	—	—	—	—	—	—	—	—	—	—	·5	·5	—	·5	1·5	·5	·5	—	·5	·5	—	—	·5	2	·5	·5	·5	1	1	·5	·5	1	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	14
9·5	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	1	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
10	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	1·5	1	·5	·5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	5
Totals	2	4	3	6·5	6·5	11	13·5	15	12·5	13·5	24	29·5	26·5	31·5	33	36·5	43·5	36·5	34·5	37·5	34	31·5	28·5	39	31	25·5	17·5	18	21·5	13	8	7	9	6	4	3·5	6	2·5	1·5	·5	—	—	—	—	—	1	729

TABLE IX. *Breadth-Thickness and Height-Thickness.*

Difference of External and Internal Breadths in mm.

Difference of External and Internal Heights in mm.	2·5	3	3·5	4	4·5	5	5·5	6	6·5	7	7·5	8	8·5	9	9·5	10	10·5	11	11·5	12	12·5	13	13·5	14	14·5	15	15·5	Totals
1·5	—	1	1	1	1	1	1	—	—	1	1	2	—	1	1	—	1	1	—	—	—	—	—	—	—	—	—	14
2	—	1	—	1	2	1	—	1	1	1	2	2	1	—	1	2	—	1	—	—	—	—	—	—	—	—	—	17
2·5	—	—	—	1	3	1	1	2	4	1	1	1	—	1	—	—	—	1	1	—	—	—	—	—	—	—	—	18
3	—	—	2	—	3	6	2	1	3	2	1	2	2	2	1	1	1	1	—	—	—	—	—	—	—	—	—	30
3·5	—	—	1	1	4	7	2	3	4	3	3	2	4	1	1	1	2	1	—	—	1	1	—	—	—	—	—	42
4	—	1	3	3	3	3	4	10	6	7	5	7	5	1	1	—	—	—	—	1	—	—	—	—	—	—	—	60
4·5	—	2	4	6	2	7	—	5	10	8	5	5	4	4	1	—	2	—	1	1	—	—	—	—	—	—	—	67
5	—	—	3	5	4	8	7	12	6	9	7	8	2	4	1	3	2	4	1	1	—	—	—	—	—	—	—	87
5·5	1	2	2	5	3	6	7	9	7	5	7	9	4	3	3	1	—	1	1	—	1	—	—	—	—	—	—	77
6	—	—	1	4	2	4	6	12	6	9	6	8	4	5	—	4	2	1	—	1	—	—	—	—	—	—	—	75
6·5	—	1	1	3	2	4	4	9	3	9	4	9	4	7	3	2	1	1	—	2	—	—	—	—	—	—	—	69
7	—	1	1	2	1	9	5	5	3	8	2	7	2	3	2	3	—	1	2	2	1	—	—	—	—	—	—	60
7·5	—	2	1	1	—	3	4	3	7	4	2	2	4	2	2	2	2	2	—	1	—	1	1	—	—	—	—	46
8	—	—	—	—	—	4	—	7	—	2	—	3	—	1	—	4	—	1	—	—	1	—	—	—	—	—	1	24
8·5	—	1	—	1	—	—	1	—	1	3	—	—	2	3	1	—	2	1	—	1	—	—	—	—	—	1	—	18
9	—	—	—	—	—	1	1	—	—	1	2	1	2	1	1	1	—	1	1	1	—	—	—	—	—	—	—	14
9·5	—	—	—	—	1	—	—	1	—	2	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	6
10	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	1	—	1	1	—	—	—	—	—	—	—	—	5
Totals	1	12	20	34	31	66	45	80	61	75	48	69	40	40	20	25	15	19	8	11	4	2	1	—	—	1	1	729

TABLE X. *External Length and External Breadth.*

External Length in mm. (Central Values).

External Breadth in mm. (Central Values).	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186
126	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	1
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	—	—	1	—	—	—	·5	·5	—	—	—	·25	·25	—	—	·5
129	·25	·25	1	—	—	1	·5	·5	—	—	·5	1	—	—	·25	2·25	·5	·5	1
130	·25	·25	1	—	—	—	1	—	·5	1	1	1	1	—	—	—	—	1	—
131	—	—	—	—	—	—	—	1	—	—	1	—	2	—	·5	1	1	·5	1
132	—	—	—	—	—	—	1	3	—	—	—	—	—	2·5	2·5	1·5	1	—	—
133	—	—	—	—	—	—	1	1·5	2·5	1	1	1·5	2	2·5	2·5	2	2·5	1·5	2·5
134	—	—	1	—	—	1	1	—	1·25	·25	1	1·5	4	2	1·75	1·5	2·75	2	1
135	1	—	—	—	—	1	·5	1·5	1·75	1·75	2·25	2·25	1·5	2	5·75	1·5	·75	·25	·75
136	—	—	—	—	—	—	1	1	—	·5	1·75	2·75	5	3·5	5	3	4·5	8	9
137	—	—	—	—	—	—	—	·5	—	1	·5	1·5	4·5	6·5	2·25	3·75	6	4·25	8·25
138	—	—	—	—	—	—	—	1	—	·5	2	4·25	3·75	4·5	9·25	6·5	4	5·75	6·5
139	—	—	—	—	—	—	—	·5	·5	3·75	4·75	2·5	5	3·5	3·75	4	2·5	5	5·25
140	—	—	—	—	—	—	—	·5	1·5	·25	1·75	·75	·75	3·25	7·5	1·25	4·75	4·5	3·75
141	—	—	—	—	—	—	—	—	—	—	—	2	1	2·75	2·75	2·5	6·75	3·75	2
142	—	—	1	—	—	1	—	—	—	—	2	2	—	2	3·5	1·5	4·5	4	3·5
143	—	—	—	—	—	—	—	—	—	—	1	3	2	1·5	3	—	—	1	4
144	—	—	—	—	—	—	—	—	—	1	—	·5	·5	1	·5	1·5	3	1	6
145	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	3	2	1	1
146	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2·25	1·75	2·5	2	—
147	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1·25	1·25	1·5	—	1·25
148	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	·5	·25
149	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	1	—
150	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	·5	—
151	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
152	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
153	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
154	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
155	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
156	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
157	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
158	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
159	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1·5	·5	4	—	—	5	6	11	8	11·5	21	26·5	36	38·5	56·5	41·5	53	48	58·5

External Breadth in mm. (Central Values).	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	Totals
126	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
127	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
128	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	5
129	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
130	1	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	10
131	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
132	1	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	13
133	2	·75	1·25	—	—	—	—	—	·5	1	—	—	—	—	—	—	—	—	29·5
134	1	—	1	1·5	1	—	—	—	—	·5	—	—	—	—	—	—	—	—	27
135	·5	—	1	2	1·5	1	1	·5	1	—	—	·25	·25	—	—	—	—	—	33·5
136	6	3·5	4	1·25	3·25	3·5	1	·5	—	—	1	·25	·25	—	—	—	—	—	69·5
137	4·5	2	7·5	1·75	·75	1·5	1	—	1	—	—	—	—	—	—	—	—	—	59
138	4·5	5	4·5	3	3·5	—	2	1	—	—	—	—	—	—	—	—	—	—	71·5
139	2·5	1·5	5	3	1	1	1	2	—	—	—	1	—	—	—	—	·5	·5	60
140	6·5	4·5	1	5	1·5	3	1·5	2	—	·5	1	2	—	—	—	—	—	—	59
141	6·5	2·5	3·5	2·5	5	3	4	1	2	1·5	·5	2	—	—	—	—	—	—	57·5
142	4	5·5	5	·75	1·75	2	2·75	3·25	·75	1·75	—	1	—	—	—	—	—	—	53·5
143	·5	—	2·25	4	2·75	1·5	·75	·5	3	1·75	·5	—	—	—	—	—	—	—	33
144	3	4	3·75	1·75	1·5	3	1·5	·25	·25	—	—	—	—	—	—	—	—	—	34
145	1·5	·5	2·5	·5	6·75	1·75	1·75	·75	2	—	—	—	—	—	1	1	—	—	30
146	2	1	2·5	1	1·25	·25	1·25	·5	·25	—	—	—	—	—	—	—	—	—	18·5
147	·25	—	·5	2	—	2	1	·75	1·75	1	—	—	·5	·5	—	—	—	—	16·5
148	·25	·5	·5	2	·5	1·5	1	—	—	—	—	—	—	—	—	1	—	—	10
149	1	·5	—	·5	—	—	—	—	—	1	—	—	—	—	—	—	—	—	6
150	—	·5	1·5	·5	2	—	—	—	1	—	—	—	1	1	—	—	—	—	9
151	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	·5
152	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	1	—	—	—	1·5
153	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
154	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
155	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
156	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
157	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
158	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
159	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
160	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
161	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	1
Totals	49	32·5	47·5	34	34	26	22	14·5	13·5	9	3·5	7	3	1·5	2	2	·5	·5	729

TABLE XI. *External Breadth and External Height.*

External Breadth in mm. (Central Values).

External Height in mm. (Central Values).

	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	Totals
120	—	—	—	—	—	—	—	—	1	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
121	—	—	—	·5	·5	—	—	—	—	·5	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2·5
122	—	—	—	—	—	—	—	—	—	·5	1	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
123	—	—	—	·5	·5	—	—	—	—	—	—	—	—	2·5	·5	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
124	—	—	—	—	—	—	—	—	1	—	—	—	1·5	1·5	·5	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·5
125	—	—	—	—	·5	1	—	1	—	1	—	1	1	·5	·5	—	1	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
126	—	—	·5	1	·5	·5	1	·5	1·5	—	2·5	·5	·5	·5	3	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14·5
127	—	—	·5	—	—	—	1·5	—	1·5	1·5	2	3·5	2·5	1·75	·75	—	—	—	2·5	1	·5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20·5
128	—	—	—	—	2	1	·5	3·5	·5	3·5	2·5	3	2	·75	2·25	1·75	·75	1·5	—	—	·5	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	28
129	·5	—	·75	1·75	1	1	3	3·5	1	1·5	8·25	5·25	3	1·5	1·5	·75	4	·75	—	1	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	41
130	·5	—	·25	·25	—	—	·5	1	2	1·5	7·75	3·75	4	4·5	3·5	—	1·75	1·25	—	1·5	1·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	35·5
131	—	—	1	1·5	—	1	1·5	4	3·5	3	5·25	4·5	5·25	6	1·5	2	5	2·5	·5	1·5	2	1	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	53
132	—	—	—	1	2	—	1	1	1	4	6·25	2·5	5·75	2·5	2·5	2	1	1·5	—	1·25	·25	·5	1·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	37·5
133	—	—	1	·5	2	—	—	2·25	4	1·75	8	5	8	3	2·75	8·75	4·75	2·75	3·5	2·25	·5	1·25	·25	1·25	—	—	—	—	1	—	—	—	—	—	—	—	64·5
134	—	—	—	—	—	2	2	·75	3	3	3	4·75	6·5	6	7·75	2·75	7·25	2·5	·25	1	2·25	1·25	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	56·5
135	—	—	1	1	—	·5	·5	2	1·5	5	4·75	5·25	3·5	4·5	5	6·5	3·5	2·75	3·25	3·5	·5	·5	·5	—	1·5	—	—	—	—	—	—	—	—	—	—	—	57
136	—	—	—	1	—	1	—	5	1	1·75	4·75	2	4·5	5	3	8	4·5	2	4	1	3	1·5	·5	—	1	—	—	—	—	—	—	—	—	—	—	—	54·5
137	—	—	—	—	—	—	1	1	2	1	4	3·5	6·75	3·25	7	8	3·5	4·5	2·5	—	1	2	2·5	—	—	—	—	—	—	—	—	—	—	—	—	—	53·5
138	—	—	—	—	1	—	—	—	·25	2·25	3	4·25	3	3·25	4	3	4	1·5	2·5	2	2·5	1·5	—	—	1	—	—	—	—	—	—	—	—	—	—	—	39
139	—	—	—	—	—	—	·25	·25	·25	·25	1	2·25	2·75	5	6	3	3	1·5	4·5	1·5	·5	—	1·5	—	—	—	—	—	—	—	—	—	—	—	—	—	33·5
140	1	—	—	—	—	—	·25	2·25	·5	1·5	1	4	4·5	1	·5	4	1·5	2	2	—	—	2	1	2·5	2	·5	·5	—	—	—	—	—	—	—	—	—	34·5
141	—	—	—	—	—	—	—	1·5	·5	—	2·5	1·5	1·5	1·5	3	1·5	4·25	1·25	1·5	4	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	25·5
142	—	—	—	1	—	—	—	—	—	—	—	1	1·5	1·25	1·25	1	·75	1·25	2·25	2·75	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
143	—	—	—	—	—	—	—	—	—	—	1	1	1	·75	·25	2	—	—	1·75	1·25	1	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10·5
144	—	—	—	—	—	—	—	—	1	—	1	—	—	—	1	—	·5	1·5	·5	2	1	·5	—	·5	·5	—	—	—	—	—	—	—	—	—	—	1	11
145	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1·5	1·5	—	—	1	—	·5	—	—	—	—	1	—	—	—	—	—	—	—	—	—	6·5
146	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1·5	·5	1·5	—	—	1	—	—	—	—	—	—	—	—	—	—	—	6·5
147	—	—	—	—	—	—	—	—	—	—	—	·5	·5	1	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	4
148	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
149	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	—	5	10	10	8	13	29·5	27	33·5	69·5	59	71·5	60	59	57·5	53·5	33	34	30	18·5	16·5	10	6	9	·5	1·5	—	1	—	—	—	—	—	—	1	729

TABLE XII. *External Length and External Height.*

External Length in mm. (Central Values).

External Height in mm. (Central Values).

	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186
120	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
121	·5	·5	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	·5
122	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	·5	—	—	—	—
123	—	—	—	—	—	—	—	—	—	—	1	—	—	—	·5	—	—	1	1
124	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	·5	2	·5	—
125	1	—	—	—	—	1	—	—	—	·25	·25	·25	1·25	1	1·5	—	—	—	—
126	—	—	—	—	—	1	—	1·5	—	·5	·5	·25	1·25	—	2·5	·5	2	1	1
127	—	—	·5	—	—	1	1	·5	—	·25	2·25	·25	2·25	·5	2·5	1	2	1·5	1
128	—	—	1·5	—	—	—	1·5	·5	—	1	2	·25	2·25	2·5	2·5	2·5	2·5	1·5	·25
129	—	—	—	—	—	1	·5	·5	2	1·5	1·5	3·5	4	4	2·25	·5	1·25	4	1·75
130	—	—	·5	—	—	—	1	·5	·5	·5	1	·5	4·5	3	2·75	3·5	1·25	4	·5
131	—	—	·5	—	—	1	1	2·5	1	·5	2·5	2·5	1·25	5·25	4	2	3·5	4·5	3
132	—	—	—	—	—	—	—	·5	1	1·5	1	1·5	3·75	2·75	4·25	5·25	1·5	2	2
133	—	—	—	—	—	—	—	1	—	—	—	1·5	2·5	1	8·25	6·5	6·25	4·5	5
134	—	—	—	—	—	—	—	1	·5	1	3	5·5	1·5	2·5	6	2·75	5·75	3·5	5
135	—	—	—	—	—	—	—	1·5	—	1	·75	5·25	1	3·5	—	5·5	5·5	3·5	7·25
136	—	—	1	—	—	—	1	—	1	1	·25	1·25	3	1·5	1	2·75	3·75	5	7·25
137	—	—	—	—	—	—	—	1	1	—	2·25	1·75	—	2	5·5	2·75	6·25	3·75	1·75
138	—	—	—	—	—	—	—	—	—	—	·25	·75	1	—	4	·5	2	1·75	5·75
139	—	—	—	—	—	—	—	—	—	1	1	—	—	3·5	2·5	—	—	2	2·5
140	—	—	—	—	—	—	—	—	1	—	—	·5	1·5	1	2	2·5	2	1·5	4
141	—	—	—	—	—	—	—	—	—	·5	·5	1	—	1	1·5	1	2·5	2	3
142	—	—	—	—	—	—	—	—	—	·5	—	—	—	·5	·25	·25	—	—	3
143	—	—	—	—	—	—	—	—	—	·5	—	—	1	1	·25	·25	2	—	1·5
144	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	·5	1
145	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	1	—	—
146	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—
147	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—
148	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
149	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1·5	·5	4	—	—	5	6	11	8	11·5	21	26·5	36	38·5	56·5	41·5	53	48	58·5

	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	Totals
120	·5	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
121	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2·5
122	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
123	—	—	—	·5	—	—	1	—	—	—	—	—	—	—	—	—	—	—	5
124	1	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·5
125	—	—	1	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	8
126	—	—	—	·5	·25	·75	1	—	—	—	—	—	—	—	—	—	—	—	14·5
127	—	—	—	2·5	·25	·25	—	—	·5	·5	—	—	—	—	—	—	—	—	20·5
128	·75	1·75	1·75	—	1	—	1	—	—	·5	·5	—	—	—	—	—	—	—	28
129	2·75	4·25	2·25	·5	·5	—	·75	1·75	—	—	—	—	—	—	—	—	—	—	41
130	4·75	1·25	1·5	·5	2	—	·25	·25	—	—	—	·5	·5	—	—	—	—	—	35·5
131	2·75	·75	5	·5	3·5	·5	2·5	1	—	·25	·25	1	—	—	—	—	—	—	53
132	2·5	·5	2	—	1	1·5	·75	·75	—	·25	1·25	—	—	—	—	—	—	—	37·5
133	5	4·5	2·5	3	4·5	1·5	2·25	1·25	·5	1	—	—	1·5	·5	—	—	—	—	64·5
134	4	3	3·5	3	1·5	·25	1·75	—	1	·5	—	—	—	—	—	—	—	—	56·5
135	2·25	1·5	1·5	5·5	3	2·25	2·25	—	1	1	1	1	—	—	—	—	—	—	57
136	5·75	1	3·5	4·5	2·25	4·25	·5	1·5	1	·5	—	—	—	—	—	—	—	—	54·5
137	4	2	7	·5	2·75	1·75	1	2·5	1·5	1	—	1·5	—	—	—	—	—	—	53·5
138	3·5	3·75	2·5	1·25	4·5	4·25	·75	1	·5	—	—	1	—	—	—	—	—	—	39
139	1·5	4·25	3	1·25	3	3·25	1·75	—	1·5	—	—	·5	—	—	—	1	—	—	33·5
140	2	1	4·5	2	1	—	3	1·5	2·5	1	—	—	—	—	—	—	—	—	34·5
141	3	·5	·5	1	1	3·5	—	2	—	1	—	—	—	—	—	—	—	—	25·5
142	2	1	2	2	—	·5	—	—	2	—	—	—	—	—	1	—	—	—	15
143	—	—	1·5	·5	—	—	1	—	—	—	—	—	—	—	—	1	—	—	10·5
144	—	1	1	1·5	·5	1	·5	—	·5	·5	·5	·5	—	—	—	—	—	—	11
145	—	—	—	·5	1	—	—	—	—	1	—	—	—	—	1	—	—	—	6·5
146	—	—	1	1	·5	—	—	1	1	—	—	—	1	—	—	—	—	—	6·5
147	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	4
148	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
149	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	1
Totals	49	32·5	47·5	34	34	26	22	14·5	13·5	9	3·5	7	3	1·5	2	2	·5	·5	729

TABLE XIII. *Internal Length and Internal Breadth.*

Internal Length in mm. (Central Values).

Internal Breadth in mm. (Central Values).	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	Totals
119	·75	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
120	—	—	—	—	—	·125	·375	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5
121	—	—	—	1	—	·125	·375	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·75	·25	—	—	—	—	—	—	—	—	—	—	—	3·5
122	—	—	—	—	—	·125	·375	—	—	—	—	—	—	—	·25	1·25	—	·5	·5	·375	·125	—	—	1	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5·5
123	—	—	—	—	—	·125	·375	—	—	·5	·5	—	—	·25	·5	·25	·375	·375	1	·625	·125	—	—	—	—	—	·5	—	—	—	·375	·125	—	—	—	—	—	—	—	—	—	—	—	6
124	—	—	—	—	—	—	—	·75	·25	1	—	—	·75	1·25	2	·875	·5	·625	1	·5	1	1	—	·25	·25	—	·5	—	—	—	·375	·125	—	—	—	—	—	—	—	—	—	—	—	13
125	—	—	—	—	—	—	—	·25	·25	—	—	·5	—	—	·125	1·5	1·125	1·375	1·375	—	·5	3	1·25	·5	·5	·25	—	—	1·5	—	1	·5	·5	·5	—	—	—	—	—	—	—	—	—	16·5
126	—	—	—	—	—	—	—	·25	·25	—	—	·5	—	·375	1	1·375	·375	1·75	4·375	1·75	1	2·25	2·25	2·5	1·5	·75	·75	—	·5	·75	·75	1	—	—	—	—	—	—	—	—	—	—	—	26
127	—	—	·5	—	—	—	—	—	—	·75	·25	·375	·375	2·375	·875	1	1·625	1·875	2·5	1·5	1·25	1·75	1·5	·5	2·75	3·75	1	·5	·5	·5	—	·125	·375	—	—	—	—	—	—	—	—	—	—	28·5
128	—	—	·5	—	—	—	—	—	—	·75	·25	·875	1·125	—	·875	·125	—	1·375	3·25	3·375	3·625	3·625	3·375	1	3·375	1·5	2·5	2	·375	·125	—	·125	·375	—	—	—	—	—	—	—	—	—	—	34·5
129	—	—	—	—	—	—	—	—	—	—	—	—	·5	1·375	2·75	2·125	4	3·625	4	4·375	4·875	4	4	6	6·375	4·75	5·75	2	·875	·625	·5	—	—	1	—	—	—	—	—	—	—	—	—	63·5
130	—	—	—	—	—	—	·5	·5	—	—	—	—	—	1·125	1·375	·75	2·75	3·375	2	1·125	2·75	5·75	4	1·375	3·25	4·125	2	1·5	1·375	3·875	2	1·5	·125	·375	—	—	—	—	—	—	—	—	—	47
131	—	—	—	—	—	—	—	—	—	—	—	—	·625	1·125	·5	1·75	3·75	4·125	2·125	4·25	4·25	4·25	5·25	8·75	6·75	4·125	4·625	1·25	4·875	3·625	·75	1·75	·375	1·625	·5	·5	1	—	—	—	—	—	—	72·5
132	—	—	—	—	—	—	—	—	—	—	—	—	·625	·875	·5	—	·25	·5	2·125	3·625	1·875	6·375	4·75	9·25	6·75	7·25	8·5	2·875	5·125	4·125	2·625	1·5	2·25	1·5	1	·75	·75	·25	—	—	—	—	—	76
133	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	3	2	1·25	1·75	3·5	2·75	3·375	7·125	9·125	4·5	4·625	4	4	6	1·875	·375	1·25	1	·375	·625	·625	1·875	—	—	—	—	—	—	67
134	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	·5	·5	·5	1·75	4·375	3·375	4·375	9·875	5·25	5·75	4·75	3·5	4·5	3·5	2·5	·25	1·625	·5	·75	·5	·375	—	·25	·75	—	—	—	61
135	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	2·375	1·875	·875	2·75	4·5	6	1·375	3·375	2·375	3·5	1·125	2·625	5·625	4·375	5·875	1·25	·875	·625	1·25	·75	·375	·125	—	—	—	—	—	54
136	—	—	—	—	—	—	—	—	—	—	—	—	·25	·75	—	·125	·375	1·125	·375	—	1·5	2·125	3·875	1·875	1·5	3·125	4	3	1·75	1·625	2·25	1·875	1	·125	1·5	1·375	·625	·875	—	—	—	—	—	37
137	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·125	2·25	1·625	1·125	1·875	2·5	3·75	2·25	1	1	1·25	·5	1·25	·125	·375	—	·25	1·25	·25	·25	—	—	—	24
138	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1·25	·875	3·125	3·25	1·75	5	·875	1·625	2·75	·5	1	3·625	1·375	1	·25	·5	·25	—	·875	·375	·25	—	—	—	31·5
139	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·75	·25	—	—	·5	—	1	3·5	·5	—	1·875	3	1·5	2·375	1·75	1	1	1	2·25	1·5	·75	·5	·375	·125	—	—	—	—	25·5
140	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	2	—	1·5	·5	·25	·625	1·5	2·25	·875	—	1·5	1·5	1	—	—	—	—	—	·25	·75	—	—	15
141	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	·625	·625	2·125	—	—	·5	·5	—	—	—	—	—	—	—	—	—	4·5
142	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	·375	·625	·125	·375	—	—	—	—	·5	—	·75	·25	—	—	—	—	—	4
143	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·25	·75	·125	·375	—	—	—	—	·5	1	—	—	—	—	—	·5	·5	5
144	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
145	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	·5	·5	—	—	·375	·125	—	—	—	—	—	—	—	2·5
146	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·375	·125	—	—	—	—	—	—	—	·5
147	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
148	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
Totals	·75	·25	1	1	—	·5	2	1·75	·75	3	1	3·25	5·25	9·5	12·75	16	20·25	25·25	27·75	32·75	36	55·25	51·5	60·75	53	50	52·25	34·25	40	32·5	26·5	16·5	13·75	10·75	9·75	6·75	6·5	4	1	1·5	·75	·5	·5	729

TABLE XIV. *Internal Breadth and Internal Height.*

Internal Breadth in mm. (Central Values).

Internal Height in mm (Central Values).	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	Totals
115	1	—	—	—	—	—	—	·25	1·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
116	—	—	—	—	—	—	—	·25	·75	—	—	—	—	—	—	·25	·25	—	—	1	—	—	—	—	—	—	—	—	—	—	2·5
117	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	·25	·5	·25	—	—	·5	—	—	—	—	—	—	—	—	—	—	2·5
118	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5	1	·5	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	3·5
119	—	—	—	—	—	1	·5	·5	·5	·5	—	1·5	1	·5	2·25	1·25	·5	·25	·25	—	—	—	—	—	—	—	—	—	—	—	10·5
120	—	—	—	—	—	—	1	—	—	1	2	·5	1·5	—	1·5	—	2	1·75	·25	·5	—	—	—	—	—	—	—	—	—	—	12
121	—	·25	·25	—	—	·5	·25	·75	1·25	1·25	3·5	1	3	·5	1·5	·5	—	1·5	·5	1	1	—	—	—	—	—	—	—	—	—	18·5
122	—	·25	·25	·25	·25	1·75	1·25	1·5	·75	·5	2·75	1	1·75	2·75	2·5	2·5	1·25	·25	—	—	1·5	—	—	—	—	—	—	—	—	—	23
123	1	—	—	·75	·25	1·75	1	2·25	·5	3·5	2·5	3·75	3	·75	1	1·5	1·75	·25	—	·5	·5	1·5	—	—	—	—	—	—	—	—	28
124	—	—	—	1·5	—	·5	—	—	·25	3·25	2	2·25	1·5	3·25	2·5	2	2·75	1·75	—	2	—	·5	—	—	·5	—	—	—	—	—	26·5
125	—	—	—	—	·25	1·25	1·75	3·75	1·75	3·5	3·75	1	5·75	4	3·75	1·75	1·5	2·75	1	2	2·5	2	—	—	·5	—	—	—	—	—	44·5
126	—	—	—	—	·25	·75	1·25	2·25	4·25	2·25	4·5	4·5	6·25	8·5	4	3	2	2	·75	·5	2·25	1·25	—	—	—	—	—	—	—	—	50·5
127	—	·5	—	·75	·75	1·25	1·75	1	2·25	3	6·75	4·75	4	6	1·5	5·75	3·75	1·25	1	·5	2·25	1·25	·25	·25	—	—	—	—	—	—	50·5
128	—	·5	·5	·25	2·5	1	·75	1·5	2·5	3·5	8·5	4·75	6·25	5	6·25	8·25	5	2·25	3·25	1·5	1·5	·5	·25	·25	—	—	—	—	·5	·5	67·5
129	—	—	1·5	—	1·25	·75	2·25	3·25	3·75	3·75	6·5	5·75	6·25	4	4·75	5·5	5	1·25	2·25	4·25	1·5	—	·5	—	·5	·5	—	—	—	—	65
130	—	—	·5	—	—	1	1·75	2·25	2·75	1	3·75	3·25	5·25	6·75	3·5	4·75	4·25	1	1·25	2	2·5	·5	·5	·5	1	—	—	—	—	—	50
131	—	—	·5	1	—	—	1	3·25	1·75	2	4·5	2	7·25	8	7	2·75	2·5	1·75	2	1·25	1	2·5	—	1	·5	—	—	—	—	—	53·5
132	—	—	—	—	·25	1·25	—	·75	1·75	·5	2	6	5·75	7·25	6·5	3·25	4·75	2·25	2·25	2	2	·5	—	—	—	—	—	—	—	—	49
133	—	—	—	—	·25	·25	·5	2	1	·75	2·75	1·5	5·25	5	6·75	1·5	4	2·75	·75	3·75	·75	1	—	1	·5	—	—	—	—	—	42
134	—	—	—	—	—	—	·5	—	·25	1·75	2	1	4·75	2·25	4·75	4·75	1	5	1·5	2·75	2·25	1·25	·25	1	·5	—	1·5	—	—	—	39
135	—	—	—	1	—	—	—	—	·25	1·25	1·5	—	·5	·5	1·25	2·75	1	3	2·5	2	1	·25	·75	—	—	—	·5	—	—	—	20
136	—	—	—	—	—	—	1	—	—	·75	2·75	1·5	1·5	2	·5	2·5	3	1	1·5	1	2	·5	2	—	—	—	·5	·5	—	—	24·5
137	—	—	—	—	—	—	—	—	—	·5	·5	—	·5	2·5	·5	1·5	3·75	2·25	2·75	·25	—	—	—	—	—	—	—	—	—	—	15
138	—	—	—	—	—	—	—	—	—	—	—	1	1	3	1·5	1·5	1·25	1·25	·25	1·75	·75	·25	—	—	—	—	—	—	—	—	13·5
139	—	—	—	—	—	—	—	—	—	—	1	—	—	·5	2	1·75	·75	·5	—	—	·25	·25	—	—	1	—	—	—	—	—	8
140	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·75	1·75	1	—	—	—	—	—	—	—	—	—	—	—	—	3·5
141	—	—	—	—	—	—	—	·25	·25	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5
142	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
143	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	2	1·5	3·5	5·5	6	13	16·5	26	28·5	34·5	63·5	47	72·5	76	67	61	54	37	24	31·5	25·5	15	4·5	4	5	·5	2·5	·5	·5	·5	729

To face p. 122

TABLE XV. *Internal Length and Internal Height.*

Internal Length in mm. (Central Values).

Rows: Internal Height in mm. (Central Values).

	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	Totals
115	·75	·25	—	—	—	—	—	—	—	—	—	—	·125	·375	—	—	—	—	·25	·75	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
116	—	—	—	—	—	—	—	—	—	—	—	—	·125	·375	—	—	—	—	·25	·25	—	·5	·75	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2·5
117	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	1	·25	·5	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2·5
118	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	1	—	·25	·75	·25	·25	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3·5
119	—	—	1	—	—	—	—	·75	·25	—	—	1	—	—	·25	·25	—	·5	—	·75	·75	—	1	—	—	1·25	1	·75	—	·25	·75	—	—	—	—	—	—	—	—	—	—	—	—	10·5
120	—	—	—	—	—	—	—	—	—	—	—	—	·125	·625	1	1·5	·75	1	·25	2·75	·25	1·25	·5	—	—	·5	—	·5	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	12
121	—	—	—	—	—	·125	·375	·25	·25	—	—	—	1·5	·5	·5	1·25	3·125	2·375	·5	·25	·5	1	2	1	·125	1·375	·25	1·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	18·5
122	—	—	—	—	—	·25	·75	·25	·25	·5	—	—	1·375	·125	1·5	2·625	1·25	1·5	·875	1·5	2·25	·25	·75	1·25	1·625	1·125	—	·75	·75	—	—	—	1·25	·25	—	—	—	—	—	—	—	—	—	23
123	—	—	—	—	—	·125	·375	—	—	1·25	·25	—	—	·125	1·375	3·375	·625	2·5	2·25	1·375	1·375	4	·75	1·5	2·25	·5	—	1·25	1·5	·25	—	—	·5	·375	·125	—	—	—	—	—	—	—	—	28
124	—	—	—	—	—	—	—	—	—	—	—	·375	·125	·125	·375	1·5	4·375	1·5	1	1·5	1·875	3·25	1·625	1·75	2	1·125	·5	·75	1·75	·5	—	—	—	·375	·125	—	—	—	—	—	—	—	—	26·5
125	—	—	—	—	—	—	—	—	—	—	—	·375	·25	2·5	2·125	·25	1·75	3·5	3·125	1·875	3·25	2	3·625	4·125	3·625	1·375	4·5	·75	1·125	·875	1	1·5	·5	—	·125	·375	—	—	—	—	—	—	—	44·5
126	—	—	—	—	—	—	·5	·5	—	—	—	·25	·375	1·75	1·125	1	·875	2	2·375	3·25	4·625	5·75	3·625	2·75	4·625	1·875	2·25	1·875	3·375	2·125	1·375	·25	—	—	·125	·375	—	—	—	·125	·375	—	—	50·5
127	—	—	—	—	—	—	—	—	—	—	—	·25	·25	·75	1·25	1·75	1	2·375	2·5	2·125	3·25	5·5	5·5	6	3·125	2·75	2·125	3·875	2·875	1·125	1·625	—	—	—	—	—	—	—	—	·125	·375	—	—	50·5
128	—	—	—	—	—	—	—	—	—	·5	·5	·5	—	·5	·75	·5	1·875	1·75	2·875	4	5·125	4·25	4·875	9·375	7·875	4·875	6·25	3·375	3·25	2·25	1·5	·75	—	—	—	—	—	—	—	—	—	—	—	67·5
129	—	—	—	1	—	—	—	—	—	·375	·125	·5	—	·875	1·125	·375	1·75	2·25	1·75	1·375	2·25	8·25	5·375	6·75	5·5	6·25	4·75	3·125	3·375	1·5	1·5	1·75	·625	1·125	·375	—	—	1	—	—	—	—	—	65
130	—	—	—	—	—	—	—	—	—	·375	·125	—	—	·375	·625	·75	1·5	1	·625	1·125	3	5·25	2·375	4·875	3·125	3·25	5	1·625	1·125	2·875	2	2·125	2·625	1·125	·625	2	·5	—	—	—	—	—	—	50
131	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	·375	·25	1·375	2·125	1·125	2·625	2·5	7·25	3	3·875	5·25	3·25	3·25	3·875	2·375	2·125	2·375	2·125	2·25	—	1·375	·625	—	—	—	—	—	53·5
132	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	2	1·125	2·25	2·375	3·625	4·875	4·75	6·625	4·5	2·75	2·375	2·375	2·5	1·875	·625	·375	·5	·5	1·625	·375	—	—	—	—	—	49
133	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	·375	·75	1·375	1·875	·625	1·875	2·625	2·5	2·5	3·5	2·75	4	2·375	4·5	2·625	3·125	2	·75	·75	—	—	·375	·5	·125	—	—	—	—	42
134	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	·375	·25	·375	1·125	·75	·25	2·125	3·875	2·75	2·125	2·5	4·5	1·875	1·75	4·375	3·5	1·875	·75	·875	·875	·5	·5	·875	·125	—	—	—	—	39
135	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	1·75	·25	·875	2·125	3·625	1·125	1·625	2·375	1·625	1·125	·5	·375	1·125	·5	·125	·375	—	—	—	—	—	20
136	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·75	1·375	1·875	1	1·5	1·5	1	1·125	4·5	1·875	1·875	·625	1·75	2	1	·25	—	—	—	—	—	—	—	24·5
137	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	·375	·125	·875	2·25	2·375	1·5	·125	1·375	2·75	1·75	—	—	—	—	—	·375	·125	·25	·25	—	—	—	15
138	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·875	·625	1·75	·5	1·25	·5	·875	·625	1·5	—	—	—	—	—	—	—	1	1·5	·75	1·125	·125	·25	·25	—	—	—	13·5
139	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	·625	1	·25	—	—	—	—	—	·375	·875	·25	—	—	·375	1·125	—	—	·5	·5	—	·25	·75	—	·5	·5	8
140	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	·5	·875	·125	—	—	—	·125	·375	—	·25	·75	—	—	—	—	—	—	—	3·5
141	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	·75	·25	—	—	—	—	—	—	—	1·5
142	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
143	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	·75	·25	1	1	—	·5	2	1·75	·75	3	1	3·25	5·25	9·5	12·75	16	20·25	25·25	27·75	32·75	36	55·25	51·5	60·75	53	50	52·25	34·25	40	32·5	26·5	16·5	13·75	10·75	9·75	6·75	6·5	4	1	1·5	·75	·5	·5	729

TABLE XVI. *External Length and Internal Length.*

Internal Length in mm. (Central Values).

Rows: External Length in mm. (Central Values).

	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	Totals
168	·375	·125	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5
169	·375	·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
170	—	—	—	1	—	·25	1·25	·5	—	—	—	·75	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
171	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
172	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
173	—	—	—	—	—	—	—	—	—	1	—	·5	1·5	·5	·5	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
174	—	—	—	—	—	—	—	·5	·5	·75	·25	—	·75	·75	·75	·5	·25	—	—	—	·75	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
175	—	—	—	—	—	—	—	—	—	1·25	·75	1	—	·375	·5	1·875	2·75	·5	·25	·75	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11
176	—	—	—	—	—	—	—	—	—	—	—	—	—	1·375	·5	1·375	·75	·75	1·75	·375	·625	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
177	—	—	—	—	—	—	—	·375	·125	—	—	1	—	·375	·625	·5	·75	2·25	·5	1·375	·125	1·5	1·875	·125	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11·5
178	—	—	—	—	—	·25	·75	·375	·125	—	—	—	·25	3·125	2·625	·5	3·875	2·875	2·75	·5	·5	—	·375	·875	·375	·875	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	21
179	—	—	—	—	—	—	—	—	—	—	—	—	·5	·25	1·125	·125	2·125	4·125	2·5	1	3·75	3·25	·75	1·875	1·75	1·875	1·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	26·5
180	—	—	—	—	—	—	—	—	—	—	—	—	1·75	1·25	·875	1·125	1·375	3·875	4	3·75	4·375	3·125	4·375	2·75	2·375	·75	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	36
181	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	2·25	3·25	2·25	1·625	2·25	3·5	4·5	5·75	4·875	3·375	1·25	1·75	1·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	38·5
182	—	—	—	—	—	—	—	—	—	—	—	—	—	·625	2·25	3·25	2·375	2·75	4·5	5·5	3·75	10	4·75	4·75	4·375	2·125	1·875	1·375	1·875	·375	—	—	—	—	—	—	—	—	—	—	—	—	—	56·5
183	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5	2·875	2·625	3·5	3·125	4	6·75	5	4·75	1·75	3	2·125	·125	·375	—	—	—	—	—	—	—	—	—	—	—	—	—	41·5
184	—	—	—	—	—	—	—	—	—	—	—	—	·25	·75	·75	1·5	·75	2·125	3·125	2·25	2·75	6·25	5·125	8·5	4·5	5·25	4	3·125	1	·75	·25	—	—	—	—	—	—	—	—	—	—	—	—	53
185	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	·5	1·5	2	3·5	6·625	5·625	8	4·75	3·375	4·125	4	·5	2·25	·75	—	—	—	—	—	—	—	—	—	—	—	—	48
186	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1·75	1·25	·375	2·875	5·75	6·875	6·5	7·375	8·5	2	5·25	3·5	2·875	1·625	—	—	—	—	—	—	—	—	—	—	—	58·5
187	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	1·5	—	·25	·75	·875	2·875	6·125	5·25	6·5	6·125	3·875	1·875	6	3·25	2·625	·625	—	—	—	—	—	—	—	—	—	—	—	49
188	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·625	1·25	4	2·25	4	2·75	2·125	4·5	2·75	2·25	2·5	1·75	·75	—	—	—	—	—	—	—	—	—	—	—	32·5
189	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2·375	2	2·5	1·375	1·625	2·5	5·75	5·625	1	3·75	6	4·25	2·5	3·75	1·25	1·25	—	—	—	—	—	—	—	—	47·5
190	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2·25	1·25	·5	·25	2	1·75	5	5·375	3·375	1·625	3·25	1·875	1	·75	1·25	1·375	·125	·75	·25	—	—	—	—	—	34
191	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	·75	·75	1·5	3·875	2·875	3·875	3	3·25	2·75	1·875	2	·5	·25	1·25	1·5	1·875	1·125	—	—	—	—	—	34
192	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	2·5	1·25	3	·625	3·125	3·25	1·25	2·75	5·25	·25	·75	·125	·875	·125	·375	—	—	—	—	—	26
193	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1·5	—	2·625	2·5	4·5	2·375	2·625	·875	1	1	—	1	—	1	—	—	—	—	—	22
194	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·75	—	·5	—	2·875	2·125	·125	·625	1·375	1·625	3·5	·75	—	—	—	—	—	—	—	14·5
195	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·875	2·125	1·25	2·25	·625	1·75	1·625	1	1	·875	·125	—	—	—	—	—	13·5
196	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	·25	1·625	·125	·5	·5	·625	1·875	1·5	—	—	1·125	·375	—	—	—	—	—	9
197	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	1	—	1·5	—	·375	·125	—	—	—	—	—	—	—	3·5
198	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·75	·75	—	1	—	·5	1	·875	·375	1·75	—	—	—	—	—	—	7
199	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	·75	·75	—	—	—	—	—	—	—	—	—	—	·125	·375	—	—	3
200	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	·375	·5	·5	1·5
201	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	·5	·5	—	—	—	2
202	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	·75	·25	—	—	—	—	2
203	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	·375	—	—	—	·5
204	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·125	·375	—	—	—	·5
Totals	·75	·25	1	1	—	·5	2	1·75	·75	3	1	3·25	5·25	9·5	12·75	16	20·25	25·25	27·75	32·75	36	55·25	51·5	60·75	53	50	52·25	34·25	40	32·5	26·5	16·5	13·75	10·75	9·75	6·75	6·5	4	1	1·5	·75	·5	·5	729

TABLE XVII. *External Length and Internal Breadth.*

External Length in mm. (Central Values).

Rows: Internal Breadth in mm. (Central Values).

	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	Totals
119	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
120	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5
121	—	—	1	—	—	—	—	—	—	1	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	3·5
122	—	—	·5	—	—	—	·25	·25	·25	·25	—	—	—	—	—	·5	1·5	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5·5
123	—	—	·5	—	—	·5	·25	1·25	·25	·25	—	—	—	·5	·5	·5	·5	—	—	—	—	—	·5	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	6
124	—	—	—	—	—	1·5	1	—	—	1·75	·75	1	—	1·5	1·5	1	1	1	—	—	—	—	·5	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	13
125	—	—	—	—	—	—	·5	·5	·5	·25	·75	1·5	·5	2·75	1	·75	·5	1	2	1	—	·5	·25	·25	—	·5	·5	—	1	—	—	—	—	—	—	—	—	16·5
126	—	—	—	—	—	—	·5	1	1	—	1·5	·5	3·25	3	1·5	2·25	1	1·5	2·5	2·5	—	1·5	·25	1·75	·5	—	—	—	—	—	—	—	—	—	—	—	—	26
127	·5	—	·5	—	—	—	1	3	2	—	·5	1·25	2·5	·25	2·75	2·75	1·25	·5	2	1·5	·25	1	1·5	2·5	·5	—	—	·25	·25	—	—	—	—	—	—	—	—	28·5
128	·5	—	·5	—	—	1·5	1·5	—	—	—	—	1·5	2·5	3·75	3·75	2·25	3·5	3·75	3·25	·75	1·5	1·5	1·5	·5	—	—	—	·25	·5	—	—	—	—	—	—	—	—	34·5
129	—	—	—	—	—	·5	1	3	2	1	3·5	1·75	4·5	3·5	6	4·5	4·75	9	6·5	2·25	2·25	1·5	2	·5	1	1	1	—	—	·5	—	—	—	—	—	—	—	63·5
130	—	—	1	—	—	—	—	·5	·5	1·5	1·5	2	2·25	2·25	5	4·25	3·5	3·5	3·5	3·25	2·25	2·25	1·5	2·5	2	1	·5	—	—	·5	—	—	—	—	—	—	—	47
131	—	—	—	—	—	—	—	1·5	1·5	1	3·5	3	6·5	3·5	4	3·75	2	4·25	7·25	7·75	4	7·5	2	3·5	1·5	·5	1·5	2	—	—	·5	—	—	—	—	—	—	72·5
132	—	—	—	—	—	—	—	—	—	·5	·5	3·5	6	3·5	3·5	3·75	5·5	4·75	4·5	5·25	3·75	7·5	4·5	2·25	4·25	3·5	2·5	2	2	1	1	·5	—	—	—	—	—	76
133	—	—	—	—	—	·5	—	—	—	1	4	·5	1·75	6·75	5·5	1·5	5·25	4·75	6·5	5·75	2·5	2	4·25	1·75	3·75	3·5	1	1	·75	·25	2·5	—	—	—	—	—	—	67
134	—	—	—	—	—	·5	—	—	—	1	1·5	3·5	·25	3·25	6·25	2·5	6·25	5·5	3	4	4	2·25	4·75	2·25	1·75	3·5	1	·5	1·25	·75	·5	—	—	—	—	·5	·5	61
135	—	—	—	—	—	—	—	—	—	1	2	3·75	1·75	1·75	5	1·5	5·25	1	5·5	5·5	2·5	4·5	1·5	4	1·5	·75	1·75	1·75	·25	—	1·5	—	—	—	—	—	—	54
136	—	—	—	—	—	—	—	—	—	—	—	·75	·75	·25	4·75	3·25	3·25	2	3·25	1·25	2	5·25	·5	2	2·75	1·25	1·75	·25	·25	—	·5	—	—	—	1	—	—	37
137	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	1·5	1·25	1·25	1·75	3·25	·5	3·25	2·5	1·75	2·5	1·5	·5	·5	·5	—	—	—	—	·5	—	—	—	24
138	—	—	—	—	—	—	—	—	—	—	—	1·5	3	1	3·25	2·25	1·75	·25	2	2·5	3·5	3	·5	·5	·5	1·5	·25	2·25	1	—	—	—	—	·5	·5	—	—	31·5
139	—	—	—	—	—	—	—	—	—	1	—	—	—	1	·75	·25	2·5	1	2	1	1·5	2	1·5	4	1	·5	1·25	2·5	·25	—	—	1	—	—	·5	—	—	25·5
140	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1·5	2	1	·5	·5	—	1·5	2·5	1	2	—	·25	·25	—	—	·5	·5	—	—	—	—	15
141	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	1·5	1	—	—	—	—	·5	—	—	—	—	1	—	—	—	4·5
142	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	1	1	—	—	—	·5	—	·5	—	—	—	—	—	—	—	—	4
143	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	1	—	·5	·5	—	—	·5	·5	—	·5	—	—	—	—	—	1	—	—	—	—	5
144	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
145	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	2·5
146	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	·5
147	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	·5
148	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	·5
Totals	1·5	·5	4	—	—	5	6	11	8	11·5	27	26·5	36	38·5	56·5	41·5	53	48	58·5	49	32·5	47·5	34	34	26	22	14·5	13·5	9	3·5	7	3	1·5	2	2	·5	·5	729

TABLE XVIII. *External Length and Internal Height.*

External Length in mm. (Central Values).

Rows: Internal Height in mm. (Central Values).

	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	Totals
115	·5	·5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
116	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2.5
117	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	·25	·25	·75	·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2·5
118	—	—	—	—	—	—	—	—	—	—	—	·25	1·25	—	—	·5	·75	·25	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3·5
119	1	—	—	—	—	—	—	1	—	·5	·5	—	—	1	1	—	·5	—	·5	·5	·5	—	·5	·5	·5	2	—	—	—	—	—	—	—	—	—	—	—	10·5
120	—	—	—	—	—	—	—	—	—	—	1	—	—	2	2·5	—	2	·5	·5	—	—	1	1	—	—	·5	·5	·25	·25	—	—	—	—	—	—	—	—	12
121	—	—	—	—	—	1	1	·5	—	—	2·5	—	·75	·25	2	1	2	2	1	2	—	·5	—	1	·5	—	—	·25	·25	—	—	—	—	—	—	—	—	18·5
122	—	—	·5	—	—	1·5	1·5	1	—	·5	1·5	·5	3·75	1·25	1·5	1·5	1	—	·5	1	—	1·5	1·25	1·25	·5	—	—	—	·25	·25	·25	·25	—	—	—	—	—	23
123	—	—	·5	—	—	·5	1	—	·5	1	—	·5	2	4	4·5	2	·5	2	·75	1·25	2·25	·5	2	1·25	—	—	—	—	·25	·25	·25	·25	—	—	—	—	—	28
124	—	—	·5	—	—	—	—	—	—	—	1	2	3·5	2	3	1·5	2	·5	·75	1·5	4·25	·75	·25	1·5	—	1·25	·25	—	—	—	—	—	—	—	—	—	—	26·5
125	—	—	·5	—	—	—	—	·25	·25	1·25	4·75	1	4·5	3	2·75	2·25	2	3·75	1·75	3·25	1·5	3·75	—	3·5	—	2·25	1·25	—	·5	·5	—	—	—	—	—	—	—	44·5
126	—	—	1	—	—	·5	1	·75	1·75	·25	1·75	3·75	·5	2·75	5·25	2·25	3·5	4·25	·75	3·5	3·5	5·25	1·75	3	—	·5	1·5	·25	·25	—	·5	·25	·25	—	—	—	—	50·5
127	—	—	—	—	—	·5	·5	1·5	·75	1·25	—	2·75	2	3·75	3·5	6·75	4	5·75	4·25	3·75	·75	1·25	1·75	1	·5	1	·75	·75	·75	—	·5	·25	·25	—	—	—	—	50·5
128	—	—	—	—	—	—	—	2	·75	2·25	·25	5·5	1·25	3·75	4·5	5	6·5	5·75	6·75	4·25	4·25	4·25	2	3·25	2	·5	·25	·5	·5	—	·5	1	—	—	—	—	—	67·5
129	—	—	1	—	—	1	—	1	·5	1·5	1·75	3·5	4·25	3·25	3·75	6·25	5	5·75	5·5	3·5	2	1·75	3	3·5	1·75	1·5	1·5	·75	·75	·5	·5	—	—	—	—	—	—	65
130	—	—	—	—	—	—	—	1·25	1·25	·5	·75	2·25	1·5	3	3·25	3	1·5	2·25	3	4·5	1·5	3·5	3	3·25	3·25	3	·5	1·75	·75	·5	1	—	—	—	—	—	—	50
131	—	—	—	—	—	—	—	·75	1·25	—	·25	1·25	1·5	1·5	2·5	1	6·25	4	6·75	4·5	·5	4	2	1·5	4·5	2·5	2·25	1·25	2	·5	1	—	—	—	—	—	—	53·5
132	—	—	—	—	—	—	1	—	—	—	·5	1	1·5	2	5	2·5	5·75	2·5	4·75	3	1·75	3·5	2·25	4	3·75	1·25	·75	·75	—	·5	1	—	—	—	—	—	—	49
133	—	—	—	—	—	—	—	1	·5	·5	2·25	1	2·25	1	2·75	·75	3	2	2·5	4	2·25	4·5	2·75	1	2·5	2·5	1·5	·5	·5	—	—	—	—	—	·5	—	—	42
134	—	—	—	—	—	—	—	—	·5	1	1·25	1	·25	1	3·5	1·5	1	2·75	4	2·75	5·5	3·25	1·25	1·5	1·75	·75	1·25	1·75	·5	—	—	—	—	—	1	—	—	39
135	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·75	·25	—	1·25	5	2·75	1	2·75	1·75	·5	1	—	·25	1·25	—	—	—	—	—	—	·5	—	—	20
136	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	1·5	·5	3·75	1·75	—	3·5	3	1	2·5	—	—	1	—	·5	·5	1	—	1	—	—	—	24·5
137	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5	—	1	1	1·75	3·5	·25	1	1	1·5	·5	—	1	—	·25	·25	—	—	—	—	·5	—	—	—	15
138	—	—	—	—	—	—	—	—	—	1	·5	—	1·5	1·5	·5	·5	·5	·25	·25	—	—	—	1	—	—	1	1	1·25	1·25	—	1	—	—	·5	—	—	—	13·5
139	—	—	—	—	—	—	—	—	—	—	·5	—	·5	—	·5	—	1	—	—	—	—	·5	—	—	·5	·5	1	1	—	—	—	—	1	—	—	·5	·5	8
140	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	1·5	—	—	—	—	·5	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	3·5
141	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5
142	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
143	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1·5	·5	4	—	—	5	6	11	8	11·5	21	26·5	36	38·5	56·5	41·5	53	48	58·5	49	32·5	47·5	34	34	26	22	14·5	13·5	9	3·5	7	3	1·5	2	2	·5	·5	729

To face p. 122

TABLE XIX. *Internal Breadth and External Height.*

Internal Breadth in mm. (Central Values).

External Height in mm. (Central Values).	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	Totals
120	—	—	—	—	—	—	—	—	1	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
121	1	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	2·5
122	—	—	—	—	—	—	—	—	·5	—	—	—	—	·5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
123	—	·5	·5	—	—	—	—	—	—	—	1	—	—	—	1·75	·25	—	—	—	1	—	—	—	—	—	—	—	—	—	—	5
124	—	—	—	—	—	—	—	—	—	—	—	1·5	·5	1·5	1·25	·75	1	—	—	—	—	—	—	—	—	—	—	—	—	—	6·5
125	—	—	—	—	—	·5	—	·5	1	1·5	1	—	—	·5	1	·5	1	—	—	·5	—	—	—	—	—	—	—	—	—	—	8
126	—	—	—	—	—	1·75	·75	·75	·25	·25	2·25	1	2·5	·5	·5	1	—	2	·5	·5	—	—	—	—	—	—	—	—	—	—	14·5
127	—	—	—	—	—	1·25	1	1	·5	2	3	1·25	1·5	·5	1	1·5	1·5	1	—	1·5	1·5	·5	—	—	—	—	—	—	—	—	20·5
128	—	—	—	·5	·5	2	·75	3·25	1·25	2·75	2·25	1·75	3	1·5	1	3	·5	·75	·25	1	1·5	·5	—	—	—	—	—	—	—	—	28
129	·5	—	—	2	·5	·5	3	1·25	2·25	5	5	4·5	4·5	4	—	2	1·75	·5	·25	1·5	1	—	—	—	1	—	—	—	—	—	41
130	·5	—	—	·5	·5	—	—	·25	1·25	2·25	2·25	3·75	7·25	4·75	5·75	·5	2·25	1·75	—	—	·5	1·5	—	—	—	—	—	—	—	—	35·5
131	—	—	—	—	1	1	2	3	2	3·25	8·5	3·5	3·75	6·25	3·5	3·25	4	1·5	1·5	1·5	2·5	1	—	—	—	—	—	—	—	—	53
132	—	—	—	·5	·5	·75	·75	2	2·5	1·5	4·25	2·75	4·25	5·25	2·25	3·75	2	2·5	—	1	·5	·5	—	—	—	—	—	—	—	—	37·5
133	—	—	—	—	·5	1·75	2·25	2·5	2	3·25	9·75	5	5·75	5·25	4	6	4·75	1	1·75	1·5	2·5	2·5	·5	·5	·25	·25	—	—	·5	·5	64·5
134	—	—	—	—	1·5	·5	·5	2	1	2·75	4·75	4·25	9	2·75	6	8	4·25	2	2·25	2·5	1·5	·5	—	—	·25	·25	—	—	—	—	56·5
135	—	1	1	—	·75	·75	1	3	4	3	3·5	3·75	3·5	4	5·25	5·5	3·25	3·25	2·5	4	2·5	1	—	·5	—	—	—	—	—	—	57
136	—	—	2	—	·25	1·25	·5	1·5	4	1	3·5	2·5	5	5·75	7·25	4	5·75	3·25	1·5	2·5	1	—	—	·5	·5	—	1	—	—	—	54·5
137	—	—	—	1	—	—	1	1	2	·25	1·25	6	7	6	8	5·5	6	1·5	2·5	·5	2·5	1	—	—	·5	—	—	—	—	—	53·5
138	—	—	—	—	—	—	—	1	·5	2·25	2·25	1·75	4·75	8	4·5	1·75	1·75	·75	1·75	3·5	1·5	1·5	1·5	—	—	—	—	—	—	—	39
139	—	—	—	—	—	·5	—	1	·5	·5	1·5	1·25	1·75	6·5	6	3·75	1·25	3·75	·75	1·5	2·5	·5	—	—	—	—	—	—	—	—	33·5
140	—	—	—	1	—	·5	1	1·5	·5	1·5	4	1	2·5	3·5	2·5	1	1·25	·75	2	3	1	2	—	1·5	1·5	—	1	—	—	—	34·5
141	—	—	—	—	—	—	1	—	·5	1	1	·5	3·5	3	·5	3	2·75	5·75	2	—	—	—	—	1	—	—	—	—	—	—	25·5
142	—	—	—	—	—	—	1	—	—	·5	·5	—	1·5	·25	·25	·5	3	1	3	2	1·5	—	—	—	—	—	—	—	—	—	15
143	—	—	—	—	—	—	—	—	—	—	1	1	·5	1·75	·75	1·5	—	1·5	1	—	·5	·75	·25	—	—	—	—	—	—	—	10·5
144	—	—	—	—	—	—	—	·5	·5	—	1	—	—	—	—	·5	3	1·5	·5	1	—	·25	1·25	—	—	—	·5	·5	—	—	11
145	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5	·5	·5	2	—	—	1	—	—	1	—	—	—	—	—	—	—	6·5
146	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	·5	—	1	—	—	1	1	—	—	—	—	—	—	—	—	6·5
147	—	—	—	—	—	—	—	—	—	—	—	—	—	1	·5	1	·5	—	—	—	—	—	—	—	1	—	—	—	—	—	4
148	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
149	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	1·5	3·5	5·5	6	13	16·5	26	28·5	34·5	63·5	47	72·5	76	67	61	54	37	24	31·5	25·5	15	4·5	4	5	·5	2·5	·5	·5	·5	729

TABLE XX. *Internal Breadth and Breadth Thickness.*

Internal Breadth in mm. (Central Values).

Difference of External and Internal Breadths in mm. (Central Values).	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	Totals
2·5	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
3	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	6	—	—	2	—	1	—	—	·5	·5	—	—	—	—	12
3·5	—	—	—	—	—	·5	·5	—	—	—	—	1·5	1·5	4·5	3	2	2·5	1	1·5	1·5	—	—	—	—	—	—	—	—	—	—	20
4	—	—	—	2	—	—	2	1	—	1	3	—	1	2·5	4	4	2·5	1	1	4	2	1	—	1	—	—	1	—	—	—	34
4·5	—	—	—	—	1	1·5	1	2	2·5	2	2·5	2	5·5	3·5	2	·5	1	1·5	1	1	·5	—	—	—	—	—	—	—	—	—	31
5	—	—	—	—	—	2	2	1	2	3	4	3·5	11·5	6·5	6·5	7	3·5	1·5	—	4	6	—	—	—	1	—	1	—	—	—	66
5·5	—	—	—	—	—	—	·5	1	2	2·5	4	3·5	6	7·5	5	5·5	2·5	·5	·5	·5	2	1	·5	—	—	—	—	—	—	—	45
6	—	—	—	—	—	3	·5	2·5	6·5	3·5	4	6	3	8·5	8·5	11	7	4·5	2·5	3	2	2	—	1·5	·5	—	—	—	—	—	80
6·5	—	—	—	1·5	·5	—	·5	2	2	2	5·5	7	5·5	6	4	3·5	4	4·5	3·5	2	2·5	1	·5	·5	1·5	—	—	—	·5	·5	61
7	—	—	1	—	—	1	—	1	3	2	8·5	3	11·5	10	8·5	8	5·5	3	2	2	1	3	—	—	1	—	—	—	—	—	75
7·5	—	—	—	1	1	·5	2·5	2·5	1·5	2	5·5	2·5	3·5	3·5	3·5	3	2	3·5	3·5	2	1·5	2	·5	·5	—	—	—	—	—	—	48
8	—	—	1	—	—	—	—	3	3	6·5	7·5	3	6	3·5	8·5	6·5	4·5	4	4	3·5	2·5	2	—	—	—	—	—	—	—	—	69
8·5	—	—	—	—	—	1	1·5	2	2	4·5	4	1	4	6	4	2	2	2	1·5	·5	—	1	1	—	—	—	—	—	—	—	40
9	—	1·5	·5	—	2	—	—	—	—	—	5	3·5	4·5	2	3	1·5	4	4·5	2	2·5	1·5	—	1	·5	·5	—	—	—	—	—	40
9·5	1	—	—	—	—	—	·5	·5	—	1	1	1	2	2·5	2	1·5	1·5	2·5	1	1·5	·5	—	—	—	—	—	—	—	—	—	20
10	—	—	—	1	—	—	1	2	—	2	3	2	1	6	1	1·5	3	·5	—	—	—	1	—	—	—	—	—	—	—	—	25
10·5	1	—	—	—	—	1	—	2·5	1	1	1·5	—	2	1	2	—	·5	·5	—	·5	·5	—	—	—	—	—	—	—	—	—	15
11	—	—	—	—	·5	1·5	·5	1·5	—	—	1	3	2	2	—	·5	1·5	1	—	—	3	—	1	—	—	—	—	—	—	—	19
11·5	—	—	—	—	1	1	·5	·5	—	·5	1·5	1·5	1·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
12	—	—	1	—	—	—	2	1	1	—	1	2	—	—	1	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	11
12·5	—	—	—	—	—	—	1	—	1	·5	·5	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	1·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
13·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	1
14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
15·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	1
Totals	2	1·5	3·5	5·5	6	13	16·5	26	28·5	34·5	63·5	47	72·5	76	67	61	54	37	24	31·5	25·5	15	4·5	4	5	·5	2·5	·5	·5	·5	729

TABLE XXI. *External Diametral Product and Capacity.*

External P, Length × Breadth × Height in cm.³ (Sub-ranges).

Capacity in cm.³ (Central Values).	260—270	270—280	280—290	290—300	300—310	310—320	320—330	330—340	340—350	350—360	360—370	370—380	380—390	390—400	400—410	410—420	420—430	430—440	440—450	450—460	Totals
1025	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
1075	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1125	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
1175	—	—	3	2	3	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	10
1225	—	—	4	2	4	5	3	—	1	—	—	—	—	—	—	—	—	—	—	—	19
1275	—	—	—	4·5	12·5	8	5·5	8	1	1	—	—	—	—	—	—	—	—	—	—	40·5
1325	—	—	—	·5	6·5	18	24·5	16	8	4	—	—	—	—	—	—	—	—	—	—	77·5
1375	—	—	—	—	7	14	29	25·5	16·5	13·5	2	1	—	—	—	—	—	—	—	—	108·5
1425	—	—	—	—	—	6	20	26	34·5	26·5	19·5	4	2	—	—	—	—	—	—	—	138·5
1475	—	—	—	—	—	2	4	16	28	28·5	26	7	5	1	—	—	—	—	—	—	117·5
1525	—	—	—	—	—	—	2	4·5	15	26·5	21	10	11	1·5	—	—	—	1	—	—	92·5
1575	—	—	—	—	—	—	—	1	2	6	12	20	14·5	6·5	3	1	—	—	—	—	66
1625	—	—	—	—	—	—	—	1	—	3	4·5	6	6·5	4	3	—	—	—	—	—	28
1675	—	—	—	—	—	—	—	—	—	—	2	2	4	6·5	5	2	1	—	—	—	22·5
1725	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	1	—	1·5
1775	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	2
1825	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1875	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1925	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1975	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2025	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
Totals	1	—	8	10	34	54	89	98	106	109	87	50	43	20	11	4	1	1	2	1	729

TABLE XXII. *Internal Diametral Product and Capacity.*

Internal P, Length × Breadth × Height in cm.³ (Sub-ranges).

Capacity in cm.³ (Central Values).	200—210	210—220	220—230	230—240	240—250	250—260	260—270	270—280	280—290	290—300	300—310	310—320	320—330	330—340	340—350	350—360	360—370	370—380	Totals
1025	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
1075	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1125	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
1175	—	—	2	3	4	1	—	—	—	—	—	—	—	—	—	—	—	—	10
1225	—	—	1	2	5	9	1	—	—	—	1	—	—	—	—	—	—	—	19
1275	—	—	—	—	4·5	11·5	17·5	6	1	—	—	—	—	—	—	—	—	—	40·5
1325	—	—	—	—	·5	10·5	27·5	27	10	2	—	—	—	—	—	—	—	—	77·5
1375	—	—	—	—	1	4	20	36	32	13·5	2	—	—	—	—	—	—	—	108·5
1425	—	—	—	—	—	1	7	22·5	51	35	20	2	—	—	—	—	—	—	138·5
1475	—	—	—	—	—	—	1	2·5	24	43	32	13	2	—	—	—	—	—	117·5
1525	—	—	—	—	—	—	—	2	6	22·5	34·5	22	5·5	—	—	—	—	—	92·5
1575	—	—	—	—	—	—	—	—	1	4	13·5	18·5	23	6	—	—	—	—	66
1625	—	—	—	—	—	—	—	—	—	1	3	9·5	5·5	9	—	—	—	—	28
1675	—	—	—	—	—	—	—	—	—	—	—	2	2·5	9	8	1	—	—	22·5
1725	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	1	—	—	—	1·5
1775	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2
1825	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1875	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1925	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1975	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2025	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	1	—	4	6	16	37	74	96	125	121	106	67	39	24	10	2	—	1	729

ON THE DISTRIBUTION OF THE RATIO OF MEAN TO STANDARD DEVIATION IN SMALL SAMPLES FROM NON-NORMAL UNIVERSES.

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INTRODUCTION.

LET $\bar{X}$ and s be the mean and the standard deviation respectively of a sample of n drawn from a universe having mean M and standard deviation σ. Let x be the deviation, $\bar{X} - M$, of the mean of a sample from the mean of the universe. The ratio $z = x/s$ (or $t = z\sqrt{n-1}$) plays an extremely important part in a number of statistical problems such as determining the probability that the mean of the sample does not deviate from the mean of the universe by more than a stipulated amount, comparing two mean values, and finding the sampling errors of regression coefficients*. The distribution of z for samples of n from a normal universe has been completely determined—originally by "Student," who applied it to the first of the problems just mentioned—and tables of the distribution for various values of n have been constructed†.

The derivation of this distribution is based upon the following assumptions:

1. That the means of samples of n are normally distributed with standard deviation $\sigma/\sqrt{n}$.

2. That the distribution of standard deviations, s, is given by

$$f(s)\,ds = \frac{n^{(n-1)/2}\, s^{n-2}\, e^{-ns^2/2\sigma^2}}{2^{(n-3)/2}\, \sigma^{n-1}\, \Gamma\left(\frac{n-1}{2}\right)}\, ds,$$

in which $f(s)\,ds$ is the probability, except for infinitesimals of higher order than ds, that a value of s will fall within the range $(s, s + ds)$.

* See R. A. Fisher, "Applications of 'Student's' distribution," *Metron*, Vol. v. (1925), pp. 90—104; "Statistical Methods for Research Workers," *passim.*

† "Student," "The probable error of a mean," *Biometrika*, Vol. VI. (1908-9), pp. 1—25 (see also *Tables for Statisticians and Biometricians*, pp. xliii—xlv, 36); "Tables for estimating the probability that the mean of a unique sample of observations lies between $-\infty$ and any given distance of the mean of the population from which the sample is drawn," *Biometrika*, Vol. XI. (1915—17), pp. 414—7; "New tables for testing the significance of observations," *Metron*, Vol. v. (1925), pp. 105—8, 118—20. R. A. Fisher, *loc. cit.*; also "Note on Dr Burnside's recent paper on Errors of Observation," *Proceedings of the Cambridge Philosophical Society*, Vol. XXI. (1923), pp. 655—8.

3. That x and s are independent.

While these assumptions have been justified for the case of a normal sampled population*, they do not hold otherwise, and we do not know how well "Student's" probability integral applies. It has been shown in some interesting experiments by Shewhart and Winters† that it applies—as is to be expected from theory—to normal universes, and that for certain types of non-normal universe it gives better results than does the normal probability integral (i.e. than assuming z to be normally distributed). However, these same experiments indicate that it fails to a degree sufficient to warrant further extension of theory.

The failure seems to be due chiefly to the correlation which always exists between x and s in samples from a non-normal universe. If the Pearsonian β's of the universe satisfy the relation $\beta_2 - \beta_1 - 3 = 0$, the regression of the variance, s^2, on x is linear; in other cases the regression is well represented by a second order parabola‡. For points in the $\beta_1\beta_2$-plane lying above the line $\beta_2 - \beta_1 - 3 = 0$§, the parabola in the (x, s^2)-plane is concave upward (that is, its branches are directed in the positive sense of the s^2-axis), while for points lying below this line the parabola is concave downward. (See Fig. 3, p. 132.) For the sake of definiteness, let us consider the latter case. For large numerical values of x the value of s^2 tends to be smaller than its mean value, which denotes that s is smaller on the average, and that $|z| = |x|/s$ tends to be larger. As a consequence, a greater number of z's would be expected to lie outside a certain value of $|z|$ than in the case of a normal universe. On the other hand, for values of x near zero the value of s^2, and also that of s, have a tendency to be larger, causing the values of $|z|$ to be smaller. The effect of this would be to make a greater number of z's lie inside a certain value than when the sampling is from a normal universe. The actual existence of the first effect was verified by the Shewhart-Winters' experiments. Both effects are shown theoretically in the present paper.

Several types of universe have been investigated, but the rectangular has been studied in greatest detail, as it is the simplest from the standpoint of the method employed. The rectangular distribution may be regarded as a special case of a considerable number of Pearsonian types‖; it is sufficiently different from the normal distribution to test whether such a difference causes an appreciable departure from the theory which is based upon an assumption of normality

* In addition to the references already cited, see R. A. Fisher, "Frequency distribution of the values of the correlation coefficient in samples from an indefinitely large population," *Biometrika*, Vol. x. (1914—15), pp. 507—21; and Karl Pearson, "On the distribution of the standard deviations of small samples," *Biometrika*, Vol. x. (1914—15), pp. 522—9.

† See W. A. Shewhart and F. W. Winters, "Small samples—new experimental results," *Journal of the American Statistical Association*, Vol. xxiii. (1928), pp. 144—53. See also "Sophister," "Discussion of small samples drawn from an infinite skew population," *Biometrika*, Vol. xx^{A}. (1928), pp. 389—423.

‡ See J. Neyman, "On the correlation of the mean and the variance in samples drawn from an 'infinite' population," *Biometrika*, Vol. xviii. (1926), pp. 401—13.

§ It is assumed that the positive direction is to the right on the β_1-axis and upward on the β_2-axis.

‖ See T. L. Kelley, *Statistical Method*, p. 128.

in the sampled population; moreover, it is one of the types of universe employed by Shewhart and Winters in their experiments.

Besides the distribution of z, the distributions of various statistical parameters (mean, median, range, extreme average, greatest variate and least variate) of samples from a rectangular universe are discussed. It is shown that it makes little difference in the distributions of these parameters whether the sampled population is continuous or discrete.

Rectangular Universe.

If with the quantity X_i ($i = 0, 1, \ldots, m-1$) is associated the probability p_i ($\Sigma p_i = 1$), the chance that a random sample of n X's will contain n_0 X_0's, n_1 X_1's, ..., n_{m-1} X_{m-1}'s ($n_0 + n_1 + \ldots + n_{m-1} = n$) is

$$\frac{n!}{n_0!\, n_1! \ldots n_{m-1}!} p_0^{n_0} p_1^{n_1} \ldots p_{m-1}^{n_{m-1}},$$

which is the general term in the multinomial expansion of

$$(p_0 + p_1 + \ldots + p_{m-1})^n.$$

Let us apply this to samples of 4 from an infinite* rectangular distribution of five classes. Suppose that the variate X may assume the values 0, 1, 2, 3, 4. Then, since the probability that X will have a specified value is 1/5, we see by the foregoing that the probability of getting a sample in which all the digits are alike is 1/625; the probability of getting a specific sample in which three are alike, such as 0004, is 4/625; the probability of getting a specific sample in which two are alike of one kind and two alike of another, e.g. 1122, is 6/625; of getting a specific sample of two alike and two different, e.g. 1124, is 12/625; and of getting a specific sample in which all are different is 24/625. Thus we know the probability of obtaining a specified sample, and it is of course a simple matter to calculate the value of $z = (\bar{X} - M)/s$ for the sample. The distribution of z, which is symmetric, may be seen in Table I.

We can now compare our results with those for a normal universe. Perhaps the best way to do this is by plotting on probability paper†. This is done in Fig. 1. The smooth curve shows the cumulated probability for random samples of 4 from a normal universe‡, while the series of irregular steps shows the corresponding probability for samples of 4 from the rectangular universe of five classes. It is seen that although for the larger numerical values of z the broken line lies on the concave side of the curve, it quite often crosses it for smaller values of $|z|$, say $|z| < 1$.

* For a description of what is meant by sampling from an infinite population: see A. E. R. Church, "On the means and squared standard-deviations of small samples from any population." *Biometrika*, Vol. XVIII. (1926), p. 323.

† For an explanation of probability paper see G. C. Whipple, *Vital Statistics*, Chapter XII.

‡ Values were obtained from *Tables for Statisticians and Biometricians*, p. 36.

TABLE I.

Probability of z for Samples of 4 from Rectangular Universe of five Classes.

z (Exact)	z (Decimal)	625 × Probability of z	Cumulated Probability for $-z$	Cumulated Probability for $+z$
0	0	85	·5680	·5680
$\sqrt{51}/51$	·1400	12	·4320	·5872
$\sqrt{35}/35$	·1690	24	·4128	·6256
$\sqrt{3}/9$	·1925⁻	16	·3744	·6512
$\sqrt{19}/19$	·2294	12	·3488	·6704
$\sqrt{11}/11$	·3015+	24	·3296	·7088
1/3	·3333	18	·2912	·7376
$\sqrt{5}/5$	·4472	24	·2624	·7760
$3\sqrt{43}/43$	·4575⁻	12	·2240	·7952
$\sqrt{3}/3$	·5774	28	·2048	·8400
$3\sqrt{19}/19$	·6882	12	·1600	·8592
$\sqrt{6}/3$	·8165⁻	12	·1408	·8784
$3\sqrt{11}/11$	·9045+	12	·1216	·8976
$5\sqrt{3}/9$	·9623	4	·1024	·9040
1	1·0000	12	·0960	·9232
$\sqrt{2}$	1·4142	12	·0768	·9424
$5\sqrt{11}/11$	1·5076	12	·0576	·9616
$\sqrt{3}$	1·7321	8	·0384	·9744
$5\sqrt{3}/3$	2·8868	4	·0256	·9808
3	3·0000	6	·0192	·9904
$7\sqrt{3}/3$	4·0415+	4	·0096	·9968
∞	∞	2	·0032	1·0000

The cumulated probability is the sum of all the probabilities from $z=-\infty$ to $z=z$ inclusive. It is the probability that the mean of a random sample will not exceed (in algebraic sense) the mean of the universe by more than z times the standard deviation of the sample.

Tables II and III give the probabilities for samples of 3 and 2 respectively from the five-class rectangular distribution. They may be compared with the tables of "Student"* for samples from a normal population.

* "Student," "Tables for estimating the probability that the mean of a unique sample of observations lies between $-\infty$ and any given distance of the mean of the population from which the sample is drawn," *Biometrika*, Vol. XI. (1915—17), pp. 414—17.

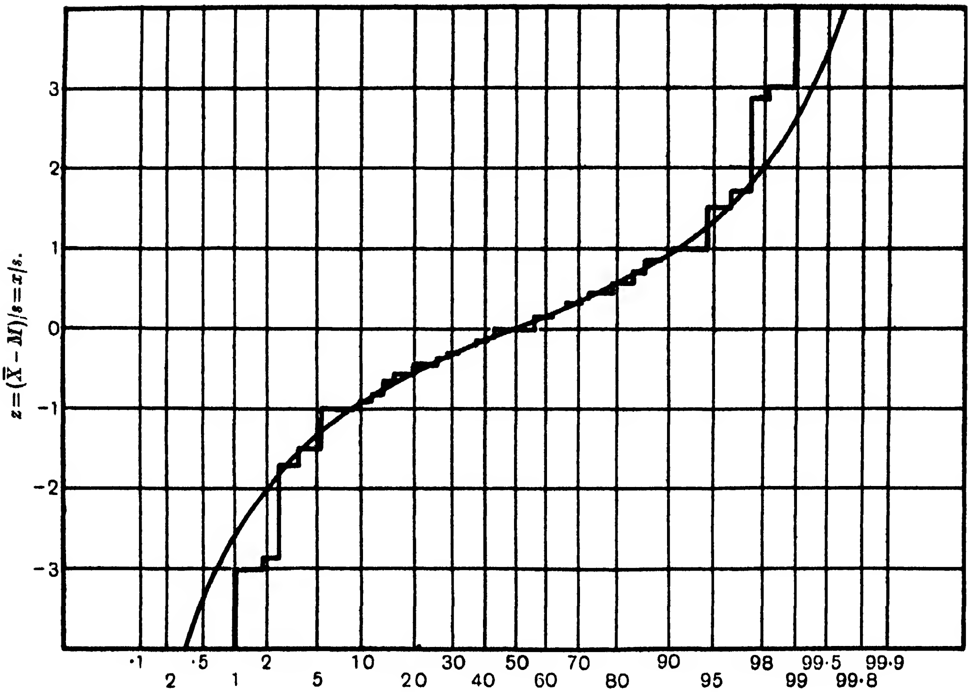


Fig. 1. Cumulated Probability of z.
The curve is for samples of 4 from a normal universe.
The steps are for samples of 4 from a rectangular universe of five classes.

TABLE II.

Probability of z for Samples of 3 from Rectangular Universe of five Classes.

z (Exact)	z (Decimal)	125 × Probability of z	Cumulated Probability	
			for $-z$	for $+z$
0	·00	19	·576	·576
$\sqrt{26}/26$	·20	6	·424	·624
$\sqrt{14}/14$	·27	6	·376	·672
$\sqrt{2}/4$	·35+	6	·328	·720
$\sqrt{14}/7$	·53	6	·280	·768
$\sqrt{2}/2$	·71	9	·232	·840
$\sqrt{6}/2$	1·22	6	·160	·888
$\sqrt{2}$	1·41	6	·112	·936
$2\sqrt{2}$	2·83	3	·064	·960
$5\sqrt{2}/2$	3·54	3	·040	·984
∞	∞	2	·016	1·000

TABLE III.

Probability of z for Samples of 2 from Rectangular Universe of five Classes.

z	25 × Probability of z	Cumulated Probability
$-\infty$	2	·08
-3	2	·16
-1	4	·32
$-1/3$	2	·40
0	5	·60
1/3	2	·68
1	4	·84
3	2	·92
∞	2	1·00

A better picture of the situation can be obtained if we employ a rectangular universe having ten classes. Suppose then that the variate X can assume the values $0, 1, 2, \ldots, 9$, with the probability of its equalling a specified digit being $1/10$. The results are shown in Tables IV and V and in Fig. 2.

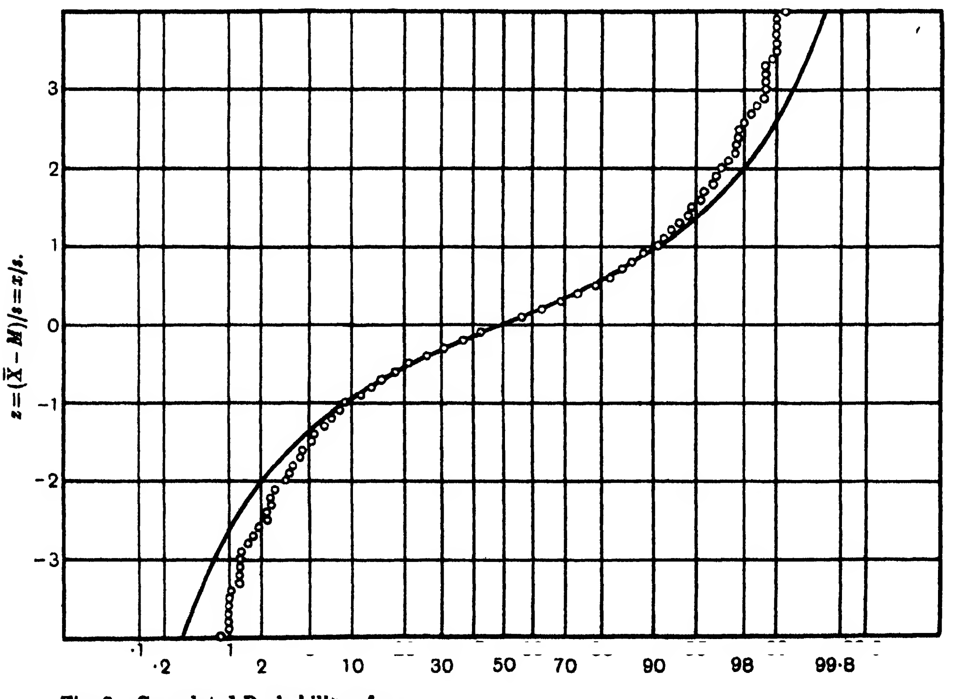


Fig. 2. Cumulated Probability of z.
The curve is for samples of 4 from a normal universe.
The small circles are for samples of 4 from a rectangular universe of ten classes.

Table V and Fig. 2 compare the probabilities, for a ten-class rectangular universe and a normal universe, that the deviation of the mean of the sample from the mean of the universe, expressed in terms of the standard deviation of the sample, will not exceed a given amount. Perhaps the most noticeable phenomenon, evident from the table rather than from the graph, is that the cumulated probability curves cross for values of z between $-0{\cdot}8$ and $-0{\cdot}7$, at $z=0$, and again between $0{\cdot}7$ and $0{\cdot}8$. This not only proves theoretically the existence of the effect

TABLE IV.

Probability of z for Samples of 4 from Rectangular Universe of ten Classes.

z	Probability
Below $-4{\cdot}25$	·0077
-4	·0022
$-3{\cdot}5$	·0026
-3	·0032
$-2{\cdot}5$	·0074
-2	·0188
$-1{\cdot}5$	·0267
-1	·0692
$-0{\cdot}5$	·2000
0	·3244
$0{\cdot}5$	·2000
1	·0692
$1{\cdot}5$	·0267
2	·0188
$2{\cdot}5$	·0074
3	·0032
$3{\cdot}5$	·0026
4	·0022
Above $4{\cdot}25$	·0077

discovered experimentally by Shewhart and Winters*, but also shows the other effect anticipated in the Introduction, viz. a greater clustering of values of z about the origin. This other effect is not marked, and Fig. 2 would indicate that for $|z| \leqq 1$ "Student's" theory, based on the hypothesis of a normal universe, is an excellent approximation in the case of a rectangular universe. The irregularity, for large numerical values of z, of the graph of the probability for the rectangular universe, is due to the paucity of such values†.

Before taking up other types of universe let us study the regression of variance on mean for the rectangular universe of five classes. The scatter diagram of variance and mean is shown in Fig. 3, from which it will be seen that the second order parabola of regression as given by Neyman‡ is in excellent agreement

* *loc. cit.*

† For the effects of the grouping see "Student," "The probable error of a mean," *loc. cit.* p. 14.

‡ J. Neyman, "On the correlation of the mean and the variance in samples drawn from an 'infinite' population," *Biometrika*, Vol. XVIII. (1926), pp. 401—18.

with the actual curve of regression. Table VI exhibits the data from which the regression chart was constructed.

OTHER TYPES OF UNIVERSE.

The other universes considered are shown in Table VII. Each has but five classes. Universe N has its first two β's (the moments are uncorrected for grouping) identical with those of a normal universe, B is the symmetric binomial $(1+1)^4$,

TABLE V*.

The Cumulated Probability of z, or Probability that the mean of a Random Sample of 4 will not exceed (in algebraic sense) the Mean of the Universe by more than z times the Standard Deviation of the Sample.

z	Cumulated Probability Rectangular Universe†		Cumulated Probability Normal Universe	
	for $-z$	for $+z$	for $-z$	for $+z$
0	·5335	·5335	·50000	·50000
·1	·4281	·5719	·43676	·56324
·2	·3649	·6351	·37595+	·62405−
·3	·3109	·6891	·31962	·68038
·4	·2621	·7409	·26912	·73088
·5	·2167	·7857	·22509	·77491
·6	·1811	·8189	·18755−	·81245+
·7	·1517	·8483	·15606	·84394
·8	·1335	·8671	·12995+	·87005−
·9	·1129	·8871	·10846	·89154
1·0	·0905	·9095	·09085−	·90915+
1·1	·0853	·9147	·07642	·92358
1·2	·0737	·9263	·06460	·93540
1·3	·0659	·9341	·05488	·94512
1·4	·0565	·9435	·04688	·95312
1·5	·0537	·9469	·04025+	·95975−
1·6	·0451	·9549	·03475+	·96525−
1·7	·0439	·9561	·03015−	·96985+
1·8	·0383	·9617	·02628	·97372
1·9	·0347	·9653	·02302	·97698
2·0	·0335	·9689	·02026	·97974
2·1	·0271	·9729	·01790	·98210
2·2	·0231	·9769	·01589	·98411
2·3	·0231	·9769	·01415+	·98585−
2·4	·0223	·9777	·01266	·98734
2·5	·0223	·9783	·01136	·98864
2·6	·0193	·9807	·01022	·98978
2·7	·0169	·9831	·00923	·99077
2·8	·0157	·9843	·00837	·99163
2·9	·0137	·9863	·00760	·99240
3·0	·0125	·9875	·00692	·99308
3·5	·0105	·9901	·00450+	·99550−
4·0	·0087	·9919	·00308	·99692

* A more detailed table is given in an Appendix to this paper.

† Having ten classes.

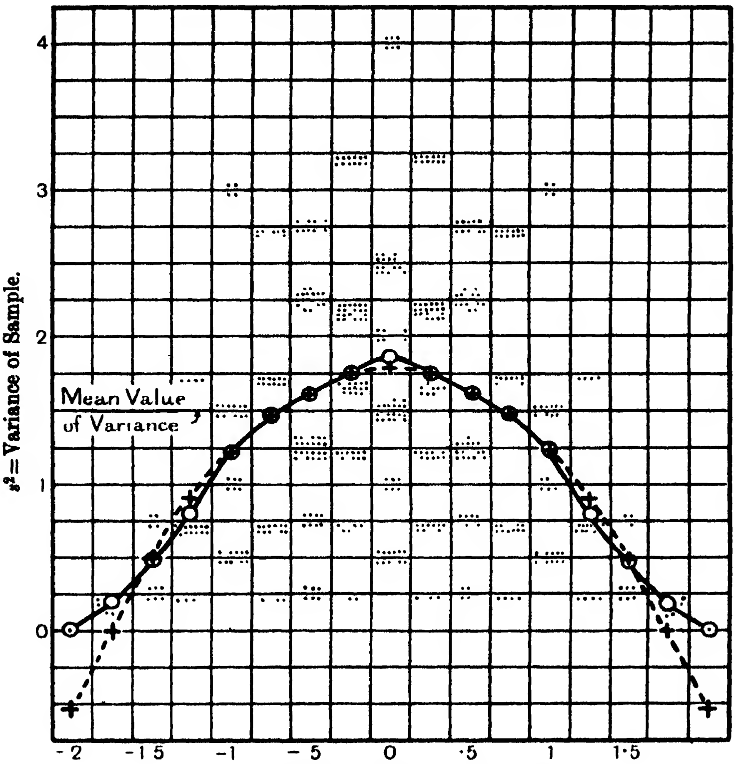


Fig. 3. Regression of Variance on Mean (Rectangular Universe of five Classes). The circles, connected by the solid curve, are the means of the columns of the scatter diagram. The crosses, connected by the dashed curve, are points on the second order parabola of regression as given by Neyman.

TABLE VI.

Regression of Variance on Mean for Samples of 4 from Rectangular Universe of five Classes.

x	$\overline{s_x^2}$	f_x	x	$\overline{s_x^2}$	f_x
0	1·8353	85	±1·25	·7875	20
± ·25	1·75	80	±1·5	·45	10
± ·5	1·6324	68	±1·75	·1875	4
± ·75	1·4567	52	±2	0	1
±1	1·2	35			

$\overline{s_x^2}$ = mean variance of column.
f_x = frequency of variances for a given column.

B' is the asymmetric binomial $(3+2)^4$. Results for samples of 4 from N, B and R (rectangular universe of five classes) are compared in Table VIII. The table can readily be completed for positive values of z. Table IX gives results for the asymmetric binomial universe.

The grouping in these universes is so coarse that results are unsatisfactory. The method, however, can be applied to any type of (discrete) sampled population and it is desirable that it be applied to other universes with more classes as it was applied to the ten-class rectangular universe.

An interesting case is that in which the sampled population contains only two classes of individuals. For such a population the relation $\beta_2 - \beta_1 - 1 = 0$ is satisfied and the variance, s^2, is a definite function of x. For example, suppose the variate X can assume the values X_0 and X_1 with the probabilities $1-p$ and p respectively. Then it can be shown* that $s^2 = (X_1 - x)(x - X_0)$.

The probability of obtaining r X_1's and $(n-r)$ X_0's in a sample of n from this universe is $\binom{n}{r} p^r (1-p)^{n-r}$. The mean of such a sample is $(1/n)[(n-r)X_0 + rX_1]$, and its deviation from $X_0 + p(X_1 - X_0)$, the mean of the universe, is

$$x = (r/n - p)(X_1 - X_0).$$

The standard deviation of the sample is $s = (1/n)\sqrt{r(n-r)}\,(X_1 - X_0)$, whence

$$z = \frac{r - np}{\sqrt{r(n-r)}}.$$

TABLE VII.

Universe	N	B	B'
X	f	f	f
0	1	1	81
1	20	4	216
2	54	6	216
3	20	4	96
4	1	1	16
Totals	96	16	625
β_1	0	0	0·04
β_2	3	2·5	2·54

* See Neyman, *loc. cit.* p. 403. The formulae for σ^2 in (9) and (10) are incorrect; they should be

$$\sigma^2 = \frac{(n-p)p}{n^2}(b-a)^2 \text{ in (9), and } \sigma^2 = (b-\bar{x})(\bar{x}-a) \text{ in (10).}$$

TABLE VIII.

Cumulated Probabilities of z for Samples of 4 from Universes N, B and R.

z	N	B	R
$-\infty$	·0019	·0039	·0032
−4·04	·0019	·0042	·0096
−3	·0019	·0056	·0192
−2·89	·0023	·0095+	·0256
−1·73	·0226	·0333	·0384
−1·51	·0228	·0377	·0576
−1·41	·0258	·0553	·0768
−1	·1084	·1113	·0960
− ·96	·1084	·1116	·1024
− ·90	·1167	·1380	·1216
− ·82	·1167	·1409	·1408
− ·69	·1179	·1526	·1600
− ·58	·2813	·2386	·2048
− ·46	·2813	·2393	·2240
− ·45	·2874	·2745−	·2624
− ·33	·2875−	·2789	·2912
− ·30	·3485+	·3503	·3296
− ·23	·3568	·3767	·3488
− ·19	·3583	·3923	·3744
− ·17	·3586	·4011	·4128
− ·14	·3586	·4018	·4320
0	·6414	·5982	·5680

TABLE IX.

Cumulated Probability of z for Samples of 4 from Universe B′.

z	Cumulated Probability	z	Cumulated Probability	z	Cumulated Probability
$-\infty$	·0146	− ·07	·5254	·78	·9032
−3·12	·0176	− ·06	·5272	·80	·9144
−2·20	·0296	·10	·5367	·84	·9152
−1·96	·0510	·12	·5536	·89	·9162
−1·27	·0540	·14	·5821	1·04	·9254
−1·03	·0781	·18	·6582	1·06	·9279
− ·89	·1351	·20	·6583	1·14	·9283
− ·85	·1993	·25+	·6625−	1·39	·9340
− ·81	·2564	·28	·6672	1·40	·9344
− ·65+	·2577	·33	·6926	1·50	·9598
− ·60	·2698	·35−	·7496	1·80	·9767
− ·49	·2805−	·36	·7500	1·98	·9792
− ·42	·3447	·40	·7669	1·99	·9796
− ·35	·3449	·44	·7711	2·19	·9797
− ·32	·3734	·50+	·7787	2·66	·9847
− ·27	·3841	·54	·7790	3·80	·9848
− ·21	·3859	·57	·8551	3·81	·9851
− ·20	·4715−	·60	·8692	4·97	·9852
− ·12	·5182	·70	·8693	∞	1·0000

The probability that the mean of a sample will not exceed (in algebraic sense) the mean of the universe by more than z times the standard deviation of the sample is the incomplete binomial moment of order zero,

$$\sum_{k=0}^{r} \binom{n}{k} p^k (1-p)^{n-k}.$$

THE DISTRIBUTIONS OF VARIOUS STATISTICAL PARAMETERS IN SAMPLES FROM A RECTANGULAR UNIVERSE.

Mean. The distribution of means from the five-class rectangular universe is shown in Table X. As usual, x is the deviation of the mean of the sample from the mean of the universe; p is the probability. It is found that the third differences are constant, except for discontinuities at $x=0$, $\pm 0{\cdot}25$. Changing the unit by the substitution $x=5\xi$, in order to compare with certain results of Irwin* for a continuous rectangular universe, and fitting with cubic curves, we have

$$\frac{p(\xi)}{\cdot 05} = \begin{cases} (8/3)(1{\cdot}02 - 0{\cdot}12\,|\xi| - 24\xi^2 + 48\,|\xi|^3) \text{ for } \xi \leq 1/4, \\ (16/3)(0{\cdot}99 - 5{\cdot}98\,|\xi| + 12\xi^2 - \;\; 8\,|\xi|^3) \text{ for } \xi \geqq 1/4. \end{cases}$$

(The decimal values are exact.) The value $\cdot 05$ is the class interval. Compare these equations with those obtained by Irwin's method for a continuous rectangular universe†, viz.

$$\frac{y}{d\xi} = \begin{cases} (8/3)(1 - 24\xi^2 + 48\,|\xi|^3) \text{ for } |\xi| \leq 1/4, \\ (16/3)(1 - 6\,|\xi| + 12\xi^2 - 8\,|\xi|^3) \text{ for } |\xi| \geqq 1/4. \end{cases}$$

It should be stated that the values for the probabilities in the continuous universe are obtained by integration. For purposes of comparison with the distribution of means from a discrete universe they are shown in Table X in the column headed $\int y\,d\xi$.

TABLE X.

Distribution of Means of Samples of 4 from a Rectangular Universe.

x	ξ	p	$\int y\,d\xi$
0	0	·1360	·1327
$\pm$ ·25	$\pm$·05	·1280	·1257
$\pm$ ·5	$\pm$·1	·1088	·1075⁻
$\pm$ ·75	$\pm$·15	·0832	·0829
$\pm$1	$\pm$·2	·0560	·0567
$\pm$1·25	$\pm$·25	·0320	·0336
$\pm$1·5	$\pm$·3	·0160	·0173
$\pm$1·75	$\pm$·35	·0064	·0074
$\pm$2	$\pm$·4	·0016	·0023
$\pm$2·25	$\pm$·45	0	·0003
$\pm$2·5	$\pm$·5	0	·0000

p = probability of given value of x (or ξ) for a discrete universe of five classes.

$\int y\,d\xi$ = probability, for a continuous universe, that x (or ξ) will fall in the given class interval.

* J. O. Irwin, "On the frequency distribution of the means of samples from a population having any law of frequency with finite moments, with special reference to Pearson's Type II," *Biometrika*, Vol. xix. (1927), pp. 226—39.

† See Irwin, *loc. cit.* p. 238.

TABLE XI.

Distribution of Medians of Samples of 3 from Rectangular Universe of ten Classes.

Median	ξ	Probability
0	$-\cdot45$	·028
1	$-\cdot35$	·076
2	$-\cdot25$	·112
3	$-\cdot15$	·136
4	$-\cdot05$	·148
5	·05	·148
6	·15	·136
7	·25	·112
8	·35	·076
9	·45	·028

TABLE XII.

Distribution of Ranges of Samples of 4 from a Rectangular Universe.

Range W	p	$\int \phi_2(W)\, dW$
0	·0010	·0004$\frac{1}{8}$
1	·0126	·0115
2	·0400	·0388
3	·0770	·0757
4	·1164	·115
5	·1510	·1495
6	·1736	·172
7	·1770	·1753
8	·1540	·1522
9	·0974	·0955
10	—	·0140$\frac{3}{16}$

p = probability of given range for discrete universe of ten classes.

$\int \phi_2(W)\, dW$ = probability, for a continuous universe, that range will fall in the given class interval.

The distribution of means of samples of 3 for the discrete universe is fitted by the quadratic

$$\frac{p(\xi)}{1/15} = \begin{cases} (9/4)(1\cdot019 - 12\xi^2) \text{ for } |\xi| < 1/6, \\ (27/8)(0\cdot995 - 4|\xi| + 4\xi^2) \text{ for } |\xi| > 1/6. \end{cases}$$

The variable ξ has the values 0, $\pm 1/15$, $\pm 2/15$, $\pm 3/15$, $\pm 4/15$, $\pm 5/15$, $\pm 6/15$. Irwin's results for samples of 3 from a continuous universe are

$$\frac{y}{d\xi} = \begin{cases} (9/4)(1 - 12\xi^2) \text{ for } |\xi| \leqq 1/6, \\ (27/8)(1 - 4|\xi| + 4\xi^2) \text{ for } |\xi| \geqq 1/6. \end{cases}$$

For samples of 2 the result is the same for discrete and continuous universes, viz.

$$p(\xi)/0\cdot1 = y/d\xi = 2(1 - 2|\xi|).$$

It is worthy of note that the discontinuities in the polynomial representations occur at the same points in the discrete universe and in the continuous universe.

*Median**. Suppose we have a universe in which the variate takes values between 0 and 1, all such values being equally probable. Take a point X in the interval (0, 1). The probability that the variate will lie in the interval $(0, X)$ is X and in $(X, 1)$ is $1 - X$. Consider $2k+1$ values of the variate, $X_1, X_2, \ldots, X_{2k+1}$. The probability that one of these values will fall in the interval $(X, X+dX)$ is

* Cf. E. S. Pearson and N. K. Adyanthāya, "The distribution of frequency constants in small samples from symmetrical populations," *Biometrika*, Vol. xx^A. (1928), pp. 356—60.

$(2k+1)\,dX$. The probability that k of the remaining values will lie in $(0, X)$ and the other k in $(X, 1)$ is $\binom{2k}{k} X^k (1-X)^k$. Consequently the probability that the median will fall in the interval $(X, X+dX)$ is

$$(2k+1)\binom{2k}{k} X^k (1-X)^k\, dX = \frac{(2k+1)!}{(k!)^2} X^k (1-X)^k\, dX.$$

Let us shift the origin to the centre of the interval by letting $X = \xi + 1/2$. Then $X^k (1-X)^k = (1/4 - \xi^2)^k$, and

$$P_k(\xi) = \frac{(2k+1)!}{(k!)^2} (1/4 - \xi^2)^k\, d\xi$$

is the probability that the median will lie in the interval $(\xi, \xi + d\xi)$.

For samples of 3 from the discrete rectangular universe of ten classes the medians are distributed as shown in Table XI. The distribution is perfectly fitted by the parabola

$$p(\xi)/0{\cdot}1 = 1{\cdot}495 - 6\xi^2.$$

If $2k+1 = 3$, then $k = 1$, and the distribution of medians of samples from the continuous rectangular universe is given by

$$1{\cdot}5 - 6\xi^2.$$

*Range**. The probability distribution of ranges for samples of 4 from the rectangular distribution of ten classes is shown in Table XII in the column headed p. If the first two groups (range 0 and range 1) are combined, the distribution is perfectly fitted by the cubic

$$p(W)/0{\cdot}001 = -12W^3 + 120W^2 - 2W + 20,$$

in which W = range.

Table XII also gives, for comparison, the probability distribution of ranges for samples of 4 from a continuous rectangular universe. This has been determined by Neyman and E. S. Pearson†.

Their formula (xxxviii) on page 210 is

$$\phi_2(W) = n(n-1)(W^{n-2}/w^{n-1})(1 - W/w),$$

in which W is the range of the sample and w the range of the universe. In our case $n = 4$, $w = 10$, and for $\phi_2(W)\,dW$ we get

$$\phi_2(W)\,dW = 0{\cdot}012\,W^2(1 - 0{\cdot}1\,W)\,dW.$$

By a simple integration we find the probability of each class.

* Cf. E. S. Pearson and N. K. Adyanthāya, "The distribution of frequency constants in small samples from symmetrical populations," *Biometrika*, Vol. xx^A. (1928), pp. 356—60; also the article by Neyman and E. S. Pearson referred to in the next footnote.

† J. Neyman and E. S. Pearson, "On the use and interpretation of certain test criteria for purposes of statistical inference," *Biometrika*, Vol. xx^A. (1928), pp. 175—240, 263—94.

The substitution $W = 10\xi$ enables us more readily to compare the two equations. In the notation employed for means we have

$$p(\xi)/0{\cdot}1 = -12\xi^3 + 12\xi^2 - 0{\cdot}02\xi + 0{\cdot}02,$$

$$y/d\xi = -12\xi^3 + 12\xi^2.$$

Extreme Average. By extreme average is meant the mean value of the greatest and least variates in a sample. The distribution of extreme averages for a sample of 4 from the rectangular universe of five classes is shown in Table XIII. If E is the value of the extreme average, then the substitution $E = 5\xi + 2$ reduces the range of the extreme average, which is the same as that of the universe, to a unit interval centred on $E = 2$. The distribution is then fitted exactly by the cubic

$$p(\xi)/0{\cdot}1 = -32\,|\xi|^3 + 48\xi^2 - 23{\cdot}84\,|\xi| + 3{\cdot}92.$$

Compare this with the equation

$$f(\xi)/d\xi = -32\,|\xi|^3 + 48\xi^2 - 24\,|\xi| + 4$$

for the distribution of the extreme averages of samples of 4 from a continuous rectangular universe, which may be deduced from a formula given by Dodd*.

TABLE XIII.

Distribution of Extreme Averages for Samples of 4 from Rectangular Universe of five Classes.

Extreme Average E	ξ	625 × Probability	Probability p
0	− ·4	1	·0016
·5	− ·3	14	·0224
1	− ·2	51	·0816
1·5	− ·1	124	·1984
2	0	245	·3920
2·5	·1	124	·1984
3	·2	51	·0816
3·5	·3	14	·0224
4	·4	1	·0016

Greatest Variate and Least Variate. The distribution of greatest variate and that of least variate can be obtained from the correlation table of these two parameters (Table XIV). The entire table is for a ten-class universe, the part set off by dotted lines is for a five-class universe.

The regression surface is fitted (except for the diagonal of 1's) by the equation $f(\xi, \eta) = 12(\xi - \eta)^2 + 2$. If the table be made symmetric after the manner described by Tippett†, the lack of fit along the diagonal will be overcome.

* E. L. Dodd, "Functions of measurement under general laws of error," *Skandinavisk Aktuarietidskrift*, Vol. v. (1922), pp. 133—58. The formula for a sample of n from a continuous rectangular universe of range unity is $f(\xi)/d\xi = 2^{n-1}n\,(1/2 - |\xi|)^{n-1}$.

† *Biometrika*, Vol. XVII. p. 381.

TABLE XIV.

Simultaneous Frequency Distribution of Greatest and Least Variates (Rectangular Universe).

Least Variate.

Greatest Variate.	0	1	2	3	4	5	6	7	8	9
0	1	—	—	—	—	—	—	—	—	—
1	14	1	—	—	—	—	—	—	—	—
2	50	14	1	—	—	—	—	—	—	—
3	110	50	14	1	—	—	—	—	—	—
4	194	110	50	14	1	—	—	—	—	—
5	302	194	110	50	14	1	—	—	—	—
6	434	302	194	110	50	14	1	—	—	—
7	590	434	302	194	110	50	14	1	—	—
8	770	590	434	302	194	110	50	14	1	—
9	974	770	590	434	302	194	110	50	14	1
Totals	3439	2465	1695	1105	671	369	175	65	15	1
Mean	7·09	7·34	7·58	7·82	8·07	8·31	8·54	8·75+	8·93	9

The entire table is for a ten-class universe, the part set off by dotted lines is for a five-class universe.

General Conclusions. A number of statistical parameters (mean, median, range, extreme average, greatest variate and least variate) in samples from a rectangular universe are distributed in polynomials, which are apparently, except in the case of greatest variate and least variate, of degree one less than the number in the sample.

There appears to be little difference in the distributions of these variates when the universe is continuous and when it is discrete.

GEOMETRIC METHODS.

The distribution of means of samples from a continuous rectangular universe has been derived by Hall* by considering a sample of n as a point in space of n dimensions. The sampled population is represented by a unit hypercube. (See Fig. 4.) Let P be the point representing the sample $(X_1, X_2, \ldots, X_n)$; let OI be the diagonal of the hypercube from the origin to the point I, each of whose coordinates is unity; and let PM be the perpendicular from P upon OI. Then the mean, $\bar{X}$, and the standard deviation, s, of the sample are OM/OI and MP/OI respectively. Hall's method consists of ingeniously finding the content of the $(n-1)$-dimensional region cut from the hypercube by the hyperplane $\Sigma X_i = n\bar{X}$.

* Philip Hall, "The distribution of means of samples of size N drawn from a population in which the variate takes values between 0 and 1, all such values being equally probable," *Biometrika*, Vol. XIX. (1927), pp. 240—4.

In an Editorial Note* on the paper of Hall (and the paper of Irwin referred to above), E. S. Pearson has called attention to the fact that the frequency of a given value of the standard deviation of a sample could be obtained by integrating the density (unity for a rectangular universe) throughout the region for which MP is constant, which is a hypercylinder with axis OI and cross-section an $(n-1)$-dimensional hypersphere.

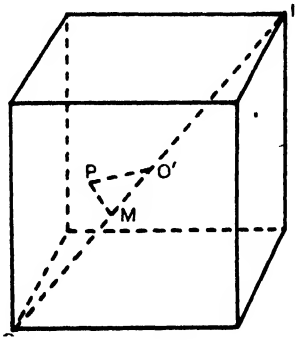


Fig. 4.

Since the mean of the rectangular universe may be represented by O', the centre of the hypersphere, the deviation of the mean of the sample from the mean of the universe may be represented by $O'M/OI$. Thus the ratio z is represented by $O'M/MP = \cot\alpha$, where $\alpha = \angle MO'P$. The region for which z is constant may be described as a hypercone with vertex O' and axis coinciding with OI, and the frequency distribution of z could be obtained if we could find the content of that part of the "surface" of this hypercone bounded by the hypercube†. An endeavour to do this seems to lead to extremely complicated integration even in the case of three dimensions, but the distribution of z, and also that of s, the standard deviation, are derived for $n=2$ in the hope that they may offer some clue to the forms of the distributions for larger values of n.

For $n=2$ (i.e. samples of 2) the hypercube degenerates into a square (see Fig. 5), and it is only necessary to find the length of the line AB for the frequency of s, and the length of the line $O'C$ for the frequency of z.

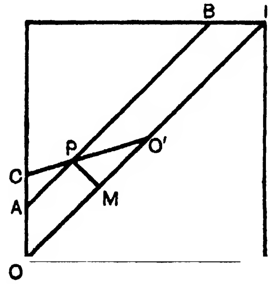


Fig. 5.

* *Biometrika*, Vol. XIX. (1927), pp. 244—6.

† See Neyman and E. S. Pearson, *loc. cit.*

We find that $$AB = OI - 2MP = \sqrt{2}\,(1 - 2s).$$

The area of the unit square is

$$2\int_{MP=0}^{\sqrt{2}/2} AB\,d(MP) = \int_{s=0}^{1/2} 2\sqrt{2}\,(1-2s)\,\sqrt{2}\,ds = \int_0^{1/2} 4\,(1-2s)\,ds.$$

Therefore the distribution of standard deviations of samples of 2 from a continuous rectangular universe is

$$f(s)\,ds = 4\,(1-2s)\,ds.$$

Applying the law of sines to the triangle $OO'C$, we find that

$$\frac{O'C}{1/\sqrt{2}} = \frac{\sin(\pi/4)}{\sin(3\pi/4 - \alpha)} = \frac{1/\sqrt{2}}{\sin(3\pi/4)\cos\alpha - \cos(3\pi/4)\sin\alpha},$$

whence $$O'C = \tfrac{1}{2}\operatorname{cosec}(\alpha + \pi/4).$$

The area of the unit square is

$$4\int_0^{\pi/2} \tfrac{1}{2}\,(O'C)^2\,d\alpha = 2\int_0^{\pi/2} \tfrac{1}{4}\operatorname{cosec}^2(\alpha + \pi/4)\,d\alpha.$$

The distribution of α is given by

$$\phi(\alpha)\,d\alpha = \tfrac{1}{4}\operatorname{cosec}^2(\alpha + \pi/4)\,d\alpha = \frac{d\alpha}{2\,(\sin\alpha + \cos\alpha)^2},$$

and the distribution of $z\ (= -\cot\alpha)$ is

$$f(z)\,dz = \phi(\alpha)\,d\alpha = \frac{dz}{2\,(1 - |z|)^2}.$$

The cumulated probability of z is

$$-\frac{1}{2\,(1+z)} \text{ for } z \leqq 0, \quad 1 - \frac{1}{2\,(1-z)} \text{ for } z \geqq 0.$$

It will be found that these distributions of s and z for samples of 2 from a continuous rectangular universe give very good fits to the corresponding distributions for a discrete rectangular universe.

APPENDIX. DETAILED TABLE V. (See p. 131.)

Probability Distribution of z (Mean of Sample − Mean of Universe) ÷ (Standard Deviation of Sample) for Samples of 4 from Rectangular Universe having ten Classes.

(f = 10,000 × probability of z)

z	f	Cumulated Probability for $-z$	Cumulated Probability for $+z$	z	f	Cumulated Probability for $-z$	Cumulated Probability for $+z$	z	f	Cumulated Probability for $-z$	Cumulated Probability for $+z$
0	670	·5335	·5335	·195	24	·3733	·6291	·391	36	·2657	·7379
·059	12	·4665	·5347	·196	48	·3709	·6339	·4	30	·2621	·7409
·065	24	·4653	·5371	·199	12	·3661	·6351	·402	24	·2591	·7433
·066	12	·4629	·5383	·204	12	·3649	·6363	·408	36	·2567	·7469
·072	24	·4617	·5407	·211	24	·3637	·6387	·420	24	·2531	·7493
·073	12	·4593	·5419	·213	24	·3613	·6411	·424	48	·2507	·7541
·075	36	·4581	·5455	·224	72	·3589	·6483	·433	4	·2459	·7545
·076	24	·4545	·5479	·229	48	·3517	·6531	·436	48	·2455	·7593
·078	12	·4521	·5491	·236	36	·3469	·6567	·437	24	·2407	·7617
·080	24	·4509	·5515	·241	24	·3433	·6591	·449	12	·2383	·7629
·082	24	·4485	·5539	·247	28	·3409	·6619	·451	24	·2371	·7653
·085	36	·4461	·5575	·250	6	·3381	·6625	·452	12	·2347	·7665
·087	48	·4425	·5623	·254	12	·3375	·6637	·458	12	·2335	·7677
·090	36	·4377	·5659	·262	48	·3363	·6685	·459	24	·2323	·7701
·093	24	·4341	·5683	·265	12	·3315	·6697	·473	12	·2299	·7713
·097	36	·4317	·5719	·267	48	·3303	·6745	·482	12	·2287	·7725
·101	60	·4281	·5779	·271	12	·3255	·6757	·483	24	·2275	·7749
·110	36	·4221	·5815	·280	36	·3243	·6793	·485	48	·2251	·7797
·115	28	·4185	·5843	·286	30	·3207	·6823	·487	36	·2203	·7833
·122	12	·4157	·5855	·289	8	·3177	·6831	·5	24	·2167	·7857
·123	12	·4145	·5867	·290	36	·3169	·6867	·503	48	·2143	·7905
·125	6	·4133	·5873	·298	24	·3133	·6891	·507	24	·2095	·7929
·130	36	·4127	·5909	·302	48	·3109	·6939	·514	12	·2071	·7941
·136	24	·4091	·5933	·305	12	·3061	·6951	·530	12	·2059	·7953
·140	24	·4067	·5957	·312	36	·3049	·6987	·535	12	·2047	·7965
·141	24	·4043	·5981	·314	24	·3013	·7011	·548	24	·2035	·7989
·143	12	·4019	·5993	·316	24	·2989	·7035	·549	36	·2011	·8025
·147	24	·4007	·6017	·329	60	·2965	·7095	·555	24	·1975	·8049
·151	12	·3983	·6029	·338	36	·2905	·7131	·562	24	·1951	·8073
·153	12	·3971	·6041	·346	28	·2869	·7159	·577	56	·1927	·8129
·154	24	·3959	·6065	·348	24	·2841	·7183	·586	24	·1871	·8153
·158	24	·3935	·6089	·354	12	·2817	·7195	·588	24	·1847	·8177
·162	12	·3911	·6101	·358	24	·2805	·7219	·594	12	·1823	·8189
·167	18	·3899	·6119	·366	24	·2781	·7243	·603	24	·1811	·8213
·169	24	·3881	·6143	·367	12	·2757	·7255	·610	12	·1787	·8225
·171	48	·3857	·6191	·371	48	·2745	·7303	·611	12	·1775	·8237
·183	48	·3809	·6239	·382	12	·2697	·7315	·612	12	·1763	·8249
·192	28	·3761	·6267	·385	28	·2685	·7343	·622	36	·1751	·8285
·625	12	·1715	·8297	·970	36	·1013	·9023	1·789	24	·0407	·9617
·631	12	·1703	·8309	·974	12	·0977	·9035	1·809	12	·0383	·9629
·640	12	·1691	·8321	·980	24	·0965	·9059	1·859	24	·0371	·9653
·651	36	·1679	·8357	·988	36	·0941	·9095	1·983	12	·0347	·9665
·653	24	·1643	·8381	1·021	12	·0905	·9107	2	24	·0335	·9689
·667	30	·1619	·8411	1·039	4	·0893	·9111	2·021	4	·0311	·9693
·671	24	·1589	·8435	1·061	12	·0889	·9123	2·041	24	·0307	·9717
·673	12	·1565	·8447	1·066	12	·0877	·9135	2·065	12	·0283	·9729

APPENDIX. DETAILED TABLE V—*Continued.* (See p. 131.)

(f = 10,000 × probability of z)

z	f	Cumulated Probability for $-z$	Cumulated Probability for $+z$	z	f	Cumulated Probability for $-z$	Cumulated Probability for $+z$	z	f	Cumulated Probability for $-z$	Cumulated Probability for $+z$
·677	12	·1553	·8459	1·068	12	·0865	·9147	2·111	12	·0271	·9741
·688	12	·1541	·8471	1·106	12	·0853	·9159	2·117	16	·0259	·9757
·696	12	·1529	·8483	1·109	24	·0841	·9183	2·121	12	·0243	·9769
·700+	24	·1517	·8507	1·147	12	·0817	·9195	2·309	8	·0231	·9777
·704	12	·1493	·8519	1·155	8	·0805	·9203	2·5	6	·0223	·9783
·707	48	·1481	·8567	1·172	24	·0797	·9227	2·502	12	·0217	·9795
·722	4	·1433	·8571	1·179	12	·0773	·9239	2·524	12	·0205	·9807
·734	24	·1429	·8595	1·183	24	·0761	·9263	2·683	24	·0193	·9831
·743	24	·1405	·8619	1·206	24	·0737	·9287	2·714	12	·0169	·9843
·750	6	·1381	·8625	1·225	24	·0713	·9311	2·858	12	·0157	·9855
·763	12	·1375	·8637	1·250	6	·0689	·9317	2·887	8	·0145	·9863
·768	12	·1363	·8649	1·260	24	·0683	·9341	2·982	12	·0137	·9875
·770	16	·1351	·8665	1·333	30	·0659	·9371	3·317	12	·0125	·9887
·8	6	·1335	·8671	1·336	24	·0629	·9395	3·464	8	·0113	·9895
·802	48	·1329	·8719	1·344	12	·0605	·9407	3·5	6	·0105	·9901
·808	36	.1281	·8755	1·347	16	·0593	·9423	3·919	12	·0099	·9913
·812	12	·1245	·8767	1·373	12	·0577	·9435	4	6	·0087	·9919
·845	24	·1233	·8791	1·432	12	·0565	·9447	4·041	4	·0081	·9923
·855	12	·1209	·8803	1·443	4	·0553	·9451	4·522	12	·0077	·9935
·857	12	·1197	·8815	1·455	12	·0549	·9463	4·619	4	·0065	·9939
·866	8	·1185	·8823	1.5	6	·0537	·9469	4·950	12	·0061	·9951
·873	24	·1177	·8847	1·501	4	·0531	·9473	5·191	4	·0049	·9955
·894	24	·1153	·8871	1·508	12	·0527	·9485	6	6	·0045	·9961
·904	12	·1129	·8883	1·521	24	·0515	·9509	6·351	4	·0039	·9965
·905	12	·1117	·8895	1·540	12	·0491	·9521	7·071	12	·0035	·9977
·907	4	·1105	·8899	1·547	4	·0479	·9525	7·506	4	·0023	·9981
·911	36	·1101	·8935	1·581	24	·0475	·9549	8	6	·0019	·9987
·918	12	·1065	·8947	1·606	12	·0451	·9561	8·660	4	·0013	·9991
·949	24	·1053	·8971	1·732	20	·0439	·9581	9·815	4	·0009	·9995
·962	16	·1029	·8987	1·768	12	·0419	·9593	∞	5	·0005	1·0000

ALBINISM IN DOGS.

By KARL PEARSON and C. H. USHER.

The long research on the experimental breeding of dogs initiated in 1905 by the late Edward Nettleship, and the authors of the present memoir*, and still carried on in connection with the Galton Laboratory, is not yet ripe for complete publication. It has been protracted first on account of the heavy expense of keeping at the same time a large number of dogs, and secondly owing to the grave difficulties of the Great War, during which only a few dogs could be preserved for starting afresh when times permitted. This involved still closer inbreeding than we should have otherwise employed. The experiment has gone so far, however, that it seems possible to report on one aspect of the investigation, that of albinism in dogs. Our original stock consisted of Albino Pekinese and pure-bred black Pomeranians. These albino Pekinese had white coats, the hairs of which contained no pigment granules whatever; the eyes of all of them were characterised by strong red reflex, and marked photophobia. Indeed, on more than one occasion too long exposure to brilliant sunshine has produced something of the nature of a collapse in the dog, who has had to be carried home. The sight of these albinos has varied a good deal, some being so short-sighted that they were apt to run against obstacles, and many, if not all, found difficulty in catching pieces of biscuit thrown to them, which was accomplished promptly by their coloured brethren. This type of albino we shall term the Dondo or White Albino, it is the commonest of all our albinos.

It has not been found possible, in the course of our twenty years' experiments, to obtain dogs having albinotic eyes, and hairs of the coat which contain melanine pigment granules. Even light fawn coats, the hairs of which contain *very* few pigment granules, cannot be associated with albinotic eyes. On the other hand, we have on a few occasions obtained dogs with white coats, and fully pigmented eyes; but even in some of these few cases the tendency of the coat was to become pale yellow as the dog approached maturity†. No pigment granules are to be found in these yellow hairs, they appear to be a stage towards the paler fawns in which granules are scarcely to be found except of course in the black "points," if such are present.

But there is one coat which has all the appearance of "some" colour, although it lacks entirely pigment granules, with which albinotic eyes can be associated. This colour may be described as something like pale millboard, although it has in certain aspects a tint not unlike the sky background of many photographic prints. Weldon

* I gladly acknowledge the help that my share of the work has received from occasional grants from the Royal Society Government Grant Committee. K.P.

† The dogs with black eyes and white coats have in our experience been found to be very delicate, the critical time being when the puppy-coat is cast.

termed a similar skin colour in his mice "lilac" or pale blue grey and found it possible to associate it with albinotic eyes*. This "lilac" shades off into pale chocolate, such as one reaches by biting off the end of a stick of that delicacy, and led one of us to describe the dogs *Hans* and *Grethel* when puppies as of a scraped chocolate colour (see Plate V). I do not think any distinction can be made between the "lilac" and "scraped chocolate" coats; it depends, in part, on the lighting by which they are seen, but more on the length of time since the last moult. The colouring is not due to pigment granules, but is of a diffused character, and possibly is due to lipochromes; it is like in nature to that of certain rather rare types of human red hair, and of some chestnut hair in horses, which carry no pigment granules. This coat colour differs essentially from the cream or faint yellow tinge which Dondos acquire when much in the sun and lose as rapidly in winter or indoors; a similar colouring occurs with the hair of human albinos under like conditions. All the dogs we have bred with the "lilac" coat have had albinotic eyes. They form the second type of albino dog and we have termed them Cornaz Spaniels, in memory of the distinguished Belgian ophthalmologist. The Cornaz Spaniel has a whole coloured coat, no dark "points" of any kind†, and the pink nose pad and flesh of the pure white Dondo. Both Dondo and Cornaz have, as a rule, "spectacle marks" of a brown colour. These are formed by exudation from the lachrymal glands, and this is caused, possibly, by some weakness of the eyes. The "spectacle marks" stain the finger when touched, and can be washed off the coat; they increase with exposure to sunlight, and will practically disappear in the case of a bitch shut up with her litter. They are due to some oily form of exuded pigment, much like what is discharged from the pores between the toes of these same albinos. A study of this exuded pigment would be of interest, but it has nothing to do with the absence of melanine pigment granules, which constitutes the essential feature of albinism of the coat.

We now reach the remaining canine albinotic type; it would be called a piebald, if the colouring were black and white. Actually the colouring is "Cornaz" and white, and we prefer to call it a skewbald‡ (see Plate VI). The reader must be warned at once that a skewbald albino is not the hybrid of a Cornaz and a Dondo. The Cornaz white skewbald has, in our experience, invariably albinotic eyes, and it appears in a litter, which, whether albinotic or coloured in its members, has other members piebald or skewbald. One of the remarkable facts about a skewbald albino is this: Two puppies are born in the same litter and there is nothing whatever to distinguish between them; both are markedly skewbald albinos. The first moult comes, and one of these puppies will lose its "Cornaz" coloured areas, cease to be skewbald and remain during life a pure white Dondo; the other remains a skewbald after its first moult, and if it does so is a skewbald for ever. We have no explanation to offer of this phenomenon, it is an extreme case of

* *Biometrika*, Vol. XI. Appendix, p. 45 and *Records*, pp. 2 *et seq.*

† By "points" we mean a darker shade on ears, head, vertebral column, etc. A Cornaz may sometimes have a white shirt front, or a white belly.

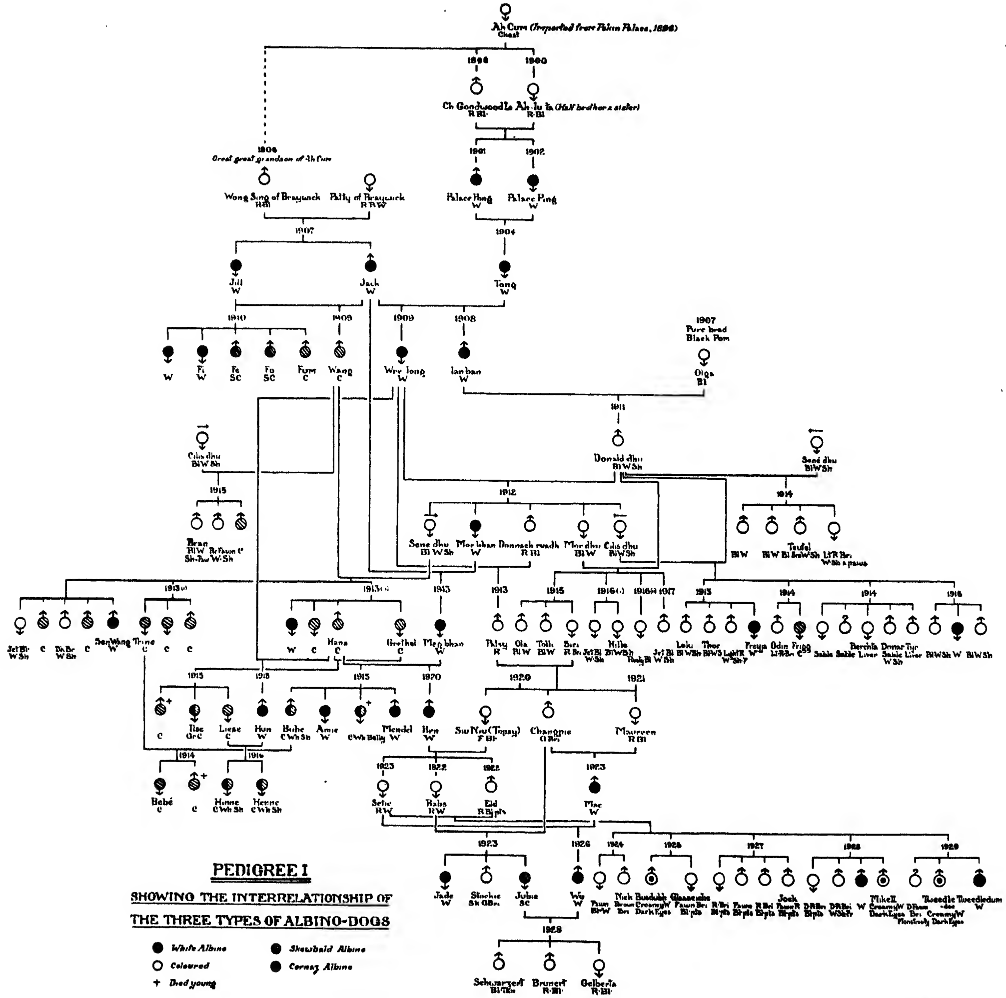
‡ By "piebald" or "skewbald," we do not understand the existence merely of a white shirt front, white belly or white paws, but parti-colouring stretching up to or even across the vertebral column.

the fact that it is not feasible to describe accurately the coat colour of a dog till after the first moult*. We have now to consider these three types, (i) the Dondo, or pure white albino, (ii) the Cornaz Spaniel, or "lilac" albino, and (iii) the Skewbald Albino, and to ask whether they may be looked upon as true breeding distinct types, or whether one can arise from the other; further, whether any of them can originate dogs with normal eyes, or pigmented coats.

Let us start from our foundation stock, *Tong* (called *Tong I* later), *Jack* and *Jill* (see Plate I). These three were all white Pekinese albinos or dondos, with a tendency for the coat to become "creamy" in parts when it was old, or the dog much in the sun. *Tong* had two litters by *Jack* (not reproduced entirely in the pedigree which accompanies this paper). In the first she had five all white albinos, of whom: *Tong II* has only had coloured litters by coloured dogs but albinism has reappeared in her grandchildren; *Mairi bhan* was mated with her brother only; *Ian ban* mated with a black Pomeranian bitch *Olga* produced a black dog *Donald dhu* shown in our Pedigree I, and various black dogs with white shirt fronts; *Ian ban* mated with his sister *Mairi bhan* sired a litter of six albinos all male; these were all white albinos with the usual tendency to cream seasonally; of this litter *Dugald ban* mated with a black dog with white shirt front, offspring of *Ian ban* and *Olga* the black Pomeranian, gave among coloured puppies a white albino, *Donald ban* ("Idiot"), who sired for an albino Pekinese *Spook* a litter of five Dondos or white albinos, the third dog *Hamish ban* of the three survivors was not mated. The second litter of *Tong I* by *Jack* consisted of two male albinos *Wee Ling* and *Wee Choo*, of whom no offspring, and *Wee Tong* a female (see the pedigree), all white albinos. Thus far the record of breeding albinos from *Jack* and *Tong* shows nothing at all inconsistent with white albinism being a recessive character in the Mendelian sense.

We now turn to the matings of *Jack* with his sister *Jill*. These two, dog and bitch, produced five litters; there was nothing to distinguish *Jack* from *Jill* in coat colour, they were white albinos with the usual creamy appearance. They have both, of course, the same ancestry, entirely of coloured dogs, which does not link up with that of *Tong I* until we come to *Ah Cum*, who was imported from the Pekin Palace in 1896 (see Pedigree I), and may now be seen in the British Museum (Natural History). His coat in life was chestnut or bright sable red. We naturally expected from *Jack* and *Jill* a repetition of the same results as from *Jack* and *Tong I*. The first litter, however, gave two albino males of which one who died at birth was said to be darkish down the back, and the second *Wang* (see pedigree) was our first Cornaz albino. Nettleship described the coat as a "rather dark cream," but Pearson was very familiar with the dog as he came to him repeatedly for mating purposes and afterwards into his ownership; the colour of the coat was the "lilac" of Weldon's albino mice. *Wang* was the first Cornaz bred by us. Mated with *Sené dhu*, a black

* One is often obliged to describe the coat-colour of a puppy dying at birth or shortly after. This is done by a knowledge of what different puppy-coats normally become after first moult. An inexperienced observer will report a litter of three "black" puppies, which one knows full well will be red brindles after the first moult.



PEDIGREE I

SHOWING THE INTERRELATIONSHIP OF THE THREE TYPES OF ALBINO-DOGS

- White Albino
- Coloured
- \+ Died young
- Skewbald Albino
- Cornaz Albino

To face p. 146

dog with a white shirt front, the offspring of *Wee Tong,* a pure Pekinese white albino, and *Donald dhu,* a Pompek (see pedigree), he sired two puppies *Hans* and *Grethel,* our most characteristic Cornaz Spaniels; they both had the "bruised" or "scraped" chocolate coats, so characteristic of this sub-species of albino (see Plate V).

The second litter of *Jack* and *Jill* was even more convincing in showing that in dogs white albino × white albino does not always breed true. The offspring were five albinos as far as the eyes were concerned; the first two, bitches (one unnamed and *Fi*), had white coats, slightly tinged with cream, the next two *Fe* and *Fo* (see Plate II) were dogs and both skewbald albinos, that is to say, distinctly parti-coloured, the two colours being the white of white albino, and the grey or pale buff of the Cornaz; the fifth dog *Fum* was a Cornaz, slightly darker than his brother *Wang.* This second litter of *Jack* and *Jill* showed that dogs rated as white albinos would not necessarily breed true, but could produce both skewbald albinos and Cornaz albinos. *Jill's* third litter by *Jack* contained an albino which died at birth, probably but not certainly a white albino, and *Patty* (see Plate I) who was distinctly skewbald at birth, but ultimately the coloured patches became whiter than the coat of either of her parents at 5 months old; at a year she must be called a white albino. *Jill's* fourth litter by *Jack* contained three dogs and a bitch, the first dog *Fo II,* a skewbald Cornaz at birth, became ultimately a pure white albino, the second *Lo* would be best described as a Cornaz with white markings, the third dog was a white albino and the fourth, the bitch, would probably have been a Cornaz. *Jill's* fifth litter consisted of four puppies, three bitches and a dog; the first *Peggy* an albino, "light coat with some pure white," was imperfectly described by Nettleship; the next two bitches died at seven days and three weeks respectively of age, and Nettleship described them simply as albinos: the fourth, a male, was probably a Cornaz without white shirt front.

The *Jack* and *Jill* matings gave litters which were unfortunate in the matter of survival*, probably it was due to the close inbreeding. Only two dogs *Wang* and *Patty* (see Plate I)—possibly *Peggy* who was given away at eight weeks and lost sight of—survived to maturity; the former pair were well known to one of us; they were respectively a Cornaz and a white albino. There is no doubt therefore that white albino mated with white albino can produce all three types of albinos.

Now let us return to *Jack.* We mated him with another white albino bitch *Mor bhan* extracted from *Donald dhu,* the Pompek, and *Wee Tong,* the pure white albino Pekinese. The litter consisted of five white albino or dondo puppies (see Plate IV), all developing into the customary white adults with the usual cream tendency. Accordingly mating *Jack* with an extracted albino did not seem to give any greater chance, than when he was mated with *Tong I,* of a variation of the usual rule of white albino × white albino giving pure white albinos. Of the litter in question

* The descriptions were also not as complete as was desirable. Nettleship was an ardent Mendelian; he was distressed and perplexed by these litters. Unfortunately he gave away as puppies, dogs which it was of the utmost importance to preserve. We have only a few photographs of these litters, a painting of *Fe* and one or two skins.

Willie and *Rab* were very fine specimens of white albino dogs or dondos, *Lassie* and *Meg bhan* quite fair dondo bitches, the fifth, an albino dog, *Tam*, died at four months, from the effects of a kick from a pony. *Meg bhan* had seven fertile matings. The first was with *Larry*, the produce of *Wee Tong* and *Donnach ruadh, Wee Tong's* son, a red dog with black points, *Larry* being a dondo; this litter consisted of five of the usual type of white tending to cream albinos, *Banshee, Hob, Imp, Pixie* and *Nixie*, no Cornaz in the litter. Her next litter was sired by *Hans* the Cornaz albino and produced a bitch, who was Cornaz with a white belly, and was nosed out of the nest, *Bube*, a Cornaz, of uniform "scraped" chocolate coat, and with a very small white shirt front (when last seen his colour was lighter and resembled that of *Wang*), a bitch *Amie*, a white albino, still of a creamy tinge, and a dog *Mendel*, a pure white albino. Thus we see that a pure white albino, who was not descended from *Jill*, but who was descended from *Jack* through *Wee Tong*, could produce, mated with a Cornaz, both white and Cornaz albinos. For her third litter *Meg* was mated with the red dog *Donnach ruadh* and she had only albinos, two Cornaz dogs and one dondo bitch *Morse*. In her fourth litter *Meg* had for mate the dondo *Imp*, and there resulted a dondo bitch that died almost at birth. For the fifth and sixth litters her mate was *Bruno* a light red dog with black markings, the son of the Cornaz albino *Hans* and of *Norah** a brindle red bitch with black points. These two matings produced eleven dogs; one light red brindle male, a dark brindle male (*Jof*), a dark brindle female (*Pet*), a black male (belly a very dark brindle) with one white paw and brindle bracelets on forepaws who died at once, and another brindle male (*Woodrow*); the remaining six, albinos, consisted of three white albinos *Kit, Haig* and *Korni* all sufficiently "creamy" to show something like white shirt fronts, and two dondo bitches without sufficient cream to show white shirt fronts. Thus the mating of a dondo bitch with a red dog of albino descent from *Jack* did not provide us with any Cornaz albinos.

Lastly, *Meg* was again mated with the Cornaz *Hans*; she bred only two puppies, one, a male, died at once, it showed a white shirt front on a creamy coat, but would probably have become a white albino; the second, a dondo *Ben*, who has been one of our best pure white albinos. *Meg* shortly afterwards had an abortive litter, became sickly and had to be destroyed.

We may now turn back to the more closely Cornaz matings. *Wang* and the black bitch *Sené dhu* besides giving rise to the Cornaz albinos *Hans* and *Grethel*, had in 1913 as offspring the bitch *Trine*, another Cornaz and two other Cornaz puppies. *Hans*, the Cornaz, mated with *Wee Tong*, the pure Pekinese dondo, gave rise to a pure white albino *Hun* (1915). *Hun* was mated with the Cornaz bitch *Liese*, and the latter's litter consisted of two Cornaz bitches, born in 1916, *Hinne* and *Henne*, both of whom had white shirt fronts. Meanwhile *Trine* mated with *Bube* gave birth to two Cornaz puppies, one of which died early unnamed, and the other was *Pontina* ("Bebé")†.

* *Norah* was the daughter of *Donnach ruadh* and his litter sister *Giorsal ruadh*, the first red dogs to reappear after the mating of dondos and Pomeranians.

† She was a whole colour Cornaz, except for a white shirt front.

It might be thought in view of these results that it would be easy to establish a true breed of Cornaz Spaniels, but we have not really succeeded in doing so, and this for two reasons: first, the greater number of our Cornaz spaniels were born during the war, when it was impossible for us to keep more than a few dogs, and secondly, the Cornaz bitches have proved less healthy and less fertile than even the dondo bitches. At present we have not a single pure Cornaz in the kennels; and, perhaps, the only hope of recovering them lies in the bitches *Jade* and *Juby*, now growing old, to whom we shall return shortly.

The reader must allow us to pass now for a little into the fringe of our stock of coloured dogs. *Wee Tong*, the pure Pekinese dondo, mated with her son *Donnach ruadh* gave rise in 1913 to a litter of which one member was an extremely fine red dog with black points, *Patsy*. Meanwhile the black Pompek, *Donald dhu*, mated with his daughter by *Wee Tong*, namely *Mor dhu*, a black dog with white points, gave rise to a red bitch with black points, *Siri* (see Pedigree I). *Patsy* and *Siri* helped largely to maintain our stud during the war years and to renew it afterwards. In 1920 they had a litter which contained among other coloured dogs: *Siu Niu* ("Topsy"), a fawn bitch with black points and *Changpie*, a grey brindle. In the litter of *Patsy* and *Siri* of the following year appeared *Maureen*, red with black points, a veritable Brunhilda, a very strong, fierce and bad-tempered bitch. With considerable difficulty and even danger to her spouse, she was mated with her brother *Changpie* and the litter consisted of two dogs, a white albino of the usual type, *Mac*, and a red dog with black points, *Jamie*. Meanwhile the dondo *Ben* had been mated twice with *Topsy*; in 1922 there was a litter of three red puppies, of whom we are only concerned with the dog *Eld*, red with black points and the bitch *Babs*, red with white shirt front. The latter was a small dog like her mother. In 1923 in a litter of two, appeared the bitch *Setie*, a fawn-red with white shirt front. For size and length of leg she takes after her maternal aunt *Maureen*, but psychically is of the very opposite temperament. Her litters have been of great interest. Thrice mated with *Eld*, a litter brother of *Babs*, and therefore *Setie's* full, although not litter, brother, she has given rise to white dogs with pigmented eyes. These, however, do not largely concern us at present. *Setie* in 1926 was mated with *Mac* and had a son *Wu*, a typical white albino; there is nothing in coat or eyes to distinguish *Wu* from any other of the dondos produced, since the original cross with the black Pomeranians*. Returning to *Babs*, she was mated in 1923 with her own father, *Changpie*, or we mated a red bitch with a grey brindle; the result was one of the most remarkable litters we have had. It consisted of three puppies, one dog *Stockie* and two bitches *Jade* and *Juby*. All three puppies at birth and till the change of puppy-coat were *markedly* and undeniably skewbald. *Stockie* is white with grey-brindle patches as dark as his father's, *Changpie's* coat. He is in no respect an albino, but the skewbaldism extends to his eyes and he has heterochromia of the irides. A fuller account of him will be provided on another occasion. Meanwhile in *Jade* and *Juby* we had reached again our skewbald Cornaz dogs, but

* They are larger dogs than the pure-bred Pekinese albinos, and not as graceful as these, or indeed as the Cornaz Spaniels.

no longer as product of two albinos, but of two coloured dogs. Unfortunately *Jade* lost her skewbaldism at first moult. She would now be described as an ordinary white albino or dondo bitch. *Juby*, on the other hand, is a typical skewbald albino, who has retained her "Cornaz" patches for six years. As we may naturally anticipate, *Jade* and *Juby*, being bitches with the Cornaz coat-tending and closely inbred, are neither concupiscent nor prolific. But *Juby*, the skewbald albino, has been mated with the white albino *Wu*, the son of her half-brother *Mac*. Both *Juby* and *Wu* have the usual albinotic eyes, in both cases with marked red reflex. *Juby* only differs from her sister *Jade*, in that the latter has lost, while the former has retained, during life, the skewbald patches of Cornaz colour. Now by mating two dondos together, or mating two Cornaz spaniels with full Cornaz colour, or by mating a dondo with a Cornaz spaniel, we have never got a coloured dog in the litters, but by the mating of *Wu* and *Juby*, dondo with skewbald Cornaz, a remarkable litter has resulted wherein all the puppies are *coloured*. The first, a male, *Schwarzert*, has black body colour with a white paw and certain tan markings, the second, a male, *Brunert*, has red body colour with black markings, while the third, the bitch *Gelberta*, has a somewhat brighter red for body colour and black points. The sire, dam and offspring as puppies are represented in the accompanying Plate VIII from the standpoint of colour rather than of form. *Juby* has had, through her mother, a very varied and continuous flow of albinotic ancestors; *Changpie's* albinotic ancestry is further removed and less intense, it goes back to *Jack* and *Tong*, but not to *Jill*. It cannot, however, be said that *Juby* has more coloured ancestry than *Jack*, *Jill* or *Tong*, nor can her coat be described in other terms than that of the skewbald albinos *Fe* and *Fo*. Our Plate VII gives her photographs together with two of *Wu*. She is just what we have described her to be, a skewbald Cornaz.

Before we enter a little more fully into the question of Cornaz pigmentation, we should like to remark on three points. Our first point is that the excessive inbreeding, which we have been driven to by the Great War, and the great cost of maintaining a canine stud of adequate numbers, does appear to increase the feebleness of the dogs and emphasise their infertility and the heavy mortality in the litters, especially of the albinotic dogs. Secondly, this very inbreeding appears to increase the interest of the progeny; the closer the inbreeding the more likely we seem to get interesting results, e.g. *Jack* and *Jill*, *Hans* and *Grethel*, *Setie* and *Eld*, *Babs* and *Changpie*, *Juby* and *Wu*. Rare factors which would not otherwise be reenforced are thus brought to light. Thirdly, but this is only a surmise, which cannot be adequately maintained from the evidence adduced in this paper, the first litter of a pair of dogs is more likely to produce varieties than later litters. On this and other grounds*, which have come to light in dog-breeding experiments,

* One we may mention here: the spermatogonia form a community of living organisms; it is contrary to experience that such a community should not exhibit *individuality* among its members. Jennings believed he had reached such a state of affairs in his *Paramecium* experiments, but his further researches on rhizopods, where he had a wider range of observable characters, convinced him, as the observations of Johannsen on *Phaseolus vulgaris* and Hanel on *Hydra grisea* (see *Biometrika*, Vol. II.

we are not yet prepared to accept an atom-like identity in the spermatogonia of an organism, i.e. that an identical factorial formula is all they contribute to inheritance. We have been impressed with the need of supposing that there is a certain amount of individuality in the ultimate source of the germ cells, and that during the life of an organism these individualities may be present at different periods in different proportions, and this population of cells be even subject to some form of selection before the development of the spermatozoa, and the spermatozoa be again intensely selected before they reach individual ova, which in their turn have passed through various stages of selection.

Whether piebaldism be determined by a unit factor or not, we know that the extent of white in the coat is a continuous character inherited as stature or cubit in man. Those who have been in close touch with many dogs will, we think, agree with us when we say no two body colours appear exactly alike, you may look through dozens of skins and find individuality in the shade of them all. The classification into reds, fawns, red brindles, grey brindles, and so forth is broad and superficial, there is really individuality in every case, and what is more, that individuality is not a "fluctuating variation"; there is less difference between the "red," say, individualities in brethren than between the "reds" of cousins. Just as the various types of chestnut in hackneys are individually inherited, and a single factor for chestnut only serves to screen this, so it is with coat colour in dogs.

We preface our remarks on albinism in dogs with the above statement, because there are undoubtedly grades in the tint of Cornaz Spaniels. It would be impossible to confuse the coats of *Hans* and *Ben* or of *Grethel* and *Wee Tong*, but within the albinotic type the intensity of the Cornaz colour varies, and we have seen that in skewbald puppies like *Patty* and *Jade*, it may disappear, and as adults they may be merely white albinos or dondos. Even a Dondo may show at times considerable "creamy" areas, though we do not think such tints can be confused with the "scraped" chocolate of *Hans* or *Grethel*. Further the dog, who would be distinctly classed as a Cornaz Spaniel, may not be whole colour like *Bube*, but have a white shirt front like *Hinne* or *Henne*, which is far from being the full skewbaldism of *Fe* or *Juby*. If, as in the case of the early writers on albinism, we look upon that condition as an arrest of development, we should say that a Dondo denotes a complete arrest from the earliest development before birth, and that the various forms of skewbald Cornaz and complete Cornaz mark arrested attempts at pigmentation development; this is not out of accord with what we know of the microscopic examination of the Cornaz hair, to be discussed later. If we put therefore on one side the white "points" and the skewbaldism of the Cornaz, or attribute them, Mendelian fashion, to two factors independent of albinism as they occur in coloured dogs as well as albinos, we are left with two forms of albinism—the Dondo and the

pp. 499—503, Vol. VII. pp. 378—381) had already convinced Pearson, that the hypothesis of uniformity, i.e. absence of individuality, in the hereditary mechanism of any organism has a high degree of improbability.

Cornaz. Let us try and see whether it is feasible to elucidate the relationship of Cornaz and Dondo on a two-factor hypothesis. Let us suppose white dominant (D) and Cornaz recessive (R)*, then however much we should like to take *Jack* and *Tong* as (DD)'s owing to their invariable production of Dondos, this is impossible, because in order that *Jill* should be white as she was, she must have been a (DR) and thus *Jack* and *Jill* could only give (DD)'s and (DR)'s or no Cornaz spaniels. *Jack* and *Jill* must accordingly be treated as both heterozygotes or (DR)'s, and thus it would be possible for them to produce Dondos and Cornaz spaniels. *Jack* and *Tong's* progeny were solely Dondos, hence it seems reasonable to suppose that *Tong* was a (DD). *Donald dhu* although a black dog had a factor for albinism due to his father *Ian ban*, the nature of this factor it is difficult to determine, but if it contained Cornaz it is surprising that all the offspring of his daughter *Mor bhan* with *Jack* were Dondos. In fact of all the albinos—and there are many born from the offspring of *Donald dhu*—not one was a Cornaz except those resulting from the mating of *Sené dhu* with a Cornaz *Wang*. Here we seem to meet with a result which needs much explanation. For while we have assumed Cornaz to be recessive to white, it appears dominant to black, for all the offspring of this mating, *Trine, Hans* and *Grethel*, are not only Cornaz in appearance but appear to be homozygotic Cornazs (RR) for their litters are all Cornazs. If as seems probable by the nature of their offspring, *Wee Tong*, *Ian ban*, *Mor bhan* and *Meg bhan* were all homozygous Dondos, then the mating of *Hans* and *Meg bhan* should have given nothing but Dondos, whereas it gave two Cornazs as well as Dondos like *Ben* and *Mendel*. Further, if Dondo was dominant to Cornaz, then the result of the *Hun* and *Liese* mating may be looked on by some with suspicion. If we treat *Meg bhan* as heterozygous, then *Ben* and *Bube* are possible theoretically as brothers, but *Ben* must also be heterozygous. Now *Juby* must, being Cornaz, be a homozygous recessive; hence a second factor for Cornaz must have been handed down to *Changpie*, and so we are ultimately again forced to consider that *Ian ban* or *Wee Tong* had a factor for Cornaz. We are led to suggest, indeed, that all the Dondos are heterozygous—a not uncommon suggestion to make, when we find as in the inheritance of anomalies or diseases in man, that it is impossible to interpret the pedigree, if we suppose any individual to carry only dominant factors.

We now turn to some of the descendants of *Donald dhu*, who while he was descended from *Jack*, had no connection except through *Jack* with *Wang*. He was chiefly mated with his own daughters *Cilis dhu*, *Mor dhu* and *Sené dhu* out of *Wee Tong*. *Cilis dhu* was a black bitch with white shirt front. *Donald dhu* was mated with *Cilis*; and in the first litter (1913) there were four offspring, *Loki* and *Thor* black dogs, an unnamed red dog and *Freyja* a white albino bitch, the hairs of whose coat showed no diffused pigment and no granules, though one or two of the more creamy hairs showed a very faint brown diffused pigment. She was of the usual dondo type. The second litter (February 1914) contained only two puppies, *Odin* a fine brindle sable dog and *Frigg*. We described this puppy at three months as having a white

* The inverse hypothesis that Cornaz is dominant will clearly not suffice, for then *Jack* and *Jill* would have to be pure recessives, which will clearly not work.

coat slightly tinted with cream, but "not a Cornaz spaniel, mere ordinary albino." But the bitch underwent a complete change when she lost her puppy-coat. On October 30, 1914, we described her as nearly whole colour, bruised chocolate, a real Cornaz Spaniel, very good coat with long hair and underhung jaw like her father *Donald dhu*. She grew very fat, failed to be mated and died March 1915; the post-mortem showed the intestines almost choked with fat. Her skin, preserved, is that of a *very* dark Cornaz spaniel; she had the usual albinotic eyes. This is the only case we know in which a dog, born apparently a Dondo, has afterwards developed a Cornaz coat, although the reverse transition has occurred more frequently*. In her third litter sired by *Donald dhu*, *Cilis* produced only coloured dogs, five in number, three sable and two liver coloured, and into their description we need not enter at present. *Cilis* in 1915 was mated with *Wang* and produced only three puppies, one *Bran* a black dog with white shirt front and white left-hand fore-paw, and two puppies which died on the day after birth, one a fawn and the other an albino, most probably Cornaz. Lastly, *Cilis* had a fifth litter of three by *Donald dhu* in 1916, all of whom she overlaid; it consisted of two black puppies, and a whole white albino bitch. It is thus clear that a black bitch mated with a black dog can produce albinos of both types. *Donald dhu* was again mated in 1913 with his aunt *Wee Tong*, the pure-bred Pekinese dondo. In the litter of three all were albinos; namely *Dargo*, a Cornaz Spaniel with white shirt front and paws, the rest of the body uniform lilac, *Sora*, a dog of perfectly whole Cornaz coat, and no sign of white markings, and *Minona*, a white albino bitch with slight creamy touches. *Dargo*, *Sora* and *Minona* had all the usual albinotic eyes. We see therefore that a white albino or Dondo crossed with a black dog can give either a Dondo or a Cornaz albino. Further, of the 14 offspring of two Pompeks, *Cilis* and *Donald dhu*, four were black with white shirt fronts like their parents; five had sable coats, one was liver, one light red and three were albinos, namely two Dondos and one Cornaz.

Crossed with his daughter *Sené dhu*, *Donald dhu* had in 1914 four offspring: *Teufel* a black dog with a small white shirt front, two black dogs with white markings, and a light red brindle bitch with white shirt front and white paws. Thus in this litter there was not a single albino. We cannot, however, assume *Sené* to have had no factor for albinism, because crossed with *Wang* the Cornaz albino, she had in 1913 a litter of three: the bitch *Trine*, a Cornaz, and a Cornaz bitch and dog, who died quite young. Crossed again with *Wang* in 1915, she had a litter of five, one bitch and four dogs. The bitch was jet black with a white shirt front, the four males were three of them Cornaz and the fourth was a dark brindle with white front.

The third black-coated sister of *Cilis* and *Sené*, namely *Mor dhu*, declined to be mated until she was three years old. She was then crossed with her sire *Donald dhu* and produced in 1915 a litter of three: *Ola*, a black dog with white front, *Tolli*, a similarly coated dog, and *Siri*, a red brindled bitch (see Pedigree I). In early 1916, *Mor dhu* gave two further puppies to *Donald dhu*, *Hille* a black bitch with a white

* There exist several more or less well authenticated reports of human albinos acquiring some pigmentation. *Frigg's* case may throw light on these.

shirt front, and a second jet black* bitch with a like front. In late 1916 and in 1917 *Mor dhu* had single puppy litters; in the former year's late litter a jet black bitch with white shirt front, and in the latter year a dog of jet black coat with a similar shirt front. The infertility of *Mor dhu*—whether due to the difficulty of procuring a good meat diet for dogs during the war, or, as we suspect, to some personal anomaly†—was so great, that in the course of five years she had only seven puppies. The whole of these were of the usual Pompek type except *Siri*, who was a red brindle. The existence of *Siri* showed that she was not a dominant black, but could like her sisters *Cilis* and *Sené* produce red puppies. It would have been interesting to know what would have happened had she been mated with *Wang*, but her fertility was exhausted before this could take place. She may have carried no factor for albinism, but of this we cannot be sure. The only white hairs she carried formed a small patch on her belly, invisible as she ran about, so that she looked like a black pure-bred Pekinese. Her absence of white may possibly be correlated with the fact that she bred to *Donald dhu*, her father, far more black dogs, than her sisters *Cilis dhu* and *Sené dhu* did. The actual distribution of colours of offspring is shown in the accompanying Table.

Mated with *Wang* (Cornaz)					Name of Bitch	Mated with *Donald dhu* (Pompek)					
Total	Cornaz	White Albino	Red	Black		Black	Red	"Sable"	White Albino	Cornaz	Total
—	—	—	—	—	*Mor dhu*	6	1	—	—	—	7
3	1	—	1	1	*Cilis dhu*	4	2	5‡	2	1	14
12	8	2	1	1	*Sené dhu*	3	1	—	—	—	4

‡ Brown with more reddish tinge than is to be found in the natural fur of the sable, but distinguishable from dark red. Not unlike, but far from wholly like, the coat of a brown retriever.

It is clear from this table that the Pompek *Donald dhu* and his two daughters *Cilis dhu* and *Sené dhu* all carried a factor for albinism. It is less certain about *Mor dhu*, but we might also have supposed her sister *Sené dhu* to carry no such factor, had she not been mated with *Wang*. Theoretically, *Donald dhu*, being the F_1 generation from albino Pekinese (RR) and black Pomeranian (DD), should have a formula of the type (DR), so that when crossed again with the pure albino (RR), his offspring should be (DR) or (RR). Thus, if D signifies "some" colour, this would cover red as well as black, and there would be nothing anomalous in the observed offspring of *Donald dhu* and an albino Pekinese. According to this theoretical view all *Donald dhu's* daughters must also have the formula (DR) and mated with their father should have given 75 °/₀ coloured and 25 °/₀ albinotic offspring, the observed numbers were 22 to 3. Again these daughters crossed by *Wang* (RR) should give 50 °/₀ albinotic dogs, the actual numbers were 4 to 11. The odds are about 14 to 1

* We use the term "jet" to distinguish the pure black coat from the rusty black which is not uncommon with Pompeks.

† She was a most unsatisfactory mother, paying no regard whatever to her offspring.

against such an excess of coloured puppies in the first, and 27 to 1 against such an excess of albinos in the second form of crossing. There should be on the simple Mendelian theory applied no interdependence of the two sets of results, and accordingly the odds against the combined result following such a simple Mendelian theory are very considerable, i.e. 378 to 1; and these odds would still be considerable, i.e. 94 to 1, if instead of taking the *excesses* of coloured and albinotic puppies, we took for calculating our odds deviations whether in excess or defect.

It may be asked why we should endeavour to apply any such simple Mendelian theory. The answer is that albinism is still cited in elementary (and not only elementary) textbooks of genetics as an illustration of this simple form of Mendelian theory.

In order to meet a difficulty in mice very similar to what we have found in dogs, Bateson in 1903* introduced a somewhat different notation and definitions. He speaks of an albino (pink-eyed) gamete G, this we suppose corresponds to our white albino with pink eyes, and G' which is described as a colour-bearing pink-eyed gamete. Then according to Bateson the homozygotes GG and $G'G'$ will all have pink eyes, but the former will have white coats, and the latter some colour in the coat. The heterozygotes (GG') will, according to Bateson, have dark eyes and some colour in the coats. In other words, it would appear as if some colour in the coats dominated the eye colour in the heterozygotes ($G'G$), but not the eye colour in the homozygotes ($G'G'$) when these are crossed. This as Weldon pointed out at the time is a somewhat curious interpretation of dominance; it also leaves very vague what is meant by "some colour" in the coat. The Cornaz spaniel no more than the white Pekinese albino has any pigment granules in its coat; the "some colour" is diffused pigment, and whether this pigment is creamy yellow as it occurs in some white albinos at certain seasons, or grey as in the Cornaz, there are no granules. Pigment granules do occur in the eyes which Bateson calls "pink" of both dogs and men. It is clear that our Cornaz albinos must be homozygotes ($G'G'$), for if they were heterozygotes ($G'G$) they would not have "pink" eyes. Now if the reader will look at the lowest generation on Pedigree I he will see a mating between a white albino ("pink-eyed") (GG) *Wu*, and a "pink-eyed" Cornaz with "some colour in the coat" ($G'G'$) *Juby*. The result was three offspring with heavily pigmented eyes and coats with plenty of pigment granules in the hairs (GG'), according to Bateson's theory. Thus this remarkable result in dogs (and mice) is in accord with Bateson's theory. But alas! the theory fails to accord at many other points with observation. According to the theory every Cornaz must be a homozygote and Cornaz × Cornaz give only Cornaz, but Cornaz × White Albino should give dark-eyed offspring with some colour in the coat. Let us look at some of the results: *Hans* and *Grethel* were very typical Cornazs, they produced in 1915 three definite Cornazs; *Trine* and *Bube* again were quite typical Cornazs, they produced in 1914 two Cornazs—these results are all in accordance with Bateson's view. But *Hans*, a Cornaz, mated with a white

* *Nature*, Vol. LXVII. p. 462. For the controversy which followed, see p. 512 (Weldon), p. 585 (Bateson), p. 610 (Weldon), and Vol. LXVIII. p. 33 (Bateson and Weldon).

albino *Meg bhan*, i.e. $(G'G') \times (GG)$, gave in 1915 *Amie* and *Mendel*, two white albinos (GG) and *Bube* and another dog, two Cornazs, one with a white shirt front and the other with a white belly. All four should have had coloured coats and pigmented eyes! In 1915 *Hans* $(G'G') \times$ *Wee Tong* (GG) produced *Hun* a white albino (GG) and *Hans* mated with *Meg bhan* (GG) in 1920 a typical white albino *Ben*. Further *Hun*, a white albino mated with *Liese*, a Cornaz bitch gave rise to a litter of two Cornaz bitches, *Hinne* and *Henne*, with white shirt fronts instead of dogs with coloured eyes. It is clear therefore that Bateson's $[G, G']$ hypothesis will not suffice to describe the relations of Cornazs and white albinos in dogs; yet our Cornaz spaniels seem to accord closely with Weldon's pink-eyed "lilac" mice. Bateson wrote* of his $[G, G']$ hypothesis in 1903 that: "Anyone conversant with Mendelian phenomena can now predict the eye colour of the future offspring of the various unions with approximate accuracy." Discarding the $[G, G']$ hypothesis as unworkable, it would be of interest to have a prediction as to the nature of the offspring which *Gelberta* may produce when mated with *Schwarzert* or *Brunert*.

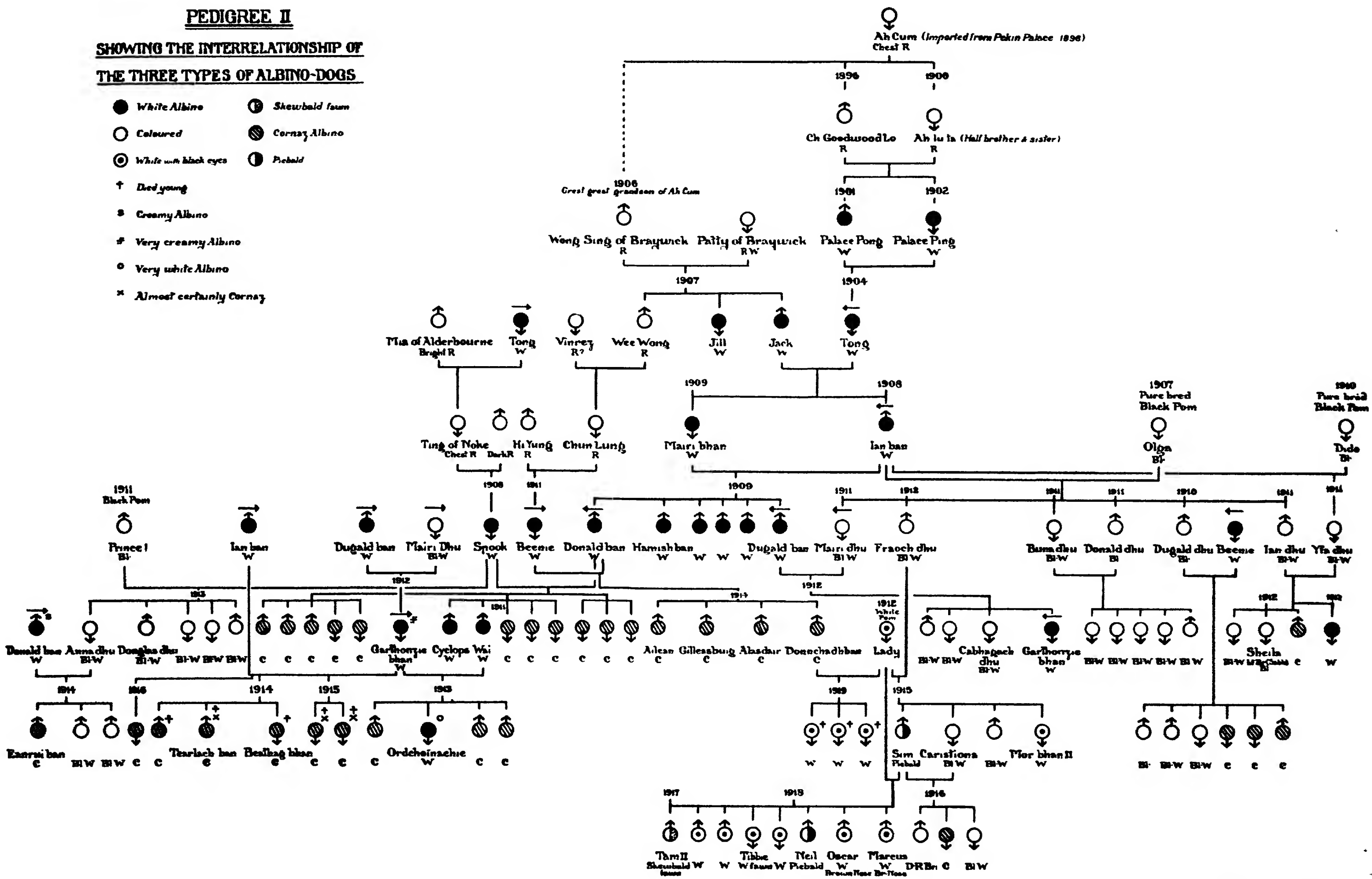
It will be seen that even on Bateson's hypothesis the inheritance of pigment in the coat and of pigment in the eye are not independent characters; nor are diffused pigment and granular pigment independent of each other. Two animals with albinotic eyes can give rise to offspring with deeply pigmented eyes, and this possibility depends on there being diffused pigment in the coat of one of them. We have never got a pigmented eye in the offspring of two Dondos or white albinos, and only as the reader will perceive very rarely from a Cornaz albino and a white albino. But as it appears possible to obtain a Cornaz from two white albinos, and from a Cornaz and a white albino offspring with heavily pigmented coat and eyes, it would seem within the range of possibility to obtain pigmented offspring from a race of white albinos, at any rate in dogs. We say a possibility, because when this has come about in our breeding, both the white albino *Wu* and the Cornaz piebald *Juby* were *extracted* albinos, i.e. their parents in both cases were coloured dogs. It is conceivable in this matter that an extracted albino, especially when the ancestry contains a number of dogs with heavily pigmented coats, may not have the same germinal constitution as pure-bred albinos.

To obtain some idea of the exact relation of eye pigment to hair pigment, attempts were made to obtain dogs with white albinotic coats and pigmented eyes. Various attempts were made to attain this end, but Pearson personally was unsuccessful in any experimental mating *directed* to this end. Only one mating, that of two red dogs, non-litter brother and sister from the same parents, gave, as it were by chance, the desired result. *Topsy*, a light fawn bitch with black points, crossed by the white albino *Ben* gave birth in separate litters to the red bitch *Setie* and the red dog *Eld*. There seemed nothing remarkable about this pair, but they were mated together to see whether a dominant red dog had been by any chance reached. All that was expected from the mating was the usual mixture of red dogs and white albinos with possibly a fawn. Judge of our surprise, when what we had failed to procure of purpose arrived indirectly. The record of the matings is the following:

* *Nature*, Vol. LXVII. p. 462.

PEDIGREE II

SHOWING THE INTERRELATIONSHIP OF THE THREE TYPES OF ALBINO-DOGS



To face p. 157

In the five litters sired by *Eld* (1924, 1926, 1927, 1928 and 1929) *Setie* has given birth to 15 puppies:

White Albinos	White coats and dark eyes	Red brindles	Fawns and Fawn brindles	Brown brindle
2	3	4	5	1

This is another illustration that one cannot take albinism and colour as allelomorphic. The three dogs with dark eyes and "white" coats were all males; as puppies they had coats as white as the whitest albinos, but on losing the puppy-coat they became creamy or yellowish in patches. The possibility of perpetuating a breed of white-coated dogs with black eyes could not be attempted, because all three were males, and *Busdubh* and *Mike II* died when about a year and about four months old respectively, in both cases suddenly. *Tweedledee* is still alive. *Setie*, however, was mated in early 1927 with her own son *Nick*, but with no success as far as white-coated, dark-eyed offspring go; she had one brindle fawn dog, and besides *Alpha* and *Gamma*, dogs, and *Beta*, a bitch, all three brown red with black points and white shirt fronts. Further, mated with her uncle *Changpie*, a grey brindle, in 1925, she had two puppies, one a grey brindle dog *Seeta*, and the other a red brindle bitch. To test whether she had a factor for albinism she was mated in early 1926 with her double cousin the white albino *Mac*, the result was a litter of three, a white albino *Wu*, a Cornaz albino and a fawn with black points, *Feng*, all dogs. Omitting her mating with *Mac* as an albino, *Setie*, a coloured dog carring a factor for albinism, has been mated with three coloured dogs *Eld*, *Nick* and *Changpie*, all carrying a factor for albinism. She has had by them 21 puppies, of which *two* only were albinos, the odds here are about 12 to 1, or only in about 13 trials would the result once fall so far short of a Mendelian quarter. These odds are not very great, but they are in the same direction as those previously reached for the mating of two coloured dogs each carrying a factor for albinism, and confirm the view that albinism in dogs is not a simple Mendelian recessive, i.e. the white albinotic coat is not recessive to a coat with "some colour in the hair."

We may remark that the hair of the white coats of our three dark-eyed dogs does not seem to differ in any character from that of very creamy white albinos.

We now turn to the dogs bred by C. H. Usher in Aberdeen. While they started from the same foundation stock as those bred by Pearson, their aim was somewhat different. Much more Pom blood was introduced through the two black Pom bitches *Olga* and *Dido* and the black Pom sires *Prince* and *Drum Chief*, but also through the white Pom *Lady* used with a view of getting ultimately a Pompek with pure white coat and black eyes.

A study of Pedigree II indicates that Usher's stock of Albino Pekinese takes its origin in *Ah Cum*. *Mairi bhan* and *Ian ban*, non-litter brother and sister, were offspring of *Jack* and *Tong*. *Spook*, a bitch that had the whitest coat of all our dondos

and was the worst of all our mothers, few of her offspring surviving, was the granddaughter of *Tong* by *Mia of Alderbourne*, through coloured parents. *Beenie*, a Dondo bitch, was a granddaughter of *Wee Wong* through coloured parents. *Wee Wong* was a red dog, litter brother to *Jack* and *Jill*. *Wee Wong* must have had a factor for Cornaz albinism, like his nephew *Wang*, because *Beenie* gave rise to Cornaz albinos. A noteworthy difference will be found between Pedigree I and Pedigree II. While there are quite a considerable number of red and fawn dogs in Pedigree I, only about 2 °/₀ of the dogs bred by Usher appearing in this pedigree are red or fawn. Of course, neither of the pedigrees now published contains anything like the whole number of dogs bred by either Usher or Pearson, the lines have been selected to illustrate the manner in which albinos occur; nevertheless the fact remains that Usher, breeding in more Pom blood and using more his F_1 generation of Pompeks, failed to obtain the variety of coloured dogs appearing in the London stud. In the present Pedigree II the coloured dogs are, first, the bitch *Sheila*, light brown coat of various intensities of brown, called by Usher shades of sable, and with brown hairs on vertebral column tipped with very dark black, so that she seems to have black "curls" down her back; secondly, a dark red brindle puppy, the offspring of *Sim* and *Caristiona*, and lastly among the somewhat weird offspring of *Sim* and *Lady*, a skewbald fawn and white dog, and a white dog with fawn markings; these are all. The other coloured dogs in this pedigree are black with white markings, i.e. Pompeks. Such dogs have almost disappeared from Pearson's stud*, presumably as a result of emphasising the Pekinese element.

Usher's results from the offspring of *Jack* and *Tong* were at first remarkably in accordance with the simple Mendelian theory. Two white albinos interbred gave rise to white albinos *Mairi bhan* and *Ian ban*. These interbred gave rise to a litter of six albinos, all white. *Ian ban*, mated with the pure bred black Pomeranian bitches *Olga* and *Dido*, sired seven Pompeks, black dogs with some white markings. *Dido* mated with *Donald ban*, also a white albino Pekinese, produced five typical Pompeks all black coated with white shirt fronts. Simple Mendelism seemed to go well, our "dominants" black Pomeranians bred true, our recessives white albino Pekinese bred true and the F_1 generation had most of their coat black, if they added indeed the white shirt front unknown to the dominant Pomeranians. Only two exceptions have occurred to this rule, the black Pomeranian *Prince I*, got out of the white albino *Beenie* two dogs; *Coinneach ruadh* (*Kenneth*) whose coat was chocolate, and a second unnamed dog with like coat; the remaining three dogs of the litter were the usual Pompeks. *Kenneth* failed to breed and the matter could not be carried further. So far, so good. Before we turn to the F_2 generation, we may note that Usher mated his white albino *Donald ban* with two other white albino bitches. *Spook* was descended from our albino *Tong*, and *Beenie* from *Wee Wong*, the brother of *Jack* and *Jill*; both *Spook* and *Beenie* traced their descent through coloured dogs. *Donald ban* and *Beenie* mated provided four dogs all Cornaz, not a single white albino! *Donald ban* and *Spook* mated resulted in eleven Cornaz and only two

* I believe the only black dogs now living bred by me are *Jet*, born in 1922, and *Schwarzert*, born in 1928, both extracted blacks and differing in many respects from Pompeks. K.P.

white albinos. It does not seem possible to suppose Cornaz albinism an allelomorph to white albinism! The white albino *Beenie* crossed with the Pompek *Dugald dhu* should have produced 50 °/₀ Pompeks and 50 °/₀ white albinos; actually she had three black puppies with white markings and three Cornaz, not white, albinos. Crossing members of the F_1 generation we find a new series of difficulties arise, if we endeavour to apply any simple Mendelian formula. A not unreasonable or impossible result was obtained when *Buna dhu* was mated with *Donald dhu*, the litter consisted of five black dogs, all with white shirt fronts; if we suppose that these were all Pompeks, or heterozygotic in character, for the dominants were whole black, the odds against this occurrence would be 31 to 1, and it might of course occur. The two matings of the Pompeks *Ian dhu* with *Yfa dhu* cause more difficulty, for the albinism of both only arises from the *Jack* × *Tong* stock, yet we find in their progeny one white albino, one Cornaz albino, a Pompek and the light brown (sable) bitch *Sheila*, with long black tips to the hairs on top of her saddle. Clearly the conception of black allelomorphic to white albino is not adequate and colour factors for reds, red brindles, etc., unknown to the Pomeranian, come in through the albino Pekinese. Turning now to the matings of albino Pekinese with Pompek, we have already referred to the litter of *Dugald dhu* and *Beenie*; in a litter of *Dugald ban* and *Mairi dhu* there were three Pompeks and a white albino *Garthonzie bhan*. The latter mated in 1913 with *Wai* (also a white albino out of *Spook* by *Donald ban*, both pure Pekinese white albinos), produced three Cornaz dogs and a white albino bitch *Ordchoinachie*, thus confirming the view that white albinos can give rise to Cornaz albinos. If we suppose Cornaz albinism can arise through the dilution of albinism by the Pomeranian black, then *Garthonzie's* descent from *Mairi dhu* and *Olga*, the latter's Pomeranian mother, can assist the hypothesis. On the other hand we have to note that *Spook* and *Donald ban*, both pure white Pekinese albinos, obtained Cornaz offspring without any dilution by Pomeranian blood. *Ian ban* and *Garthonzie bhan* gave in three litters six Cornaz and not a single white albino. *Spook* mated with *Prince I*, a black Pomeranian, gave rise to a litter of five Pompeks, also black with white shirt fronts. One of these *Anna dhu*, mated with a pure Pekinese white albino *Donald ban*, had a litter of two Pompeks and one Cornaz. Thus Usher's albinos tended during the war years, much like Pearson's, to be Cornaz rather than pure white, and both were subject to a very heavy death-rate, apparently greater in the case of the Cornaz than in that of the white albino.

We may now stay to consider an attempt to obtain directly pigmented eyes with a Pekinese habit. In 1915 Usher mated a pure white Pomeranian with a Pompek of the F_1 generation, *Fraoch dhu*, see Pedigree II. The litter was noteworthy; all members had dark eyes. *Sim* was a piebald black and white with white hair; *Caristiona*, of the usual Pompek type; the third puppy, a dog, was also a Pompek, mostly black with thin white stripe on throat, chest and belly; the last puppy, *Mor bhan II*, was white with slight traces of cream, she had on the left ear brown hairs with some black, and across the rump a pale grey band. *Caristiona* mated with her brother *Sim* gave birth to a dark red brindle dog, a Cornaz and a Pompek, black with white forepaws. Thus *Sim* and *Caristiona* both contained a factor for albinism

of the eye, but the normal dark eye dominated when *Lady* was mated with either a hybrid or a pure albino. *Sim* was mated with his mother *Lady* in the war years 1917 and 1918, and this bitch had puppies: *Tam II*, a skewbald white and fawn dog, perhaps the use of the term skewbald is too definite—he had a white coat with light fawn patches, not very clearly marked off from the white, fawn-coloured ears, and a grey right front paw; two dogs and a bitch had white coats and pigmented eyes; a bitch *Tibbie* was all white except for fawn-coloured ears; *Neil* was a piebald like his father *Sim*, while *Oscar* and *Marcus* were all white, with brown noses. All the eyes of the eight puppies were dark.

Finally in 1919 *Lady* was covered by *Donnchadh ban*, a Cornaz albino, and she had three white puppies, two bitches and one dog, all of which died in the first fortnight. Their skins have not all the same degree of whiteness, one being almost Cornaz colour. Usher's descriptions, macroscopic and microscopic of the eyes, seem to indicate that they all would have had "dark" eyes*. *Lady* had dark brown irides, but they were not examined after death, though those of the black Pomeranian bitches *Dido* and *Olga* were. It is difficult to compare a puppy's eye with that of a fully grown dog, but it is just possible that the albinism of the father *Donnchadh ban* may have had some influence on the pigmentation of the offspring. Still the evidence seems in favour of terming these dogs "white and dark eyed." Unfortunately *Lady* was never mated with a pure white albino Pekinese, and her offspring with the Cornaz albino did not survive. The experiments with *Lady* were carried on during the war years 1915—1919, during a part of which time Usher was in Salonika, and they had to be abruptly terminated. They left the problem of whether it would be feasible to obtain a pure white Pekinese coat and skeletal form combined with a dark eye unsettled. Pearson's *Busdubh*, *Mike* and *Tweedledee* are nearer to the conditions of skeletal form, but the coat is like that of the white Pekinese in hair colour, and tends as the puppies grow older to show yellowish patches, it is not pure white, and this is exactly the trouble with pure-bred "white" Pekinese with dark eyes.

* No. 1. Died at three days. Eyes closed. Both eyeballs translucent, iris blue grey. In opened eyeball iris and ciliary body black, fundus very pale. *Microscopical Examination:* Retinal epithelium well pigmented, stroma of iris and ciliary body contains numerous faintly pigmented chromatophores, which have a pale brown colour. Choroid in some parts shows no pigmentation, in other parts some of the cells are distinctly pigmented.

No. 2. Died at thirteen days. Eyes only partly open. Iris dark blue; deep red translucency of eyeball, when held up towards the light. Eyes frozen and opened; posterior surface of iris dark brown; ciliary body black; fundus pale brown. *Microscopical Examination:* Retinal epithelial layers on back of iris, the pigment layer of ciliary body and the hexagonal cells of retina darkly pigmented. In iris stroma are numerous lightly pigmented chromatophores, which do not appear to have received their full complement of pigment granules; lightly pigmented chromatophores in smaller numbers are present in the ciliary body and choroid, also a few at sclero-corneal margin.

No. 3. Dark iris and fundus. Eyes frozen and opened; iris, ciliary body and fundus black. *Microscopical Examination:* Darkly pigmented epithelial layers of retina, pigment layer of ciliary body and hexagonal layer of retina; stroma of iris and ciliary body and choroid are darkly pigmented, much darker than in No. 2.

A piece of skin from the head, placed in 10 °/₀ formalin, embedded, cut and examined microscopically, showed no pigment.

As the reader has been previously warned the present paper does not deal at length with the 500 and more dogs which have been bred. It endeavours solely to illustrate the types of albinism occurring and their relations to each other. Even thus many problems arise, which cannot at present be solved. Perhaps one of the most important of these is the question whether all Cornaz albinos are equivalent. We do not think the amount of cream in the coat of a white Pekinese albino has the slightest gametic importance; it varies so with the season of the year, and the interval since change of coat. On the other hand the intensity of diffused pigment in the coat of the Cornaz albino is very considerable and if a number of skins of these dogs be taken a fairly continuous scale can be reached running from a dog like *Wang* up to *Frigg*. Some might indeed link *Wang* on to the creamy white albinos, and suggest that there is only one kind of albinism, and that it varies in intensity. The skewbald Cornaz and the Cornaz with white shirt front give, however, a contrast which cannot be put on one side; there is an essential difference between the two areas. Further, our breeding experiments seem to indicate that: (i) two dondos, or white albino dogs, never give rise to dogs with dark eyes, but (ii) they can give rise to Cornaz albinos. Again (iii) we have never known a Cornaz mated with a Cornaz produce puppies with dark eyes, they have only albino puppies, white or Cornaz; but (iv) we have known a case in which a Cornaz skewbald, crossed by a white albino, produced dogs with pigmented coats and dark eyes.

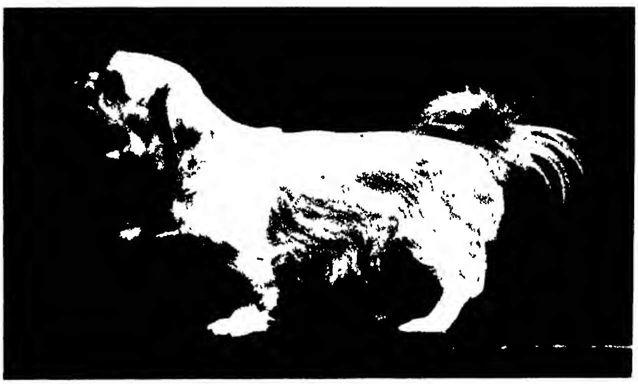
Our "Cornaz" dog seems exactly in accordance with what Weldon in his mice-breeding experiments (see *Biometrika*, Vol. XI. Appendix, "Records of Mice-Breeding Experiments") termed pale-blue-grey ("lilac") and represented by the letter f. These mice had always pink eyes (p), and might be either whole-coloured (Weldon's 6), or mixed white and grey in patches of different sizes (Weldon's 1 to 5), i.e. like our skewbalds and our Cornazs with white shirt front or white markings. Weldon made a very large number of matings of pink-eyed lilac mice, his mice being, like our Cornaz albinos, extracted albinos. In all cases $(p, f) \times (p, f)$ bred true, and this whatever the hybrid generation they appeared in. He also bred white albino mice with the pink-eyed lilac mice. From these crosses he obtained as a rule mice with *dark* eyes and colour in the coat, even black (matings 2 H. 108, 2 H. 156, see *loc. cit.* pp. 20 and 22) and chocolate, but they might be also fawn-yellow or wild colour. Occasionally, however, he obtained from this cross pink-eyed, and not dark-eyed, mice with fawn or "lilac" coats (see mating 3 H. 31, *loc. cit.* p. 25). All this corresponds to what we have found for dogs, only the cross Cornaz albino with white albino seems less frequently than in mice to give coloured coats and dark eyes, and more often pink-eyed offspring. Of course the explanation may be that our pure white albinos had a latent Cornaz factor, or at least that some of them, like *Jill*, had.

In this respect, perhaps, our Cornaz albino dogs corresponded to some extent more to Weldon's Japanese waltzing mice; these mice were skewbald, fawn and white, with albinotic eyes. The hairs of seven such mice have been examined, and in five of them the fawn hairs showed only diffused pigment and no pigment

granules. In two cases, however, the coloured patches showed to the naked eye a dusky hue and examined under a lens the patches were seen to consist of two kinds of hairs, the usual fawn and a darker kind. The former microscopically examined showed no pigment granules, the latter contained a certain number of pigment granules. The Japanese Waltzers bred true to their fawn patch and albinism, but when crossed with white albino mice, the offspring showed in part dark eyes and in part pink eyes, the former by far the most frequent. The pink eyes appear to have arisen when the Waltzer was mated by an extracted albino, i.e. by an albino very possibly having Waltzer blood in its ancestry. If we consider the waltzing albino mice really very similar in constitution to the lilac albino mice, i.e. when mating together as giving Waltzer albinos, and when mated with white albino mice as usually giving dark-eyed mice with colour in the coat, but pink-eyed mice if the white albino were an "extracted" one, possibly having a factor for the albinism of the Japanese Waltzer, we can perhaps see some light on the problem of why the Cornaz crossed with the white albino does not always give a coloured dog with dark eyes. The white albino dog may carry a factor for Cornaz, and when mated with a Cornaz give an albino litter instead of one of coloured dogs with dark eyes.

Our results indicate that there is considerable correspondence between the results for dogs and mice with regard to albinism. The correspondence is not complete—the lilac albino mouse crossed with the white albino mouse far more often produces coloured mice with dark eyes than the Cornaz albino dog crossed with the white albino produces coloured dogs with dark eyes—still the strange phenomenon exists in both dogs and mice. Albinotic eyes in both parents and an absence of granular pigment in the coats of both are not sufficient to indicate that the offspring may not have dark eyes and pigment granules in the coat hairs. A further point of difference between albinotic dogs and albinotic mice seems to lie in this: the mating of albinotic white mice, as far as my experience goes, always leads to albinotic white mice. The mating of two pure-bred Dondos, white Pekinese albinos, may on the other hand lead to Cornaz Pekinese albinos, i.e. to dogs with as markedly albinotic eyes as their parents, but with pale buff or light-grey patches of hair on the coat. As a rule, but perhaps not invariably, these patches contain only diffused pigment. These Cornaz albino dogs correspond closely to Weldon's "lilac" albino mice, or indeed to Japanese waltzing albinos, and when crossed with pure white albinos can give dogs with dark eyes and coloured coat. Thus a descent from pure white albino dogs to dogs with coloured coat and dark eyes seems feasible. On the other hand, the production of a "Cornaz albino mouse" —a fawn Japanese Waltzer or a "lilac" albino—from albinotic white mice has not been recorded. One point may be noted here: the white albino dog—and the eyes of many have now been examined—has, like man, eyes which exhibit all the usual albinotic characteristics, the marked red reflex, the photophobia, and the defective sight, but, like adult man's albinotic eye, in all the cases yet examined, it is not absolutely free from pigment. On the other hand, the albinotic white mouse has

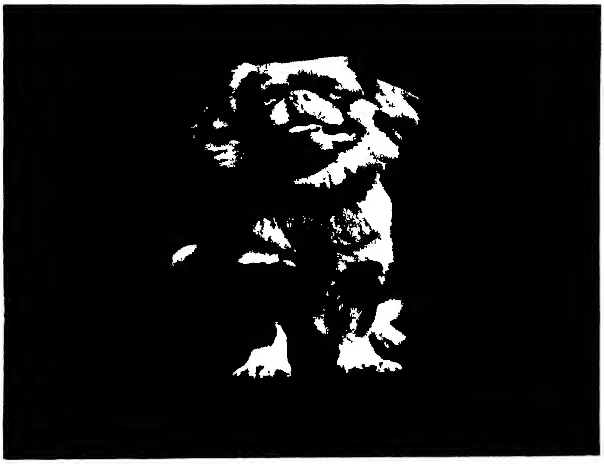
(i) Pure-bred Pekinese albino *Jack*.



(ii) Pure-bred Pekinese albino *Jill*.

(iii) Pure-bred Pekinese albino *Patty*.

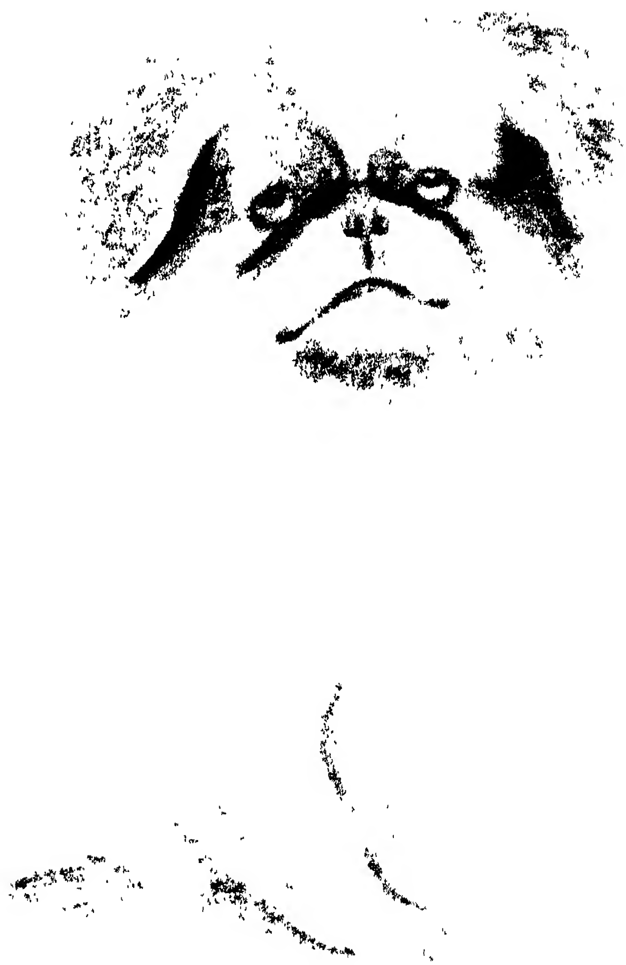
Dondos, or pure-bred Pekinese albinos.



(i) *Fo*, showing contrast of white paws with pale buff body colour.

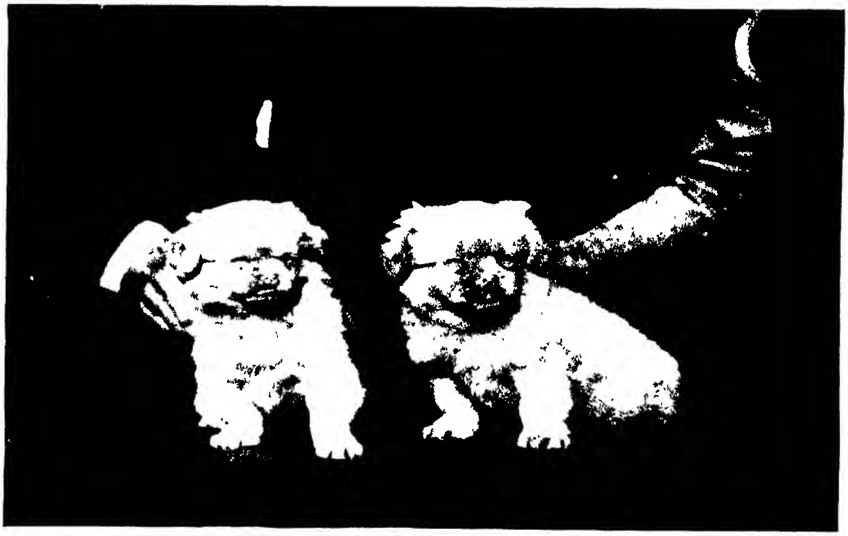
(ii) *Fo*, showing white patches against pale buff body colour.

(iii) *Fe*, skewbald Cornaz and *Fo*, Cornaz albino as puppies.
Cornaz albinos from pure-bred Pekinese white albinos.



Fe, skewbald Cornaz albino, aged $5\frac{1}{2}$ months, white and buff, bred by E. Nettleship. From pure-bred Pekinese albino (Dondo) *Jill* × pure-bred Pekinese albino (Dondo) *Jack*.

(i) Pure white albinos, young dogs.

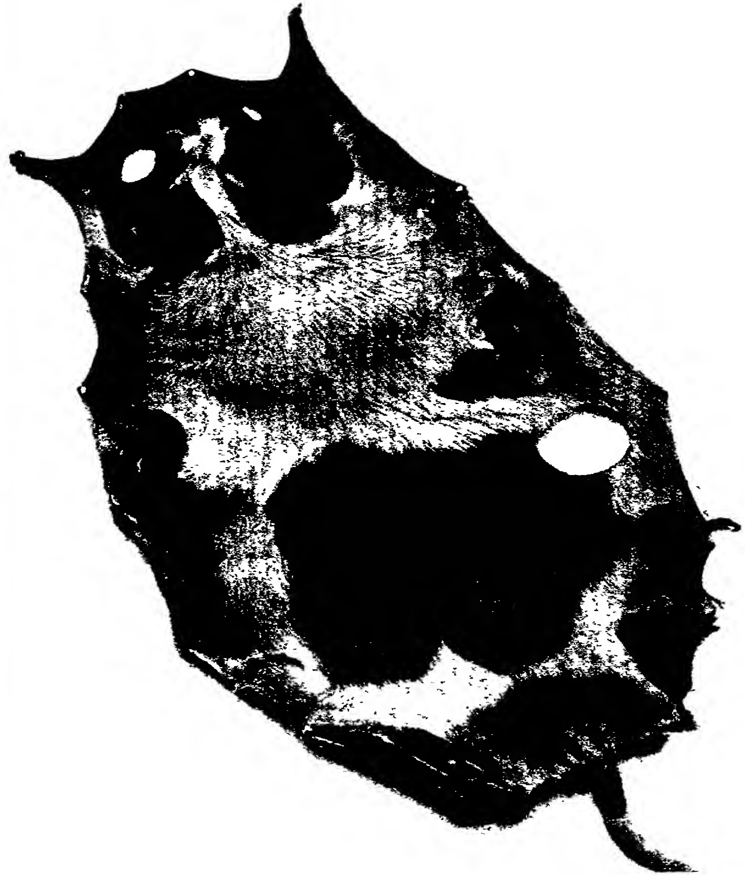


(ii) Pure white albino puppies.
Litters from extracted albino × pure Pekinese albino.

(i) Cornaz albinos, *Hans* and *Grethel*, showing grey (lilac) body colour and white forepaws and shirt fronts.

(ii) Extracted Cornaz albinos, product of the mating of a pure bred Pekinese Cornaz (*Wang*) with a black bitch (from white Pekinese albino × black Pompek, the latter a product of white Pekinese albino × black Pomeranian).

Skin of piebald puppy from Dondo × Pompek. Eyes and hair with many pigment granules.



Skin of skewbald Cornaz puppy with albinotic eyes, pale buff patches with diffused pigment and a few granules.

(i)

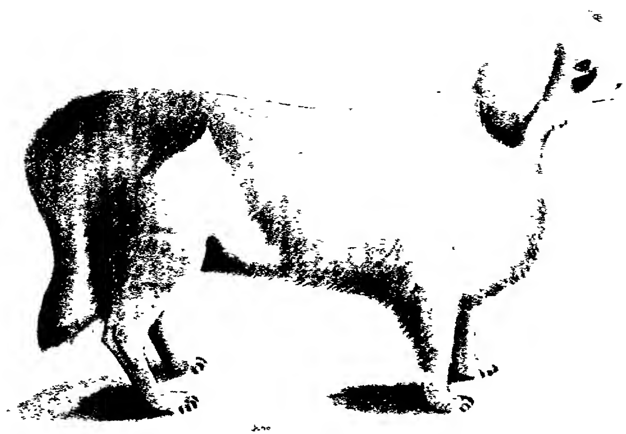
(ii)

Juby, skewbald Cornaz albino, coat white with grey patches; the face and saddle patches can be distinguished, also the pure white on left thigh in (ii).

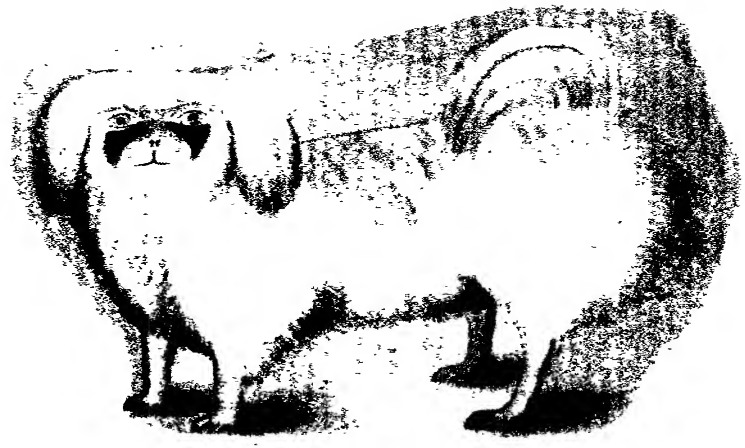
(iii)

(iv)

Wu, pure white extracted albino. Both *Juby* and *Wu* have very marked red reflexes, and pink mucous membrane of nose.



(i) ***Juby***, skewbald Cornaz albino bitch.



(ii) ***Wu***, pure white albino or Dondo (extracted).

(iii) ***Schwarzert.***

(iv) ***Gelberta.***

(v) ***Brunert.***

Colour diagram to indicate colour results of mating a Dondo with a skewbald Cornaz albino.

an eye which stands almost unique among albinotic eyes of animals, in that it has been impossible hitherto to find traces of granular pigment associated with it*. Japanese albino mice and "lilac" mice have a small amount of pigment in the eye†. It would seem therefore that the pure white-coated albino mouse is more differentiated from the Cornaz albinotic mouse (such as the Japanese or lilac albinotic mouse) than the pure-bred white albino Pekinese, the Dondo, is from the Cornaz albino Pekinese, both the latter having some amount, if small, of pigment in the eye.

This may possibly account for the transition from pure albinism (no pigment granules anywhere) to normal pigmentation (granules in skin, hair and eye) being more feasible in the dog than in the mouse.

* See Pearson, Nettleship and Usher: *A Monograph on Albinism*, p. 365, Case VI and p. 377, Case IX.

† *Loc. cit.* p. 364, Case V and pp. 375 *et seq.*, Cases IV—IX.

ON THE DISTRIBUTION OF THE FIRST PRODUCT MOMENT-COEFFICIENT, IN SAMPLES DRAWN FROM AN INDEFINITELY LARGE NORMAL POPULATION.

By KARL PEARSON, G. B. JEFFERY AND ETHEL M. ELDERTON.

LET the origin be taken at the mean of the bivariate population and let that population be defined by σ_1, σ_2 and ρ. Let the sample be defined by m_1, m_2, Σ_1, Σ_2 and r. Then it is known that the distribution surface of m_1, m_2 is independent of that for Σ_1, Σ_2 and r, so that we need not consider m_1, m_2 further. The surface for Σ_1, Σ_2 and r is given by*

$$z = z_0 e^{-\frac{n}{2(1-\rho^2)}\left(\frac{\Sigma_1^2}{\sigma_1^2} - \frac{2r\rho\Sigma_1\Sigma_2}{\sigma_1\sigma_2} + \frac{\Sigma_2^2}{\sigma_2^2}\right)} \Sigma_1^{n-2}\Sigma_2^{n-2}(1-r^2)^{\frac{1}{2}(n-4)} \quad \text{......(i)},$$

where n is >2 and r lies between -1 and $+1$.

Now take

$$\Sigma_1 = \sigma_1\sqrt{\frac{2(1-\rho^2)}{n}}\,x, \quad \Sigma_2 = \sigma_2\sqrt{\frac{2(1-\rho^2)}{n}}\,y,$$

and consider the integral

$$I_0 = A_0\int_{-1}^{+1} dr \int_0^\infty dx \int_0^\infty dy\, e^{-(x^2-2r\rho xy+y^2)}(xy)^{n-2}(1-r^2)^{\frac{1}{2}(n-4)},$$

where

$$A_0 = C\left\{\frac{2\sigma_1\sigma_2(1-\rho^2)}{n}\right\}^{n-1} \quad \text{..........................(ii)}.$$

Change the variables of integration to u, x and y, where $u = rxy$, and we have

$$I_0 = A_0\int_{-\infty}^{+\infty} du\, e^{2\rho u}\iint e^{-(x^2+y^2)}(x^2y^2-u^2)^{\frac{1}{2}(n-4)}\,xy\,dx\,dy,$$

the double integral being taken over the area above and to the right of the hyperbola $xy = |u|$.

Let $F(u)$ be an even function of u defined for $u>0$ by

$$F(u) = \iint_{xy>u} e^{-(x^2+y^2)}(x^2y^2-u^2)^{\frac{1}{2}(n-4)}\,xy\,dx\,dy.$$

Transform to coordinates $\xi = xy$, $\eta = x^2+y^2$,

$$F(u) = \int_{\xi=u}^\infty\int_{\eta=2\xi}^\infty e^{-\eta}\frac{(\xi^2-u^2)^{\frac{1}{2}(n-4)}}{(\eta^2-4\xi^2)^{\frac{1}{2}}}\,\xi\,d\xi\,d\eta.$$

Making further substitutions, $\eta = 2\xi\cosh\phi$ and $\xi = u\cosh\psi$, this becomes

$$F(u) = u^{n-3}\int_{\psi=0}^\infty\int_{\phi=0}^\infty e^{-2hu\cosh\psi\cosh\phi}\sinh^{n-3}\psi\cosh\psi\,d\psi\,d\phi.$$

* R. A. Fisher, *Biometrika*, Vol. x. p. 510.

Now the Bessel function of the second kind and imaginary argument is given by*

$$K_0(w) = \int_0^\infty e^{-w\cosh\phi}\,d\phi, \qquad w > 0.$$

Hence

$$F(u) = u^{n-3}\int_0^\infty K_0(2u\cosh\psi)\sinh^{n-3}\psi\cosh\psi\,d\psi.$$

Further we have†

$$\int_0^\infty K_0(2u\cosh\psi)\sinh^{2\lambda+1}\psi\cosh\psi\,d\psi = \frac{\Gamma(\lambda+1)\,K_{\lambda+1}(2u)}{2u^{\lambda+1}} \quad \text{......(iii)}$$

and thus

$$F(u) = \tfrac{1}{2}\Gamma(\tfrac{1}{2}n-1)\,u^{\frac{1}{2}(n-2)}\,K_{\frac{1}{2}n-1}(2u) \quad \text{...............(iv)},$$

for $u > 0$, while for $u < 0$, $F(u)$ is defined by $F(-u) = F(u)$. Hence

$$I_0 = \tfrac{1}{2}\Gamma(\tfrac{1}{2}n-1)\,A_0\int_0^{+\infty}(e^{2\rho u}+e^{-2\rho u})\,u^{\frac{1}{2}n-1}K_{\frac{1}{2}n-1}(2u)\,du \quad \text{...(iv)}^{bis}.$$

Accordingly the curve of frequency of $2u$ which is such that

$$v = 2u = \frac{n}{1-\rho^2}\,\frac{p_{11}}{\sigma_1\sigma_2} \quad \text{..............................(v)}$$

is

$$y = \frac{A_0\Gamma(\frac{1}{2}n-1)}{2^{\frac{1}{2}n-1}}\,e^{\rho v}v^{\frac{1}{2}n-1}K_{\frac{1}{2}n-1}(v) \quad \text{..................(vi)}.$$

To complete this curve we have to find A_0. If N be the total number of samples,

$$N = \int_{-\infty}^{+\infty} y\,dv = \frac{A_0\Gamma(\frac{1}{2}n-1)}{2^{\frac{1}{2}n-1}}\int_0^\infty \frac{e^{\rho v}+e^{-\rho v}}{2}\,v^{\frac{1}{2}n-1}K_{\frac{1}{2}n-1}(v)\,dv$$

$$= \frac{A_0\Gamma(\frac{1}{2}n-1)}{2^{\frac{1}{2}n-1}}\int_0^\infty \cosh\rho v\,v^{\frac{1}{2}n-1}K_{\frac{1}{2}n-1}(v)\,dv \quad \text{......(vii)}.$$

We will proceed shortly to the determination of this integral, but we will take first a rather more general form. We require the moment-coefficients of the distribution of p_{11}. If we obtain these coefficients about the fixed origin $p_{11}=0$, it is easy by the usual reduction formula to obtain them when they are calculated about the mean of the individual samples. If we are seeking the mth moment-coefficient about zero, we must include the term $(r\Sigma_1\Sigma_2)^m$ under the signs of the triple integration. In other words we need

$$I_m = \left(\frac{2\sigma_1\sigma_2(1-\rho^2)}{n}\right)^m A_0\int_{-\infty}^{+\infty} e^{2\rho u}\,u^m F(u)\,du$$

$$= A_m\int_0^\infty (e^{2\rho u}+(-1)^m e^{-2\rho u})\,u^m F(u)\,du \quad \text{.................(viii)},$$

where

$$A_m = C\left(\frac{2\sigma_1\sigma_2(1-\rho^2)}{n}\right)^{m+n-1} \quad \text{.....................(viii)}^{bis}.$$

* G. N. Watson, *Theory of Bessel Functions*, p. 172 (5).

† Watson, *loc. cit.* p. 417 (6). The result (iii) follows from putting in Watson's formula (6) $a=2$, $z=u$, $t=u\cosh\psi$, and remembering that $K_{-\nu}(2u)=K_\nu(2u)$; for which see Watson, p. 79.

To evaluate I_m consider the relation*

$$\int_0^\infty e^{2\rho u} u^\lambda K_\mu(2u)\,du = \frac{\sqrt{\pi}\,\Gamma(\lambda-\mu+1)\,\Gamma(\lambda+\mu+1)}{2^{\lambda+\frac{3}{2}}\,\Gamma(\lambda+\frac{3}{2})(1-\rho^2)^{\lambda+\frac{1}{2}}} \times F(-\mu+\tfrac{1}{2}, \mu+\tfrac{1}{2}, \lambda+\tfrac{3}{2}, \tfrac{1}{2}(1+\rho)) \quad \text{......(ix).}$$

This gives

$$I_m = \tfrac{1}{4} A_m \sqrt{\pi} \frac{\Gamma(\frac{1}{2}n-1)\,\Gamma(n+m-1)\,\Gamma(m+1)}{\Gamma(\frac{1}{2}(n+1)+m)(1-\rho^2)^{\frac{1}{2}(n-1)+m}} f_m(\rho),$$

where

$$f_m(\rho) = \left(\frac{1+\rho}{2}\right)^{\frac{1}{2}(n-1)+m} F(-\tfrac{1}{2}(n-3), \tfrac{1}{2}(n-1), \tfrac{1}{2}(n+1)+m, \tfrac{1}{2}(1+\rho))$$
$$+(-1)^m \left(\frac{1-\rho}{2}\right)^{\frac{1}{2}(n-1)+m} F(-\tfrac{1}{2}(n-3), \tfrac{1}{2}(n-1), \tfrac{1}{2}(n+1)+m, \tfrac{1}{2}(1-\rho)) \quad \text{.....(x).}$$

Considering the function $f_m(\rho)$ and noting that†

$$\frac{d}{d\rho} F(\alpha, \beta, \gamma, \tfrac{1}{2}(1+\rho)) = \frac{\gamma-1}{1+\rho}\left(F(\alpha, \beta, \gamma-1, \tfrac{1}{2}(1+\rho)) - F(\alpha, \beta, \gamma, \tfrac{1}{2}(1+\rho))\right),$$

$$\frac{d}{d\rho} F(\alpha, \beta, \gamma, \tfrac{1}{2}(1-\rho)) = -\frac{\gamma-1}{1-\rho}\left(F(\alpha, \beta, \gamma-1, \tfrac{1}{2}(1-\rho)) - F(\alpha, \beta, \gamma, \tfrac{1}{2}(1-\rho))\right),$$

we obtain by straightforward differentiation,

$$\frac{d}{d\rho} f_m(\rho) = \frac{1}{2}\left(\frac{n-1}{2}+m\right) f_{m-1}(\rho) \quad \text{....................(xi).}$$

Again putting $m=-1$,

$$f_{-1}(\rho) = \left(\frac{1+\rho}{2}\right)^{\frac{1}{2}(n-3)} F(-\tfrac{1}{2}(n-3), \tfrac{1}{2}(n-1), \tfrac{1}{2}(n+1), \tfrac{1}{2}(1+\rho))$$
$$- \left(\frac{1-\rho}{2}\right)^{\frac{1}{2}(n-3)} F(-\tfrac{1}{2}(n-3), \tfrac{1}{2}(n-1), \tfrac{1}{2}(n+1), \tfrac{1}{2}(1-\rho))$$
$$= \left(\frac{1+\rho}{2}\right)^{\frac{1}{2}(n-3)} \left(\frac{1-\rho}{2}\right)^{\frac{1}{2}(n-3)} F(1, 0, \tfrac{1}{2}(n-1), \tfrac{1}{2}(1+\rho))$$
$$- \left(\frac{1-\rho}{2}\right)^{\frac{1}{2}(n-3)} \left(\frac{1+\rho}{2}\right)^{\frac{1}{2}(n-3)} F(1, 0, \tfrac{1}{2}(n-1), \tfrac{1}{2}(1-\rho)),$$

using the Euler transformation, or

$$f_{-1}(\rho) = 0 \quad \text{..................................(xii).}$$

Now
$$\frac{\Gamma(\gamma-\alpha-\beta)\,\Gamma(\gamma)}{\Gamma(\gamma-\alpha)\,\Gamma(\gamma-\beta)} = F(\alpha, \beta, \gamma, 1);$$

hence, putting $\rho = 1$,

$$f_m(1) = \frac{\Gamma(\frac{1}{2}(n+1)+m)\,\Gamma(\frac{1}{2}(n-1)+m)}{\Gamma(n+m-1)\,\Gamma(m+1)} \quad \text{..............(xiii).}$$

* Watson, *loc. cit.* p. 388 (7), with proper interchange of symbols.

† Forsyth, *A Treatise on Differential Equations*, 1885, p. 195.

Putting $m=0$ in (xi),

$$\frac{d}{d\rho}f_0(\rho)=\frac{1}{2}\frac{n-1}{2}f_{-1}(\rho)=0$$

by (xii), thus $f_0(\rho)$ is independent of ρ, or putting $\rho=1$, by (xiii),

$$f_0(\rho)=\frac{\Gamma(\frac{1}{2}(n+1))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)\,\Gamma(1)} \quad \text{.....................(xiv).}$$

Hence putting $m=1$ in (xi) we have

$$\frac{d}{d\rho}f_1(\rho)=\frac{1}{2}\frac{n+1}{2}f_0(\rho)=\frac{1}{2}\frac{n+1}{2}\frac{\Gamma(\frac{1}{2}(n+1))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)\,\Gamma(1)},$$

and integrating $$f_1(\rho)=\frac{1}{2}\frac{\Gamma(\frac{1}{2}(n+3))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)\,\Gamma(1)}\rho \quad \text{.....................(xv),}$$

the constant being clearly zero, for from the expression for $f_m(\rho)$,

$$f_1(0)=0.$$

Again, from (xi),

$$\frac{d}{d\rho}f_2(\rho)=\frac{1}{2}\frac{n+3}{2}f_1(\rho)=\frac{1}{2^2}\frac{\Gamma(\frac{1}{2}(n+3))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)}\rho,$$

or $$f_2(\rho)=\frac{1}{2^2}\frac{\Gamma(\frac{1}{2}(n+3))}{\Gamma(n-1)}\Gamma(\tfrac{1}{2}(n-1))\frac{\rho^2}{2}+\text{const.}$$

To determine the constant, put $\rho=1$ and use (xiii); we have

$$\frac{\Gamma(\frac{1}{2}(n+5))\,\Gamma(\frac{1}{2}(n+3))}{\Gamma(n+1)\,2\,.\,1}=\frac{1}{8}\frac{\Gamma(\frac{1}{2}(n+5))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)}+\text{const.}$$

or $$\text{const.}=\frac{\Gamma(\frac{1}{2}(n+5))\,\Gamma(\frac{1}{2}(n-1))}{8\,\Gamma(n-1)}\frac{1}{n}.$$

Thus $$f_2(\rho)=\frac{1}{8}\frac{\Gamma(\frac{1}{2}(n+5))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)}\left(\rho^2+\frac{1}{n}\right) \quad \text{...............(xvi).}$$

Now use (xi) again, putting $m=3$, integrating and remembering that $f_m(0)=0$ when m is *odd*; we have

$$f_3(\rho)=\frac{1}{16}\frac{\Gamma(\frac{1}{2}(n+7))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)}\left(\frac{\rho^3}{3}+\frac{\rho}{n}\right) \quad \text{............(xvii).}$$

Finally, from (xi) once more we have

$$f_4(\rho)=\frac{1}{32}\frac{\Gamma(\frac{1}{2}(n+9))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)}\left(\frac{\rho^4}{12}+\frac{\rho^2}{2n}\right)+\text{const.},$$

where to determine the constant we use (xiii) or

$$\frac{\Gamma(\frac{1}{2}(n+9))\,\Gamma(\frac{1}{2}(n+7))}{\Gamma(n+3)\,4\,.\,3\,.\,2}=\frac{1}{32}\frac{\Gamma(\frac{1}{2}(n+9))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)}\frac{n+6}{12n}+\text{const.}$$

Thus $$f_4(\rho)=\frac{1}{384}\frac{\Gamma(\frac{1}{2}(n+9))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)}\left(\rho^4+\frac{6}{n}\rho^2+\frac{3}{n(n+2)}\right)$$

......(xviii).

We can now find the corresponding I_m's from (x),

$$I_0 = \tfrac{1}{4} A_0 \sqrt{\pi} \frac{\Gamma(\tfrac{1}{2}n - 1)\,\Gamma(n-1)}{\Gamma(\tfrac{1}{2}(n+1))(1-\rho^2)^{\frac{1}{2}(n-1)}} \frac{\Gamma(\tfrac{1}{2}(n+1))\,\Gamma(\tfrac{1}{2}(n-1))}{\Gamma(n-1)}$$

$$= \tfrac{1}{4} A_0 \sqrt{\pi} \frac{\Gamma(\tfrac{1}{2}n - 1)\,\Gamma(\tfrac{1}{2}(n-1))}{(1-\rho^2)^{\frac{1}{2}(n-1)}},$$

or applying the duplication formula:

$$\Gamma(\tfrac{1}{2}n - 1)\,\Gamma(\tfrac{1}{2}(n-1)) = \sqrt{\pi}\,\Gamma(n-2)/2^{n-3} \quad \text{...............(xix)},$$

$$I_0 = \tfrac{1}{4} A_0 \frac{\pi\,\Gamma(n-2)}{2^{n-3}(1-\rho^2)^{\frac{1}{2}(n-1)}} \quad \text{........................(xx)}.$$

Now by (iv)^{bis},
$$\int_0^\infty \tfrac{1}{2}(e^{\rho v} + e^{-\rho v})\, v^{\frac{1}{2}n-1} K_{\frac{1}{2}n-1}(v)\,dv$$

$$= \frac{I_0}{A_0} \frac{2^{\frac{1}{2}n-1}}{\frac{1}{2}\Gamma(\tfrac{1}{2}n-1)},$$

and hence by (xx),

$$= \frac{\pi\,\Gamma(n-2)}{2^{\frac{1}{2}n-1}\,\Gamma(\tfrac{1}{2}n-1)(1-\rho^2)^{\frac{1}{2}(n-1)}} \quad \text{...............(xxi)}.$$

Turning back to (vii) we find

$$N = \frac{A_0 \pi\,\Gamma(n-2)}{2^{n-2}(1-\rho^2)^{\frac{1}{2}(n-1)}},$$

or

$$A_0 = \frac{N 2^{n-2}(1-\rho^2)^{\frac{1}{2}(n-1)}}{\pi\,\Gamma(n-2)} \quad \text{........................(xxii)},$$

and therefore by (viii)^{bis},

$$A_m = \frac{N 2^{n-2}(1-\rho^2)^{\frac{1}{2}(n-1)}}{\pi\,\Gamma(n-2)} (2\sigma_1\sigma_2(1-\rho^2))^m \quad \text{.. ....(xxiii)}.$$

The curve for the frequency distribution of $v = \dfrac{n}{1-\rho^2}\dfrac{p_{11}}{\sigma_1\sigma_2}$ is now known from (vi) using (xix), namely,

$$y = \frac{N(1-\rho^2)^{\frac{1}{2}(n-1)}}{\sqrt{\pi}\, 2^{\frac{1}{2}n-1}\,\Gamma(\tfrac{1}{2}(n-1))} e^{\rho v} \{v^{\frac{1}{2}n-1} K_{\frac{1}{2}n-1}(v)\} \quad \text{.........(xxiv)}.$$

Here the function of v in curled brackets is not to change sign with v but the power of the exponential does so. Such is the curve which has to be traced for samples of various sizes (n) from populations of various correlations ρ*.

We next turn to I_1. By (x) and (xv),

$$I_1 = \tfrac{1}{4} A_1 \sqrt{\pi} \frac{\Gamma(\tfrac{1}{2}n-1)\,\Gamma(n)}{\Gamma(\tfrac{1}{2}(n+3))(1-\rho^2)^{\frac{1}{2}(n+1)}} \frac{\Gamma(\tfrac{1}{2}(n+3))\,\Gamma(\tfrac{1}{2}(n-1))}{2\Gamma(n-1)} \rho,$$

or

$$\frac{I_1}{I_0} = \frac{A_1}{A_0} \frac{\rho(n-1)}{2(1-\rho^2)} = \frac{2\sigma_1\sigma_2(1-\rho^2)}{n} \frac{\rho(n-1)}{2(1-\rho^2)}.$$

* For the special case $\rho = 0$, or of sampling from uncorrelated material,

$$y = \frac{N}{\sqrt{\pi}\, 2^{\frac{1}{2}n-1}\,\Gamma(\tfrac{1}{2}(n-1))} v^{\frac{1}{2}n-1} K_{\frac{1}{2}n-1}(v).$$

But I_1/I_0 = mean value of p_{11} in samples or

$$\bar{p}_{11} = \left(1 - \frac{1}{n}\right) \sigma_1\sigma_2\rho = \left(1 - \frac{1}{n}\right) P_{11} \quad \text{.................(xxv)},$$

where P_{11} is the value in the sampled population*.

We proceed to find I_2/I_0. By (x) and (xvi),

$$I_2 = \tfrac{1}{4} A_2 \sqrt{\pi} \frac{\Gamma(\frac{1}{2}n-1)\,\Gamma(n+1)\,2}{\Gamma(\frac{1}{2}(n+5))(1-\rho^2)^{\frac{1}{2}(n+3)}} \frac{1}{8} \frac{\Gamma(\frac{1}{2}(n+5))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)} \left(\rho^2 + \frac{1}{n}\right),$$

$$\frac{I_2}{I_0} = \frac{1}{4} \frac{n(n-1)}{(1-\rho^2)^2} \left(\rho^2 + \frac{1}{n}\right) \frac{A_2}{A_0} = \left(1 - \frac{1}{n}\right)\left(\rho^2 + \frac{1}{n}\right) \sigma_1^2\sigma_2^2,$$

$$\sigma^2_{p_{11}} = \frac{I_2}{I_0} - \bar{p}^2_{11} = \frac{n-1}{n^2}(1+\rho^2)\,\sigma_1^2\sigma_2^2 \quad \text{.................(xxvi)}.$$

Since $v = \dfrac{n}{1-\rho^2}\dfrac{p_{11}}{\sigma_1\sigma_2}$, we have

$$\bar{v} = (n-1)\frac{\rho}{1-\rho^2} \quad \text{..........................(xxvii)},$$

$$\sigma_v = \sqrt{n-1}\,\frac{\sqrt{1+\rho^2}}{1-\rho^2} \quad \text{......................(xxviii)},$$

which are the values used in dealing with the frequency curves.

Next we obtain from (x) and (xvii), using (xix),

$$\frac{I_3}{I_0} = \frac{3}{32}\frac{A_3}{A_0}\frac{(n+1)\,n\,(n-1)}{(1-\rho^2)^3}\left(\tfrac{1}{3}\rho^3 + \frac{1}{n}\rho\right),$$

or

$$_{p_{11}}\mu_3' = 3\sigma_1^3\sigma_2^3 \frac{n^2-1}{n^2}\left(\tfrac{1}{3}\rho^3 + \frac{\rho}{n}\right) \quad \text{....................(xxix)}.$$

Hence by the formula

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3,$$

and remembering $\mu_2' = I_2/I_0$, $\mu_1' = I_1/I_0$, we find after reduction,

$$_{p_{11}}\mu_3 = \frac{2}{n^2}\left(1 - \frac{1}{n}\right)\rho\,(\rho^2+3)\,\sigma_1^3\sigma_2^3 \quad \text{.....................(xxx)}.$$

We are now in a position to determine $_{p_{11}}\beta_1$.

$$_{p_{11}}\beta_1 = \frac{(_{p_{11}}\mu_3)^2}{(_{p_{11}}\mu_2)^3} = \frac{4}{n-1}\frac{\rho^2(\rho^2+3)^2}{(1+\rho^2)^3} \quad \text{.................(xxxi)}.$$

The function of ρ^2 on the right attains its maximum when $\rho^2 = 1$ and then $_{p_{11}}\beta_1 = 8/(n-1)$. We see accordingly that there can be sensible skewness, or β_1 of the order 0·1, when the size of the sample is of order 100. For samples of the order 400, β_1 would be of the order ·02, and could be treated for practical purposes as negligible. For the smallest samples ($n \rightarrow 2$ (!)), and high correlation ($\rho \rightarrow 1$ (!)), we can have $_{p_{11}}\beta_1 \rightarrow 8$, which is a very high value indeed.

* This is a special case of the value of $\bar{p}_{11}$ given some time back by K. Pearson for sampling out of a limited population N, with *any* form of distribution (not necessarily normal), i.e.

$$\bar{p}_{11} = \frac{1-1/n}{1-1/N} P_{11}.$$

Turning to I_4 we have from (x), (xviii) and (xix),

$$I_4 = \tfrac{1}{4} A_4 \frac{\sqrt{\pi}\,\Gamma(\frac{1}{2}n-1)\,\Gamma(n+3)\,4.3.2}{\Gamma(\frac{1}{2}(n+9))(1-\rho^2)^{\frac{1}{2}(n+7)}} \frac{1}{384} \frac{\Gamma(\frac{1}{2}(n+9))\,\Gamma(\frac{1}{2}(n-1))}{\Gamma(n-1)} \times \left(\rho^4 + \frac{6}{n}\rho^2 + \frac{3}{n(n+2)}\right),$$

$$\frac{I_4}{I_0} = \frac{1}{16}\frac{A_4}{A_0}\frac{(n+2)(n+1)n(n-1)}{(1-\rho^2)^4}\left(\rho^4 + \frac{6}{n}\rho^2 + \frac{3}{n(n+2)}\right),$$

$$_{p_{11}}\mu_4' = \sigma_1^4\sigma_2^4\left(1+\frac{2}{n}\right)\left(1+\frac{1}{n}\right)\left(1-\frac{1}{n}\right)\left(\rho^4 + \frac{6}{n}\rho^2 + \frac{3}{n(n+2)}\right) \quad \text{..............(xxxii).}$$

We must now transfer to the mean by aid of the formula

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4,$$

remembering that $$_{p_{11}}\mu_s' = I_s/I_0.$$

We find $$_{p_{11}}\mu_4 = 3\sigma_1^4\sigma_2^4\left(1-\frac{1}{n}\right)\frac{1}{n^3}((n+1)(1+\rho^2)^2 + 8\rho^2) \quad \text{.........(xxxiii).}$$

Hence we deduce

$$_{p_{11}}\beta_2 = \frac{_{p_{11}}\mu_4}{_{p_{11}}\mu_2^2} = 3\left(\frac{n+1}{n-1} + \frac{8}{n-1}\frac{\rho^2}{(1+\rho^2)^2}\right) \quad \text{............(xxxiv)}$$

$$= 3 + \frac{6}{n-1}\left(1 + \frac{4\rho^2}{(1+\rho^2)^2}\right) \quad \text{....................(xxxiv)}^{bis}.$$

$_{p_{11}}\beta_2$ reaches its maximum, 15, when $n=2$ and $\rho=1$. Its minimum is attained when $n \to \infty$ and it then takes the normal value 3.

Equations (xxvi), (xxx), (xxxi), (xxxiii) and (xxxiv) have been reached by Mr J. Pepper as special cases for the normal surface distribution from his far more general expressions for the moments of p_{11}, when the sampling is made from any form of distribution*. The novelty of the present method is that the same results are obtained, by starting from the actual distribution curve, when the sampling is made from a bivariate normal surface.

The question now arises as to when we shall arrive at an adequate distribution curve for p_{11} by applying (xxv), (xxvi), (xxxi) and (xxxiv) to fit a four-moment leptokurtic curve to the data, i.e. how closely does the Bessel-function curve (xxiv) fit a Pearson curve of Types IV or VI? The following illustrations indicate that for practical purposes the labour of computing the ordinates of the Bessel-function curve may be avoided for fairly moderate-sized samples.

We start with samples of 50, and compare the true Bessel-function curve with the Pearson type curve for $\rho = 0$ and $\rho = {\cdot}6$. These are shown in Figs. 1 and 2, and it will be seen that the accordance is excellent. We next passed to a sample of 30 for $\rho = 0$, and the fit is again excellent, and if for $\rho = 0$, it will be so for higher values of ρ (see Fig. 3). Accordingly we proceeded to lower still further the size of the sample and reduced n to 22. For $\rho = {\cdot}9$ and ${\cdot}6$ (Figs. 4 and 5) the accordance is all that could be desired. But when we pass to lower values of the correlation, $\rho = {\cdot}3$ and $\rho = {\cdot}1$ (Figs. 6 and 7) the agreement while for most purposes statistically adequate is losing its merits. We can therefore conclude that below $n = 22$, the

* Dr Wishart has also given the values of β_1 and β_2 for the distribution of p_{11} in *Biometrika*, Vol. xx^A. p. 42.

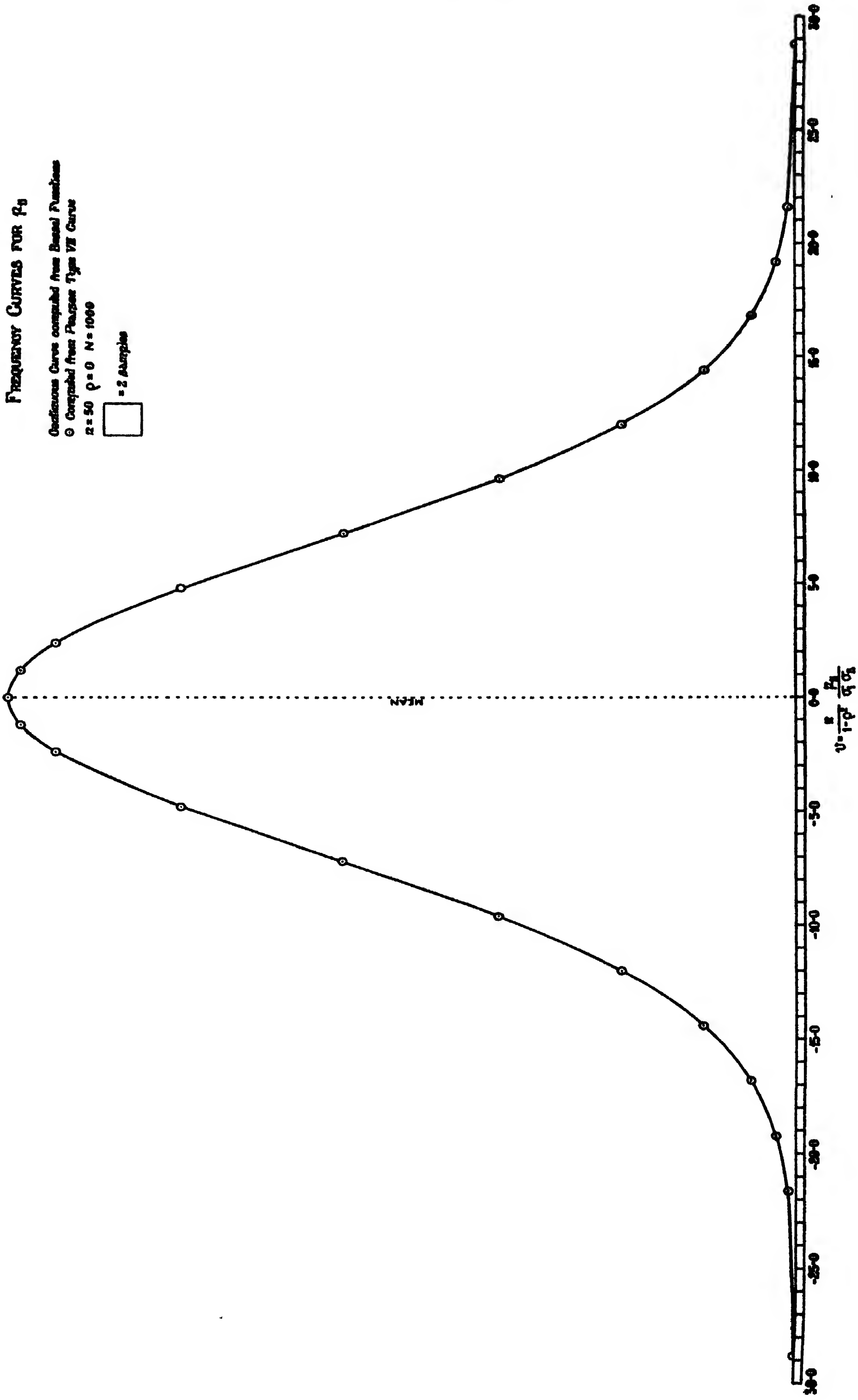

FREQUENCY CURVES FOR r_{12}
Continuous Curve computed from Bessel Functions
⊙ Computed from Pearson Type VII Curve
$n = 50$ $\rho = 0$ $N = 1000$
= 2 Samples
MEAN
-30·0 -25·0 -20·0 -15·0 -10·0 -5·0 0·0 5·0 10·0 15·0 20·0 25·0 30·0
$v = \frac{n}{1-\rho^2}\frac{r_{12}}{\sigma_1\sigma_2}$

Fig. 1.

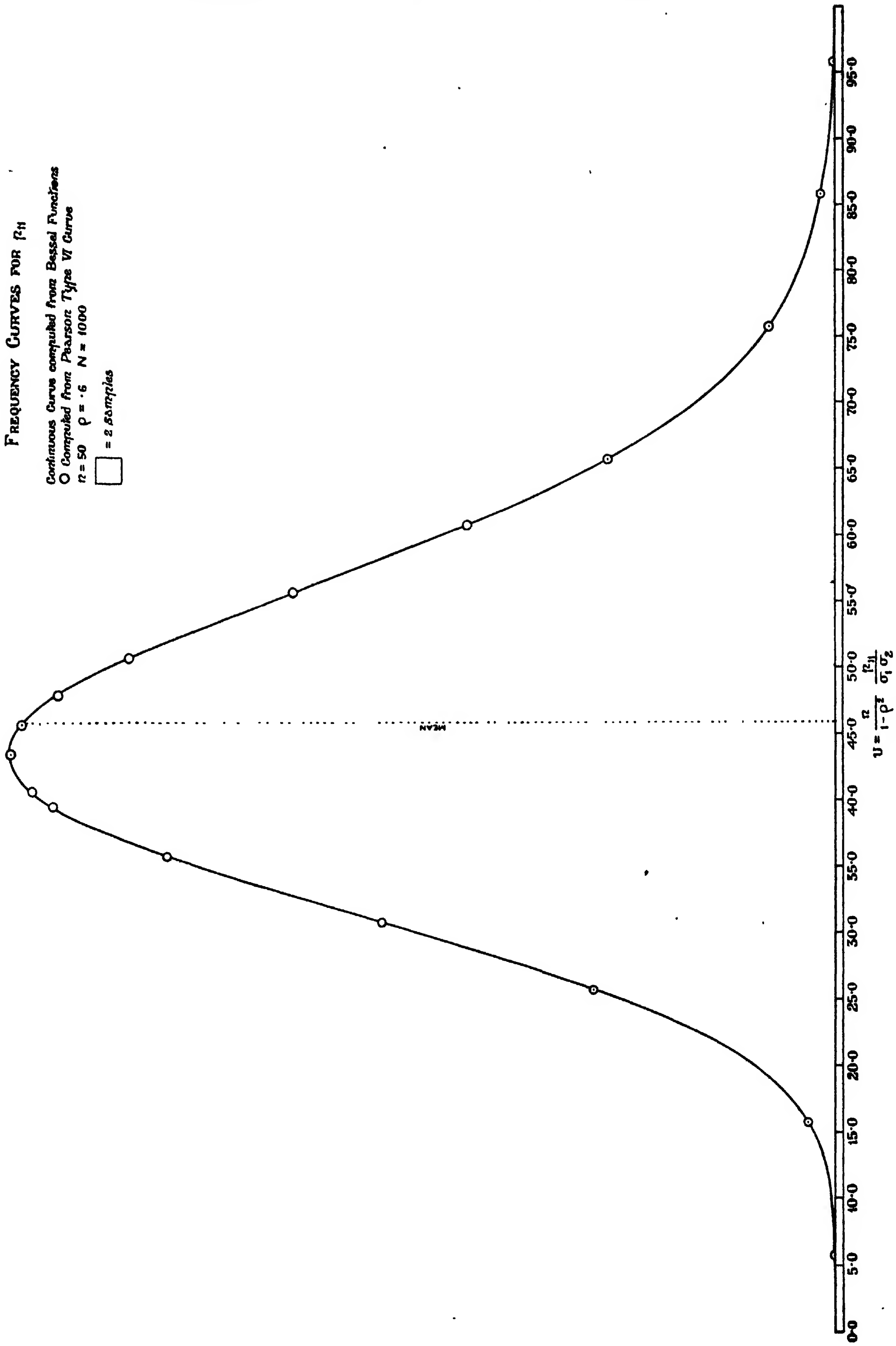

Frequency Curves for r_{11}
Continuous Curve computed from Bessel Functions
O Computed from Pearson Type VI Curve
$n = 50$ $\rho = \cdot 6$ $N = 1000$
= 2 samples
MEAN
0·0 5·0 10·0 15·0 20·0 25·0 30·0 35·0 40·0 45·0 50·0 55·0 60·0 65·0 70·0 75·0 80·0 85·0 90·0 95·0
$u = \frac{n}{1-\rho^2} \frac{r_{11}}{\sigma_1 \sigma_2}$

Fig. 2.

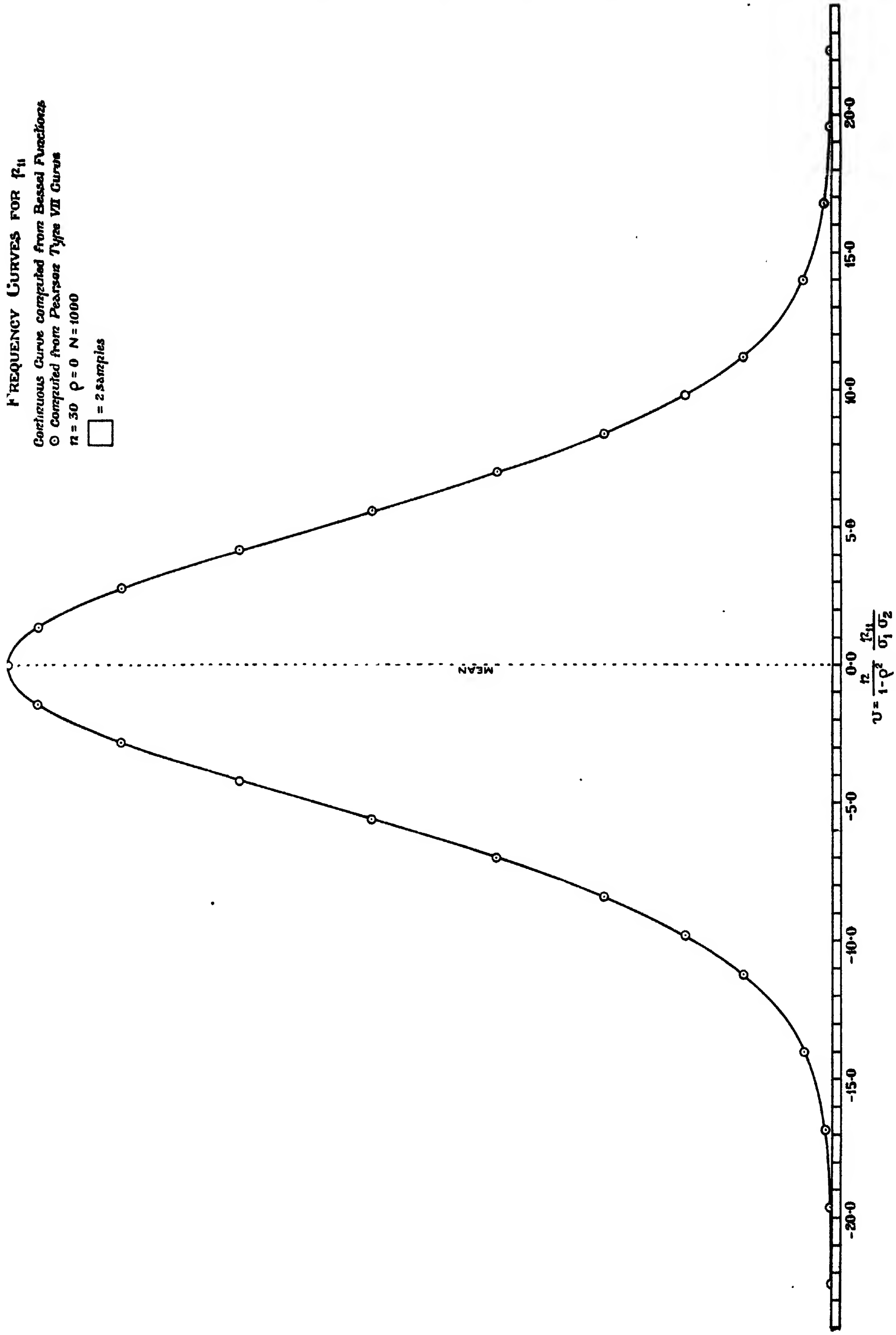
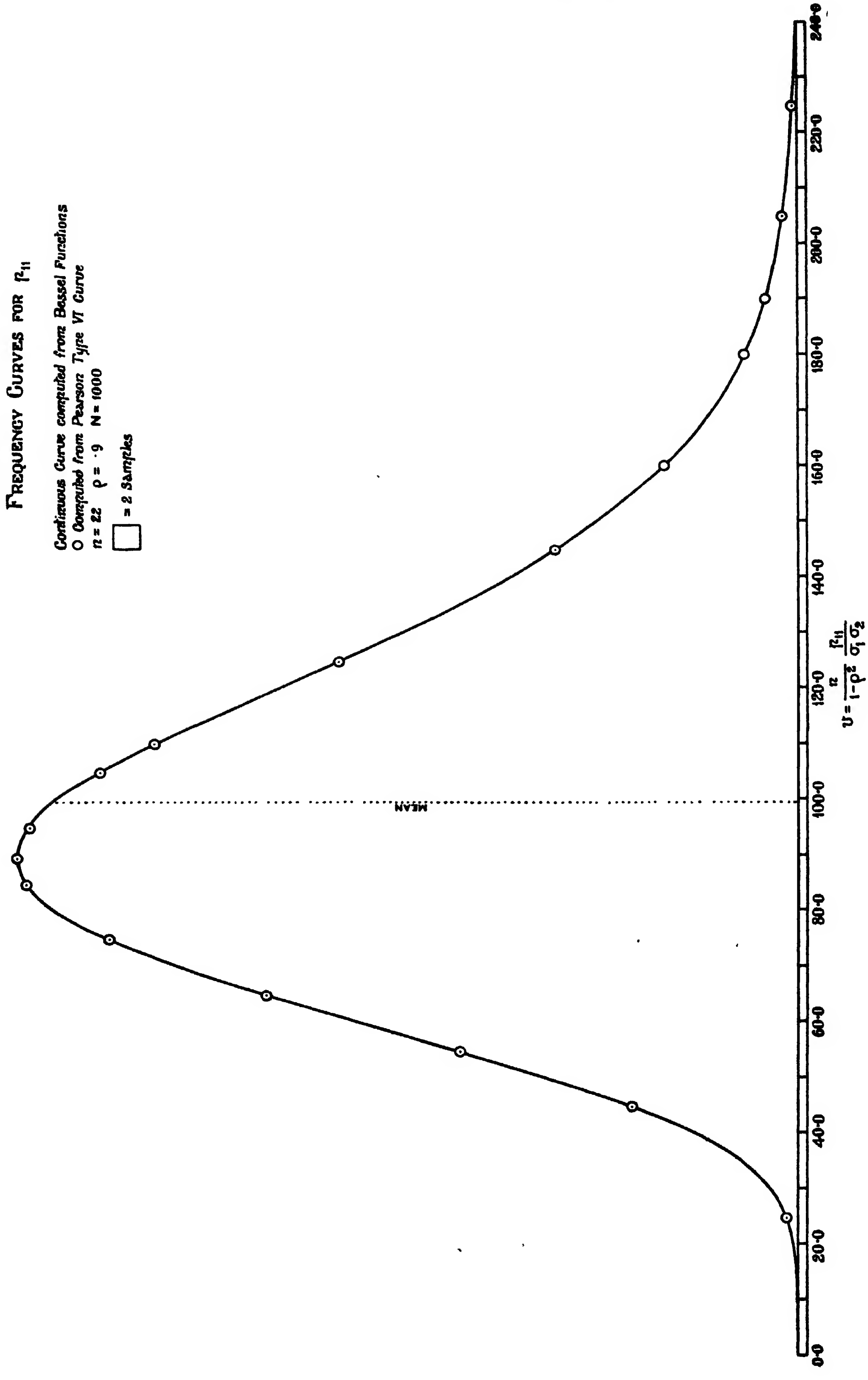

FREQUENCY CURVES FOR r_{11}
Continuous Curve computed from Bessel Functions
⊙ Computed from Pearson Type VII Curves
$n = 30$ $\rho = 0$ $N = 1000$
= 2 samples
MEAN
20·0
15·0
10·0
5·0
0·0
-5·0
-10·0
-15·0
-20·0
$v = \frac{n}{1-\rho^2} \frac{r_{11}}{\sigma_1 \sigma_2}$

Fig. 3.



Frequency Curves for r_{11}
Continuous Curve computed from Bessel Functions
O Computed from Pearson Type VI Curve
$n = 22$ $\rho = \cdot 9$ $N = 1000$
= 2 Samples
MEAN
0·0
20·0
40·0
60·0
80·0
100·0
120·0
140·0
160·0
180·0
200·0
220·0
240·0
$u = \frac{n}{1-\rho^2}\frac{r_{11}}{\sigma_1\sigma_2}$

Fig. 4.

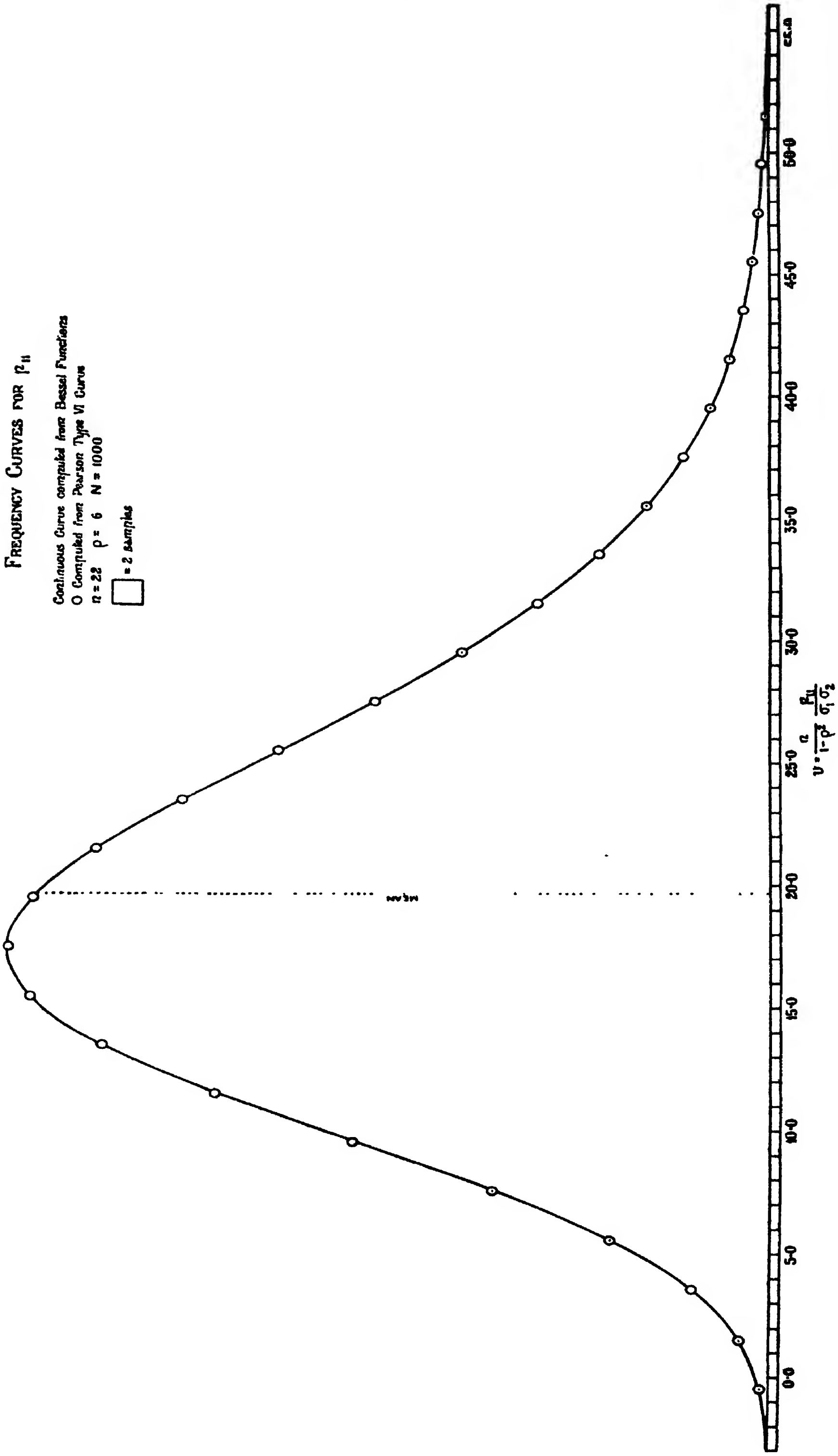
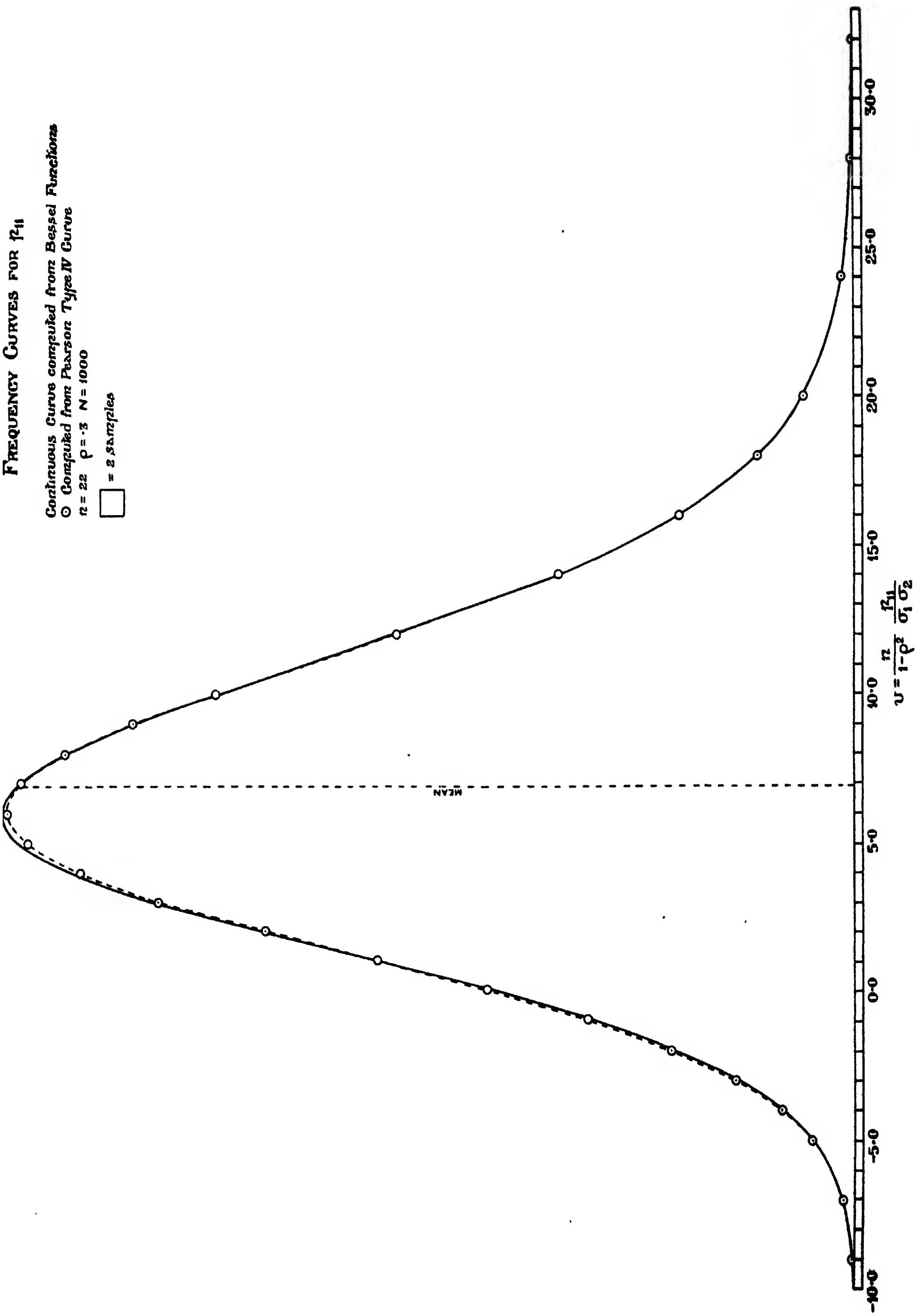

FREQUENCY CURVES FOR p_{11}
Continuous Curve computed from Bessel Functions
O Computed from Pearson Type VI Curve
$n = 22 \quad \rho = \cdot 6 \quad N = 1000$
= 2 samples
Mean
0·0
5·0
10·0
15·0
20·0
25·0
30·0
35·0
40·0
45·0
50·0
55·0
$v = \frac{n}{1-\rho^2} \frac{p_{11}}{\sigma_1 \sigma_2}$

Fig. 5.



FREQUENCY CURVES FOR p_{11}
Continuous Curve computed from Bessel Functions
⊙ Computed from Pearson Type IV Curve
$n = 22$ $\rho = \cdot 3$ $N = 1000$
= 2 samples
MEAN
-10·0
-5·0
0·0
5·0
10·0
15·0
20·0
25·0
30·0
$v = \frac{n}{1-\rho^2}\frac{p_{11}}{\sigma_1\sigma_2}$

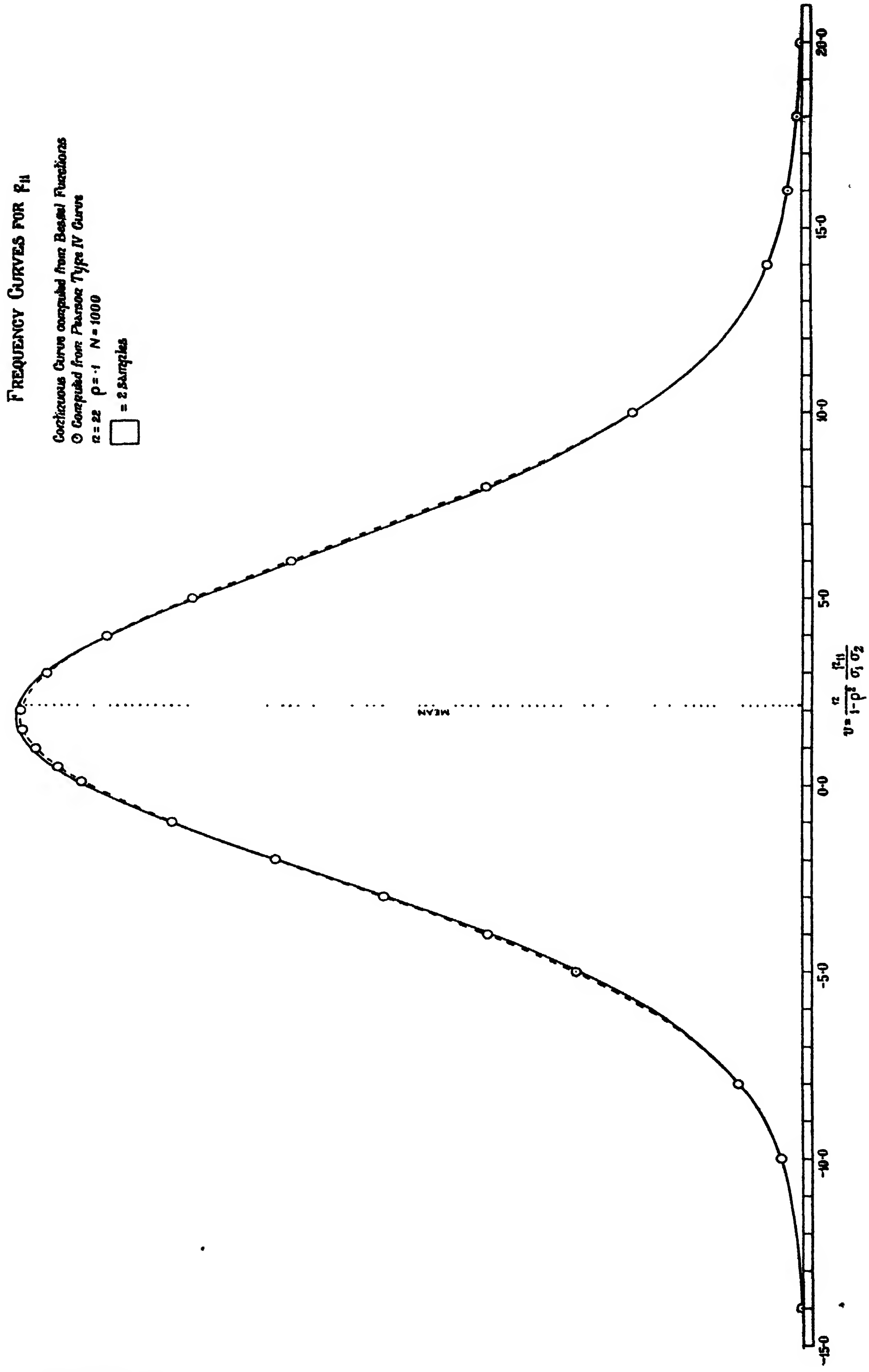

FREQUENCY CURVES FOR r_{11}
Continuous Curve computed from Bessel Functions
⊙ Computed from Pearson Type IV Curve
$n = 22 \quad \rho = \cdot 1 \quad N = 1000$
= 2 samples
MEAN
$v = \frac{n}{1-\rho^2}\,\frac{r_{11}}{\sigma_1 \sigma_2}$
−15·0
−10·0
−5·0
0·0
5·0
10·0
15·0
20·0

Fig. 7.

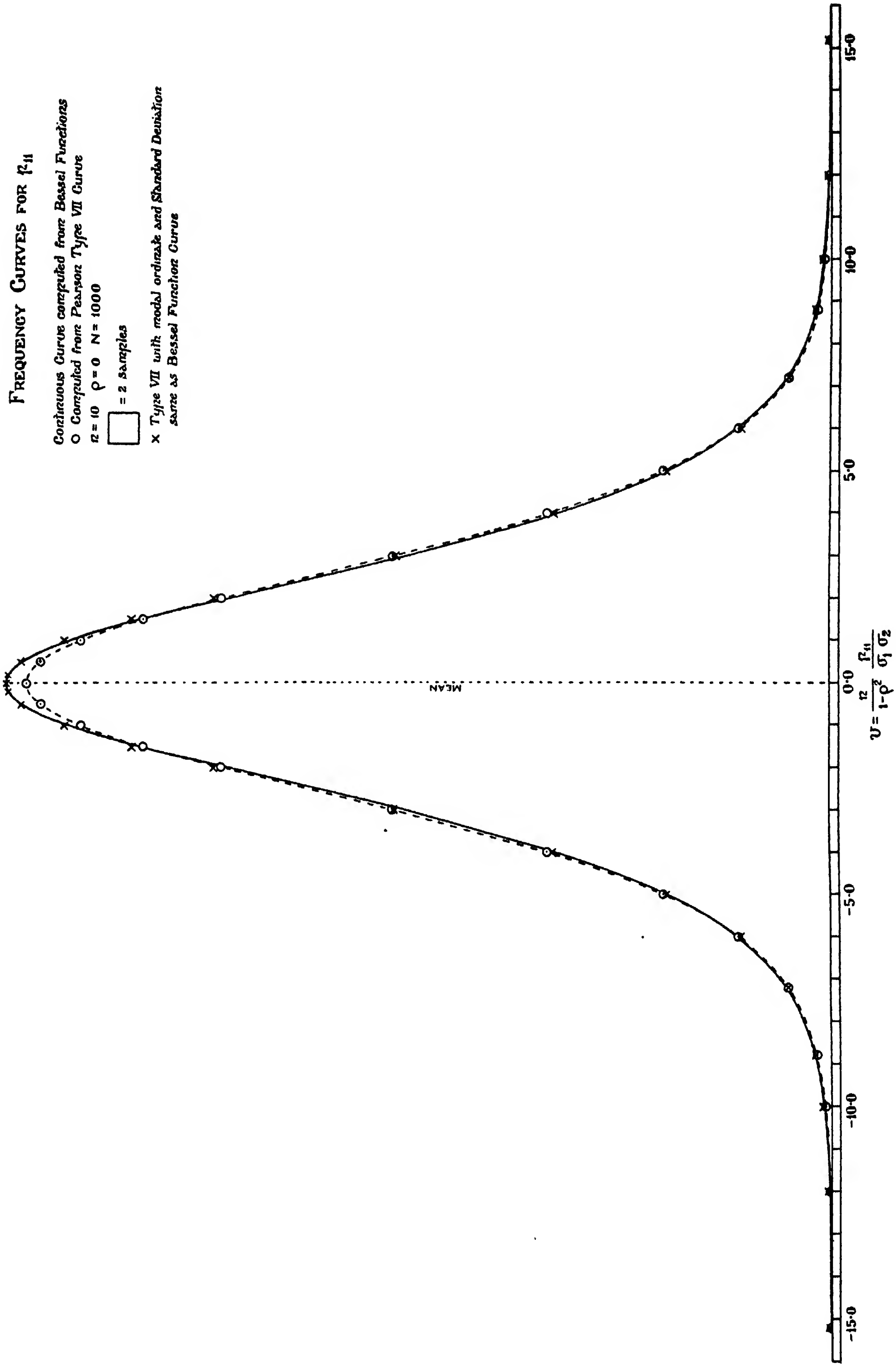

FREQUENCY CURVES FOR r_{11}
Continuous Curve computed from Bessel Functions
o Computed from Pearson Type VII Curve
$n = 10$ $\rho = 0$ $N = 1000$
= 2 samples
× Type VII with modal ordinate and Standard Deviation same as Bessel Function Curve
MEAN
15·0
10·0
5·0
0·0
-5·0
-10·0
-15·0
$v = \frac{n}{1-\rho^2}\frac{r_{11}}{\sigma_1\sigma_2}$

Fig. 8.

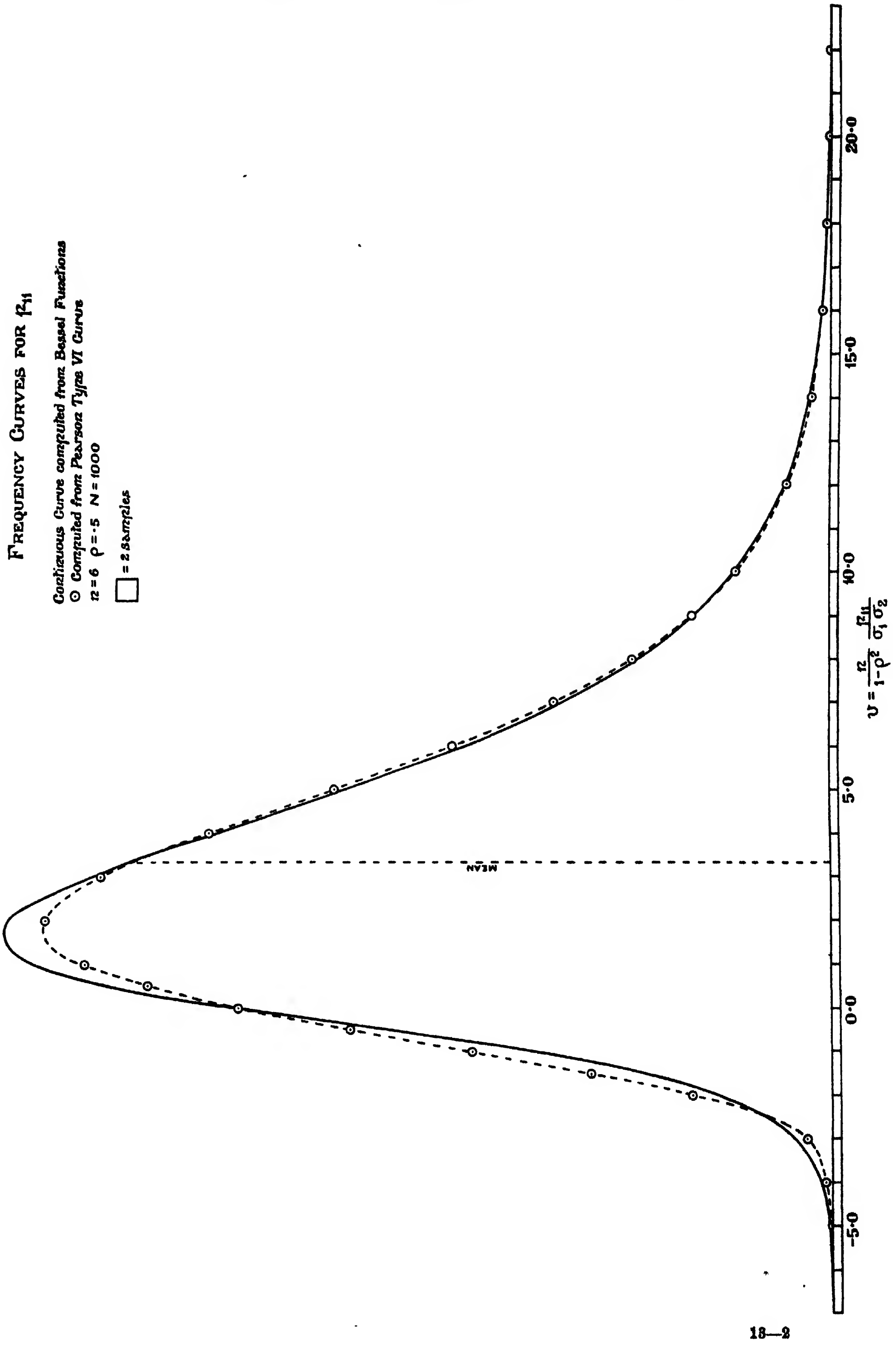

FREQUENCY CURVES FOR r_{11}
Continuous Curve computed from Bessel Functions
⊙ Computed from Pearson Type VI Curve
$n = 6$ $\rho = \cdot 5$ $N = 1000$
= 2 samples
MEAN
-5·0
0·0
5·0
10·0
15·0
20·0
$v = \frac{n}{1-\rho^2} \frac{r_{11}}{\sigma_1 \sigma_2}$

Fig. 9.

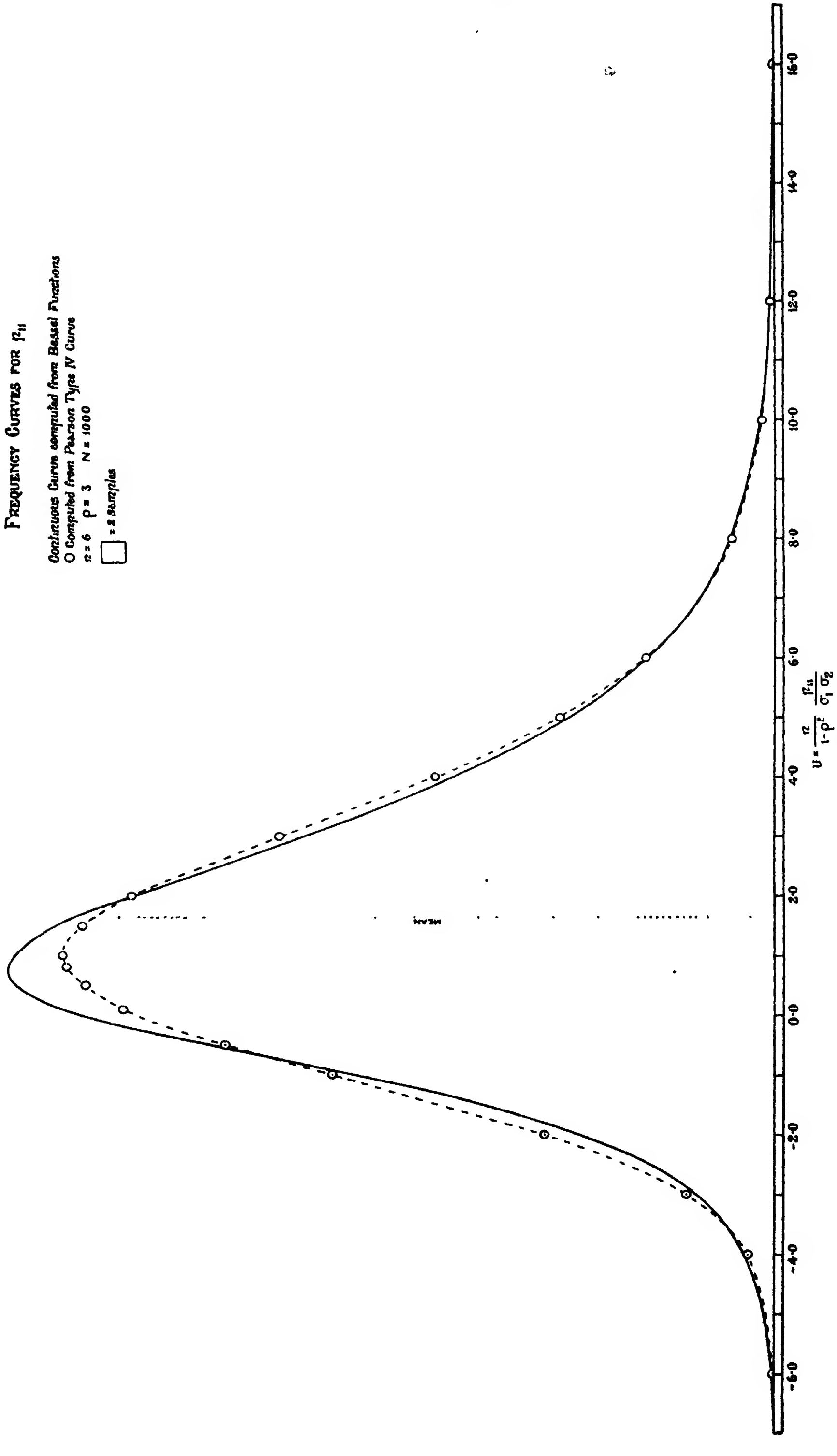

FREQUENCY CURVES FOR p_{11}
Continuous Curve computed from Bessel Functions
O Computed from Pearson Type IV Curve
$n = 6$ $\rho = 3$ $N = 1000$
= 2 Samples
MEAN
-6·0 -4·0 -2·0 0·0 2·0 4·0 6·0 8·0 10·0 12·0 14·0 16·0
$U = \frac{n}{1-\rho^2} \frac{p_{11}}{\sigma_1 \sigma_2}$

Fig. 10.

Pearson curve with the same mean, standard deviation, and constants β_1 and β_2 as the Bessel-function curve will not suffice to describe the distribution of p_{11}. Illustrations of this are provided in Fig. 8 for $n=10$ and $\rho=0$, and still more effectively in Figs. 9 and 10 corresponding to $n=6$, for $\rho=\cdot 5$ and $\cdot 3$. It is feasible, however, to get an adequate description of the distribution of p_{11} or v by a Pearson curve at least down to $n=10$, if we drop the equality of β_2 for the Pearson curve and the p_{11} distribution and make our conditions the equality of the ordinates at $x=0$, and of the standard deviations. This is illustrated for the case of $\rho=0$ and $n=10$ in Fig. 8. But the wiser course seems to be to table the ordinates of the $\rho=0$ curves for samples of 2 to 25, and only use the Pearson-curve method for samples greater than 25. When ρ is not zero the ordinates of the asymmetrical curve can be obtained by multiplying by the factor

$$n(1-\rho^2)^{\frac{1}{2}n-\frac{3}{2}}e^{\rho v}.$$

The Tables now provided in the Appendix for the small samples has meant much work in computation because notwithstanding the labours of Professor Watson adequate tables of $K_m(v)$ for our purposes are not in existence*. It was then a great comfort when Professor Watson provided us with a suitable asymptotic formula for some of our purposes, the first term sufficing in many cases. This formula is as follows:

$$K_m(mt)=\sqrt{\frac{\pi}{2m}}\frac{e^{-m\sqrt{1+t^2}}}{(1+t^2)^{\frac{1}{4}}}\left(\frac{1+\sqrt{1+t^2}}{t}\right)^m\sum_{s=0}^{s=\infty}\frac{\Gamma(s+\frac{1}{2})}{\Gamma(\frac{1}{2})}\frac{A_s(-1)^s}{(\frac{1}{2}m\sqrt{1+t^2})^s} \quad\text{......(xxxv)},$$

where

$$A_0=1,\quad A_1=\frac{1}{8}-\frac{5}{24(1+t^2)},\quad A_2=\frac{3}{128}-\frac{77}{576(1+t^2)}+\frac{385}{3456}\frac{1}{(1+t^2)^2}.$$

This may be written in the more compact form, if $\tan\theta=v/m=t$,

$$K_m(v)=\sqrt{\frac{\pi}{2m}}e^{-\frac{m}{\cos\theta}}\sqrt{\cos\theta}\,(\cot\tfrac{1}{2}\theta)^m\sum_{s=0}^{s=\infty}\frac{\Gamma(s+\frac{1}{2})}{\Gamma(\frac{1}{2})}\left(\frac{2}{m}\right)^s A_s(-1)^s\cos^s\theta \quad\text{......(xxxv)}^{bis},$$

where $A_0=1,\quad A_1=\frac{1}{8}-\frac{5}{24}\cos^2\theta,\quad A_2=\frac{3}{128}-\frac{77}{576}\cos^2\theta+\frac{385}{3456}\cos^4\theta.$

Confining our attention to the first term we have for our frequency curve from (xxiv),

$$y=y_0e^{m(\rho t-\sqrt{1+t^2})}(1+\sqrt{1+t^2})^m/(1+t^2)^{\frac{1}{4}} \quad\text{............(xxxvi)},$$

where $$m=\tfrac{1}{2}n-1.$$

* The highest value tabled is $K_{10}(v)$, and we require $K_{\frac{1}{2}n-1}(v)$, or the largest possible sample n is one of 22. Even then the existing table for $K_{10}(v)$ provides only a small portion of the rise of the frequency curve for p_{11}; we do not get near the modal value and *a fortiori* do not get the fall of the curve. For $m=10$, we can use the recurrence formula

$$K_{m+1}(v)=K_{m-1}(v)+\frac{2m}{v}K_m(v),$$

and the tables of $e^vK_0(v)$ and $e^vK_1(v)$ on pp. 698—713 of Professor Watson's book. But these only carry us up to $v=16$, a point short of the mode for the case of a sample of 22. With samples of 50 and upwards, the labour involved in the use of the recurrence formula becomes prohibitive. It practically amounts to computing new tables of $K_m(v)$, and extending the existing tables of $K_0(v)$ and $K_1(v)$ far beyond the range of the argument provided as v may run to 90 or even 200, while the frequency of p_{11} is still quite sensible.

Taking logarithmic differentials, the mode of this curve is given by

$$m\left(\rho - \frac{t}{\sqrt{1+t^2}}\right) + \frac{mt}{\sqrt{1+t^2}}\frac{1}{1+\sqrt{1+t^2}} - \frac{1}{2}\frac{t}{\sqrt{1+t^2}} = 0,$$

or, if $\tan\theta = t$, and $\check{\theta}$ corresponds to mode,

$$\tan\tfrac{1}{2}\check{\theta} = \rho - \frac{\sin 2\check{\theta}}{4m} \quad \text{.......................(xxxvii)}.$$

This equation admits of easy solution by successive approximations if m be fairly large. For example, to a first approximation,

$$\tan\tfrac{1}{2}\check{\theta} = \rho, \text{ or } \check{t} = \tan\check{\theta} = 2\rho/(1-\rho^2);$$

hence to a second approximation,

$$\tan\tfrac{1}{2}\check{\theta} = \rho - \frac{\rho(1-\rho^2)}{m(1+\rho^2)^2},$$

which gives us

$$\check{t} = \tan\check{\theta} = \frac{2\rho}{1-\rho^2}\left(1 - \frac{1}{m(1+\rho^2)}\right),$$

or

$$\check{v} = \frac{2\rho}{1-\rho^2}\left(\tfrac{1}{2}n - 1 - \frac{1}{1+\rho^2}\right) \quad \text{.................(xxxviii)}.$$

As an illustration consider a sample of 22 from a population with correlation ρ of ·6; we find

$$\check{v} = 17{\cdot}3713.$$

Now by Equations (xxvii), (xxviii), (xxxi) and (xxxiv)*bis* we have for the *exact* values,

$$\bar{v} = 19{\cdot}6875, \quad \sigma_v = 8{\cdot}350{,}243,$$
$$\beta_1 = {\cdot}307{,}755, \quad \beta_2 = 3{\cdot}508{,}156.$$

Hence by the Pearson-curve formula for the mode

$$\check{v} = \bar{v} - \frac{\sigma_v\sqrt{\beta_1}(\beta_2+3)}{2(5\beta_2 - 6\beta_1 - 9)} \quad \text{.....................(xxxix)},$$

we find $\check{v} = 17{\cdot}3301,$

or there is less than $\frac{1}{4}$ % difference in the values given by (xxxviii) and (xxxix).

As we have neglected terms of the order $1/m^2$ in obtaining (xxxviii), it is quite possible that (xxxix) really gives the modal value of v, with closer approximation than $\frac{1}{4}$ °/₀. It is clear, however, that (xxxix) will be of sufficient accuracy for most statistical purposes, when the sample is as large as 22.

But the reader must not expect to get equal agreement when ρ is small. Thus when $\rho = 0{\cdot}1$, (xxxviii) gives $\check{v} = 1{\cdot}8202$, but from the exact equations as before,

$$\bar{v} = 2{\cdot}121{,}212, \quad \sigma_v = 4{\cdot}651{,}951,$$
$$\beta_1 = {\cdot}016{,}750, \quad \beta_2 = 3{\cdot}296{,}918,$$

whence from (xxxix) $\check{v} = 1{\cdot}8645,$

or, the two values differ by somewhat over 2 °/₀ in the position of the mode.

It is worth while considering whether this arises from not having gone far enough with the solution of (xxxvii) by approximation, or from our determination of the mode being based only on the first term of Professor Watson's series. Now the actual solution of (xxxvii) is $\breve{\theta} = 10^\circ\, 24'{\cdot}719$ which gives $\breve{v} = 1{\cdot}8375$, which differs from $\breve{v} = 1{\cdot}8645$ by a little less than $1{\cdot}5\,^\circ/_\circ$. This is not very serious for most statistical purposes, but it indicates that (xxxvii) and (xxxix) for low values of ρ and small samples cannot be brought closer into accord without considering higher order terms in one or both of them.

If we take into account the A_1 term in Professor Watson's formula and develop the equation for the mode as far as terms in $1/m^2$, we have

$$\tan \tfrac{1}{2}\breve{\theta} = \rho - \frac{\sin 2\breve{\theta}}{4m}\left(1 + \frac{\cos \breve{\theta}}{4m}(5\cos^2\breve{\theta} - 1)\right) \quad \text{......(xxxvii)}^{bis}$$

instead of (xxxvii). Now it is clear if ρ be small, e.g. $= 0{\cdot}1$, $\breve{\theta}$ will be small and accordingly $\cos\breve{\theta}$ approach unity, and the corrective term accordingly will be positive, approaching $\frac{1}{16}$. Thus $\breve{\theta}$ will be slightly reduced instead of increased by this first correction. It is not till $\breve{\theta} =$ about $63^\circ\, 25'$ that the term in $1/m^2$ changes sign, or roughly when ρ is about 0·72. The agreement of (xxxvii) and (xxxix) in our first illustration may very likely be associated with the relative smallness of the term in $1/m^2$ when $\rho = 0{\cdot}6$ notwithstanding the smallness of m. Unless ρ be fairly large and m of the order of 50 or more, it does not seem probable that a satisfactory formula for the mode or the most probable value of p_{11} can be found from (xxxv).

The true mode $\breve{v}$ would be determined by

$$\rho + m/\breve{v} + K_m'(\breve{v})/K_m(\breve{v}) = 0 \quad \text{.......................(xl).}$$

In order to obtain a reasonable value for $\breve{v}$ by tabling, we return to Equation (xxiv) and write it in the form

$$y = N(1-\rho^2)^{\frac{1}{2}(n-1)}\, e^{\rho v}\, T_{\frac{1}{2}n-1}(v) \quad \text{....................(xli).}$$

Here the function $T_{\frac{1}{2}n-1}(v) = T_m(v)$ does not contain ρ, and accordingly, if tabled, will be the same for samples of the same size n whatever be the value of ρ. When $\rho = 0$, then $y = NT_{\frac{1}{2}n-1}(v)$ will give the frequency curve of p_{11} for a sample of size n, by simply multiplying by the number N of samples. Our Table in the Appendix gives the values of $T_m(v)$ and $\log T_m(v)$ for $v = 0$ to $v = 120$ and for $m = 0$ to 11·5, the latter by intervals of 0·5, or from $n = 2$ to $n = 25$ in the case of sampling*. The interval of v in order to make the table publishable had to be varied. Thus from $m = 0$ to 5·5 the table proceeds by intervals of 0·1 as the argument v passes from 0 to 4·0; from $m = 6$ to 11·5, the argument increases by intervals of 0·5; for all values of m, the argument alters by 0·5 from 4·0 to 16·0, by 2·0 from 16·0 to 40·0 and by 5·0 from 40·0 to 120·0.

* It is easy to deduce that

$$T_{\frac{1}{2}}(v) = \tfrac{1}{2}e^{-v}, \quad T_{\frac{3}{2}} = \tfrac{1}{4}e^{-v}(1+v), \quad T_{\frac{5}{2}}(v) = \tfrac{1}{16}e^{-v}(v^2+3v+3), \text{ etc.}$$

The successive $T_{n+\frac{1}{2}}$ may be found from the formula

$$T_{n+\frac{3}{2}}(v) = \frac{v^2}{4n(n+1)}T_{n-\frac{1}{2}}(v) + \frac{2n+1}{2(n+1)}T_{n+\frac{1}{2}}.$$

Clearly $$T_m(v) = \frac{1}{\sqrt{\pi}} \frac{1}{2^m} \frac{1}{\Gamma(m+\frac{1}{2})} v^m K_m(v) \quad \text{.................(xlii)}$$

and since the value of $v^m K_m(v)$ when $v=0$ is $2^{m-1}\Gamma(m)$, it follows that

$$T_m(0) = \frac{1}{2\sqrt{\pi}} \frac{\Gamma(m)}{\Gamma(m+\frac{1}{2})} \quad \text{........................(xliii).}$$

The reduction formula, from which the Appendix Table has been computed (the highest function $T_m(v)$ being independently computed in order to act as a check), is as follows:

$$T_{m+1}(v) = \frac{v^2}{4m^2-1} T_{m-1}(v) + \frac{2m}{2m+1} T_m(v) \quad \text{..............(xliv).}$$

Making use of the formula*

$$K_{m-1}(v) + K_{m+1}(v) = -2K_m'(v),$$

the equation (xl) becomes

$$\rho + \frac{m}{\check{v}} = \frac{1}{2} \frac{K_{m-1}(\check{v}) + K_{m+1}(\check{v})}{K_m(\check{v})},$$

but $$\frac{T_{m+1}(v)}{T_m(v)} = \frac{v}{2m+1} \frac{K_{m+1}(v)}{K_m(v)}.$$

Hence $$\rho + \frac{m}{v} = \frac{1}{2} \frac{2m+1}{\check{v}} \frac{T_{m+1}(\check{v})}{T_m(\check{v})} + \frac{1}{2} \frac{\check{v}}{2m-1} \frac{T_{m-1}(\check{v})}{T_m(\check{v})},$$

or, using (xliv), $$\rho = \frac{\check{v}}{2m-1} \frac{T_{m-1}(\check{v})}{T_m(\check{v})} \quad \text{..........................(xlv).}$$

Accordingly if a table be formed of the expression on the right of (xlv) for each value of v and m, it will give the value of ρ for which, with the corresponding value of m, the value of v gives the mode. Consequently by backward interpolation from this table we can find for a given value of m and a given value of ρ the modal value of the ordinate or $\check{v}$. We have provided the needful entries in the third column of each section of the table of $T_m(v)$. Thus the calculation of the true modal value now becomes fairly easy.

We reach a specially interesting case when $\rho = 0$, or we are sampling from uncorrelated material. In this case the distribution curve for p_{11} is symmetrical about $\bar{v} = 0$, while we have from (xxviii), (xxxi) and (xxxiv)^{bis},

$$\sigma_v^2 = n-1, \quad {}_{p_{11}}\beta_1 = 0, \quad {}_{p_{11}}\beta_2 = 3 + \frac{6}{n-1}.$$

The appropriate curve is

$$y = y_0 \frac{1}{\left(1+\frac{v^2}{a^2}\right)^q},$$

where
$$q = \tfrac{1}{2}(5\beta_2 - 9)/(\beta_2 - 3) = \tfrac{1}{2}(n+4),$$
$$a^2 = \sigma^2 2\beta_2/(\beta_2 - 3) = n^2 - 1,$$
$$y_0 = \frac{N}{\sqrt{2\pi}\sigma} \frac{\Gamma(p)}{\sqrt{(p-\frac{3}{2})}\,\Gamma(p-\frac{1}{2})} = \frac{N}{\sqrt{(n^2-1)\pi}} \frac{\Gamma(\frac{1}{2}(n+4))}{\Gamma(\frac{1}{2}(n+3))}.$$

* Watson, *Theory of Bessel Functions*, p. 79.

Thus the equation to the required curve having the same first four moments as the frequency curve for p_{11} is

$$y = \frac{N}{\sqrt{\pi(n^2-1)}} \frac{\Gamma(\frac{1}{2}(n+4))}{\Gamma(\frac{1}{2}(n+3))} \frac{1}{\left(1+\frac{v^2}{n^2-1}\right)^{\frac{1}{2}(n+4)}} \quad \text{.........(xlvi).}$$

The true curve of frequency is

$$y = \frac{N}{\sqrt{\pi}\, 2^{\frac{1}{2}n-1}\,\Gamma(\frac{1}{2}(n-1))} v^{\frac{1}{2}n-1} K_{\frac{1}{2}n-1}(v) \quad \text{...........(xlvii).}$$

We shall consider later for what value of n these two curves for statistical purposes become practically identical.

Another point which it is of interest to consider is the relation of the distributions of p_{11} to those of r. In *Biometrika*, Vol. XI. pp. 379—403, are given the frequency curves for various distributions of r in the case of samples of various sizes. The β_1 and β_2 of such distributions have been tabled.

Now $p_{11} = \Sigma_1 \Sigma_2 r$, hence

$$v = \frac{n}{1-\rho^2} \frac{\Sigma_1 \Sigma_2}{\sigma_1 \sigma_2} r,$$

or, v would be proportional to r, if we neglected the variation of Σ_1, Σ_2 or treated $\Sigma_1 \Sigma_2/\sigma_1\sigma_2$ as a constant. This is what Dr Wishart has done* when he replaces the true tetrad

$$r_{su} r_{tv} - r_{tu} r_{sv}$$

by

$$\frac{\Sigma_s \Sigma_t \Sigma_u \Sigma_v}{\sigma_s \sigma_t \sigma_u \sigma_v} (r_{su} r_{tv} - r_{tu} r_{sv}),$$

and supposes the variation of the latter to be sensibly the same as the variation of the former, i.e. he is neglecting the variation of the standard deviations of the samples.

Now the value of v only differs from that of $p_{11}/\sigma_1\sigma_2$ by a constant and we can at once compare the β_1 and β_2 of v or $p_{11}/\sigma_1\sigma_2$ with those of r by means of Equations (xxxi) and (xxxiv)^{bis}. Table I on p. 186 gives the comparative values.

The values of β_2 for $\rho = 0$ indicate clearly the difference of the r and $\frac{\Sigma_1 \Sigma_2}{\sigma_1 \sigma_2} r$ distributions. The former has a platykurtic and the latter a leptokurtic curve and the distributions approach the normal curve from opposite sides, the difference between them being very marked for small samples. When we reach $\rho = \cdot 6$, however, both curves have become leptokurtic—the transition for the r-distribution takes place before $\rho = \cdot 45$—and the β_2 for the r-curve rises with great rapidity and reaches remarkable values for high correlations in the sampled population. As the β_2 for the r-curve rises much above that for the v-curve so does its skewness. An examination of the column for $\rho = \cdot 9$ will show how little can be inferred from the distribution of $\frac{\Sigma_1 \Sigma_2}{\sigma_1 \sigma_2} r$ as to the distribution of r. It is not, however, only the shapes of the curves as judged from β_1, β_2 which diverge widely; the position and spread

* *British Journal of Psychology*, Vol. XIX. p. 183.

TABLE I.

β_1 and β_2 for r and v.

Size of Sample		$\rho=0\cdot0$		$\rho=\cdot3$		$\rho=\cdot6$		$\rho=\cdot9$	
		r	v	r	v	r	v	r	v
5	β_1	·0000	·0000	·3077	·6636	1·7207	1·6157	13·0290	1·8847
	β_2	2·0000	4·5000	2·4201	4·9954	4·4027	5·6678	21·7579	5·9835
10	β_1	·0000	·0000	·2317	·2949	1·2002	·7181	5·7475	·8376
	β_2	2·4545	3·6667	2·8292	3·8869	4·4598	4·1857	13·6667	4·3260
15	β_1	·0000	·0000	·1745	·1896	·8473	·4616	3·0956	·5385
	β_2	2·6250	3·4286	2·9265	3·5701	4·1375	3·7622	8·8548	3·8524
20	β_1	·0000	·0000	·1386	·1397	·6464	·3401	2·0603	·3968
	β_2	2·7143	3·3158	2·9623	3·4201	3·9055	3·5616	6·8681	3·6281
25	β_1	·0000	·0000	·1146	·1106	·5203	·2693	1·5334	·3141
	β_2	2·7692	3·2500	2·9788	3·3326	3·7453	3·4446	5·8584	3·4972
50	β_1	·0000	·0000	·0611	·0542	·2611	·1319	·5004	·1539
	β_2	2·8824	3·1224	2·9991	3·1629	3·3891	3·2178	3·8731	3·2435
100	β_1	·0000	·0000	·0315	·0268	·1303	·0653	·3115	·0761
	β_2	2·9400	3·0606	3·0021	3·0806	3·1974	3·1078	3·5790	3·1205

as determined by the means and standard deviations are very different when there is considerable correlation, say above 0·4 or 0·5, in the sampled population. The means and standard deviations given in Table II illustrate this. For those upon whom a graph has a greater impressional value than a table, Figs. 11, 12 and 13 are provided; they correspond to samples of 20, 10, and 5 from populations with correlations of 0·6, 0·0 and 0·9 respectively. These values were originally selected to determine a proper range of argument to give to our v in the Table of $T_m(v)$*.

* It may be suggested that the divergence of the two curves would be less if we took much larger samples. That this is not so the following values of the standard deviations indicate for $\rho=0\cdot5$:

	Size of Sample		
	50	100	400
Standard Deviation of r	·108,620	·075,897	·037,612
$\frac{p_{11}}{\sigma_1\sigma_2}$	·156,525	·111,243	·055,832

In each case the standard deviation of $\frac{p_{11}}{\sigma_1\sigma_2}$ is about 50 °/$_\circ$ increase on that of r.

TABLE II.

Means and Standard Deviations of r and $\frac{\Sigma_1\Sigma_2}{\sigma_1\sigma_2}r$.

Size of Sample	$\rho=0\cdot0$		$\rho=0\cdot3$		$\rho=0\cdot6$		$\rho=0\cdot9$	
	r	$\frac{\Sigma_1\Sigma_2}{\sigma_1\sigma_2}r$	r	$\frac{\Sigma_1\Sigma_2}{\sigma_1\sigma_2}r$	r	$\frac{\Sigma_1\Sigma_2}{\sigma_1\sigma_2}r$	r	$\frac{\Sigma_1\Sigma_2}{\sigma_1\sigma_2}r$
5 Mean	·0000	·0000	·2671	·2400	·5480	·4800	·8687	·7200
5 S.D.	·5000	·4000	·4740	·4176	·3858	·4665	·1748	·5381
10 Mean	·0000	·0000	·2850	·2700	·5776	·5400	·8887	·8100
10 S.D.	·3000	·3000	·3103	·3132	·2355	·3499	·0832	·4036
15 Mean	·0000	·0000	·2903	·2833	·5858	·5667	·8932	·8500
15 S.D.	·2673	·2494	·2470	·2604	·1828	·2909	·0602	·3356
20 Mean	·0000	·0000	·2928	·2850	5896	·5700	·8951	·8550
20 S.D.	·2294	·2179	·2113	·2275	·1543	·2542	·0493	·2932
25 Mean	·0000	·0000	·2943	·2880	·5918	·5760	·8962	·8640
25 S.D.	·2041	·1960	·1875	·2046	·1359	·2285	·0427	·2636
50 Mean	·0000	·0000	·2972	·2940	·5960	·5880	·8982	·8820
50 S.D.	·1429	·1400	·1306	·1462	·0933	·1633	·0284	·1884
100 Mean	·0000	·0000	·2986	·2970	·5980	·5940	·8991	·8910
100 S.D.	·1005	·0995	·0917	·1039	·0650	·1160	·0195	·1339

A word may be said here as to the method of using the Table. What we usually need is the distribution curve of $p_{11}/\sigma_1\sigma_2$ or $(1-\rho^2)\,v/n$. What is tabled is, to each value of the argument v, the value of the function

$$f(v)=\frac{1}{\sqrt{\pi}}\frac{1}{2^m}\frac{1}{\Gamma(m+\frac{1}{2})}v^m K_m(v)$$

$$=T_m(v) \text{ of our equation (xlii).}$$

Here m is to be obtained from the size of sample n by the relation $m=\frac{1}{2}n-1$.

But the actual frequency curve of v, if there be a correlation ρ and N samples, is

$$y_v=N(1-\rho^2)^{m+\frac{1}{2}}e^{\rho v}T_m(v).$$

Hence to obtain the ordinates of the v-curve we must multiply $T_m(v)$ taken from the Appendix Table by $N(1-\rho^2)^{m+\frac{1}{2}}e^{\rho v}$. This can be done by using the values of $\log T_m(v)$, and adding the logs of $N(1-\rho^2)^{m+\frac{1}{2}}$ and $\rho\log e\times v$; or often more simply

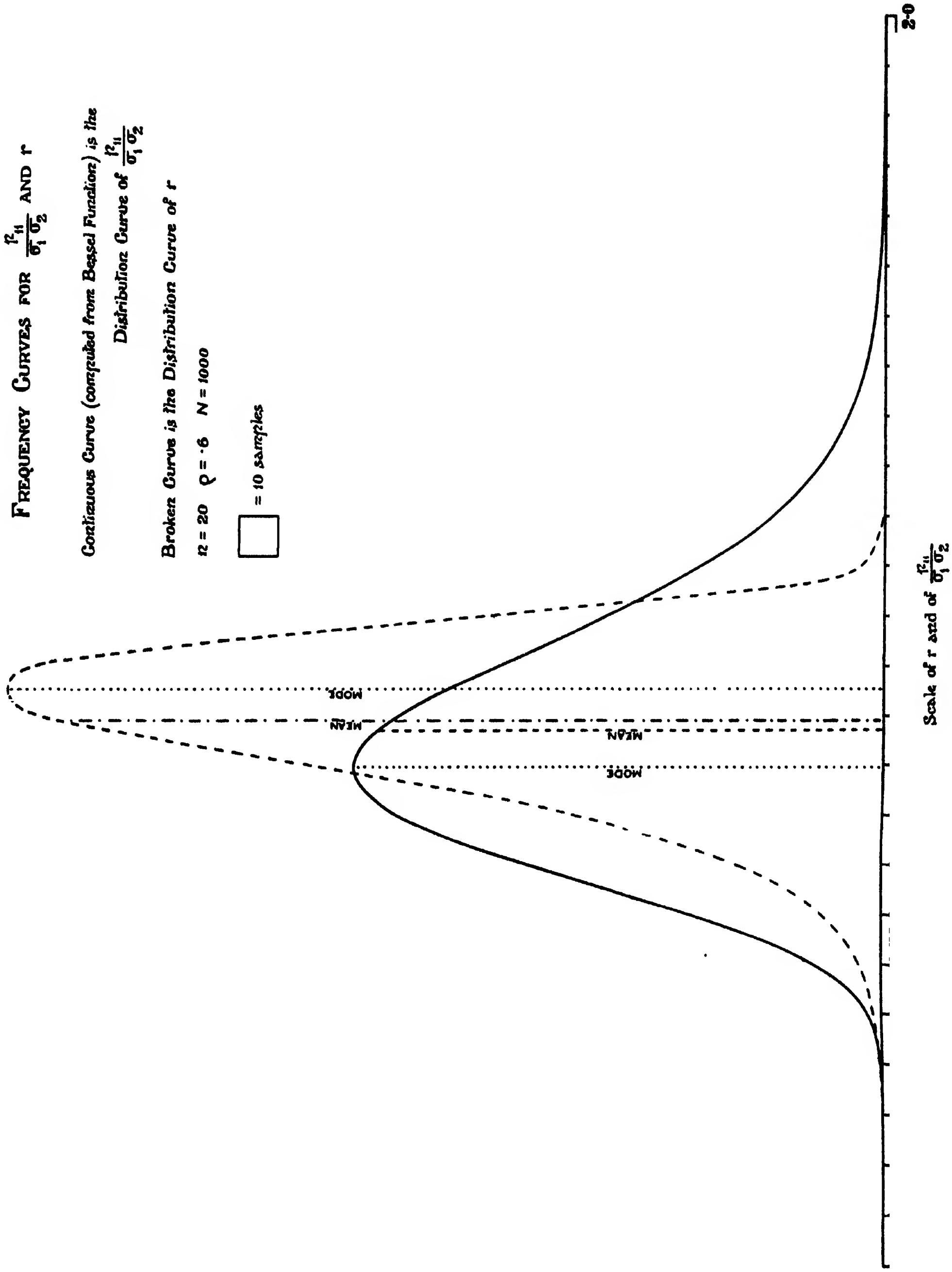
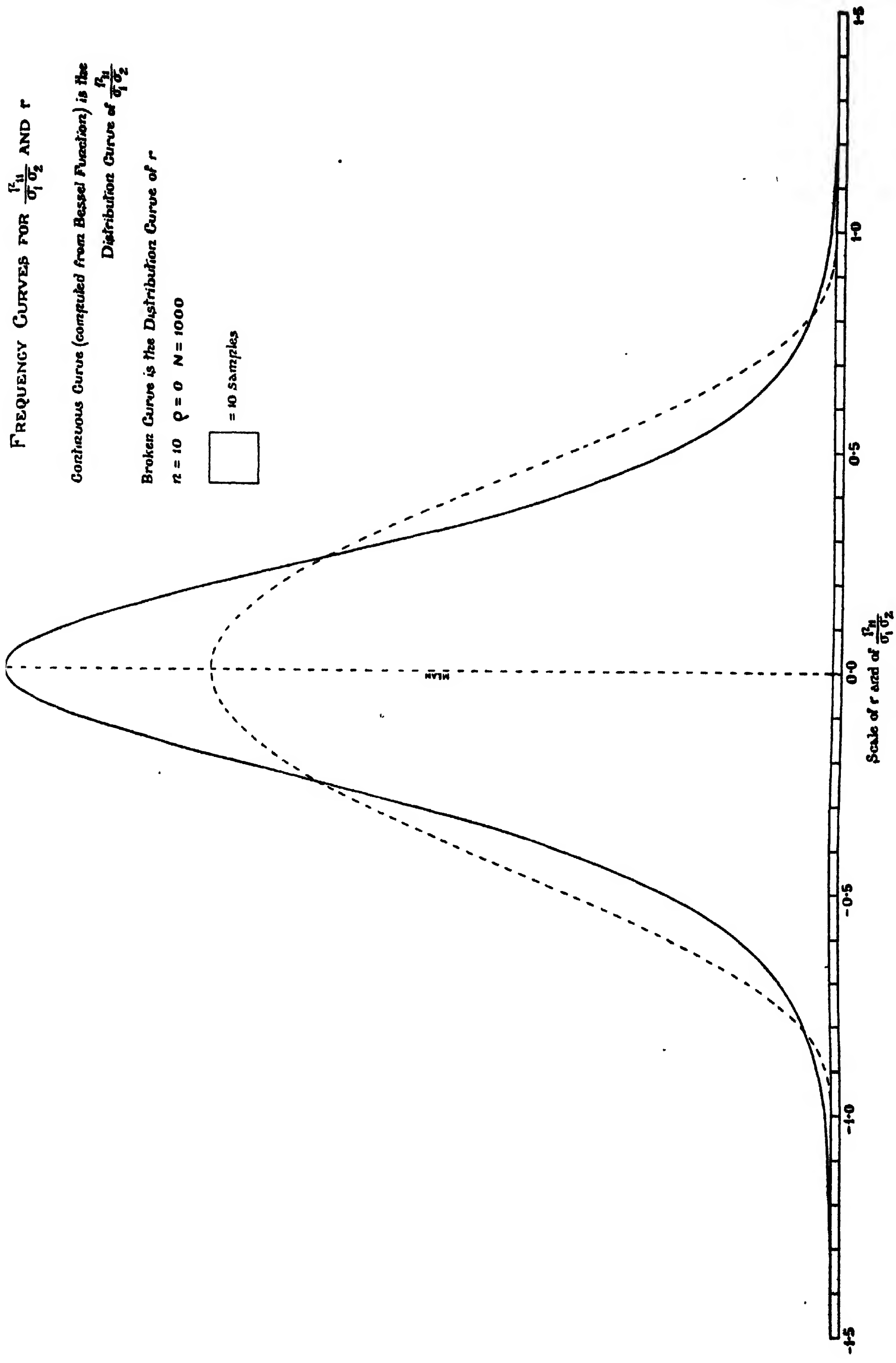
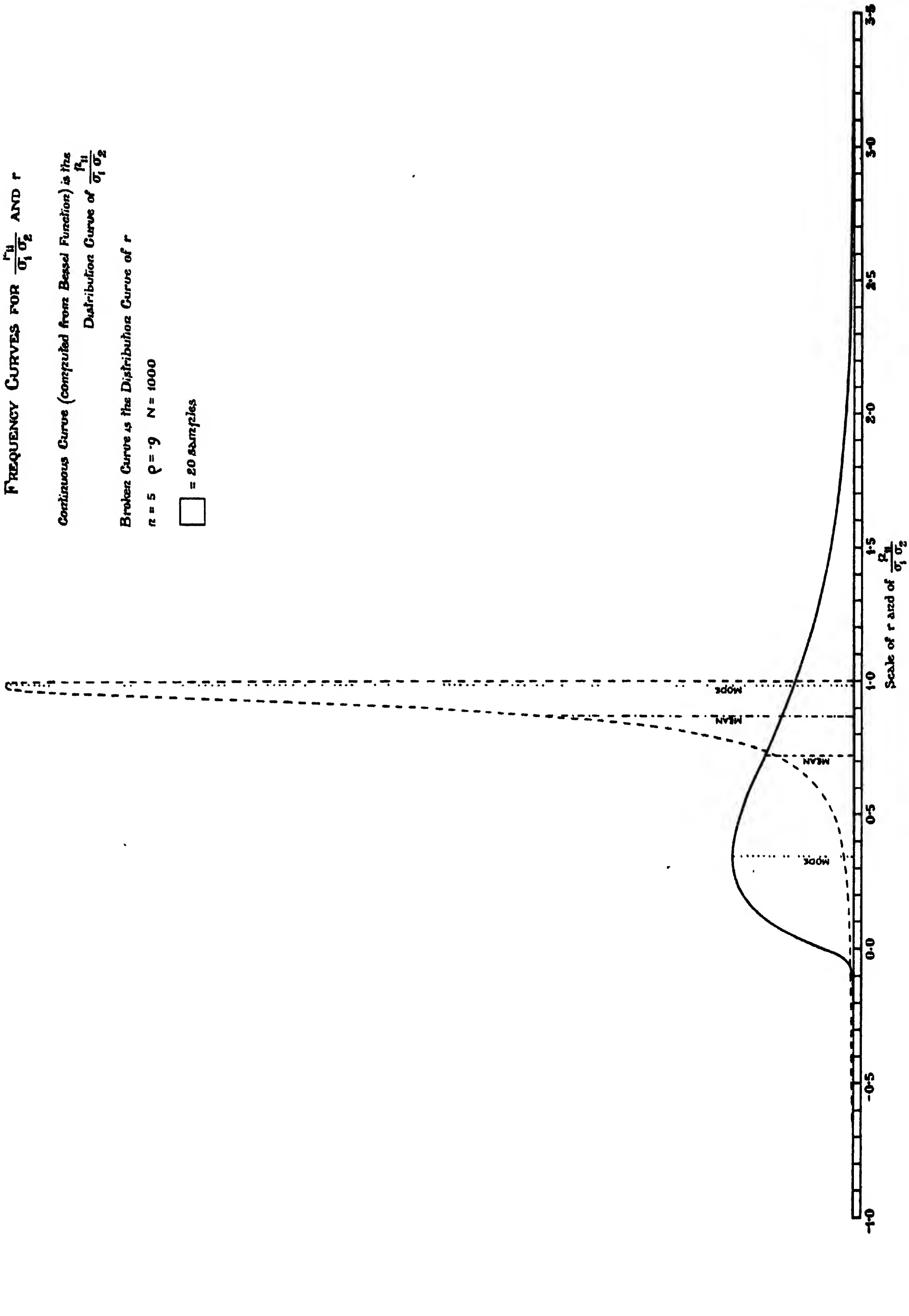

Frequency Curves for $\frac{\mu_{11}}{\sigma_1 \sigma_2}$ and r
Continuous Curve (computed from Bessel Function) is the Distribution Curve of $\frac{\mu_{11}}{\sigma_1 \sigma_2}$
Broken Curve is the Distribution Curve of r
n = 20 ρ = ·6 N = 1000
= 10 samples
MODE
MEAN
MEAN
MODE
2·0
Scale of r and of $\frac{\mu_{11}}{\sigma_1 \sigma_2}$

Fig. 11.



FREQUENCY CURVES FOR $\frac{r_{11}}{\sigma_1\sigma_2}$ AND r
Continuous Curve (computed from Bessel Function) is the Distribution Curve of $\frac{r_{11}}{\sigma_1\sigma_2}$
Broken Curve is the Distribution Curve of r
$n = 10$ $\rho = 0$ $N = 1000$
= 10 samples
MEAN
-1·5
-1·0
-0·5
0·0
0·5
1·0
1·5
Scale of r and of $\frac{r_{11}}{\sigma_1\sigma_2}$

Fig. 12.


FREQUENCY CURVES FOR $\frac{p_{11}}{\sigma_1 \sigma_2}$ AND r
Continuous Curve (computed from Bessel Function) is the Distribution Curve of $\frac{p_{11}}{\sigma_1 \sigma_2}$
Broken Curve is the Distribution Curve of r
$n = 5$ $\rho = \cdot 9$ $N = 1000$
= 20 samples
MODE
MEAN
MEAN
MODE
−1·0
−0·5
0·0
0·5
1·0
1·5
2·0
2·5
3·0
3·5
Scale of r and of $\frac{p_{11}}{\sigma_1 \sigma_2}$

by using Newman and Glaisher's Tables of e^x and e^{-x} in the *Cambridge Philosophical Transactions**. Writing R for $p_{11}/(\sigma_1\sigma_2)$ we need the ordinates y_R of the frequency curve for R. Clearly R and v must have their elements of frequency the same, or

$$y_R dR = y_v dv = y_v \frac{n}{1-\rho^2} dR,$$

or

$$y_R = \frac{n}{1-\rho^2} y_v.$$

Thus

$$y_R = N \times n(1-\rho^2)^{m-\frac{1}{2}} e^{\rho v} T_m(v).$$

Accordingly, to get the frequency distribution of R, we have to plot to the values of $R = (1-\rho^2)v/n$, the values of y_R, or the $T_m(v)$ of our Table multiplied by $N \times n \times (1-\rho^2)^{m-\frac{1}{2}} e^{\rho v}$. Of course v must be given both negative and positive values in the factor $e^{\rho v}$, but the value of $T_m(v)$ is the same for v positive or negative. Thus by aid of the Table of $T_m(v)$, it is relatively easy to obtain, whatever be the value of ρ, the distribution curve of $R = p_{11}/\sigma_1\sigma_2$, for small samples, i.e. $n < 25$. When $n > 25$, the Pearson curve obtained from the $\bar{v}$, σ_v, ${}_v\beta_1$ and ${}_v\beta_2$ of Equations (xxvii), (xxviii), (xxxi) and (xxxiv)^{bis} will be amply sufficient to describe the distribution.

From our Tables I and II, and again from the examples we have given graphically (Figs. 11—13), it seems impossible to predict from the distribution of a product-moment what is likely to be the distribution of a correlation-coefficient. It is equally—or rather more—unlikely that we can learn anything about the distribution of a function of correlation-coefficients from the distribution of the corresponding product-moments. For example, it is highly improbable that the distribution of tetrads, $r_{su}r_{tv} - r_{tu}r_{sv}$, will have even approximately the same distribution curve as

$$\frac{p_{11}(su)\,p_{11}(tv) - p_{11}(tu)\,p_{11}(sv)}{\sigma_s\sigma_t\sigma_u\sigma_v} = \frac{\Sigma_s\Sigma_t\Sigma_u\Sigma_v}{\sigma_s\sigma_t\sigma_u\sigma_v}(r_{su}r_{tv} - r_{tu}r_{sv}).$$

Of course we may start *ab initio* with either of these expressions, but having done so, we cannot apply the standard deviation of the one to the consideration of goodness of fit of theory to observation in the case of the other. The extreme leptokurtosis of the p_{11} curves, while the r-curves are of limited range and of much less variation (see our Table II), suggests that for practical purposes it would be advantageous to use correlation-coefficient tetrads rather than the product-moment tetrads. There are other interesting differences between the r and $p_{11}/\sigma_1\sigma_2$ curves. The r-curves have all negative skewness, i.e. the mode is greater than the mean when a true mode exists, but in the case of the $p_{11}/\sigma_1\sigma_2$ curves the skewness is positive or the mode is less than the mean. For samples of two the r-curve consists of two concentrated frequency lumps ($r = \pm 1$), for samples of three of U-curves, and for samples of four J-like curves—all these without true modes. In the corresponding cases for p_{11} we have always continuous curves; in the case of samples of two we have a double J-curve; if ρ be not zero we have two unequal J-curves, set back to back along the asymptote; in the case of samples of three we have, if ρ be not zero, a double

* Vol. XIII. 1883, pp. 145—272.

exponential curve, consisting of two unequal exponential curves of the same maximum height at $p_{11} = 0$. Hence for samples of two and three no modal values are given in our Table for the different values of ρ. For samples of three the value of the ordinate at the vertical for $v = 0$ of the exponential curves is $\cdot 5$.

Dr Wishart*, as we have said, has obtained the standard deviation σ_P of the product-moment tetrad; if σ_F be the standard deviation of the correlation-coefficient tetrad, Dr Wishart really assumes that

$$\sigma_P/(\sigma_s\sigma_t\sigma_u\sigma_v)$$

is interchangeable with σ_F. But in doing this he is neglecting the variability of $\Sigma_s\Sigma_t\Sigma_u\Sigma_v$, and our present discussion shows that in a like case the variability of $p_{11}/\sigma_1\sigma_2$ is much greater than that of r. Dr Wishart has also overlooked the fact that if the sample values are inserted in the formula for σ_F, since these values differ from the sampled population values by terms of the order $1/\sqrt{N}$, it is idle to introduce the terms in $1/N$, whether they be those he has himself found or those suggested by Spearman and Holzinger, in order to correct the first approximation†. Clearly using $\sigma_P \div \sigma_s\sigma_t\sigma_u\sigma_v$ instead of σ_F will give a much larger variability to the tetrad. Dr Wishart‡ applies it to Holzinger's data and states that "the discrepancy of the observations from the two-factor theory...is seen to disappear" when his value of σ_P is used; and again: "The extraordinarily close agreement...may be regarded as fortuitous, but we can at least say that in one example we have obtained striking confirmation of the two-factor theory." The "discrepancy"§ is of course likely to disappear if σ_P is essentially greater than σ_F, and we fear that emphasis must be laid on the word "fortuitous," if an imperfect theory is applied to link up an hypothesis with observation. Dr Wishart has suggested a new manner of dealing with the two-factor problem, namely, by using product-moment tetrads instead of correlation-coefficient tetrads, on the ground that the standard deviation of the former tetrad is accurately known. At first sight this appears to have advantages, but when we remember that we have to use in the formulae the sampled population values, and therefore terms of the second order cannot be accurately determined at all, and when we note further the extreme variability of product-moments (divided of course by σ-products) as compared with correlation-coefficient variabilities, we see that the proposed method has its disadvantages. Further, we cannot admit that a formula thus obtained is in any way applicable to correlation-coefficient tetrads as Dr Wishart applies it. We must *de novo* compute the product-moment tetrads. Professor Spearman apparently holds Dr Wishart's

* *The British Journal of Psychology*, Vol. XIX. p. 183.

† See Pearson and Moul, "The Mathematics of Intelligence," *Biometrika*, Vol. XIX. p. 251.

‡ *Loc. cit.* p. 188.

§ We would draw attention to the fact that in the paper by Pearson and Moul it was *not* asserted that there *was* a discrepancy between theory and observation; the difference was shown to be 2·11 times its probable error, and the conclusion drawn that this did not indicate either "a very good or very bad accordance between theory and observation." What it certainly does not justify is the remark of Spearman and Holzinger that "in this case at any rate every one of the abilities can be resolved into two independent factors, the one being always specific and the other throughout common" (*The British Journal of Psychology*, Vol. XVI. p. 88).

formula when applied to correlation-coefficient tetrads to be wholly satisfactory, and to include "*all* the terms in $\frac{1}{n^2}$"; he fails to realise that the variability of F must be essentially less than that of P*.

Apart from the illustration given above of the dangers which may attend the application of the standard error of one quantity to another which bears some resemblance to it, the present paper appears to us to indicate that the distribution of product-moments, while in many cases less skew than that of correlation-coefficients, has in numerous instances such a wide range and low modal frequency that it cannot replace the direct consideration of the correlation-coefficient. At the same time the distribution of the product moment-coefficient introduces into statistics the study of functions based on the Bessel functions of imaginary argument. In a paper by Dr Fisher†, Bessel functions have been also introduced in the case of the distribution of the multiple correlation-coefficient. It is interesting to note how, as statistical theory advances, even on the basis of simple normal distributions, we call into requisition more and more functions familiar in physics.

As a general statement of the results of this paper we may say that the distribution of p_{11} has been ascertained theoretically when the sampling is from a normal population, and tables have been provided for tracing the curve of distribution of p_{11}, up to samples of 25. Further, it has been shown that good fits are obtained for samples greater than 25 by use of a Pearson curve with the appropriate moment-coefficients. This gives us confidence in holding that for samples in excess of 25 the distribution of p_{11}, with the use of the more general moments of that coefficient determined for any form of sampled population by Mr Pepper, will also be effectively described by a Pearson curve.

* See *The British Journal of Psychology*, Vol. XIX. p. 101.

† *R.S. Proc.* A, Vol. CXXI. p. 663.

TABLE OF THE PRODUCT MOMENT T_m FUNCTION.

COMPUTED BY ETHEL M. ELDERTON.

THE figures in round brackets in the T_m columns give the number of zeros between the decimal place and the first integer recorded. The logarithms of T_m have been obtained, not from the six-figure value of T_m in the T_m column, but from the fuller computed value before it was cut down.

The third column headed ρ gives the value of the argument v, for which, with that value of ρ, the argument v would be the mode $\tilde{v}$ of the distribution curve.

If $v = \dfrac{n}{1-\rho^2}\dfrac{p_{11}}{\sigma_1\sigma_2}$, then we have for the distribution curve of v:

$$y_v = N(1-\rho^2)^{m+\frac{1}{2}} e^{\rho v} T_m(v),$$

or

$$\log y_v = \log N + (m+\tfrac{1}{2})\log(1-\rho^2) + \rho v \log_e + \log T_m(v).$$

Here $T_m(v)$ and its logarithm do not change sign with v. Thus the ordinates of the curve of distribution of p_{11} in samples of size n, ($m = \frac{1}{2}n - 1$), are easily computed from the log T_m column.

When n is equal to or exceeds 25, the distribution curve is adequately given by a Pearson curve with the Mean, S.D., and the β_1 and β_2 determined by Equations (xxv), (xxviii), (xxxi) and (xxxiv).

v	n = 2, (m = 0) T_m	log T_m	ρ	n = 3, (m = ·5) T_m	log T_m	ρ
0	∞	∞	No true mode, the distribution curve has an infinite ordinate at v = 0, and consists of two unlike J-curves placed back to back	·500000	1̄·6989700	No true mode, the distribution curve has a finite ordinate, ½N (1 − ρ²), at v = 0, and consists of two unlike exponential curves placed back to back
·1	·772560	1̄·8879323		·452419	1̄·6555406	
·2	·557903	1̄·7465586		·409365	1̄·6121111	
·3	·436868	1̄·6403499		·370409	1̄·5686817	
·4	·354766	1̄·5499416		·335160	1̄·5252522	
·5	·294252	1̄·4687190		·303265	1̄·4818228	
·6	·247493	1̄·3935629		·274406	1̄·4383933	
·7	·210250	1̄·3227360		·248293	1̄·3949629	
·8	·179956	1̄·2551653		·224664	1̄·3515344	
·9	·154931	1̄·1901385		·203285	1̄·3081050	
1·0	·134016	1̄·1271574		·183940	1̄·2646755	
1·1	·116375	1̄·0658592		·166436	1̄·2212461	
1·2	·101384	1̄·0059708		·150597	1̄·1778166	
1·3	(1) 885690	2̄·9472817		·136266	1̄·1343872	
1·4	(1) 775578	2̄·8896256		·123298	1̄·0909577	
1·5	(1) 680564	2̄·8328691		·111565	1̄·0475283	
1·6	(1) 598279	2̄·7769034		·100948	1̄·0040988	
1·7	(1) 526791	2̄·7216384		(1) 913418	2̄·9606694	
1·8	(1) 464514	2̄·6669989		(1) 826494	2̄·9172399	
1·9	(1) 410129	2̄·6129210		(1) 747843	2̄·8738105	
2·0	(1) 362535	2̄·5593504		(1) 676676	2̄·8303810	
2·1	(1) 320805	2̄·5062406		(1) 612282	2̄·7869516	
2·2	(1) 284152	2̄·4535508		(1) 554016	2̄·7435221	
2·3	(1) 251910	2̄·4012458		(1) 501294	2̄·7000927	
2·4	(1) 223509	2̄·3492945		(1) 453590	2̄·6566633	
2·5	(1) 198458	2̄·2976695		(1) 410425	2̄·6132338	
2·6	(1) 176338	2̄·2463466		(1) 371368	2̄·5698044	
2·7	(1) 156785	2̄·1953040		(1) 336028	2̄·5263749	
2·8	(1) 139483	2̄·1445223		(1) 304050	2̄·4829455	
2·9	(1) 124161	2̄·0939842		(1) 275116	2̄·4395160	
3·0	(1) 110579	2̄·0436737		(1) 248935	2̄·3960866	
3·1	(2) 985319	3̄·9935769		(1) 225246	2̄·3526571	
3·2	(2) 878376	3̄·9436805		(1) 203811	2̄·3092276	
3·3	(2) 783381	3̄·8939729		(1) 184416	2̄·2657982	
3·4	(2) 698945	3̄·8444433		(1) 166866	2̄·2223688	
3·5	(2) 623852	3̄·7950817		(1) 150987	2̄·1789393	
3·6	(2) 557031	3̄·7458793		(1) 136619	2̄·1355099	
3·7	(2) 497539	3̄·6968275		(1) 123618	2̄·0920804	
3·8	(2) 444548	3̄·6479187		(1) 111854	2̄·0486510	
3·9	(2) 397325	3̄·5991455		(1) 101210	2̄·0052234	
4·0	(2) 355224	3̄·5505018		(2) 915782	3̄·9617921	
4·5	(2) 203714	3̄·3090204		(2) 555450	3̄·7446449	
5·0	(2) 117491	3̄·0700058		(2) 336897	3̄·5274976	
5·5	(3) 680772	4̄·8330017		(2) 204339	3̄·3103504	
6·0	(3) 395976	4̄·5976685		(2) 123938	3̄·0932031	
6·5	(3) 231071	4̄·3637459		(3) 751720	4̄·8760561	
7·0	(3) 135217	4̄·1310303		(3) 455941	4̄·6589086	
7·5	(4) 793157	5̄·8993592		(3) 276542	4̄·4417614	
8·0	(4) 466231	5̄·6686009		(3) 167731	4̄·2246141	
8·5	(4) 274566	5̄·4386473		(3) 101734	4̄·0074670	

v	n = 4, (m = 1·0) T_m	log T_m	ρ	n = 5, (m = 1·5) T_m	log T_m	ρ
0	·318310	1̄·5028502	·00000	·250000	1̄·3979400	·00000
·1	·313658	1̄·4964558	·24631	·248830	1̄·3959032	·09091
·2	·304048	1̄·4829422	·36698	·245619	1̄·3902623	·16667
·3	·291826	1̄·4651236	·44910	·240766	1̄·3815950	·23077
·4	·278121	1̄·4442332	·51023	·234612	1̄·3703502	·28571
·5	·263631	1̄·4209961	·55808	·227449	1̄·3568840	·33333
·6	·248823	1̄·3958908	·59679	·219525	1̄·3414833	·37500
·7	·234021	1̄·3692547	·62890	·211049	1̄·3243828	·41176
·8	·219451	1̄·3413374	·65602	·202198	1̄·3057769	·44444
·9	·205272	1̄·3123292	·67928	·193121	1̄·2858286	·47368
1·0	·191593	1̄·2823796	·69948	·183940	1̄·2646755	·50000
1·1	·178488	1̄·2516086	·71720	·174757	1̄·2424354	·52381
1·2	·166002	1̄·2201135	·73289	·165657	1̄·2192093	·54545
1·3	·154161	1̄·1879751	·74688	·156706	1̄·1950849	·56522
1·4	·142975	1̄·1552611	·75944	·147958	1̄·1701389	·58333
1·5	·132443	1̄·1220288	·77078	·139456	1̄·1444383	·60000
1·6	·122554	1̄·0883270	·78108	·131233	1̄·1180422	·61538
1·7	·113292	1̄·0541979	·79048	·123311	1̄·0910032	·62963
1·8	·104635	1̄·0196783	·79909	·115709	1̄·0633680	·64286
1·9	(1) 965607	2̄·9848003	·80700	·108437	1̄·0351785	·65517
2·0	(1) 890414	2̄·9495919	·81431	·101501	1̄·0064723	·66667
2·1	(1) 820499	2̄·9140783	·82107	(1) 949037	2̄·9772833	·67742
2·2	(1) 755582	2̄·8782814	·82736	(1) 886425	2̄·9476421	·68750
2·3	(1) 695379	2̄·8422213	·83321	(1) 827135	2̄·9175766	·69697
2·4	(1) 639611	2̄·8059157	·83867	(1) 771103	2̄·8871122	·70588
2·5	(1) 588004	2̄·7693806	·84378	(1) 718244	2̄·8562718	·71429
2·6	(1) 540294	2̄·7326305	·84857	(1) 668462	2̄·8250768	·72222
2·7	(1) 496225	2̄·6956786	·85308	(1) 621651	2̄·7935466	·72973
2·8	(1) 455551	2̄·6585369	·85732	(1) 577696	2̄·7616991	·73684
2·9	(1) 418038	2̄·6212161	·86132	(1) 536476	2̄·7295506	·74359
3·0	(1) 383466	2̄·5837265	·86510	(1) 497871	2̄·6971172	·75000
3·1	(1) 351623	2̄·5460771	·86868	(1) 461754	2̄·6644110	·75610
3·2	(1) 322312	2̄·5082763	·87208	(1) 428003	2̄·6314469	·76190
3·3	(1) 295347	2̄·4703320	·87530	(1) 396494	2̄·5982367	·76744
3·4	(1) 270552	2̄·4322514	·87836	(1) 367106	2̄·5647915	·77273
3·5	(1) 247766	2̄·3940411	·88127	(1) 339721	2̄·5311218	·77778
3·6	(1) 226834	2̄·3557073	·88405	(1) 314223	2̄·4972377	·78261
3·7	(1) 207614	2̄·3172558	·88669	(1) 290501	2̄·4631483	·78723
3·8	(1) 189973	2̄·2786919	·88922	(1) 268449	2̄·4288622	·79167
3·9	(1) 173788	2̄·2400205	·89164	(1) 247963	2̄·3943876	·79592
4·0	(1) 158945	2̄·2012464	·89395	(1) 228945	2̄·3597321	·80000
4·5	(1) 101386	2̄·0059791	·90418	(1) 152749	2̄·1839775	·81818
5·0	(2) 643720	3̄·8714096	·91260	(1) 101069	2̄·0046188	·83333
5·5	(2) 407138	3̄·6097421	·91965	(2) 664100	3̄·8222337	·84615
6·0	(2) 256670	3̄·4093747	·92565	(2) 433782	3̄·6372712	·85714
6·5	(2) 161361	3̄·2077992	·93081	(2) 281895	3̄·4500871	·86667
7·0	(2) 101200	3̄·0051786	·93530	(2) 182376	3̄·2609686	·87500
7·5	(3) 633351	4̄·8016443	·93924	(2) 117530	3̄·0701503	·88235
8·0	(3) 395644	4̄·5973051	·94273	(3) 754791	4̄·8778267	·88889
8·5	(3) 246746	4̄·3922508	·94584	(3) 483237	4̄·6841605	·89474

n = 6, (m = 2·0) T_m	log T_m	ρ	n = 7, (m = 2·5) T_m	log T_m	ρ	v
·212207	1̄·3267589	·00000	·187500	1̄·2730013	·00000	0
·211680	1̄·3256804	·04939	·187188	1̄·2722786	·03323	·1
·210137	1̄·3225031	·09646	·186261	1̄·2701226	·06593	·2
·207657	1̄·3173456	·14053	·184742	1̄·2665646	·09774	·3
·204335	1̄·3103419	·18148	·182662	1̄·2616487	·12844	·4
·200275	1̄·3016264	·21939	·180064	1̄·2554264	·15789	·5
·195581	1̄·2913272	·25444	·176992	1̄·2479530	·18605	·6
·190355	1̄·2795638	·28686	·173494	1̄·2392857	·21288	·7
·184691	1̄·2664460	·31686	·169622	1̄·2294814	·23841	·8
·178679	1̄·2520740	·34465	·165423	1̄·2185960	·26267	·9
·172401	1̄·2365392	·37044	·160947	1̄·2066836	·28571	1·0
·165930	1̄·2199243	·39442	·156241	1̄·1937960	·30759	1·1
·159333	1̄·2023044	·41674	·151350	1̄·1799827	·32836	1·2
·152668	1̄·1837480	·43757	·146316	1̄·1652903	·34808	1·3
·145988	1̄·1643171	·45704	·141177	1̄·1497632	·36681	1·4
·139338	1̄·1440683	·47526	·135970	1̄·1334431	·38462	1·5
·132756	1̄·1230531	·49235	·130728	1̄·1163686	·40154	1·6
·126275	1̄·1013184	·50840	·125481	1̄·0985771	·41765	1·7
·119924	1̄·0789076	·52351	·120255	1̄·0801028	·43299	1·8
·113726	1̄·0558599	·53774	·115074	1̄·0609786	·44760	1·9
·107699	1̄·0322116	·55117	·109960	1̄·0412344	·46154	2·0
·101858	1̄·0079962	·56387	·104930	1̄·0208991	·47484	2·1
(1) 962153	2̄·9832442	·57589	·100000	1̄·0000000	·48753	2·2
(1) 907787	2̄·9579841	·58728	(1) 951832	2̄·9785605	·49967	2·3
(1) 855544	2̄·9322423	·59809	(1) 904912	2̄·9566062	·51128	2·4
(1) 805458	2̄·9060429	·60835	(1) 859327	2̄·9341586	·52239	2·5
(1) 757545	2̄·8794086	·61812	(1) 815153	2̄·9112389	·53303	2·6
(1) 711804	2̄·8523603	·62742	(1) 772443	2̄·8878666	·54323	2·7
(1) 668217	2̄·8249175	·63629	(1) 731241	2̄·8640605	·55301	2·8
(1) 626756	2̄·7970986	·64475	(1) 691573	2̄·8398381	·56241	2·9
(1) 587382	2̄·7689203	·65284	(1) 653455	2̄·8152159	·57143	3·0
(1) 550046	2̄·7403988	·66057	(1) 616893	2̄·7902095	·58010	3·1
(1) 514694	2̄·7115488	·66797	(1) 581880	2̄·7648338	·58844	3·2
(1) 481265	2̄·6823843	·67506	(1) 548407	2̄·7391027	·59647	3·3
(1) 449695	2̄·6529183	·68185	(1) 516451	2̄·7130294	·60420	3·4
(1) 419917	2̄·6231632	·68837	(1) 485989	2̄·6866266	·61165	3·5
(1) 391860	2̄·5931309	·69464	(1) 456989	2̄·6599060	·61883	3·6
(1) 365453	2̄·5628314	·70066	(1) 429417	2̄·6328790	·62576	3·7
(1) 340624	2̄·5322757	·70645	(1) 403233	2̄·6055562	·63245	3·8
(1) 317302	2̄·5014734	·71202	(1) 378397	2̄·5779480	·63892	3·9
(1) 295416	2̄·4704337	·71738	(1) 354866	2̄·5500638	·64516	4·0
(1) 205098	2̄·3119606	·74150	(1) 255160	2̄·4068122	·67347	4·5
(1) 140824	2̄·1486770	·76185	(1) 181082	2̄·2578761	·69767	5·0
(2) 957871	3̄·9813069	·77925	(1) 127073	2̄·1040534	·71859	5·5
(2) 646284	3̄·8104234	·79429	(2) 883055	3̄·9459880	·73684	6·0
(2) 433000	3̄·6364874	·80743	(2) 608423	3̄·7842057	·75290	6·5
(2) 288320	3̄·4598752	·81899	(2) 416046	3̄·6191415	·76712	7·0
(2) 190940	3̄·2808977	·82925	(2) 282592	3̄·4511592	·77982	7·5
(2) 125839	3̄·0998147	·83842	(2) 190794	3̄·2805655	·79121	8·0
(3) 825745	4̄·9168459	·84665	(2) 128121	3̄·1076220	·80149	8·5

v	$n = 2, (m = 0)$ T_m	$\log T_m$	ρ	$n = 3, (m = \cdot 5)$ T_m	$\log T_m$	ρ
9·0	(4) 161960	$\overline{5}$·2094084	No true mode, the distribution curve has an infinite ordinate at v=o, and consists of two unlike J-curves placed back to back	(4) 617049	$\overline{5}$·7903197	No true mode, the distribution curve has a finite ordinate at v = o, and consists of two unlike exponential curves placed back to back
9·5	(5) 956772	$\overline{6}$·9808085		(4) 374259	$\overline{5}$·5731724	
10·0	(5) 565957	$\overline{6}$·7527834		(4) 227000	$\overline{5}$·3560252	
10·5	(5) 335180	$\overline{6}$·5252781		(4) 137682	$\overline{5}$·1388780	
11·0	(5) 198722	$\overline{6}$·2982449		(5) 835085	$\overline{6}$·9217307	
11·5	(5) 117935	$\overline{6}$·0716428		(5) 506505	$\overline{6}$·7045835	
12·0	(6) 700545	$\overline{7}$·8454357		(5) 307211	$\overline{6}$·4874362	
12·5	(6) 416478	$\overline{7}$·6195919		(5) 186333	$\overline{6}$·2702890	
13·0	(6) 247790	$\overline{7}$·3940833		(5) 113016	$\overline{6}$·0531417	
13·5	(6) 147532	$\overline{7}$·1688850		(6) 685480	$\overline{7}$·8359945	
14·0	(7) 878972	$\overline{8}$·9439750		(6) 415764	$\overline{7}$·6188472	
14·5	(7) 524002	$\overline{8}$·7193328		(6) 252174	$\overline{7}$·4017000	
15·0	(7) 312566	$\overline{8}$·4949412		(6) 152951	$\overline{7}$·1845528	
15·5	(7) 186545	$\overline{8}$·2707834		(7) 927696	$\overline{8}$·9674055	
16·0	(7) 111390	$\overline{8}$·0468456		(7) 562676	$\overline{8}$·7502583	
18·0	(8) 142246	$\overline{9}$·1530407		(8) 761499	$\overline{9}$·8816693	
20·0	(9) 182751	$\overline{10}$·2618589		(8) 103058	$\overline{9}$·0130804	
22·0	(10) 235944	$\overline{11}$·3728088		(9) 139473	$\overline{10}$·1444914	
24·0	(11) 305860	$\overline{12}$·4855221		(10) 188757	$\overline{11}$·2759025	
26·0	(12) 397850	$\overline{13}$·5997191		(11) 255454	$\overline{12}$·4073135	
28·0	(13) 519017	$\overline{14}$·7151814		(12) 345720	$\overline{13}$·5387245	
30·0	(14) 678790	$\overline{15}$·8317356		(13) 467881	$\overline{14}$·6701355	
32·0	(15) 889696	$\overline{16}$·9492418		(14) 633208	$\overline{15}$·8015466	
34·0	(15) 116838	$\overline{16}$·0675850		(15) 856954	$\overline{16}$·9329576	
36·0	(16) 153699	$\overline{17}$·1866710		(15) 115976	$\overline{16}$·0643687	
38·0	(17) 202497	$\overline{18}$·3064182		(16) 156957	$\overline{17}$·1957797	
40·0	(18) 267153	$\overline{19}$·4267606		(17) 212418	$\overline{18}$·3271907	
45·0	(20) 169769	$\overline{21}$·2298592		(19) 143126	$\overline{20}$·1557182	
50·0	(22) 108549	$\overline{23}$·0356261		(22) 964375	$\overline{23}$·9842459	
55·0	(25) 697516	$\overline{26}$·8435542		(24) 649791	$\overline{25}$·8127735	
60·0	(27) 450058	$\overline{28}$·6532683		(26) 437826	$\overline{27}$·6413011	
65·0	(29) 291396	$\overline{30}$·4644833		(28) 295005	$\overline{29}$·4698287	
70·0	(31) 189224	$\overline{32}$·2769773		(30) 198772	$\overline{31}$·2983563	
75·0	(33) 123190	$\overline{34}$·0905743		(32) 133932	$\overline{33}$·1268839	
80·0	(36) 803771	$\overline{37}$·9051322		(35) 902426	$\overline{36}$·9554115	
85·0	(38) 525454	$\overline{39}$·7205347		(37) 608050	$\overline{38}$·7839391	
90·0	(40) 344101	$\overline{41}$·5366856		(39) 409701	$\overline{40}$·6124666	
95·0	(42) 225686	$\overline{43}$·3535041		(41) 276054	$\overline{42}$·4409942	
100·0	(44) 148225	$\overline{45}$·1709217		(43) 186004	$\overline{44}$·2695218	
105·0	(47) 974721	$\overline{48}$·9888803		(45) 125328	$\overline{46}$·0980460	
110·0	(49) 641696	$\overline{50}$·8073294		(48) 844456	$\overline{49}$·9265770	
115·0	(51) 422888	$\overline{52}$·7184089		(50) 568990	$\overline{51}$·7551046	
120·0	(53) 278953	$\overline{54}$·4455310		(52) 383382	$\overline{53}$·5836322	

v	$n = 4, (m = 1\cdot 0)$ T_m	$\log T_m$	ρ	$n = 5, (m = 1\cdot 5)$ T_m	$\log T_m$	ρ
9·0	(3) 153659	$\overline{4}$·1865572	·94862	(3) 308525	$\overline{4}$·4892897	·90000
9·5	(4) 955628	$\overline{5}$·9802887	·95114	(3) 196486	$\overline{4}$·2933317	·90476
10·0	(4) 593609	$\overline{5}$·7735001	·95342	(3) 124850	$\overline{4}$·0063879	·90909
10·5	(4) 368332	$\overline{5}$·5662396	·95549	(4) 791673	$\overline{5}$·8985458	·91304
11·0	(4) 228322	$\overline{5}$·3585477	·95739	(4) 501051	$\overline{5}$·6998819	·91667
11·5	(4) 141404	$\overline{5}$·1504608	·95914	(4) 316565	$\overline{5}$·5004635	·92000
12·0	(5) 875005	$\overline{6}$·9420105	·96074	(4) 199687	$\overline{5}$·3003496	·92308
12·5	(5) 541034	$\overline{6}$·7332246	·96223	(4) 125775	$\overline{5}$·0995927	·92593
13·0	(5) 334294	$\overline{6}$·5241290	·96360	(5) 791115	$\overline{6}$·8982398	·92857
13·5	(5) 206417	$\overline{6}$·3147448	·96488	(5) 496973	$\overline{6}$·6963325	·93103
14·0	(5) 127377	$\overline{6}$·1050925	·96607	(5) 311823	$\overline{6}$·4939086	·93333
14·5	(6) 785579	$\overline{7}$·8951900	·96719	(5) 195435	$\overline{6}$·2910017	·93548
15·0	(6) 484232	$\overline{7}$·6850534	·96823	(5) 122361	$\overline{6}$·0876428	·93750
15·5	(6) 298330	$\overline{7}$·4746973	·96921	(6) 765340	$\overline{7}$·8838595	·93939
16·0	(6) 183711	$\overline{7}$·2641352	·97013	(6) 478274	$\overline{7}$·6796772	·94118
18·0	(7) 263062	$\overline{8}$·4200584	·97332	(7) 723424	$\overline{8}$·8593929	·94737
20·0	(8) 374530	$\overline{9}$·5734868	·97589	(7) 108211	$\overline{8}$·0342697	·95238
22·0	(9) 530746	$\overline{10}$·7248864	·97801	(8) 160394	$\overline{9}$·2051892	·95652
24·0	(10) 749203	$\overline{11}$·8745996	·97979	(9) 235946	$\overline{10}$·3728124	·96000
26·0	(10) 105412	$\overline{11}$·0228891	·98130	(10) 344864	$\overline{11}$·5376472	·96296
28·0	(11) 147897	$\overline{12}$·1699606	·98260	(11) 501294	$\overline{12}$·7000025	·96552
30·0	(12) 207004	$\overline{13}$·3159781	·98374	(12) 725216	$\overline{13}$·8604673	·96774
32·0	(13) 289118	$\overline{14}$·4610745	·98473	(12) 104479	$\overline{13}$·0190305	·96970
34·0	(14) 403050	$\overline{15}$·6053593	·98561	(13) 149967	$\overline{14}$·1759948	·97143
36·0	(15) 560949	$\overline{16}$·7489232	·98639	(14) 214556	$\overline{15}$·3315404	·97297
38·0	(16) 779548	$\overline{17}$·8918428	·98710	(15) 306065	$\overline{16}$·4858143	·97436
40·0	(16) 108189	$\overline{17}$·0341831	·98773	(16) 435456	$\overline{17}$·6389446	·97561
45·0	(19) 772404	$\overline{20}$·8878446	·98907	(18) 329190	$\overline{19}$·5174461	·97826
50·0	(21) 548146	$\overline{22}$·7388964	·99015	(20) 245916	$\overline{21}$·3907861	·98039
55·0	(23) 387106	$\overline{24}$·5878297	·99103	(22) 181941	$\overline{23}$·2599315	·98214
60·0	(25) 272276	$\overline{26}$·4350000	·99177	(24) 133537	$\overline{25}$·1256009	·98361
65·0	(27) 190859	$\overline{28}$·2807116	·99240	(27) 973515	$\overline{28}$·9883426	·98485
70·0	(29) 133400	$\overline{30}$·1251556	·99293	(29) 705642	$\overline{30}$·8485846	·98592
75·0	(32) 930062	$\overline{33}$·9685117	·99340	(31) 508941	$\overline{32}$·7066674	·98684
80·0	(34) 647023	$\overline{35}$·8109197	·99381	(33) 365482	$\overline{34}$·5628665	·98765
85·0	(36) 449256	$\overline{37}$·6524935	·99417	(35) 261461	$\overline{36}$·4174075	·98837
90·0	(38) 311407	$\overline{39}$·4933279	·99449	(37) 186414	$\overline{38}$·2704780	·98901
95·0	(40) 215527	$\overline{41}$·3335015	·99478	(39) 132506	$\overline{40}$·1222355	·98958
100·0	(42) 148964	$\overline{43}$·1730824	·99504	(42) 939319	$\overline{43}$·9728132	·99010
105·0	(44) 102832	$\overline{45}$·0121279	·99527	(44) 664240	$\overline{45}$·8223253	·99057
110·0	(47) 709067	$\overline{48}$·8506873	·99549	(46) 468673	$\overline{47}$·6708700	·99099
115·0	(49) 488431	$\overline{50}$·6888036	·99568	(48) 330014	$\overline{49}$·5185326	·99138
120·0	(51) 336136	$\overline{52}$·5265144	·99586	(50) 231946	$\overline{51}$·3653875	·99174

v	$n = 6, (m = 2\cdot 0)$ T_m	$\log T_m$	ρ	$n = 7, (m = 2\cdot 5)$ T_m	$\log T_m$	ρ
9·0	(3) 539732	$\overline{4}$·7321780	·85408	(3) 856156	$\overline{4}$·9325527	·81081
9·5	(3) 351537	$\overline{4}$·5459717	·86083	(3) 569576	$\overline{4}$·7555514	·81930
10·0	(3) 228226	$\overline{4}$·3583656	·86699	(3) 377387	$\overline{4}$·5767860	·82707
10·5	(3) 147734	$\overline{4}$·1694807	·87262	(3) 249119	$\overline{4}$·3964065	·83420
11·0	(4) 953725	$\overline{5}$·9794231	·87780	(3) 163885	$\overline{4}$·2145404	·84076
11·5	(4) 614166	$\overline{5}$·7882858	·88257	(3) 107474	$\overline{4}$·0313033	·84683
12·0	(4) 394595	$\overline{5}$·5961516	·88699	(4) 702744	$\overline{5}$·8467973	·85246
12·5	(4) 252984	$\overline{5}$·4030939	·89109	(4) 458262	$\overline{5}$·6611137	·85769
13·0	(4) 161874	$\overline{5}$·2091784	·89490	(4) 298081	$\overline{5}$·4743342	·86256
13·5	(4) 103387	$\overline{5}$·0144640	·89845	(4) 193434	$\overline{5}$·2865323	·86711
14·0	(5) 650180	$\overline{6}$·8190040	·90177	(4) 125249	$\overline{5}$·0977743	·87137
14·5	(5) 419610	$\overline{6}$·6228461	·90488	(5) 809320	$\overline{6}$·9081205	·87537
15·0	(5) 266706	$\overline{6}$·4260333	·90780	(5) 521946	$\overline{6}$·7176254	·87912
15·5	(5) 169280	$\overline{6}$·2286058	·91055	(5) 336000	$\overline{6}$·5263391	·88266
16·0	(5) 107300	$\overline{6}$·0305997	·91313	(5) 215927	$\overline{6}$·3343067	·88599
18·0	(6) 171163	$\overline{7}$·2334109	·92214	(6) 362664	$\overline{7}$·5595043	·89764
20·0	(7) 268636	$\overline{8}$·4291645	·92946	(7) 596446	$\overline{8}$·7755714	·90713
22·0	(8) 416039	$\overline{9}$·6191342	·93552	(8) 964110	$\overline{9}$·9841265	·91501
24·0	(9) 637197	$\overline{10}$·8042739	·94062	(8) 153601	$\overline{9}$·1863934	·92166
26·0	(10) 966763	$\overline{11}$·9853199	·94498	(9) 241724	$\overline{10}$·3833194	·92734
28·0	(10) 145496	$\overline{11}$·1628517	·94874	(10) 376403	$\overline{11}$·5756539	·93226
30·0	(11) 217437	$\overline{12}$·3373341	·95202	(11) 580757	$\overline{12}$·7639948	·93656
32·0	(12) 322957	$\overline{13}$·5091454	·95490	(12) 888866	$\overline{13}$·9488363	·94034
34·0	(13) 477087	$\overline{14}$·6785974	·95746	(12) 135077	$\overline{13}$·1305827	·94370
36·0	(14) 701375	$\overline{15}$·8459505	·95974	(13) 203973	$\overline{14}$·3095728	·94670
38·0	(14) 102665	$\overline{15}$·0114243	·96179	(14) 306262	$\overline{15}$·4860926	·94939
40·0	(15) 149694	$\overline{16}$·1752055	·96364	(15) 457495	$\overline{16}$·6603860	·95183
45·0	(17) 119744	$\overline{18}$·0782525	·96757	(17) 386977	$\overline{18}$·5876849	·95700
50·0	(20) 941119	$\overline{21}$·9736444	·97074	(19) 319811	$\overline{20}$·5048932	·96118
55·0	(22) 729136	$\overline{23}$·9680792	·97334	(21) 259348	$\overline{22}$·4138824	·96461
60·0	(24) 558221	$\overline{25}$·7468062	·97551	(23) 207037	$\overline{24}$·3160474	·96749
65·0	(26) 423106	$\overline{27}$·6264495	·97736	(25) 163101	$\overline{26}$·2124556	·96993
70·0	(28) 317060	$\overline{29}$·5023725	·97895	(27) 127040	$\overline{28}$·1039420	·97203
75·0	(30) 237181	$\overline{31}$·3750800	·98033	(30) 979879	$\overline{31}$·9911724	·97386
80·0	(32) 175785	$\overline{33}$·2449808	·98154	(32) 749352	$\overline{33}$·8746857	·97546
85·0	(34) 129542	$\overline{35}$·1124102	·98261	(34) 568754	$\overline{35}$·7549248	·97688
90·0	(37) 949832	$\overline{38}$·9776470	·98356	(36) 428803	$\overline{37}$·6322577	·97814
95·0	(39) 693306	$\overline{40}$·8409251	·98442	(38) 321361	$\overline{39}$·5069938	·97928
100·0	(41) 504015	$\overline{42}$·7024431	·98519	(40) 239550	$\overline{41}$·3793955	·98030
105·0	(43) 365065	$\overline{44}$·5623706	·98588	(42) 177700	$\overline{43}$·2496874	·98122
110·0	(45) 263545	$\overline{46}$·4208540	·98652	(44) 131239	$\overline{45}$·1180629	·98206
115·0	(47) 189679	$\overline{48}$·2780203	·98710	(47) 965363	$\overline{48}$·9846905	·98283
120·0	(49) 136138	$\overline{50}$·1339805	·98763	(49) 707484	$\overline{50}$·8497168	·98354

v	$n = 8, (m = 3\cdot0)$ T_m	$\log T_m$	ρ	$n = 9, (m = 3\cdot5)$ T_m	$\log T_m$	ρ	$n = 10, (m = 4\cdot0)$ T_m	$\log T_m$	ρ
0	·169765	$\bar{1}$·2298489	·00000	·156250	$\bar{1}$·1938200	·00000	·145513	$\bar{1}$·1629021	·00000
·1	·169553	$\bar{1}$·2293063	·02497	·156094	$\bar{1}$·1933859	·01999	·145392	$\bar{1}$·1625402	·01666
·2	·168921	$\bar{1}$·2276826	·04976	·155627	$\bar{1}$·1920851	·03989	·145029	$\bar{1}$·1614556	·03328
·3	·167876	$\bar{1}$·2249891	·07422	·154854	$\bar{1}$·1899229	·05965	·144428	$\bar{1}$·1596509	·04982
·4	·166434	$\bar{1}$·2212428	·09822	·153783	$\bar{1}$·1869072	·07919	·143592	$\bar{1}$·1571304	·06623
·5	·164614	$\bar{1}$·2164660	·12166	·152422	$\bar{1}$·1830488	·09845	·142528	$\bar{1}$·1539002	·08250
·6	·162437	$\bar{1}$·2106843	·14449	·150786	$\bar{1}$·1783610	·11738	·141243	$\bar{1}$·1499675	·09858
·7	·159929	$\bar{1}$·2039259	·16663	·148888	$\bar{1}$·1728587	·13595	·139747	$\bar{1}$·1453411	·11444
·8	·157116	$\bar{1}$·1962208	·18808	·146743	$\bar{1}$·1665584	·15412	·138048	$\bar{1}$·1400307	·13007
·9	·154028	$\bar{1}$·1875998	·20881	·144370	$\bar{1}$·1594780	·17187	·136159	$\bar{1}$·1340470	·14544
1·0	·150694	$\bar{1}$·1780945	·22881	·141787	$\bar{1}$·1516360	·18919	·134092	$\bar{1}$·1274015	·16054
1·1	·147142	$\bar{1}$·1677361	·24809	·139012	$\bar{1}$·1430517	·20606	·131858	$\bar{1}$·1201064	·17536
1·2	·143402	$\bar{1}$·1565558	·26666	·136064	$\bar{1}$·1337448	·22247	·129472	$\bar{1}$·1121745	·18987
1·3	·139503	$\bar{1}$·1445843	·28454	·132964	$\bar{1}$·1237350	·23842	·126946	$\bar{1}$·1036186	·20408
1·4	·135472	$\bar{1}$·1318511	·30173	·129731	$\bar{1}$·1130423	·25392	·124295	$\bar{1}$·0944523	·21799
1·5	·131337	$\bar{1}$·1183855	·31828	·126382	$\bar{1}$·1016862	·26897	·121532	$\bar{1}$·0846891	·23157
1·6	·127120	$\bar{1}$·1042152	·33419	·122938	$\bar{1}$·0896867	·28356	·118670	$\bar{1}$·0743426	·24485
1·7	·122848	$\bar{1}$·0893673	·34949	·119416	$\bar{1}$·0770626	·29772	·115725	$\bar{1}$·0634265	·25781
1·8	·118541	$\bar{1}$·0738677	·36420	·115833	$\bar{1}$·0638330	·31145	·112708	$\bar{1}$·0519545	·27045
1·9	·114220	$\bar{1}$·0577393	·37836	·112206	$\bar{1}$·0500163	·32476	·109633	$\bar{1}$·0399400	·28279
2·0	·109904	$\bar{1}$·0410117	·39198	·108550	$\bar{1}$·0356305	·33766	·106512	$\bar{1}$·0273965	·29481
2·1	·105609	$\bar{1}$·0237019	·40508	·104880	$\bar{1}$·0206931	·35017	·103356	$\bar{1}$·0143372	·30654
2·2	·101352	$\bar{1}$·0058338	·41770	·101209	$\bar{1}$·0052210	·36228	·100179	$\bar{1}$·0007752	·31797
2·3	(1) 971467	$\bar{2}$·9874279	·42985	(1) 975508	$\bar{2}$·9892309	·37403	(1) 969891	$\bar{2}$·9867231	·32911
2·4	(1) 930046	$\bar{2}$·9685043	·44155	(1) 939158	$\bar{2}$·9727385	·38541	(1) 937980	$\bar{2}$·9721936	·33996
2·5	(1) 889368	$\bar{2}$·9490816	·45283	(1) 903149	$\bar{2}$·9557593	·39645	(1) 906147	$\bar{2}$·9571989	·35053
2·6	(1) 849529	$\bar{2}$·9291782	·46370	(1) 867577	$\bar{2}$·9383082	·40715	(1) 874482	$\bar{2}$·9417509	·36083
2·7	(1) 810608	$\bar{2}$·9088111	·47418	(1) 832529	$\bar{2}$·9203995	·41752	(1) 843066	$\bar{2}$·9258614	·37086
2·8	(1) 772675	$\bar{2}$·8879968	·48429	(1) 798081	$\bar{2}$·9020472	·42758	(1) 811973	$\bar{2}$·9095418	·38064
2·9	(1) 735785	$\bar{2}$·8667510	·49405	(1) 764301	$\bar{2}$·8832645	·43734	(1) 781273	$\bar{2}$·8928030	·39016
3·0	(1) 699985	$\bar{2}$·8450885	·50348	(1) 731248	$\bar{2}$·8640644	·44681	(1) 751028	$\bar{2}$·8756561	·39944
3·1	(1) 665310	$\bar{2}$·8230238	·51259	(1) 698971	$\bar{2}$·8444593	·45600	(1) 721292	$\bar{2}$·8581112	·40849
3·2	(1) 631786	$\bar{2}$·8005703	·52138	(1) 667515	$\bar{2}$·8244611	·46491	(1) 692116	$\bar{2}$·8401788	·41730
3·3	(1) 599434	$\bar{2}$·7777411	·52989	(1) 636915	$\bar{2}$·8040813	·47357	(1) 663542	$\bar{2}$·8218687	·42588
3·4	(1) 568262	$\bar{2}$·7545486	·53812	(1) 607199	$\bar{2}$·7833309	·48197	(1) 635610	$\bar{2}$·8031904	·43425
3·5	(1) 538275	$\bar{2}$·7310045	·54608	(1) 578390	$\bar{2}$·7622208	·49014	(1) 608350	$\bar{2}$·7841533	·44241
3·6	(1) 509472	$\bar{2}$·7071203	·55379	(1) 550505	$\bar{2}$·7407610	·49808	(1) 581790	$\bar{2}$·7647665	·45036
3·7	(1) 481844	$\bar{2}$·6829067	·56125	(1) 523554	$\bar{2}$·7189616	·50579	(1) 555954	$\bar{2}$·7450386	·45811
3·8	(1) 455380	$\bar{2}$·6583741	·56848	(1) 497545	$\bar{2}$·6968320	·51328	(1) 530858	$\bar{2}$·7249782	·46567
3·9	(1) 430063	$\bar{2}$·6335324	·57549	(1) 472478	$\bar{2}$·6743815	·52057	(1) 506516	$\bar{2}$·7045934	·47305
4·0	(1) 405874	$\bar{2}$·6083910	·58228	(1) 448352	$\bar{2}$·6516187	·52766	(1) 482939	$\bar{2}$·6838923	·48024
4·5	(1) 300950	$\bar{2}$·4784937	·61335	(1) 341515	$\bar{2}$·5334096	·56036	(1) 376620	$\bar{2}$·5759038	·51369
5·0	(1) 219946	$\bar{2}$·3423161	·64027	(1) 256182	$\bar{2}$·4085492	·58904	(1) 289114	$\bar{2}$·4610688	·54340
5·5	(1) 158736	$\bar{2}$·2006752	·66378	(1) 189599	$\bar{2}$·2778350	·61437	(1) 218847	$\bar{2}$·3401401	·56990
6·0	(1) 113303	$\bar{2}$·0542432	·68448	(1) 138655	$\bar{2}$·1419362	·63687	(1) 163592	$\bar{2}$·2137625	·59365
6·5	(2) 800900	$\bar{3}$·9035785	·70283	(1) 100327	$\bar{2}$·0014185	·65698	(1) 120918	$\bar{2}$·0824904	·61504
7·0	(2) 561241	$\bar{3}$·7491497	·71921	(2) 719057	$\bar{3}$·8567633	·67503	(2) 884712	$\bar{3}$·9468021	·63438
7·5	(2) 390259	$\bar{3}$·5913527	·73390	(2) 510955	$\bar{3}$·7083826	·69133	(2) 641376	$\bar{3}$·8071127	·65193
8·0	(2) 269479	$\bar{3}$·4305256	·74715	(2) 360273	$\bar{3}$·5566316	·70611	(2) 461088	$\bar{3}$·6637835	·66793
8·5	(2) 184909	$\bar{3}$·2669583	·75917	(2) 252242	$\bar{3}$·4018182	·71957	(2) 328951	$\bar{3}$·5171310	·68257

$n = 11, (m = 4\cdot5)$ T_m	$\log T_m$	ρ	$n = 12, (m = 5\cdot0)$ T_m	$\log T_m$	ρ	$n = 13, (m = 5\cdot5)$ T_m	$\log T_m$	ρ	v
·136719	$\bar{1}$·1358281	·00000	·129345	$\bar{1}$·1117496	·00000	·123047	$\bar{1}$·0900706	·00000	0
·136621	$\bar{1}$·1355179	·01428	·129264	$\bar{1}$·1114781	·01250	·122979	$\bar{1}$·0898293	·01111	·1
·136329	$\bar{1}$·1345879	·02854	·129022	$\bar{1}$·1106642	·02498	·122774	$\bar{1}$·0891058	·02221	·2
·135844	$\bar{1}$·1330397	·04275	·128620	$\bar{1}$·1093090	·03743	·122434	$\bar{1}$·0879005	·03329	·3
·135169	$\bar{1}$·1308760	·05689	·128060	$\bar{1}$·1074138	·04983	·121959	$\bar{1}$·0862151	·04433	·4
·134307	$\bar{1}$·1281001	·07093	·127345	$\bar{1}$·1049811	·06218	·121353	$\bar{1}$·0840506	·05534	·5
·133265	$\bar{1}$·1247167	·08486	·126478	$\bar{1}$·1020140	·07445	·120617	$\bar{1}$·0814092	·06629	·6
·132048	$\bar{1}$·1208298	·09866	·125463	$\bar{1}$·0985158	·08663	·119755	$\bar{1}$·0782935	·07719	·7
·130662	$\bar{1}$·1161495	·11231	·124306	$\bar{1}$·0944906	·09872	·118770	$\bar{1}$·0747060	·08801	·8
·129116	$\bar{1}$·1109786	·12579	·123011	$\bar{1}$·0899431	·11069	·117666	$\bar{1}$·0706501	·09876	·9
·127417	$\bar{1}$·1052259	·13910	·121584	$\bar{1}$·0848781	·12254	·116447	$\bar{1}$·0661292	·10942	1·0
·125574	$\bar{1}$·0988995	·15221	·120033	$\bar{1}$·0793012	·13426	·115119	$\bar{1}$·0611473	·11999	1·1
·123597	$\bar{1}$·0920077	·16513	·118364	$\bar{1}$·0732182	·14585	·113686	$\bar{1}$·0557085	·13046	1·2
·121495	$\bar{1}$·0845594	·17784	·116583	$\bar{1}$·0666353	·15728	·112155	$\bar{1}$·0498172	·14083	1·3
·119279	$\bar{1}$·0765640	·19033	·114690	$\bar{1}$·0595589	·16857	·110529	$\bar{1}$·0434780	·15108	1·4
·116958	$\bar{1}$·0680305	·20261	·112719	$\bar{1}$·0519958	·17970	·108817	$\bar{1}$·0366960	·16122	1·5
·114543	$\bar{1}$·0589687	·21466	·110650	$\bar{1}$·0439529	·19066	·107023	$\bar{1}$·0294761	·17124	1·6
·112044	$\bar{1}$·0493887	·22648	·108502	$\bar{1}$·0354373	·20146	·105154	$\bar{1}$·0218248	·18114	1·7
·109471	$\bar{1}$·0393001	·23808	·106281	$\bar{1}$·0264565	·21209	·103215	$\bar{1}$·0137444	·19091	1·8
·106835	$\bar{1}$·0287130	·24944	·103996	$\bar{1}$·0170177	·22255	·101215	$\bar{1}$·0052435	·20055	1·9
·104145	$\bar{1}$·0176373	·26058	·101655	$\bar{1}$·0071283	·23284	(1) 991578	$\bar{2}$·9963267	·21006	2·0
·101411	$\bar{1}$·0060830	·27148	(1) 992650	$\bar{2}$·9967961	·24295	(1) 970510	$\bar{2}$·9869999	·21943	2·1
(1) 986416	$\bar{2}$·9940601	·28216	(1) 968341	$\bar{2}$·9860284	·25289	(1) 949006	$\bar{2}$·9772690	·22867	2·2
(1) 958469	$\bar{2}$·9815783	·29261	(1) 943698	$\bar{2}$·9748331	·26265	(1) 927128	$\bar{2}$·9671397	·23778	2·3
(1) 930352	$\bar{2}$·9686474	·30284	(1) 918793	$\bar{2}$·9632179	·27224	(1) 904936	$\bar{2}$·9566181	·24674	2·4
(1) 902147	$\bar{2}$·9552772	·31285	(1) 893695	$\bar{2}$·9511895	·28165	(1) 882491	$\bar{2}$·9457101	·25557	2·5
(1) 873931	$\bar{2}$·9414770	·32264	(1) 868473	$\bar{2}$·9387564	·29089	(1) 859848	$\bar{2}$·9344217	·26426	2·6
(1) 845778	$\bar{2}$·9272564	·33221	(1) 843191	$\bar{2}$·9259260	·29996	(1) 837064	$\bar{2}$·9227589	·27281	2·7
(1) 817757	$\bar{2}$·9126244	·34158	(1) 817909	$\bar{2}$·9127051	·30885	(1) 814194	$\bar{2}$·9107276	·28123	2·8
(1) 789933	$\bar{2}$·8975902	·35074	(1) 792687	$\bar{2}$·8991016	·31758	(1) 791287	$\bar{2}$·8983339	·28950	2·9
(1) 762364	$\bar{2}$·8821626	·35969	(1) 767578	$\bar{2}$·8851226	·32615	(1) 768393	$\bar{2}$·8855836	·29765	3·0
(1) 735107	$\bar{2}$·8663505	·36845	(1) 742635	$\bar{2}$·8707753	·33455	(1) 745560	$\bar{2}$·8724826	·30565	3·1
(1) 708210	$\bar{2}$·8501622	·37702	(1) 717905	$\bar{2}$·8560668	·34278	(1) 722831	$\bar{2}$·8590368	·31353	3·2
(1) 681720	$\bar{2}$·8336061	·38539	(1) 693432	$\bar{2}$·8410038	·35086	(1) 700248	$\bar{2}$·8452519	·32127	3·3
(1) 655678	$\bar{2}$·8166904	·39358	(1) 669258	$\bar{2}$·8255935	·35878	(1) 677850	$\bar{2}$·8311337	·32888	3·4
(1) 630120	$\bar{2}$·7994231	·40158	(1) 645420	$\bar{2}$·8098424	·36655	(1) 655674	$\bar{2}$·8166878	·33636	3·5
(1) 605079	$\bar{2}$·7818119	·40941	(1) 621953	$\bar{2}$·7937573	·37417	(1) 633753	$\bar{2}$·8019198	·34371	3·6
(1) 580583	$\bar{2}$·7638644	·41707	(1) 598887	$\bar{2}$·7773446	·38164	(1) 612118	$\bar{2}$·7868352	·35094	3·7
(1) 556658	$\bar{2}$·7455881	·42456	(1) 576250	$\bar{2}$·7606106	·38896	(1) 590799	$\bar{2}$·7714394	·35804	3·8
(1) 533323	$\bar{2}$·7269901	·43188	(1) 554066	$\bar{2}$·7435617	·39614	(1) 569820	$\bar{2}$·7557379	·36502	3·9
(1) 510596	$\bar{2}$·7080775	·43905	(1) 532358	$\bar{2}$·7262039	·40319	(1) 549207	$\bar{2}$·7397359	·37188	4·0
(1) 406471	$\bar{2}$·6090296	·47261	(1) 431507	$\bar{2}$·6349883	·43640	(1) 452270	$\bar{2}$·6553977	·40443	4·5
(1) 318473	$\bar{2}$·5030730	·50275	(1) 344270	$\bar{2}$·5368993	·46655	(1) 366683	$\bar{2}$·5642907	·43426	5·0
(1) 245981	$\bar{2}$·3909019	·52991	(1) 270749	$\bar{2}$·4325667	·49396	(1) 293075	$\bar{2}$·4669788	·46162	5·5
(1) 187552	$\bar{2}$·2731218	·55447	(1) 210160	$\bar{2}$·3225503	·51894	(1) 231192	$\bar{2}$·3639729	·48674	6·0
(1) 141340	$\bar{2}$·1502656	·57673	(1) 161194	$\bar{2}$·2073481	·54177	(1) 180191	$\bar{2}$·2557341	·50985	6·5
(1) 105389	$\bar{2}$·0227947	·59700	(1) 122293	$\bar{2}$·0874024	·56267	(1) 138892	$\bar{2}$·1426779	·53115	7·0
(2) 778248	$\bar{3}$·8911177	·61551	(2) 918557	$\bar{3}$·9631063	·58187	(1) 105969	$\bar{2}$·0251780	·55081	7·5
(2) 569631	$\bar{3}$·7555938	·63247	(2) 683613	$\bar{3}$·8348100	·59954	(2) 800886	$\bar{3}$·9035709	·56900	8·0
(2) 413562	$\bar{3}$·6165403	·64805	(2) 504459	$\bar{3}$·7028261	·61586	(2) 600012	$\bar{3}$·7781600	·58587	8·5

v	$n = 8, (m = 3·0)$ T_m	$\log T_m$	ρ	$n = 9, (m = 3·5)$ T_m	$\log T_m$	ρ	$n = 10, (m = 4·0)$ T_m	$\log T_m$	ρ
9·0	(2) 126154	$\bar{3}$·1009019	·77010	(2) 175473	$\bar{3}$·2442111	·73187	(2) 233042	$\bar{3}$·3674334	·69601
9·5	(3) 856199	$\bar{4}$·9325749	·78010	(2) 121352	$\bar{3}$·0840454	·74315	(2) 164035	$\bar{3}$·2149364	·70838
10·0	(3) 578320	$\bar{4}$·7621684	·78927	(3) 834697	$\bar{4}$·9215287	·75354	(2) 114778	$\bar{3}$·0598579	·71980
10·5	(3) 388911	$\bar{4}$·5898507	·79772	(3) 571274	$\bar{4}$·7568443	·76313	(3) 798715	$\bar{4}$·9023919	·73038
11·0	(3) 260478	$\bar{4}$·4157706	·80552	(3) 389184	$\bar{4}$·5901552	·77202	(3) 552983	$\bar{4}$·7427117	·74021
11·5	(3) 173804	$\bar{4}$·2400603	·81274	(3) 264002	$\bar{4}$·4216078	·78026	(3) 381042	$\bar{4}$·5809729	·74935
12·0	(3) 115568	$\bar{4}$·0628379	·81945	(3) 178374	$\bar{4}$·2513319	·78794	(3) 261406	$\bar{4}$·4173156	·75789
12·5	(4) 765965	$\bar{5}$·8842090	·82571	(3) 120073	$\bar{4}$·0794452	·79511	(3) 178594	$\bar{4}$·2518660	·76587
13·0	(4) 506138	$\bar{5}$·7042688	·83154	(4) 805478	$\bar{5}$·9060536	·80181	(3) 121545	$\bar{4}$·0847388	·77335
13·5	(4) 333506	$\bar{5}$·5231030	·83700	(4) 538583	$\bar{5}$·7312530	·80809	(4) 824210	$\bar{5}$·9160381	·78037
14·0	(4) 219174	$\bar{5}$·3407895	·84212	(4) 359030	$\bar{5}$·5551306	·81399	(4) 557004	$\bar{5}$·7458585	·78698
14·5	(4) 143681	$\bar{5}$·1573988	·84692	(4) 238652	$\bar{5}$·3777657	·81954	(4) 375221	$\bar{5}$·5742867	·79320
15·0	(5) 939713	$\bar{6}$·9729952	·85145	(4) 158209	$\bar{5}$·1992308	·82477	(4) 252001	$\bar{5}$·4014021	·79907
15·5	(5) 613250	$\bar{6}$·7876374	·85572	(4) 104615	$\bar{5}$·0195923	·82971	(4) 168763	$\bar{5}$·2272771	·80463
16·0	(5) 399373	$\bar{6}$·6013792	·85975	(5) 690099	$\bar{6}$·8389111	·83438	(4) 112714	$\bar{5}$·0519789	·80988
18·0	(6) 705145	$\bar{7}$·8482784	·87385	(5) 127884	$\bar{6}$·1068170	·85076	(5) 218889	$\bar{6}$·3402247	·82838
20·0	(6) 121366	$\bar{7}$·0840957	·88538	(6) 230055	$\bar{7}$·3618313	·86421	(6) 411040	$\bar{7}$·6138845	·84361
22·0	(7) 204537	$\bar{8}$·3107720	·89498	(7) 403805	$\bar{8}$·6061713	·87544	(7) 750640	$\bar{8}$·8754318	·85638
24·0	(8) 338670	$\bar{9}$·5297765	·90311	(8) 694271	$\bar{9}$·8415289	·88496	(7) 133893	$\bar{8}$·1267589	·86722
26·0	(9) 552397	$\overline{10}$·7422512	·91006	(8) 117280	$\bar{9}$·0692247	·89313	(8) 234072	$\bar{9}$·3503868	·87655
28·0	(10) 889408	$\overline{11}$·9491008	·91609	(9) 195123	$\overline{10}$·2903084	·90023	(9) 402146	$\overline{10}$·6043843	·88466
30·0	(10) 141597	$\overline{11}$·1510547	·92136	(10) 320352	$\overline{11}$·5056281	·90644	(10) 680494	$\overline{11}$·8328240	·89177
32·0	(11) 223208	$\overline{12}$·3487089	·92601	(11) 519851	$\overline{12}$·7158787	·91192	(10) 113620	$\overline{11}$·0554556	·89806
34·0	(12) 348784	$\overline{13}$·5425570	·93014	(12) 834906	$\overline{13}$·9216373	·91680	(11) 187471	$\overline{12}$·2729335	·90366
36·0	(13) 540770	$\overline{14}$·7330124	·93384	(12) 132858	$\overline{13}$·1233877	·92116	(12) 306061	$\overline{13}$·4858080	·90868
38·0	(14) 832577	$\overline{15}$·9204244	·93716	(13) 209671	$\overline{14}$·3215387	·92509	(13) 494932	$\overline{14}$·6945456	·91320
40·0	(14) 127377	$\overline{15}$·1050914	·94017	(14) 328429	$\overline{15}$·5164412	·92865	(14) 793498	$\overline{15}$·8995456	·91729
45·0	(16) 113854	$\overline{17}$·0563485	·94656	(16) 310002	$\overline{17}$·4913642	·93623	(16) 790392	$\overline{17}$·8978424	·92602
50·0	(19) 988866	$\overline{20}$·9951377	·95171	(18) 282813	$\overline{19}$·4514994	·94235	(18) 756988	$\overline{19}$·8790888	·93308
55·0	(21) 838994	$\overline{22}$·9237590	·95597	(20) 250934	$\overline{21}$·3995600	·94740	(20) 702095	$\overline{21}$·8463961	·93892
60·0	(23) 698119	$\overline{24}$·8439298	·95953	(22) 217558	$\overline{23}$·3375756	·95164	(22) 634009	$\overline{23}$·8020955	·94382
65·0	(25) 571433	$\overline{26}$·7569656	·96256	(24) 184971	$\overline{25}$·2671034	·95524	(24) 559730	$\overline{25}$·7479784	·94799
70·0	(27) 461210	$\overline{28}$·6638986	·96517	(26) 154655	$\overline{27}$·1893649	·95835	(26) 484676	$\overline{27}$·6854517	·95158
75·0	(29) 367748	$\overline{30}$·5655498	·96743	(28) 127449	$\overline{29}$·1053354	·96105	(28) 412705	$\overline{29}$·6156398	·95471
80·0	(31) 290126	$\overline{32}$·4625865	·96942	(30) 103707	$\overline{31}$·0158063	·96343	(30) 346303	$\overline{31}$·5394557	·95746
85·0	(33) 226755	$\overline{34}$·3555565	·97119	(33) 834504	$\overline{34}$·9214283	·96553	(32) 286848	$\overline{33}$·4576512	·95990
90·0	(35) 175758	$\overline{36}$·2449158	·97276	(35) 664880	$\overline{36}$·8227433	·96740	(34) 234883	$\overline{35}$·3708522	·96207
95·0	(37) 135222	$\overline{38}$·1310467	·97416	(37) 525058	$\overline{38}$·7202070	·96908	(36) 190364	$\overline{37}$·2795858	·96402
100·0	(39) 103342	$\overline{40}$·0142756	·97543	(39) 411345	$\overline{40}$·6142067	·97059	(38) 152862	$\overline{39}$·1842996	·96578
105·0	(42) 785020	$\overline{43}$·8948806	·97658	(41) 319944	$\overline{42}$·5050736	·97197	(40) 121724	$\overline{41}$·0853774	·96737
110·0	(44) 593064	$\overline{45}$·7731018	·97763	(43) 247226	$\overline{44}$·3930940	·97322	(43) 961945	$\overline{44}$·9831504	·96883
115·0	(46) 445808	$\overline{47}$·6491480	·97859	(45) 189896	$\overline{46}$·2785153	·97436	(45) 754930	$\overline{46}$·8779065	·97016
120·0	(48) 333581	$\overline{49}$·5232015	·97947	(47) 145064	$\overline{48}$·1615582	·97541	(47) 588705	$\overline{48}$·7698976	·97138

$n = 11, (m = 4·5)$ T_m	$\log T_m$	ρ	$n = 12, (m = 5·0)$ T_m	$\log T_m$	ρ	$n = 13, (m = 5·5)$ T_m	$\log T_m$	ρ	v
(2) 298015	$\bar{3}$·4742387	·66241	(2) 369346	$\bar{3}$·5674338	·63096	(2) 445881	$\bar{3}$·6492186	·60154	9·0
(2) 213275	$\bar{3}$·3289394	·67568	(2) 268463	$\bar{3}$·4288841	·64496	(2) 328847	$\bar{3}$·5169939	·61613	9·5
(2) 151658	$\bar{3}$·1808657	·68798	(2) 193822	$\bar{3}$·2874021	·65798	(2) 240829	$\bar{3}$·3817097	·62973	10·0
(2) 107206	$\bar{3}$·0302187	·69940	(2) 139056	$\bar{3}$·1431909	·67011	(2) 175214	$\bar{3}$·2435688	·64245	10·5
(3) 753664	$\bar{4}$·8771779	·71004	(3) 991823	$\bar{4}$·9964341	·68144	(2) 126694	$\bar{3}$·1027558	·65436	11·0
(3) 527115	$\bar{4}$·7219055	·71996	(3) 703555	$\bar{4}$·8472981	·69204	(3) 910833	$\bar{4}$·9594386	·66553	11·5
(3) 366901	$\bar{4}$·5275019	·72925	(3) 496517	$\bar{4}$·6959337	·70197	(3) 651284	$\bar{4}$·8137705	·67602	12·0
(3) 254238	$\bar{4}$·4052398	·73795	(3) 348721	$\bar{4}$·5424788	·71130	(3) 463331	$\bar{4}$·6658917	·68590	12·5
(3) 175429	$\bar{4}$·2441005	·74612	(3) 243814	$\bar{4}$·3870585	·72008	(3) 328043	$\bar{4}$·5159307	·69521	13·0
(3) 120570	$\bar{4}$·0812408	·75380	(3) 169742	$\bar{4}$·2297881	·72835	(3) 231209	$\bar{4}$·3640055	·70399	13·5
(4) 825585	$\bar{5}$·9167616	·76104	(3) 117699	$\bar{4}$·0707729	·73616	(3) 162265	$\bar{4}$·2102246	·71230	14·0
(4) 563320	$\bar{5}$·7507551	·76787	(4) 813036	$\bar{5}$·9101096	·74354	(3) 113420	$\bar{4}$·0546882	·72017	14·5
(4) 383095	$\bar{5}$·5833063	·77433	(4) 559613	$\bar{5}$·7478874	·75052	(4) 789748	$\bar{5}$·8974884	·72763	15·0
(4) 259713	$\bar{5}$·4144931	·78044	(4) 383874	$\bar{5}$·5841885	·75714	(4) 547912	$\bar{5}$·7387109	·73471	15·5
(4) 175545	$\bar{5}$·2443875	·78624	(4) 262476	$\bar{5}$·4190889	·76343	(4) 378822	$\bar{5}$·5784348	·74143	16·0
(5) 356697	$\bar{6}$·5522993	·80668	(5) 557214	$\bar{6}$·7460223	·78566	(5) 838958	$\bar{6}$·9237404	·76530	18·0
(6) 698337	$\bar{7}$·8440648	·82358	(5) 113594	$\bar{6}$·0553572	·80411	(5) 177878	$\bar{6}$·2501215	·78519	20·0
(6) 132547	$\bar{7}$·1223709	·83779	(6) 223860	$\bar{7}$·3499765	·81966	(6) 363594	$\bar{7}$·5606171	·80200	22·0
(7) 245070	$\bar{8}$·3892895	·84989	(7) 428657	$\bar{8}$·6321102	·83295	(7) 720438	$\bar{8}$·8575964	·81640	24·0
(8) 443048	$\bar{9}$·6464604	·86031	(8) 800794	$\bar{9}$·9035209	·84442	(7) 138976	$\bar{8}$·1429400	·82887	26·0
(9) 785524	$\overline{10}$·8951592	·86939	(8) 146428	$\bar{9}$·1656246	·85443	(8) 261918	$\bar{9}$·4181647	·83975	28·0
(9) 136923	$\overline{10}$·1364760	·87737	(9) 262770	$\overline{10}$·4195758	·86323	(9) 483627	$\overline{10}$·6845105	·84935	30·0
(10) 235112	$\overline{11}$·3712743	·88443	(10) 463797	$\overline{11}$·6663275	·87103	(10) 877010	$\overline{11}$·9430044	·85787	32·0
(11) 398366	$\overline{12}$·6002819	·89073	(11) 806632	$\overline{12}$·9066756	·87800	(10) 156497	$\overline{11}$·1945053	·86548	34·0
(12) 666978	$\overline{13}$·8241114	·89637	(11) 138449	$\overline{12}$·1412913	·88425	(11) 275258	$\overline{12}$·4397397	·87232	36·0
(12) 110480	$\overline{13}$·0432834	·90147	(12) 234826	$\overline{13}$·3707460	·88990	(12) 477888	$\overline{13}$·6793265	·87850	38·0
(13) 181236	$\overline{14}$·2582438	·90608	(13) 394031	$\overline{14}$·5955299	·89502	(13) 819970	$\overline{14}$·9137979	·88411	40·0
(15) 190381	$\overline{16}$·2796235	·91593	(15) 436217	$\overline{16}$·6397021	·90596	(15) 956035	$\overline{16}$·9804738	·89611	45·0
(17) 191314	$\overline{18}$·2817474	·92391	(17) 459695	$\overline{18}$·6624699	·91484	(16) 105597	$\overline{17}$·0236530	·90587	50·0
(19) 185400	$\overline{20}$·2681094	·93051	(19) 465259	$\overline{20}$·6676947	·92219	(18) 111571	$\overline{19}$·0475494	·91395	55·0
(21) 174314	$\overline{22}$·2413320	·93606	(21) 455282	$\overline{22}$·6582803	·92838	(20) 113589	$\overline{21}$·0553380	·92076	60·0
(23) 159747	$\overline{24}$·2034340	·94079	(23) 432977	$\overline{24}$·6364648	·93365	(22) 112065	$\overline{23}$·0494701	·92657	65·0
(25) 143219	$\overline{26}$·1560021	·94487	(25) 401801	$\overline{26}$·6040112	·93820	(24) 107616	$\overline{25}$·0318790	·93159	70·0
(27) 125981	$\overline{28}$·1003061	·94842	(27) 365031	$\overline{28}$·5623297	·94217	(26) 100951	$\overline{27}$·0041093	·93596	75·0
(29) 108988	$\overline{30}$·0373782	·95154	(29) 325514	$\overline{30}$·5125691	·94566	(29) 927742	$\overline{30}$·9674271	·93981	80·0
(32) 929113	$\overline{33}$·9680685	·95431	(31) 285546	$\overline{32}$·4556756	·94875	(31) 837281	$\overline{32}$·9228714	·94323	85·0
(34) 781782	$\overline{35}$·8930856	·95678	(33) 246853	$\overline{34}$·3924392	·95151	(33) 743551	$\overline{34}$·8713110	·94627	90·0
(36) 650169	$\overline{37}$·8130263	·95899	(35) 210632	$\overline{36}$·3235246	·95399	(35) 650846	$\overline{36}$·8134782	·94901	95·0
(38) 535054	$\overline{39}$·7283980	·96099	(37) 177622	$\overline{38}$·2494972	·95622	(37) 562337	$\overline{38}$·7499964	·95148	100·0
(40) 436150	$\overline{41}$·6396356	·96280	(39) 148198	$\overline{40}$·1708435	·95825	(39) 480176	$\overline{40}$·6814004	·95373	105·0
(42) 352464	$\overline{43}$·5471147	·96445	(41) 122457	$\overline{42}$·0879823	·96010	(41) 405651	$\overline{42}$·6081525	·95577	110·0
(44) 282593	$\overline{45}$·4511620	·96597	(43) 100295	$\overline{44}$·0012785	·96180	(43) 339356	$\overline{44}$·5306551	·95765	115·0
(46) 224938	$\overline{47}$·3520634	·96736	(46) 814801	$\overline{47}$·9110513	·96335	(45) 281359	$\overline{46}$·4492604	·95937	120·0

v	$n=14$, $(m=6\cdot0)$ T_m	$\log T_m$	ρ	$n=15$, $(m=6\cdot5)$ T_m	$\log T_m$	ρ
0	·117586	$\bar{1}$·0703556	·00000	·112793	$\bar{1}$·0522821	·00000
·5	·116128	$\bar{1}$·0649366	·04985	·111520	$\bar{1}$·0473530	·04534
1·0	·111886	$\bar{1}$·0487750	·09879	·107805	$\bar{1}$·0326394	·09001
1·5	·105234	$\bar{1}$·0221544	·14606	·101942	$\bar{1}$·0083519	·13343
2·0	(1) 967170	$\bar{2}$·9855029	·19110	(1) 943661	$\bar{2}$·9748160	·17513
2·5	(1) 869657	$\bar{2}$·9393478	·23355	(1) 855936	$\bar{2}$·9324415	·21480
3·0	(1) 766074	$\bar{2}$·8842705	·27326	(1) 761538	$\bar{2}$·8816915	·25225
3·5	(1) 662021	$\bar{2}$·8208718	·31020	(1) 665359	$\bar{2}$·8230560	·28742
4·0	(1) 562013	$\bar{2}$·7497462	·34445	(1) 571519	$\bar{2}$·7570307	·32032
4·5	(1) 469315	$\bar{2}$·6714649	·37613	(1) 483173	$\bar{2}$·6841024	·35102
5·0	(1) 385981	$\bar{2}$·5865664	·40542	(1) 402475	$\bar{2}$·6047385	·37961
5·5	(1) 313005	$\bar{2}$·4955515	·43250	(1) 330660	$\bar{2}$·5193815	·40624
6·0	(1) 250543	$\bar{2}$·3988818	·45754	(1) 268192	$\bar{2}$·4284455	·43102
6·5	(1) 198144	$\bar{2}$·2969800	·48072	(1) 214939	$\bar{2}$·3323152	·45410
7·0	(1) 154964	$\bar{2}$·1902321	·50220	(1) 170352	$\bar{2}$·2313464	·47561
7·5	(1) 119947	$\bar{2}$·0789895	·52214	(1) 133618	$\bar{2}$·1258663	·49567
8·0	(2) 919543	$\bar{3}$·9635719	·54067	(1) 103795	$\bar{2}$·0161764	·51440
8·5	(2) 698667	$\bar{3}$·8442702	·55793	(2) 799010	$\bar{3}$·9025520	·53192
9·0	(2) 526440	$\bar{3}$·7213487	·57403	(2) 609884	$\bar{3}$·7852474	·54832
9·5	(2) 393594	$\bar{3}$·5950485	·58907	(2) 461843	$\bar{3}$·6644946	·56369
10·0	(2) 292139	$\bar{3}$·4655890	·60314	(2) 347142	$\bar{3}$·5405074	·57812
10·5	(2) 215363	$\bar{3}$·3331705	·61634	(2) 259108	$\bar{3}$·4134813	·59169
11·0	(2) 157753	$\bar{3}$·1979765	·62872	(2) 192131	$\bar{3}$·2835965	·60446
11·5	(2) 114861	$\bar{3}$·0601730	·64037	(2) 141585	$\bar{3}$·1510184	·61650
12·0	(3) 831606	$\bar{4}$·9199174	·65134	(2) 103729	$\bar{3}$·0159007	·62787
12·5	(3) 598891	$\bar{4}$·7773476	·66168	(3) 755759	$\bar{4}$·8783833	·63861
13·0	(3) 429136	$\bar{4}$·6325947	·67145	(3) 547768	$\bar{4}$·7385967	·64878
13·5	(3) 306040	$\bar{4}$·4857783	·68069	(3) 395058	$\bar{4}$·5966612	·65841
14·0	(3) 217275	$\bar{4}$·3370092	·68944	(3) 283588	$\bar{4}$·4526884	·66755
14·5	(3) 153599	$\bar{4}$·1863894	·69774	(3) 202666	$\bar{4}$·3067816	·67623
15·0	(3) 108147	$\bar{4}$·0340137	·70562	(3) 144224	$\bar{4}$·1590370	·68448
15·5	(4) 758525	$\bar{5}$·8799697	·71311	(3) 102222	$\bar{4}$·0095440	·69234
16·0	(4) 530077	$\bar{5}$·7243392	·72024	(4) 721748	$\bar{5}$·8583858	·69982
18·0	(4) 122292	$\bar{5}$·0873994	·74559	(4) 173213	$\bar{5}$·2385796	·72653
20·0	(5) 269345	$\bar{6}$·4303083	·76681	(5) 395833	$\bar{6}$·5975124	·74896
22·0	(6) 570489	$\bar{7}$·7562471	·78480	(6) 867902	$\bar{7}$·9384709	·76805
24·0	(6) 116870	$\bar{7}$·0677045	·80025	(6) 183674	$\bar{7}$·2640466	·78448
26·0	(7) 232630	$\bar{8}$·3666661	·81365	(7) 376978	$\bar{8}$·5763164	·79876
28·0	(8) 451584	$\bar{9}$·6547386	·82538	(8) 753300	$\bar{9}$·8769679	·81129
30·0	(9) 857512	$\overline{10}$·9332404	·83573	(8) 147025	$\bar{9}$·1673901	·82236
32·0	(9) 159686	$\overline{10}$·2032659	·84493	(9) 281021	$\overline{10}$·4487391	·83221
34·0	(10) 292235	$\overline{11}$·4657329	·85316	(10) 527214	$\overline{11}$·7219871	·84104
36·0	(11) 526525	$\overline{12}$·7214189	·86056	(11) 972656	$\overline{12}$·9879592	·84899
38·0	(12) 935379	$\overline{13}$·9709876	·86726	(11) 176751	$\overline{12}$·2473610	·85619
40·0	(12) 164063	$\overline{13}$·2150106	·87335	(12) 316812	$\overline{13}$·5008010	·86273

v	$n=16$, $(m=7\cdot0)$ T_m	$\log T_m$	ρ	$n=17$, $(m=7\cdot5)$ T_m	$\log T_m$	ρ
0	·108541	$\bar{1}$·0355946	·00000	·104736	$\bar{1}$·0200973	·00000
·5	·107418	$\bar{1}$·0310755	·04158	·103735	$\bar{1}$·0159251	·03839
1·0	·104129	$\bar{1}$·0175736	·08265	·100798	$\bar{1}$·0034515	·07639
1·5	(1) 989122	$\bar{2}$·9952500	·12276	(1) 961175	$\bar{2}$·9828026	·11364
2·0	(1) 921208	$\bar{2}$·9643575	·16152	(1) 899866	$\bar{2}$·9541777	·14981
2·5	(1) 841820	$\bar{2}$·9252193	·19867	(1) 827629	$\bar{2}$·9178357	·18468
3·0	(1) 755454	$\bar{2}$·8782080	·23401	(1) 748306	$\bar{2}$·8740794	·21807
3·5	(1) 666386	$\bar{2}$·8237257	·26747	(1) 665643	$\bar{2}$·8232413	·24989
4·0	(1) 578346	$\bar{2}$·7621874	·29900	(1) 583002	$\bar{2}$·7656698	·28009
4·5	(1) 494319	$\bar{2}$·6940076	·32864	(1) 503175	$\bar{2}$·7017191	·30865
5·0	(1) 416478	$\bar{2}$·6195917	·35645	(1) 428292	$\bar{2}$·6317403	·33561
5·5	(1) 346202	$\bar{2}$·5393292	·38251	(1) 359812	$\bar{2}$·5560759	·36103
6·0	(1) 284178	$\bar{2}$·4535899	·40691	(1) 298576	$\bar{2}$·4750555	·38496
6·5	(1) 230527	$\bar{2}$·3627221	·42976	(1) 244902	$\bar{2}$·3889927	·40748
7·0	(1) 184949	$\bar{2}$·2670515	·45116	(1) 198694	$\bar{2}$·2981845	·42868
7·5	(1) 146852	$\bar{2}$·1668800	·47122	(1) 159555	$\bar{2}$·2029101	·44863
8·0	(1) 115476	$\bar{2}$·0624922	·49003	(1) 126891	$\bar{2}$·1034307	·46742
8·5	(2) 899799	$\bar{3}$·9541454	·50769	(2) 999978	$\bar{3}$·9999906	·48512
9·0	(2) 695155	$\bar{3}$·8420814	·52428	(2) 781299	$\bar{3}$·8928174	·50182
9·5	(2) 532749	$\bar{3}$·7265230	·53989	(2) 605512	$\bar{3}$·7821228	·51757
10·0	(2) 405206	$\bar{3}$·6076758	·55459	(2) 465697	$\bar{3}$·6681036	·53245
10·5	(2) 306006	$\bar{3}$·4857298	·56844	(2) 355585	$\bar{3}$·5509431	·54651
11·0	(2) 229541	$\bar{3}$·3608606	·58152	(2) 269657	$\bar{3}$·4308113	·55982
11·5	(2) 171092	$\bar{3}$·2332308	·59388	(2) 203173	$\bar{3}$·3078664	·57243
12·0	(2) 126762	$\bar{3}$·1029907	·60557	(2) 152144	$\bar{3}$·1822555	·58438
12·5	(3) 933855	$\bar{4}$·9702796	·61664	(2) 113270	$\bar{3}$·0541156	·59573
13·0	(3) 684269	$\bar{4}$·8352269	·62714	(3) 838637	$\bar{4}$·9235742	·60651
13·5	(3) 498830	$\bar{4}$·6979526	·63711	(3) 617661	$\bar{4}$·7907500	·61676
14·0	(3) 361883	$\bar{4}$·5585682	·64659	(3) 452641	$\bar{4}$·6557540	·62652
14·5	(3) 261323	$\bar{4}$·4171776	·65560	(3) 330133	$\bar{4}$·5186895	·63582
15·0	(3) 187879	$\bar{4}$·2738776	·66418	(3) 239692	$\bar{4}$·3796534	·64468
15·5	(3) 134511	$\bar{4}$·1287582	·67236	(3) 173275	$\bar{4}$·2387360	·65315
16·0	(4) 959188	$\bar{5}$·9819039	·68016	(3) 124745	$\bar{4}$·0960221	·66123
18·0	(4) 239135	$\bar{5}$·3786436	·70808	(4) 322639	$\bar{5}$·5087174	·69025
20·0	(5) 566373	$\bar{6}$·7531022	·73163	(5) 791078	$\bar{6}$·8982192	·71482
22·0	(5) 128428	$\bar{6}$·1086614	·75173	(5) 185341	$\bar{6}$·2679709	·73586
24·0	(6) 280542	$\bar{7}$·4479974	·76908	(6) 417561	$\bar{7}$·6207201	·75407
26·0	(7) 593293	$\bar{8}$·7732691	·78420	(7) 909265	$\bar{8}$·9586903	·76997
28·0	(7) 121964	$\bar{8}$·0862323	·79748	(7) 192177	$\bar{8}$·2837025	·78396
30·0	(8) 244535	$\bar{9}$·3883405	·80924	(8) 395609	$\bar{9}$·5972659	·79637
32·0	(9) 479519	$\overline{10}$·6808062	·81972	(9) 795506	$\overline{10}$·9006437	·80745
34·0	(10) 921831	$\overline{11}$·9646511	·82912	(9) 156640	$\overline{10}$·1949035	·81740
36·0	(10) 174078	$\overline{11}$·2407443	·83759	(10) 302660	$\overline{11}$·4809548	·82638
38·0	(11) 323468	$\overline{12}$·5098308	·84527	(11) 574882	$\overline{12}$·7595788	·83452
40·0	(12) 592316	$\overline{13}$·7725535	·85226	(11) 107511	$\overline{12}$·0314511	·84194

v	$n=18$, $(m=8\cdot0)$ T_m	$\log T_m$	ρ	$n=19$, $(m=8\cdot5)$ T_m	$\log T_m$	ρ	v
0	·101305	$\bar{1}$·0056315	·00000	(1) 981903	$\bar{2}$·9920686	·00000	0
·5	·100405	$\bar{1}$·0017568	·03566	(1) 973760	$\bar{2}$·9884518	·03327	·5
1·0	(1) 977613	$\bar{2}$·9901668	·07101	(1) 949793	$\bar{2}$·9776290	·06633	1·0
1·5	(1) 935323	$\bar{2}$·9709617	·10575	(1) 911341	$\bar{2}$·9596811	·09888	1·5
2·0	(1) 879633	$\bar{2}$·9443015	·13964	(1) 860475	$\bar{2}$·9347383	·13072	2·0
2·5	(1) 813572	$\bar{2}$·9103962	·17245	(1) 799784	$\bar{2}$·9029728	·16169	2·5
3·0	(1) 740448	$\bar{2}$·8694943	·20405	(1) 732135	$\bar{2}$·8645910	·19164	3·0
3·5	(1) 663549	$\bar{2}$·8218729	·23433	(1) 660427	$\bar{2}$·8198248	·22048	3·5
4·0	(1) 585903	$\bar{2}$·7678257	·26323	(1) 587387	$\bar{2}$·7689243	·24813	4·0
4·5	(1) 510101	$\bar{2}$·7076565	·29072	(1) 515406	$\bar{2}$·7121497	·27458	4·5
5·0	(1) 438197	$\bar{2}$·6416697	·31681	(1) 446443	$\bar{2}$·6497661	·29979	5·0
5·5	(1) 371677	$\bar{2}$·5701662	·34153	(1) 381978	$\bar{2}$·5820381	·32380	5·5
6·0	(1) 311486	$\bar{2}$·4934392	·36493	(1) 323018	$\bar{2}$·5092262	·34663	6·0
6·5	(1) 258090	$\bar{2}$·4117708	·38706	(1) 270137	$\bar{2}$·4315837	·36830	6·5
7·0	(1) 211559	$\bar{2}$·3254308	·40797	(1) 223540	$\bar{2}$·3493552	·38887	7·0
7·5	(1) 171662	$\bar{2}$·2346750	·42774	(1) 183136	$\bar{2}$·2627747	·40839	7·5
8·0	(1) 137958	$\bar{2}$·1397461	·44642	(1) 148616	$\bar{2}$·1720655	·42691	8·0
8·5	(1) 109868	$\bar{2}$·0408701	·46409	(1) 119520	$\bar{2}$·0774391	·44448	8·5
9·0	(2) 867486	$\bar{3}$·9382625	·48081	(2) 953007	$\bar{3}$·9790960	·46115	9·0
9·5	(2) 679396	$\bar{3}$·8321231	·49663	(2) 753745	$\bar{3}$·8772245	·47698	9·5
10·0	(2) 528007	$\bar{3}$·7226396	·51162	(2) 591565	$\bar{3}$·7720027	·49202	10·0
10·5	(2) 407368	$\bar{3}$·6099872	·52582	(2) 460891	$\bar{3}$·6635978	·50631	10·5
11·0	(2) 312126	$\bar{3}$·4943298	·53930	(2) 356588	$\bar{3}$·5521668	·51990	11·0
11·5	(2) 237586	$\bar{3}$·3758206	·55210	(2) 274067	$\bar{3}$·4378570	·53283	11·5
12·0	(2) 179723	$\bar{3}$·2546025	·56426	(2) 209318	$\bar{3}$·3208071	·54514	12·0
12·5	(2) 135148	$\bar{3}$·1308092	·57582	(2) 158908	$\bar{3}$·2011468	·55688	12·5
13·0	(2) 101057	$\bar{3}$·0045659	·58683	(2) 119949	$\bar{3}$·0789980	·56807	13·0
13·5	(3) 751604	$\bar{4}$·8759893	·59732	(3) 900483	$\bar{4}$·9544754	·57875	13·5
14·0	(3) 556146	$\bar{4}$·7451891	·60732	(3) 672491	$\bar{4}$·8276864	·58895	14·0
14·5	(3) 409513	$\bar{4}$·6122678	·61686	(3) 499726	$\bar{4}$·6987319	·59869	14·5
15·0	(3) 300138	$\bar{4}$·4773214	·62597	(3) 369579	$\bar{4}$·5677071	·60802	15·0
15·5	(3) 218998	$\bar{4}$·3404399	·63469	(3) 272083	$\bar{4}$·4347013	·61695	15·5
16·0	(3) 159114	$\bar{4}$·2017081	·64302	(3) 199434	$\bar{4}$·2997985	·62549	16·0
18·0	(4) 426386	$\bar{5}$·6318357	·67301	(4) 553014	$\bar{5}$·7427363	·65635	18·0
20·0	(4) 108112	$\bar{5}$·0338724	·69850	(4) 144848	$\bar{5}$·1609128	·68268	20·0
22·0	(5) 261465	$\bar{6}$·4174133	·72041	(5) 361286	$\bar{6}$·5578509	·70538	22·0
24·0	(6) 607057	$\bar{7}$·7832292	·73942	(6) 863767	$\bar{7}$·9363966	·72513	24·0
26·0	(6) 136019	$\bar{7}$·1336001	·75605	(6) 199010	$\bar{7}$·2988754	·74245	26·0
28·0	(7) 295393	$\bar{8}$·4704004	·77072	(7) 443821	$\bar{8}$·6472082	·75776	28·0
30·0	(8) 624007	$\bar{9}$·7951897	·78376	(8) 961607	$\bar{9}$·9829977	·77138	30·0
32·0	(8) 128611	$\bar{9}$·1092767	·79540	(8) 203046	$\bar{9}$·3075935	·78357	32·0
34·0	(9) 259281	$\overline{10}$·4137702	·80588	(9) 418930	$\overline{10}$·6221419	·79455	34·0
36·0	(10) 512409	$\overline{11}$·7096171	·81534	(10) 846494	$\overline{11}$·9276240	·80448	36·0
38·0	(11) 994563	$\overline{12}$·9976324	·82393	(10) 167836	$\overline{11}$·2248857	·81350	38·0
40·0	(11) 189899	$\overline{12}$·2785218	·83176	(11) 327085	$\overline{12}$·6305144	·82173	40·0

v	$n = 14, (m = 6\cdot0)$ T_m	$\log T_m$	ρ	$n = 15, (m = 6\cdot5)$ T_m	$\log T_m$	ρ
45·0	(14) 201327	$\overline{15}$·3039022	·88638	(14) 408904	$\overline{15}$·6116218	·87677
50·0	(16) 232949	$\overline{17}$·3672608	·89699	(16) 495369	$\overline{17}$·6949288	·88820
55·0	(18) 256825	$\overline{19}$·4096379	·90579	(18) 569635	$\overline{19}$·7555967	·89771
60·0	(20) 271938	$\overline{21}$·4344699	·91321	(20) 627065	$\overline{21}$·7973128	·90572
65·0	(22) 278236	$\overline{23}$·4444135	·91954	(22) 665171	$\overline{23}$·8229332	·91258
70·0	(24) 276418	$\overline{25}$·4415658	·92502	(24) 683461	$\overline{25}$·8347137	·91850
75·0	(26) 267676	$\overline{27}$·4276096	·92980	(26) 683076	$\overline{27}$·8344688	·92368
80·0	(28) 253465	$\overline{29}$·4039176	·93400	(28) 666312	$\overline{29}$·8236775	·92824
85·0	(30) 235300	$\overline{31}$·3716210	·93774	(30) 636154	$\overline{31}$·8035624	·93228
90·0	(32) 214619	$\overline{33}$·3316674	·94107	(32) 595862	$\overline{33}$·7751455	·93589
95·0	(34) 192688	$\overline{35}$·2848539	·94406	(34) 548642	$\overline{35}$·7392892	·93914
100·0	(36) 170554	$\overline{37}$·2318608	·94677	(36) 497426	$\overline{37}$·6967287	·94208
105·0	(38) 149029	$\overline{39}$·1732714	·94922	(38) 444729	$\overline{39}$·6480952	·94474
110·0	(40) 128704	$\overline{41}$·1095904	·95146	(40) 392586	$\overline{41}$·5939346	·94717
115·0	(42) 109966	$\overline{43}$·0412570	·95351	(42) 342549	$\overline{43}$·5347228	·94940
120·0	(45) 930371	$\overline{46}$·9686560	·95540	(44) 295717	$\overline{45}$·4708766	·95145

v	$n = 16, (m = 7\cdot0)$ T_m	$\log T_m$	ρ	$n = 17, (m = 7\cdot5)$ T_m	$\log T_m$	ρ
45·0	(14) 803560	$\overline{15}$·9050181	·86727	(13) 153206	$\overline{14}$·1852759	·85789
50·0	(15) 101869	$\overline{16}$·0080430	·87952	(15) 203137	$\overline{16}$·3077900	·87093
55·0	(17) 122127	$\overline{18}$·0868120	·88970	(17) 253788	$\overline{18}$·4044711	·88178
60·0	(19) 139718	$\overline{20}$·1452535	·89831	(19) 301633	$\overline{20}$·4794795	·89096
65·0	(21) 153608	$\overline{22}$·1864148	·90567	(21) 343596	$\overline{22}$·5360482	·89881
70·0	(23) 163196	$\overline{24}$·2127084	·91204	(23) 377345	$\overline{24}$·5767383	·90562
75·0	(25) 168296	$\overline{26}$·2260736	·91760	(25) 401433	$\overline{26}$·6036131	·91157
80·0	(27) 169081	$\overline{28}$·2280951	·92250	(27) 415297	$\overline{28}$·6183590	·91681
85·0	(29) 165990	$\overline{30}$·2200830	·92686	(29) 419152	$\overline{30}$·6223718	·92147
90·0	(31) 159637	$\overline{32}$·2031336	·93075	(31) 413828	$\overline{32}$·6168199	·92564
95·0	(33) 150720	$\overline{34}$·1781718	·93425	(33) 400581	$\overline{34}$·6026907	·92938
100·0	(35) 139955	$\overline{36}$·1459876	·93741	(35) 380914	$\overline{36}$·5808267	·93277
105·0	(37) 128014	$\overline{38}$·1072588	·94028	(37) 356412	$\overline{38}$·5519520	·93585
110·0	(39) 115497	$\overline{40}$·0625724	·94290	(39) 328620	$\overline{40}$·5166936	·93865
115·0	(41) 102906	$\overline{42}$·0124400	·94530	(41) 298950	$\overline{42}$·4755981	·94123
120·0	(44) 906379	$\overline{45}$·9573097	·94751	(43) 268624	$\overline{44}$·4291450	·94359

v	$n = 18, (m = 8\cdot0)$ T_m	$\log T_m$	ρ	$n = 19, (m = 8\cdot5)$ T_m	$\log T_m$	ρ
45·0	(13) 284069	$\overline{14}$·4534243	·84862	(13) 513288	$\overline{14}$·7103608	·83947
50·0	(15) 393730	$\overline{16}$·5951990	·86243	(15) 743309	$\overline{16}$·8711692	·85403
55·0	(17) 512394	$\overline{18}$·7096040	·87394	(16) 100719	$\overline{17}$·0031105	·86617
60·0	(19) 632443	$\overline{20}$·8010215	·88367	(18) 129057	$\overline{19}$·1107799	·87646
65·0	(21) 746213	$\overline{22}$·8728627	·89202	(20) 157674	$\overline{21}$·1977601	·88528
70·0	(23) 846904	$\overline{24}$·9278341	·89925	(22) 184883	$\overline{23}$·2668974	·89293
75·0	(25) 929219	$\overline{26}$·9681180	·90558	(24) 209166	$\overline{25}$·3204903	·89963
80·0	(27) 989693	$\overline{28}$·9955003	·91116	(26) 229309	$\overline{27}$·3604209	·90554
85·0	(28) 102674	$\overline{29}$·0114601	·91612	(28) 244484	$\overline{29}$·3882499	·91080
90·0	(30) 104049	$\overline{31}$·0172367	·92055	(30) 254264	$\overline{31}$·4052852	·91550
95·0	(32) 103247	$\overline{33}$·0138774	·92454	(32) 258603	$\overline{33}$·4126343	·91973
100·0	(34) 100526	$\overline{35}$·0022776	·92815	(34) 257776	$\overline{35}$·4112424	·92356
105·0	(37) 962068	$\overline{38}$·9832059	·93143	(36) 252303	$\overline{37}$·4019233	·92704
110·0	(39) 906419	$\overline{40}$·9573292	·93443	(38) 242875	$\overline{39}$·3853820	·93022
115·0	(41) 841838	$\overline{42}$·9252285	·93717	(40) 230268	$\overline{41}$·3622339	·93313
120·0	(43) 771638	$\overline{44}$·8874138	·93969	(42) 215287	$\overline{43}$·3330187	·93581

v	$n = 20, (m = 9\cdot0)$ T_m	$\log T_m$	ρ	$n = 21, (m = 9\cdot5)$ T_m	$\log T_m$	ρ
0	(1) 953460	$\bar{2}$·9793026	·00000	(1) 927353	$\bar{2}$·9672450	·00000
·5	(1) 946044	$\bar{2}$·9759115	·03122	(1) 920562	$\bar{2}$·9640532	·02938
1·0	(1) 924189	$\bar{2}$·9657610	·06222	(1) 900527	$\bar{2}$·9544966	·05859
1·5	(1) 889032	$\bar{2}$·9489173	·09283	(1) 868221	$\bar{2}$·9386304	·08747
2·0	(1) 842340	$\bar{2}$·9254876	·12286	(1) 825169	$\bar{2}$·9165430	·11587
2·5	(1) 786348	$\bar{2}$·8956148	·15215	(1) 773313	$\bar{2}$·8883550	·14364
3·0	(1) 723555	$\bar{2}$·8594715	·18059	(1) 714845	$\bar{2}$·8542119	·17070
3·5	(1) 656529	$\bar{2}$·8172539	·20808	(1) 652050	$\bar{2}$·8142806	·19694
4·0	(1) 587726	$\bar{2}$·7691752	·23456	(1) 587143	$\bar{2}$·7687441	·22231
4·5	(1) 519350	$\bar{2}$·7154603	·25999	(1) 522152	$\bar{2}$·7177970	·24677
5·0	(1) 453252	$\bar{2}$·6563402	·28435	(1) 458819	$\bar{2}$·6616413	·27028
5·5	(1) 390883	$\bar{2}$·5920469	·30763	(1) 398550	$\bar{2}$·6004824	·29285
6·0	(1) 333283	$\bar{2}$·5228131	·32986	(1) 342394	$\bar{2}$·5345264	·31447
6·5	(1) 281103	$\bar{2}$·4488658	·35105	(1) 291057	$\bar{2}$·4639776	·33516
7·0	(1) 234653	$\bar{2}$·3704265	·37124	(1) 244927	$\bar{2}$·3890360	·35493
7·5	(1) 193958	$\bar{2}$·2877086	·39046	(1) 204125	$\bar{2}$·3098967	·37382
8·0	(1) 158825	$\bar{2}$·2009180	·40876	(1) 168558	$\bar{2}$·2267482	·39186
8·5	(1) 128899	$\bar{2}$·1102503	·42618	(1) 137966	$\bar{2}$·1397717	·40909

v	$n = 22, (m = 10\cdot0)$ T_m	$\log T_m$	ρ	$n = 23, (m = 10\cdot5)$ T_m	$\log T_m$	ρ
0	(1) 903278	$\bar{2}$·9558214	·00000	(1) 880985	$\bar{2}$·9449686	00000
·5	(1) 897030	$\bar{2}$·9528070	·02775	(1) 875211	$\bar{2}$·9421126	·02630
1·0	(1) 878575	$\bar{2}$·9437787	·05536	(1) 858139	$\bar{2}$·9335575	·05247
1·5	(1) 848756	$\bar{2}$·9287829	·08269	(1) 830505	$\bar{2}$·9193425	·07841
2·0	(1) 808900	$\bar{2}$·9078948	·10961	(1) 793472	$\bar{2}$·8995314	·10399
2·5	(1) 760704	$\bar{2}$·8812156	·13601	(1) 748532	$\bar{2}$·8742104	·12914
3·0	(1) 706105	$\bar{2}$·8488691	·16180	(1) 697406	$\bar{2}$·8434858	·15375
3·5	(1) 647140	$\bar{2}$·8109985	·18688	(1) 641920	$\bar{2}$·8074809	·17776
4·0	(1) 585817	$\bar{2}$·7677617	·21121	(1) 583892	$\bar{2}$·7663327	·20111
4·5	(1) 523996	$\bar{2}$·7193279	·23474	(1) 525036	$\bar{2}$·7201891	·22376
5·0	(1) 463313	$\bar{2}$·6658744	·25744	(1) 466881	$\bar{2}$·6692061	·24568
5·5	(1) 405119	$\bar{2}$·6075828	·27930	(1) 410719	$\bar{2}$·6135447	·26685
6·0	(1) 350458	$\bar{2}$·5446366	·30031	(1) 357576	$\bar{2}$·5533687	·28726
6·5	(1) 300068	$\bar{2}$·4772193	·32048	(1) 308207	$\bar{2}$·4888430	·30691
7·0	(1) 254397	$\bar{2}$·4055121	·33983	(1) 263107	$\bar{2}$·4201317	·32582
7·5	(1) 213645	$\bar{2}$·3296925	·35836	(1) 222534	$\bar{2}$·3473964	·34398
8·0	(1) 177801	$\bar{2}$·2499335	·37612	(1) 186550	$\bar{2}$·2707957	·36142
8·5	(1) 146691	$\bar{2}$·1664026	·39311	(1) 155055	$\bar{2}$·1904844	·37816

v	$n = 24, (m = 11\cdot0)$ T_m	$\log T_m$	ρ	$n = 25, (m = 11\cdot5)$ T_m	$\log T_m$	ρ
0	(1) 860265	$\bar{2}$·9346322	00000	(1) 840940	$\bar{2}$·9247650	·00000
·5	(1) 854007	$\bar{2}$·9319188	·02498	(1) 835951	$\bar{2}$·9221810	·02379
1·0	(1) 839054	$\bar{2}$·9237899	·04986	(1) 821179	$\bar{2}$·9144378	·04750
1·5	(1) 813352	$\bar{2}$·9102788	·07454	(1) 797195	$\bar{2}$·9015645	·07103
2·0	(1) 778825	$\bar{2}$·8914400	·09892	(1) 764906	$\bar{2}$·8836082	·09430
2·5	(1) 736797	$\bar{2}$·8673480	·12291	(1) 725492	$\bar{2}$·8606328	·11725
3·0	(1) 688802	$\bar{2}$·8380943	·14645	(1) 680328	$\bar{2}$·8327182	·13979
3·5	(1) 636481	$\bar{2}$·8037853	·16946	(1) 630895	$\bar{2}$·7999574	·16187
4·0	(1) 581489	$\bar{2}$·7645412	·19189	(1) 578702	$\bar{2}$·7624552	·18345
4·5	(1) 525402	$\bar{2}$·7204916	·21371	(1) 525202	$\bar{2}$·7203261	·20448
5·0	(1) 469650	$\bar{2}$·6717741	·23488	(1) 471728	$\bar{2}$·6736919	·22494
5·5	(1) 415462	$\bar{2}$·6185316	·25538	(1) 419450	$\bar{2}$·6226803	·24480
6·0	(1) 363841	$\bar{2}$·5609112	·27520	(1) 369337	$\bar{2}$·5674227	·26404
6·5	(1) 315545	$\bar{2}$·4990609	·29434	(1) 322146	$\bar{2}$·5080528	·28267
7·0	(1) 271100	$\bar{2}$·4331295	·31280	(1) 278423	$\bar{2}$·4447052	·30068
7·5	(1) 230815	$\bar{2}$·3632642	·33058	(1) 238514	$\bar{2}$·3775146	·31807
8·0	(1) 194810	$\bar{2}$·2896106	·34769	(1) 202588	$\bar{2}$·3066140	·33485
8·5	(1) 163046	$\bar{2}$·2123107	·36416	(1) 170661	$\bar{2}$·2321346	·35103

v	n = 20, (m = 9·0) T_m	$\log T_m$	ρ	n = 21, (m = 9·5) T_m	$\log T_m$	ρ	n = 22, (m = 10·0) T_m	$\log T_m$	ρ
9·0	(1) 103727	$\bar{2}$·0158923	·44276	(1) 111980	$\bar{2}$·0491413	·42552	(1) 120022	$\bar{2}$·0792612	·40937
9·5	(2) 827983	$\bar{3}$·9180216	·45854	(2) 901618	$\bar{3}$·9550228	·44122	(2) 974237	$\bar{3}$·9886645	·42494
10·0	(2) 655852	$\bar{3}$·8168059	·47357	(2) 720401	$\bar{3}$·8575743	·45620	(2) 784803	$\bar{3}$·8947607	·43984
10·5	(2) 515708	$\bar{3}$·7124038	·48789	(2) 571408	$\bar{3}$·7569462	·47051	(2) 627613	$\bar{3}$·7976919	·45410
11·0	(2) 402685	$\bar{3}$·6049655	·50154	(2) 450071	$\bar{3}$·6532809	·48418	(2) 498418	$\bar{3}$·6975933	·46775
11·5	(2) 312343	$\bar{3}$·4946324	·51456	(2) 352139	$\bar{3}$·5467137	·49724	(2) 393182	$\bar{3}$·5945937	·48082
12·0	(2) 240734	$\bar{3}$·3815376	·52698	(2) 273762	$\bar{3}$·4373725	·50973	(2) 308188	$\bar{3}$·4888155	·49334
12·5	(2) 184420	$\bar{3}$·2658069	·53884	(2) 211533	$\bar{3}$·3253784	·52168	(2) 240090	$\bar{3}$·3803749	·50535
13·0	(2) 140462	$\bar{3}$·1475587	·55018	(2) 162497	$\bar{3}$·2108460	·53312	(2) 185944	$\bar{3}$·2693826	·51685
13·5	(2) 106391	$\bar{3}$·0269046	·56101	(2) 124132	$\bar{3}$·0938835	·54407	(2) 143200	$\bar{3}$·1559431	·52789
14·0	(3) 801585	$\bar{4}$·9039496	·57137	(3) 943178	$\bar{4}$·9745935	·55456	(2) 109687	$\bar{3}$·0401561	·53848
14·5	(3) 600887	$\bar{4}$·7787930	·58129	(3) 712972	$\bar{4}$·8530727	·56462	(3) 835826	$\bar{4}$·9221157	·54865
15·0	(3) 448258	$\bar{4}$·6515284	·59079	(3) 536306	$\bar{4}$·7294127	·57427	(3) 633740	$\bar{4}$·8019114	·55841
15·5	(3) 332846	$\bar{4}$·5222436	·59990	(3) 401513	$\bar{4}$·6036999	·58353	(3) 478220	$\bar{4}$·6796280	·56780
16·0	(3) 246049	$\bar{4}$·3910221	·60864	(3) 299238	$\bar{4}$·4760171	·59242	(3) 359208	$\bar{4}$·5553462	·57682
18·0	(4) 705147	$\bar{5}$·8482798	·64025	(4) 885261	$\bar{5}$·9470711	·62469	(3) 109574	$\bar{4}$·0397078	·60966
20·0	(4) 190595	$\bar{5}$·2801112	·66733	(4) 246673	$\bar{5}$·3921214	·65245	(4) 314448	$\bar{5}$·4975487	·63803
22·0	(5) 489847	$\bar{6}$·6900603	·69076	(5) 652690	$\bar{6}$·8147068	·67654	(5) 855858	$\bar{6}$·9324017	·66272
24·0	(5) 120504	$\bar{6}$·0810025	·71120	(5) 165090	$\bar{6}$·2177214	·69761	(5) 222417	$\bar{6}$·3471685	·68437
26·0	(6) 285299	$\bar{7}$·4553002	·72916	(6) 401379	$\bar{7}$·6035543	·71618	(6) 554955	$\bar{7}$·7442575	·70350
28·0	(7) 652997	$\bar{8}$·8149113	·74507	(7) 942314	$\bar{8}$·9741958	·73265	(6) 133562	$\bar{7}$·1256830	·72050
30·0	(7) 145036	$\bar{8}$·1614772	·75925	(7) 214446	$\bar{8}$·3313183	·74736	(7) 311275	$\bar{8}$·4931442	·73570
32·0	(8) 313605	$\bar{9}$·4963832	·77196	(8) 474612	$\bar{9}$·6763387	·76056	(8) 704831	$\bar{9}$·8480850	·74937
34·0	(9) 661925	$\overline{10}$·8208091	·78341	(8) 102439	$\bar{9}$·0104667	·77247	(8) 155504	$\bar{9}$·1917413	·76172
36·0	(9) 136699	$\overline{10}$·1357668	·79379	(9) 216144	$\overline{10}$·3347424	·78327	(9) 335103	$\overline{10}$·5251784	·77292
38·0	(10) 276777	$\overline{11}$·4421306	·80322	(10) 446751	$\overline{11}$·6500660	·79311	(10) 706838	$\overline{11}$·8493201	·78314
40·0	(11) 550377	$\overline{12}$·7406606	·81184	(11) 906195	$\overline{12}$·9572215	·80210	(10) 146208	$\overline{11}$·1649723	·79249
45·0	(13) 905480	$\overline{14}$·9568790	·83044	(12) 156200	$\overline{13}$·1936814	·82152	(12) 263875	$\overline{13}$·4213988	·81272
50·0	(14) 136929	$\overline{15}$·1364948	·84572	(14) 246536	$\overline{15}$·3918803	·83750	(14) 434467	$\overline{15}$·6379568	·82938
55·0	(16) 193102	$\overline{17}$·2857859	·85848	(16) 361689	$\overline{17}$·5583351	·85087	(16) 662812	$\overline{17}$·8213903	·84334
60·0	(18) 256774	$\overline{19}$·4095503	·86931	(18) 498929	$\overline{19}$·6980384	·86222	(18) 948149	$\overline{19}$·9768768	·85521
65·0	(20) 324740	$\overline{21}$·5115354	·87860	(20) 652975	$\overline{21}$·8148964	·87198	(19) 128373	$\overline{20}$·1084742	·86541
70·0	(22) 393300	$\overline{23}$·5947241	·88666	(22) 816622	$\overline{23}$·9120210	·88044	(21) 165738	$\overline{22}$·2194212	·87427
75·0	(24) 458697	$\overline{25}$·6615258	·89373	(24) 981594	$\overline{25}$·9919319	·88787	(23) 205278	$\overline{24}$·3123418	·88205
80·0	(26) 517508	$\overline{27}$·7139170	·89996	(25) 113945	$\overline{26}$·0566961	·89442	(25) 245127	$\overline{26}$·3893913	·88892
85·0	(28) 566949	$\overline{29}$·7535374	·90551	(27) 128242	$\overline{28}$·1080305	·90026	(27) 283375	$\overline{28}$·4523623	·89504
90·0	(30) 605010	$\overline{31}$·7817629	·91047	(29) 140403	$\overline{30}$·1473763	·90548	(29) 318244	$\overline{30}$·5027600	·90052
95·0	(32) 630605	$\overline{33}$·7997576	·91494	(31) 149953	$\overline{32}$·1759553	·91019	(31) 348226	$\overline{32}$·5418610	·90545
100·0	(34) 643455	$\overline{35}$·8085179	·91899	(33) 156607	$\overline{34}$·1948119	·91445	(33) 372184	$\overline{34}$·5707580	·90993
105·0	(36) 644022	$\overline{37}$·8089005	·92267	(35) 160268	$\overline{36}$·2048455	·91832	(35) 389397	$\overline{36}$·5903922	·91400
110·0	(38) 633357	$\overline{39}$·8016485	·92603	(37) 161004	$\overline{38}$·2068368	·92186	(37) 399559	$\overline{38}$·6015806	·91771
115·0	(40) 612930	$\overline{41}$·7874107	·92911	(39) 159026	$\overline{40}$·2014672	·92511	(39) 402751	$\overline{40}$·6050371	·92112
120·0	(42) 584462	$\overline{43}$·7667565	·93194	(41) 154645	$\overline{42}$·1893352	·92809	(41) 399382	$\overline{42}$·6013887	·92426

v	n = 23, (m = 10·5) T_m	$\log T_m$	ρ	n = 24, (m = 11·0) T_m	$\log T_m$	ρ	n = 25, (m = 11·5) T_m	$\log T_m$	ρ
9·0	(1) 127824	$\bar{2}$·1066119	·39422	(1) 135364	$\bar{2}$·1315036	·38000	(1) 142628	$\bar{2}$·1542054	·36663
9·5	(1) 104550	$\bar{2}$·0193229	·40963	(1) 111514	$\bar{2}$·0473278	·39522	(1) 118291	$\bar{2}$·0729513	·38166
10·0	(2) 848705	$\bar{3}$·9287566	·42441	(2) 911806	$\bar{3}$·9599023	·40986	(2) 973855	$\bar{3}$·9884942	·39613
10·5	(2) 683985	$\bar{3}$·8350467	·43859	(2) 740225	$\bar{3}$·8693637	·42393	(2) 796072	$\bar{3}$·9009522	·41007
11·0	(2) 547421	$\bar{3}$·7383212	·45219	(2) 596801	$\bar{3}$·7758295	·43746	(2) 646307	$\bar{3}$·8104390	·42350
11·5	(2) 435213	$\bar{3}$·6387022	·46524	(2) 477987	$\bar{3}$·6794157	·45046	(2) 521273	$\bar{3}$·7170649	·43643
12·0	(2) 343801	$\bar{3}$·5363069	·47777	(2) 380394	$\bar{3}$·5802333	·46296	(2) 417768	$\bar{3}$·6209354	·44888
12·5	(2) 269927	$\bar{3}$·4312465	·48979	(2) 300877	$\bar{3}$·4783890	·47498	(2) 332776	$\bar{3}$·5221520	·46087
13·0	(2) 210682	$\bar{3}$·3236274	·50134	(2) 236584	$\bar{3}$·3739846	·48654	(2) 263519	$\bar{3}$·4208123	·47243
13·5	(2) 163512	$\bar{3}$·2135504	·51243	(2) 184977	$\bar{3}$·2671173	·49767	(2) 207496	$\bar{3}$·3170096	·48356
14·0	(2) 126215	$\bar{3}$·1011119	·52309	(2) 143840	$\bar{3}$·1578801	·50838	(2) 162492	$\bar{3}$·2108332	·49429
14·5	(3) 969178	$\bar{4}$·9864035	·53334	(2) 111266	$\bar{3}$·0463615	·51868	(2) 126581	$\bar{3}$·1023689	·50464
15·0	(3) 740478	$\bar{4}$·8695119	·54320	(3) 856340	$\bar{4}$·9326461	·52861	(3) 981067	$\bar{4}$·9916986	·51461
15·5	(3) 563015	$\bar{4}$·7505201	·55269	(3) 655865	$\bar{4}$·8168143	·53818	(3) 756659	$\bar{4}$·8789003	·52424
16·0	(3) 426096	$\bar{4}$·6295073	·56182	(3) 499969	$\bar{4}$·6989433	·54740	(3) 580830	$\bar{4}$·7640491	·53353
18·0	(3) 133871	$\bar{4}$·1266865	·59515	(3) 161616	$\bar{4}$·2084853	·58113	(3) 192973	$\bar{4}$·2854973	·56760
20·0	(4) 395282	$\bar{5}$·5969066	·62404	(4) 490547	$\bar{5}$·6906804	·61049	(4) 601562	$\bar{5}$·7792806	·59736
22·0	(4) 110578	$\bar{5}$·0436703	·64928	(4) 140930	$\bar{5}$·1490043	·63621	(4) 177348	$\bar{5}$·2488263	·62351
24·0	(5) 295038	$\bar{6}$·4698786	·67147	(5) 385787	$\bar{6}$·5863477	·65889	(5) 497746	$\bar{6}$·6970076	·64664
26·0	(6) 755007	$\bar{7}$·8779508	·69111	(5) 101189	$\bar{6}$·0051340	·67901	(5) 133735	$\bar{6}$·1262457	·66720
28·0	(6) 186174	$\bar{7}$·2699197	·70860	(6) 255510	$\bar{7}$·4074082	·69697	(6) 345615	$\bar{7}$·5385927	·68559
30·0	(7) 444126	$\bar{8}$·6475059	·72428	(7) 623602	$\bar{8}$·7949077	·71308	(7) 862578	$\bar{8}$·9357984	·70211
32·0	(7) 102843	$\bar{8}$·0121761	·73838	(7) 147611	$\bar{8}$·1691184	·72761	(7) 208624	$\bar{8}$·3193639	·71703
34·0	(8) 231841	$\bar{9}$·3651895	·75115	(8) 339875	$\bar{9}$·5313191	·74077	(8) 490438	$\bar{9}$·6905844	·73057
36·0	(9) 510074	$\overline{10}$·7076335	·76275	(9) 763162	$\overline{10}$·8826168	·75274	(8) 112353	$\bar{9}$·0505848	·74290
38·0	(9) 109762	$\overline{10}$·0404535	·77333	(9) 167485	$\overline{10}$·2239759	·76367	(9) 251389	$\overline{10}$·4003461	·75417
40·0	(10) 231460	$\overline{11}$·3644753	·78303	(10) 359949	$\overline{11}$·5562409	·77370	(10) 550464	$\overline{11}$·7407290	·76451
45·0	(12) 437114	$\overline{13}$·6405951	·80402	(12) 710858	$\overline{13}$·8517829	·79544	(11) 113612	$\overline{12}$·0554242	·78697
50·0	(14) 750396	$\overline{15}$·8752903	·82135	(13) 127173	$\overline{14}$·1043942	·81342	(13) 211706	$\overline{14}$·3257330	·80557
55·0	(15) 118992	$\overline{16}$·0755185	·83589	(15) 209524	$\overline{16}$·3212338	·82851	(15) 362245	$\overline{16}$·5590019	·82121
60·0	(17) 176455	$\overline{18}$·2466333	·84825	(17) 321975	$\overline{18}$·5078225	·84137	(17) 576648	$\overline{18}$·7601110	·83455
65·0	(19) 247081	$\overline{20}$·3928386	·85890	(19) 466126	$\overline{20}$·6685034	·85244	(19) 862854	$\overline{20}$·9359371	·84604
70·0	(21) 329226	$\overline{22}$·5174936	·86815	(21) 640846	$\overline{22}$·8067534	·86208	(20) 122368	$\overline{21}$·0876681	·85605
75·0	(23) 420073	$\overline{24}$·6233244	·87627	(23) 842162	$\overline{24}$·9253955	·87054	(22) 165586	$\overline{23}$·2190229	·86485
80·0	(25) 515008	$\overline{26}$·7125724	·88345	(24) 106354	$\overline{25}$·0267548	·87803	(24) 214984	$\overline{25}$·3324067	·87264
85·0	(27) 612495	$\overline{28}$·7871026	·88985	(26) 129648	$\overline{27}$·1127673	·88470	(26) 269045	$\overline{27}$·4298245	·87958
90·0	(29) 705477	$\overline{30}$·8484831	·89558	(28) 153131	$\overline{29}$·1850620	·89068	(28) 325810	$\overline{29}$·5129646	·88581
95·0	(31) 790760	$\overline{32}$·8980446	·90075	(30) 175801	$\overline{31}$·2450221	·89607	(30) 383056	$\overline{31}$·5832620	·89142
100·0	(33) 864821	$\overline{34}$·9369262	·90543	(32) 196713	$\overline{33}$·2938329	·90096	(32) 438477	$\overline{33}$·6419465	·89651
105·0	(35) 924934	$\overline{36}$·9661106	·90969	(34) 215039	$\overline{35}$·3325167	·90541	(34) 489869	$\overline{35}$·6900796	·90115
110·0	(37) 969282	$\overline{38}$·9864502	·91359	(36) 230124	$\overline{37}$·3619616	·90948	(36) 535284	$\overline{37}$·7285839	·90539
115·0	(39) 996990	$\overline{40}$·9986908	·91716	(38) 241515	$\overline{39}$·3829442	·91321	(38) 573148	$\overline{39}$·7582666	·90928
120·0	(40) 100806	$\overline{41}$·0034874	·92045	(40) 248970	$\overline{41}$·3961473	·91665	(40) 602334	$\overline{41}$·7798376	·91287

LAPLACE, *being Extracts from Lectures delivered by Karl Pearson.*

IN preparing a course of lectures in the spring of this year on the "Life and Work of Laplace," as part of a more general series on the History of Statistics and the Theory of Probability, I was astonished to find how meagre were the French accounts of Laplace's family, boyhood, education and personal opinions. I was still more surprised to discover statements made—often to his discredit—by the smaller historians of mathematics not only in this country, but abroad and even in France itself, which were most certainly in grave error.

In the biographies we are told that he was the son of a peasant, who dwelt in Beaumont-en-Auge in the department of Calvados in Western Normandy, that he went as a day-boy to the "military school" of Beaumont-en-Auge and afterwards became a "professeur provisoire" there; and that he owed his education to "the interest excited by his lively parts in some persons of position." Rouse Ball citing no authority, and probably having none, states that Laplace was son of a labourer and that "from a pupil he became an usher in the school at Beaumont*." We are even told that the biographers' ignorance of the great mathematician's early life was due to Laplace's false shame of springing from such humble origins. We are also informed that Laplace was a time-server seeking honours and political advancement by flattering the successive French rulers—again with no evidential proof. At least, our friends, the smaller historians, might have taken the trouble to inquire whether there was a "military school" in Beaumont-en-Auge before the year 1771 when Laplace left home for Paris, they might have ascertained the position of Laplace's father, and taken the pains to find out whether Poisson's statement that Laplace was educated at the University of Caen† was or was not correct! While the facts I have now the honour of publishing and which I owe to the great kind-

* *A Short History of Mathematics*, p. 383.

† Henry VI of England, not content with his colleges at Eton and Cambridge, blessed Caen with a University in 1436; to this University, the later one was a successor. Caen was the burial place of William the Conqueror, and from the earliest date his Castle was and is still a military centre for the district. In the eighteenth century it possessed four learned societies, two of which were royal foundations. The Caen Royal Academy started its publications in 1754, five years after Laplace's birth, and, except when its schools and academies were suspended for supporting the Girondists, has continued to be a centre of art, military and civil education, justice and administration for Lower Normandy. Its streets today are adorned with the statues of Laplace, Malherbe and Élie de Beaumont. A Jesuit College, ancient seminaries and schools existed in Caen in the eighteenth century beside the University and the military school. Indeed Caen was probably in Laplace's day the most intellectually active of all the towns of Normandy. It was here that Laplace was educated and was provisionally a professor. It was here he wrote his first paper published in the *Mélanges* of the Royal Society of Turin, Tome IV. 1766–1769, at least two years before he went at 22 or 23 to Paris in 1771. Thus before he was 20 he was in touch with Lagrange in Turin. He did not go to Paris a raw self-taught country lad with only a peasant background! In 1765 at the age of sixteen Laplace left the "School of the Duke of Orleans" in Beaumont and went to the University of Caen, where he appears to have studied for five years. The "École militaire" of Beaumont did not replace the old school until 1776.

ness of M. l'Abbé Simon and M. le Comte de Colbert-Laplace, the illustrious mathematician's great-great-grandson, dispel the crude misstatements as to Laplace's family and education*, I have ventured to add to them a few extracts from my

* The following unpublished details with regard to Laplace may fitly find a place here; they are contained in a letter of M. le Comte de Colbert-Laplace to the lecturer.

Orbec, 8 Rue Grande, 16 février 1929.

Monsieur, Vous me voyez très perplexe. L'incendie qui a détruit radicalement le château de Mailloc, en 1925, a anéanti tous mes papiers de famille. C'est avec une peine, que vous pouvez imaginer, que j'ai vu disparaître la correspondance de Lagrange et de Laplace que je voulais publier (Laplace y tenait certainement beaucoup, puisqu'il avait conservé le double de ses propres lettres), et deux grands volumes de correspondance avec les savants européens, dont beaucoup d'anglais.

J'ai cependant eu le bonheur de connaître, jusqu'en 1889, ma grand'mère, née en 1813, orpheline de mère à sa naissance, Angélique de Portes, marquise de Colbert-Chabanais, qui avait été élevée par ses grandsparents Laplace de 1813 à 1827 et avait fort bien connu Laplace.

Je tâcherai, Monsieur, de coordonner mes souvenirs, mais ce sont des souvenirs de souvenirs, et par conséquent ils peuvent bien avoir été déformés. Je possédais aussi des papiers relatifs à la famille Laplace (L'orthographe variable: Delaplace, de la Place, de Laplace s'y trouvait). C'étaient des comptes de tutelle, et des papiers relatifs à la propriété du Mérisier à Beaumont-en-Auge que je possède encore. Ces papiers dont une grande partie remontaient à 17— (1720, je crois), bien difficiles à lire, m'ont paru indiquer qu'à une certaine époque la famille Laplace avait fieffé à d'autres cette propriété. Laplace l'avait racheté à la Révolution, et nous la possédons depuis à la façon moderne des propriétés. C'est ce qui me rend très indécis sur le lieu de naissance de Laplace. Dans mon enfance, le fermier montrait un cabinet sombre, et peu confortable, dans la maison du Mérisier, comme étant le lieu de naissance de Laplace. Beaumont-en-Auge a apposé une plaque de marbre sur une maison du bourg, indiquant qu'elle était la maison natale de ce savant. Je ne déciderai pas. Je puis vous dire cependant, que j'ai vu rire mon regretté père de la tradition qui faisait naître mon aïeul dans un réduit obscur, alors que la maison possédait et possède encore des chambres confortables.

Je n'ai plus les noms exacts de son père, sa mère s'appelait Sochon. J'ai lu, dans un ouvrage de la Société historique de Lisieux, qu'elle appartenait à une famille ancienne et distinguée. Mais je ne sais pas au juste ce qui peut en être de cette vanité, à laquelle ni Laplace ni aucun de nous ne pouvaient donner d'importance. Pour moi, j'ai la certitude que la famille de Laplace était une bonne famille du pays, comme il y en a beaucoup encore. Je ne sais pas si l'on peut dire que c'était une famille de cultivateurs. De 1700 à 1750, les Laplace se sont occupés de culture de la terre, mais j'ai vu par les papiers que le père de Laplace s'occupait du commerce des cidres—ce qui n'exclue pas, à la vérité, l'occupation de cultiver la terre. Ce père était Syndic de Beaumont en 17—, ce qui indique qu'il était dans une bonne situation. Il y avait un oncle curé, j'ignore s'il était du côté paternel ou maternel [paternel, Louis de Laplace. Voyez la généalogie]; et c'est parce que la famille voulait un prêtre, que le jeune Pierre-Simon Laplace fit d'abord ses premières études en ce sens....

Une publication, *Revue illustrée du Calvados*, interrompue par la guerre, a publié en 1912 ou 1913 une étude sur le prieuré de Beaumont; on y parlait du séjour de Laplace. Peut-être, M. Morière, directeur du journal Le Lenovien, rue du Bouteiller à Lisieux, possède-t-il encore des exemplaires de ce numéro. En tout cas, Monsieur l'Abbé Simon, curé de Montreuil-en-Auge, président de la Société historique de Lisieux, a écrit quelques articles sur Laplace et Beaumont.

Vous me permettrez, Monsieur, de faire appel à mes souvenirs d'enfant. Nous sommes arrivés à un point de la vie de Laplace, où la précision n'est guère possible. Par suite de quelles circonstances Laplace est-il venu à Paris où a commencé sa brillante carrière? Voici, ce que ma grand'mère me racontait vers 1884, pendant un de ces longs trajets en voiture entre Lisieux et Mailloc qu'elle faisait deux fois par semaine. Muni d'une lettre de recommandation pour d'Alembert, (j'ai oublié de qui était cette lettre) Laplace se rendit à Paris. D'Alembert, qui devait être assailli par de nombreuses lettres de ce genre, le reçut assez mal, et pour s'en débarrasser lui remit un livre assez gros, peut-être ses derniers travaux, en lui disant de revenir quand il l'aurait lu. Laplace revint quelques jours plus tard, et trouva d'Alembert encore moins aimable que la première fois, et il ne lui cacha pas qu'il trouvait impossible qu'il ait pu lire et comprendre l'ouvrage prêté. Je ne me souviens plus par quelle transition d'Alembert fut amené à interroger le jeune Laplace sur cette lecture, le résultat fut qu'il fut d'abord étonné, puis intéressé. Il passait pour certain chez moi, que d'Alembert se serait à ce moment occupé de Laplace. Je ne suis pas au courant du curriculum vitae de mon aïeul jusqu'à la Révolution. Il s'était marié probablement peu de temps auparavant. Tous mes papiers étant brûlés, je ne puis faire que de conjectures. Sa femme, mon arrière-arrière-grand'mère était d'une famille de Besançon, de magistrature; et s'appelait de Courty de Romanges—encore sur ce point j'hésite et ne suis sûr que d'une chose, c'est que l'oncle Hercule de Courty, qui par son énergie a refait la fortune de la famille, devait être le frère de Madame de Laplace—car comment [autrement] aurait-il pu être le grand-oncle de ma grand'mère? Cette famille Courty nous avait laissé des portraits de parents—qui s'appelaient de Mollerat. Je donne ces renseignements qui pourront peut-être aiguiller vos recherches.

Il a été publié, par M. Marmottan, 15 mars, 1922, un volume des lettres de Madame de Laplace à la princesse Elisa, dont elle fut dame d'honneur. Le ménage Laplace eut deux enfants: (i) *Sophie de Laplace*, née en 1787, qui épousa le marquis de Portes, dont une fille, Angélique de Portes, mariée vers 1830 au

lecture notes wherein I have endeavoured to see in its true light the accusation of time-serving made against Laplace. I had been discussing in my lecture the charge made against Laplace, namely that he did not adequately acknowledge the work of other men; that it was unlikely that he could have failed to see either Thomas Wright of Durham's *Original Theory of the Universe*, 1726, or Kant's

M[lle] de Colbert-Chabanais, mon grand-père. (ii) *Émile de Laplace*, né 15 avril, 1789, élève de polytechnique et de l'École de Metz, qui mourut général (nommé en 1843), le 27 octobre 1874. Laplace était ami de Bailly. Si mes souvenirs sont exacts, il habitait alors à Paris, dans la maison qu'on appelle pavillon de Hanôvre, chez M. Arthur (? un anglais (tout à fait sous réserve)).

Quelles étaient ses idées à cette époque? Il dut probablement penser comme ses contemporains. Les idées de 1789 devaient correspondre à son idéal de justice. Comme me l'a dit mon père, Laplace était passionné de justice, et il aurait dit que sa créance en Dieu dérivait de cette idée de justice. Mais la Révolution évolua, et il se trouva en butte aux tracasseries. Ma tante, la D[sse] de la Rochefoucauld Doudeauville, me racontait comment une visite domiciliaire avait trouvé, dans la chambre de la petite bonne qu'ils avaient, une image de piété clouée au mur, et les ennuis qui en survinrent.

Toujours est-il que Laplace et les siens quittèrent Paris, et allèrent du côté de Melun, aux Mées (si j'ai bon souvenir). J'ai eu en ma possession plusieurs certificats de civisme ou de présence, qui lui furent délivrés. J'ai oublié la date exacte de ces documents maintenant détruits par le feu.

Je crois avoir entendu dire à mon père, que Bailly fut arrêté lorsqu'il se rendait chez Laplace. Comment, dans la suite, Bonaparte fit-il connaissance de Laplace, je l'ignore, mais ce qui me paraît certain, c'est qu'il y eut des rapports d'amitié entre ces deux hommes. Je possédais des petits cadeaux du G[al] Bonaparte qui semblent prouver ce que j'avance. Une livre de café moka, un cédrat, un cachemire, des bouteilles de vin de Constance, donnés par le G[al] à son retour d'Egypte. Comment encore, après le 18 brumaire, Bonaparte nomma-t-il Laplace ministre de l'Intérieur? C'était une fonction tout à fait en dehors des aptitudes du savant. Aussi Bonaparte par un billet du. .rendait-il Laplace à ses occupations scientifiques. Je pense que Bonaparte voulait flatter l'Institut, et faire tenir provisoirement une place qu'il destinait à son frère.

Laplace avait acheté de Rewbell une maison à Arcueil. Je l'ai habitée dans mon enfance. Il y avait un parc superbe, et la maison était très grande et fort belle. Berthollet était son voisin. Ils se réunissaient souvent avec d'autres savants, et de cette société sortirent quelques travaux, qui furent publiés sous le nom de Travaux de la Société de Arcueil. On y dînait en famille, un de ces savants M. Bouvard y tenait une place spéciale. Il faisait enrager ma grand'mère, parce qu'il avait la mauvaise habitude de cracher à terre—elle le grondait, puis on disait "Allons, M. Bouvard, la main aux dames," et en boîtant, M. Bouvard offrait la main à Madame de Laplace pour passer à table, où suivant l'usage d'alors, elle servait le potage à tout le monde. Je sais que M. Magendie venait aussi à Arcueil. A la mort de ma grand'mère en 1889, ne pouvant conserver cette propriété, elle fut vendue, suivant le désir exprimé par le G[al] Laplace, aux dominicains qui possédaient déjà la maison Berthollet et y avaient fondé un collège. La loi des congrégations a fait passer cette propriété dans les mains de l'État, qui a loti le parc magnifique. Je n'y suis jamais retourné.

De souvenirs savants, je ne puis en avoir. Il faut néanmoins que je vous raconte ce que ma grand'mère avait retenue. Laplace, paraît-il, était tout à fait contraire à l'idée, que les changements de la lune ont une influence sur la température. Ma grand'mère, qui disait tenir cette opinion de lui, y tenait fermement et me rabrouait s'il m'arrivait d'exprimer l'opinion que la lune allait changer le temps.

Mon père m'a raconté aussi, que c'était ma grand'mère, âgée de 13 ou 14 ans, qui avait déterminé l'orthographe du nom. Lorsque la nouvelle que le roi Louis XVIII avait nommé Laplace marquis arriva à Arcueil, Laplace était encore au lit, et sa petite-fille en allant lui dire bonjour, reçut de lui cette demande, "Voilà, qu'ils m'ont nommé marquis, Angélique, comment va-t-il falloir que je signe?"—"Mais bon-papa, c'est tout simple, marquis de Laplace."

J'ai pu sauver de l'incendie une vieille moutre divisée en 10 heures—et le chronomètre de Borda—de mon [arrière-arrière] grand-père.

Je m'excuse, Monsieur, d'avoir mis plus de deux mois, à écrire cette lettre, mais j'ai dû voyager. Je ne recommence même pas cette lettre, écrite en rappelant des souvenirs déjà vieux et qui manque de la forme et de la concision que j'aurais voulu lui donner. On peut trouver (je n'ai plus les titres exacts) des renseignements sur Laplace dans des œuvres de MM. Biot et Arago—ce dernier s'est montré un peu dur pour le vieillard qui l'avait accueilli—et que M. Andoyer a écrit un petit livre sur Laplace. Je vous envoie le discours de M. E. Picard.

Veuillez, Monsieur, avec toutes mes excuses, agréer l'expression de ma considération la plus distinguée. A. de Colbert-Laplace.

N.B. Dans ma bibliothèque de Mailloc se trouvaient le masque de Newton et, au-dessus, l'extrait d'une lettre de M. Davy répondant à l'envoi par Laplace de ses œuvres. Il y était dit "qu'elles avaient été placées sur la même planche que celles de Newton, ne pouvant leur faire un plus grand honneur."

[I have not discovered any work of Arago on Laplace beyond his *Report*, which speaks in the highest terms of Laplace's researches. After a vain search among the Paris booksellers for Andoyer's work, I wrote to inquire directly of him, but the illness followed by the most regrettable death of that savant has hindered any reply. K.P.]

Allgemeine Naturgeschichte und Theorie des Himmels, 1755* Yet I reminded my audience that the *Mécanique céleste* did not profess to be an original memoir, but a gigantic treatise on the mechanics of the universe, in which the author not only included all that was already known in his own day, but an immense amount of new matter. Is it, I continued, more reasonable to blame Laplace than to blame Euclid, many of whose propositions must have been known long before his day? Few of the treatise writers even of last century were like D. F. Gregory or E. J. Routh, and took the trouble to assign to their original discoverers the results they made use of! In this respect we may indeed approve the words of Gregory: "It has always appeared to me that we sacrifice many of the advantages and more of the pleasures of studying any science by omitting all reference to the history of its progress." How many English text-books of Algebra have been written in which no reference is made to Newton as the discoverer of the Binomial Theorem, or to Halley who first stated the Exponential Theorem? I do not desire wholly to excuse Laplace; he was distinctly worse in this respect than Lagrange, but not worse than the bulk of French writers even up to the present date. Nor can we suppose that by omitting authors' names Laplace intended to claim their results as his own. The audience for which his work was intended was really the very one that knew well what Lagrange, Euler, Clairaut and others had accomplished. Laplace, like Ptolemy, set about writing his *Almagest*, a great work that should embrace all that was known about the heavens in its author's day. So much, and so much especially of importance was original, that readers of a later day are apt to consider it *all* original, and express anger when they find it is not so. We expect a big man in mind to be a big man in heart, and it is a pity that Laplace was not more careful to be generous† to his compeers, marking off his own from other men's contributions to the mechanics of the planetary system; he would have lost no fame by doing so. He certainly did not set out to steal, but he followed the usage—if a bad one—of his nation. His treatment of his own countryman Legendre and of our English Thomas Young shows that he was far from careful not to wound the susceptibilities of men who had made very real contributions to knowledge.

The remarks which the *Mécanique céleste* calls forth apply as strongly, if not more so, to the *Théorie analytique des Probabilités*. Laplace put together all that was known of the subject in his day, and immensely added to and developed his material. But only those intimately acquainted with what Montmort, De Moivre, the Bernoullis, Condorcet and Lagrange had achieved, can fully grasp how much he owed to them not only for fundamental principles but for suggestions for further research.

The second matter wherein Laplace has been severely criticised is in relation to political affairs. Thus we find it asserted that overmastering vanity led him to

* The result has been that the Nebular Hypothesis is almost universally attributed to Laplace, although actually he only restated and developed it.

† For the real generosity of Laplace towards his friends and pupils the reader must turn to the memoirs of Biot and Poisson, and also to the graceful picture of Madame de Laplace and her husband as hostess and host to both students and savants at Arcueil.

seek office, and "souplesse"—a flexible obsequiousness—and to praise each successive French ruler. Here again we must judge neither hurriedly nor too harshly. Let us examine matters historically, and call as well for evidence. Let us note in the first place the words of Robespierre, namely that with the new order in France there was no need for men of science; let us remember that the new rulers had suppressed the *Académie des Sciences*, had condemned Condorcet to death, and had sent Lavoisier, Laplace's friend, to the guillotine; let us recall that Lagrange, Berthollet, and possibly even Laplace, were allowed to survive because they were useful to the Republic for the purposes of its munition factories and artillery schools. Let us consider the corresponding condition of science under Soviet rule in Russia today, where the Government is "purging" the St Petersburg Academy by ejecting obnoxious Academicians and by forcing for political reasons, not solely for their scientific reputation, its own nominees on that institution.

Laplace came young to Paris, and like the much older Condorcet himself may well have thought that great results would flow from the Revolution; many men—politically wiser than Laplace—also made that error. As soon as Laplace reached Paris he developed a definite purpose for his life, namely to compile his Mechanics of the Heavens; his fame grew rapidly with the years, and he became, outside France, one of the most famous Frenchmen of his day; to fulfil his mission he had to regard governments, and in a somewhat different sense they in turn had to regard him. He was a national asset, which could be destroyed as Lavoisier was destroyed or else must be used and honoured. I find nothing remarkable in successive French governments conferring posts and dignities upon him. The less so, as there is a closer link between science and the state in France than elsewhere, and many of his dignities followed a well-established routine. Let us take an illustration which has been cited as an instance of time-serving. Laplace's *Exposition du système du monde* was published in 1796, four years after the foundation of the Republic; it was written through the days of the reign of terror, and though the revolution was running to its end Laplace could not foresee that end. Laplace dedicated his work to the Council of the Five Hundred and it concluded with the sentences·

"Let us preserve with care, let us ever increase our stock .of this mighty knowledge, the delight of thinking beings. It has rendered important services to agriculture, to navigation and to geography. But the greatest benefit of the Astronomical Sciences is the dissipation of errors born from man's ignorance of his true relations to nature, errors the more grave in that the social order ought to rest entirely on these relations. Truth and Justice we find are their immutable foundations. Let us put far from us the dangerous maxim that sometimes it is useful to deceive or enslave mankind the better to ensure its happiness. Fatal experience has shown in all ages that these sacred laws are never infringed with impunity."

I admit that it is not clear how Laplace passes from the physical side of the universe to the moral side in social matters; there may indeed be a small element of rodomontade in the later sentences, but that they should be interpreted as the

words of a time-server seems to me a gross exaggeration of the facts. Laplace was at this time organising the *École polytechnique* and presiding over the commission to report to the Council of the Five Hundred on the progress of science (1796). Twenty-eight years later when Laplace in 1824, an old man, is accused of suppressing this peroration, he wrote as follows:

"Let us preserve with care, let us ever increase our stock of this mighty knowledge, the delight of thinking beings. It has rendered important services to navigation and geography*, but its greatest benefit has been the scattering of the fears that flow from the phenomena of the heavens, the destruction of errors which flow from our ignorance of our true relations to nature; errors and fears, which will promptly be reborn, if the torch of science should ever be again extinguished."

I think it unreasonable to assert any connection between the marquisate conferred on Laplace and this change of peroration. In the course of 28 years Laplace may well have learnt that the vague use—without definition—of such terms as "truth" and "justice," terms ever-changing with the atmosphere man attaches to them—was not philosophic, and to describe these terms as "immutable bases," when their interpretations are modified by every change in public opinion, was hardly worthy of a great man of science. But the whole accusation of time-serving seems to disappear, when we examine the editions of the *Exposition*, which appeared between 1796 and 1824. In the edition of 1799, Truth and Justice are cited no longer as "foundations" but as "immutable laws." In the third edition of 1808, although Napoleon is now in command, the word *Humanity* is added to *Truth* and *Justice* as one of the "immutable laws." Thus these words can hardly be held as a catering for the applause of the Revolutionists! In the fourth edition of 1813, all these words have disappeared; thus their disappearance can have nothing to do with the later marquisate (1817). The concluding words of his *Exposition* were clearly a great trouble to Laplace and he altered them with each edition.

We may now pass to other points in his political career. Napoleon, who had been educated at the Brienne Military School, was in 1784 transferred to the *École militaire* in Paris, and doubtless came in touch with Laplace, who had been appointed professor in that school in 1773. Further Napoleon received his commission as a sub-lieutenant of the artillery in 1786, Laplace having been appointed "examinateur" of the scholars of the royal artillery corps in 1785. Remembering that Napoleon prided himself on his mathematical knowledge, it is highly probable that he felt attraction and admiration for Laplace. When Napoleon came to be Consul and ultimately Emperor, his theory of government was that of a highly organised military state; the people should have equality and justice, indeed all that was good for them, but forced on them from above, not a product of their own activities. In order to carry out this idea of a highly organised state Napoleon chose men not by their birth, but by their ability, and made them servants of the state. Thus he appointed David state painter; to Cuvier and Laplace he delegated the reconstruction of the scientific and educational institutions of France. He chose what he held to be the

* Note agriculture has disappeared! Had Laplace recognised that it would be a long time before meteorology could be reduced to a true science?

ablest men, and if his judgment failed occasionally it did not often fail, but one of those failures was his belief that a great mathematician would be *ipso facto* a great administrator. One biographer tells us without citing any authority that Laplace repeatedly *begged* Napoleon to make him Minister of the Interior. Agnes Clark with no authority writes:

"But merely scientific distinctions by no means satisfied his ambition. He aspired to the rôle of a politician, and left a memorable example of genius degraded to servility for the sake of a riband and title. The ardour of his republican principles gave place, after the 18th Brumaire [1799], to devotion towards the first Consul, a sentiment promptly rewarded with the post of minister of the interior." (*Encycl. Brit.* Vol. XIV. p. 302.) Rouse Ball writes: "It would have been well for Laplace's reputation if he had been content with his scientific work, but above all things he coveted a decoration. The skill and rapidity with which he managed to change his politics as occasion required would be amusing if they had not been so servile. As Napoleon's power increased Laplace abandoned his republican principles (which had themselves gone through numerous changes, since they had faithfully reflected the opinions of the party in power) and begged the First Consul to give him the post of minister of the interior. Napoleon who desired the support of men of science accepted the offer*." Such writing of history without a *single* reference to sources is pitiable. As far as I am aware Laplace before, during and after the Revolution never expressed anywhere his political opinions. How then did Rouse Ball discover that they had gone "through numerous changes" and "faithfully reflected the opinions of the party in power"? Such statements published by a writer of one nation about one of the most distinguished men of a second nation, and wholly unsubstantiated by references, are in every way deplorable. Where did Rouse Ball's information come from? I believe it to be merely an exaggeration of a catch-penny character from the account of Agnes Clark, one of the most superficial writers that ever obscured the history of science. At the end of her account Miss Clark gives as her authorities the *Éloge* of Fourier, the Funeral Oration of Poisson, and the Report of Arago to the French Government on the works of Laplace. I know all these writings well, there is not a single word in them that justifies her defamation of Laplace! Miss Clark says that notices of Laplace's life are scanty, so they are, but that in itself is no reason for allowing a too facile pen to give full freedom to an inventive imagination! The *Éloge* spoken by M. de Pastoret in the Chamber of Peers on April 2, 1827 refers to Laplace's political career; it has apparently been overlooked by both Agnes Clark and Rouse Ball. It speaks well of Laplace's procedure both in the Senate and Chamber of Peers, it states that his speeches on the budget, on criminal instruction, and on the export of grain were always clear and illuminating. Where again, I ask, did these smaller English historians draw their characterisation of Laplace as a time-server and a futile politician?

Probably, I believe, from an article on Laplace by Augustus De Morgan in the *Penny Encyclopaedia* of 1835. That distinguished mathematician had a fatal bent

* *A Short History of Mathematics*, 1888, p. 390.

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BIOMETRIKA

A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS

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towards damaging the scientific and moral reputations of greater mathematicians. I need only cite his treatment of both Newton and Laplace.

De Morgan says that Laplace voted in 1814 for the deposition of Napoleon. It is indeed difficult to see what else any wise and patriotic Frenchman could do. Even De Morgan fully admits this, but he says that the suppression of the dedication to Napoleon of the *Théorie analytique des Probabilités* (which appeared in the first or 1812 edition of that work), when the second appeared in 1814 after the deposition of Napoleon, is *primâ facie* evidence of ingratitude and cowardice.

Now let us read the dedication carefully. It runs:

Sire, The kindness with which your Majesty has deigned to receive the homage of my *Treatise on the Mechanics of the Heavens* has inspired me with the desire of dedicating to you this work on the Calculus of Probabilities. This delicate calculus applies to the most important questions in life, which are for the most part only problems in probability. It ought for this reason to interest your Majesty, whose genius knows so well how to appreciate and worthily encourage all that can contribute to the public illumination and prosperity. I venture to ask you to agree to this new homage dictated by the most keen gratitude, and profound sentiments of admiration and respect, with which I am, Sire, the very humble and very obedient servant and faithful subject of your Majesty, Laplace.

However we may judge of Laplace's original rendering unto Caesar of that which is Caesar's, it is perfectly clear that no publisher in 1814 could be found, or if found would have been permitted, to reprint in Paris in the year of the Emperor's deposition that dedication! What Laplace's real wishes may have been we do not know, but whether he wished to reprint it or not, the Censor would most certainly not have permitted its republication. More than once in the course of his career De Morgan has erred in his judgments, because he failed to grasp that we cannot estimate the worth of a man without an understanding study of his environment. It is the more remarkable in this case because De Morgan himself takes a sound view of Laplace's *religious* opinions:

It is sometimes stated by English writers that Laplace was an atheist. We have attentively examined every passage which has been brought in proof of this assertion, and we can find nothing which makes either for or against such a supposition. It is easy, with an hypothesis, to interpret passages of an author; but we are quite convinced that a person reading Laplace for philosophical information would meet with nothing which could either raise or solve a question as to the writer's opinions on the fundamental point of natural religion, unless it had been put into his head to look. If those who make the assertion have any private grounds for it they should produce their evidence; but the assertion, whether considered with reference to the individual, or to the public before which it is made, should not be hazarded merely because a writer who is investigating such points as can be determined by experiment and analysis does not introduce his opinions on a question which cannot be submitted to calculation. An attempt to explain how the solar system might possibly have arisen from the cooling of a mass of fluid or vapour is called atheistical, because it attempts to ascend one step in the chain of causes; the *Principia* of Newton was designated by the same term, and for a similar reason. What Laplace's opinions were we do not know; and it is not fair that a writer who, at a time of perfect license on such matters, has studiously avoided entering on the subject, should be stated as of one opinion or the other, upon the authority of a few passages of which it can only be said (as it could equally be said of most mathematical works) that they might have been written by a person of any religious or political sentiments whatever.

If De Morgan had applied his own statement as to expression of religious opinions to a consideration of Laplace's political views for which it certainly holds as fully, then undoubtedly he would not have given rise by the tone of his article to such writings as those of Agnes Clark and Rouse Ball who assert that Laplace changed his views with every change of government in France. Laplace has nowhere expressed any political views whatever, for the dedication of a book to the head of a state cannot be looked upon as an expression of political sentiments. The permission to dedicate a new and important work to the sovereign was, in that day, equivalent to the statement that the book was approved by the state and it thus formed a much desired and excellent publisher's advertisement.

That Laplace failed as Minister of the Interior may be quite true and not in the least to Laplace's discredit. What was needed in 1799 was a firm hand, and a head which would not consider minor points of justice or duty, but act promptly and forcibly. It needed a soldier, and one who would take his orders from Napoleon as from his commanding officer. That was the essential reason why Napoleon after six weeks' experience replaced Laplace by his brother Lucien Buonaparte. In times of turmoil, when the success of the Consulate hung in the balance, Laplace was no more fitted than Condorcet to take a leading part. We may consider that it was a bad mistake for Laplace to accept the office, but there is no ground to suppose that he pressed ("repeatedly begged") Napoleon to give him the post.

Years afterwards* at St Helena (*Mémoires de Sainte Hélène*) Napoleon thus described the incident: "Mathematician of the highest rank, Laplace was not long in showing himself an extremely poor administrator. From his first actions I realised that I had deceived myself. He sought everywhere for subtleties, had only problematic ideas, and carried the spirit of the 'indefinitely small' into administration." Yet if Napoleon had failed in his judgment of Laplace as an administrator, it did not modify his admiration of him as a mathematician. In 1802 Laplace dedicated to General Buonaparte the first volume of the *Mécanique céleste*, and Napoleon in thanking him spoke as a mathematician when he said that the first six months of freedom he had he should devote to reading the splendid work. After reading some chapters he wrote again regretting that circumstances had forced him into a career so far from that of science. Again three years later in 1805 when in Milan he wrote: "*La Mécanique céleste* appears to me destined to give new fame to the century in which we live." Lastly in 1812 when the *Théorie analytique des Probabilités* reached him at Witepsk on his march to Moscow, he wrote: "There was a time when I should have read with interest your Treatise on the Calculus of Probabilities. Today I must limit myself to expressing the satisfaction which I feel whenever I see you producing new works which render more perfect and advance further the first of the sciences, thus contributing to the lustre of our nation. The advancement and the perfecting of mathematics are bound up with the prosperity of the state."

* Rouse Ball calls it "Napoleon's memorandum on the subject," as if it had been given at the time as a reason for dismissal, *loc. cit.* p. 390.

I do not think these were merely formal and therefore idle expressions of thanks; they were evidence that he felt—after himself—Laplace to be in the eyes of Europe the chief honour of his nation. One other anecdote about the *Mécanique céleste* has survived. Napoleon meeting Laplace said to him: "M. Laplace, they tell me you have written this large volume on the system of the universe without ever mentioning its Creator." Laplace drew himself up and said: "I had no need of such an hypothesis." This does not sound like a time-server! Napoleon mentioned the incident to Lagrange, and the the latter exclaimed: "Ah, it is a beautiful hypothesis, because it reaches so far."

In this respect another saying of Laplace's may be cited, far less boastful than it appears: "Give me matter and I will create the universe." Laplace saw that adequate knowledge of an element of matter would like Tennyson's full understanding of the flower in the crannied wall explain all in all. Laplace's dying words show that he was no vain boaster, and how the words "Give me matter" are to be interpreted; those around his bed were recalling to him the great discoveries he had made in life, Laplace replied: "What we know is but a little thing; what we are ignorant of is immense."

The last days of Laplace have been briefly described by Fourier who writes:

He had contracted the habit of excessive application so harmful to health, so necessary when studies are profound; nevertheless he did not experience any enfeeblement until the last two years of his life. At the commencement of the illness to which he succumbed, an instant of delirium was observed with fear. The sciences still occupied his mind. He spoke with unaccustomed fire of the movements of the stars, and afterwards of a physical experiment which he said was crucial, announcing to those he believed to be present that he would soon make a communication to the Academy on these problems. His strength diminished more and more. His doctor [the famous physiologist Magendie], who by his talents and by the care which friendship inspired in him, merited Laplace's entire confidence, watched by his bedside. M. Bouvard, his collaborator and his friend, never for a moment quitted him.

"Surrounded by a beloved family, under the eyes of a wife whose tenderness had aided him to support the trials inseparable from life, whose amenity and grace had shown him the worth of domestic happiness"—the great mind parted from its mortal frame, and according to Fourier "returned to the heavens"—perhaps the most fitting place for the genius who had timed the courses of the stars in their paths.

Personally I prefer to quote the lines with which Virgil opens the second book of his *Georgics*, lines which Laplace so highly appreciated that he placed them at the head of his *Mécanique céleste*:

Ye muses, beloved beyond all else, whose sacred emblems I bear, penetrated as I am by ardent love, take me to yourselves, and show me the pathways of heaven, and the stars that traverse them*.

This the Muses accomplished for Laplace, and by their grace his genius may live immortal, even as Calypso, the fair nymph, promised immortality to Odysseus after his toils, should he be content to remain faithfully with her.

* Me vero primum, dulces ante omnia Musae,
Quarum sacra fero, ingenti perculsus amore,
Accipiant, coelique vias ac sidera monstrant. *Georgics*, II, ll. 475-7.

Mrs Somerville's final summary of the *Mécanique céleste* may be cited here, it still remains true (*Mechanism of the Heavens*, 1831, p. 2):

Tables of the motions of the planets, by which their places may be determined at any instant for thousands of years, are computed from the analytical formulae of La Place. In a research so profound and complicated, the most abstruse analysis is required, the higher branches of mathematical science are employed from the first, and approximations are made to the most intricate series. Easier methods and more convergent series may be discovered in process of time, which will supersede those now in use; but the work of La Place, regarded as embodying the results of not only his own researches, but those of so many of his illustrious predecessors and contemporaries, must ever remain, as he himself expressed it to the writer of these pages [Mary Somerville], a monument to the genius of the age in which it appeared.

It is the *Almagest* of a century ago, and Laplace's own description of it you will note is far from claiming all its results as the product of his own brain.

One word, I think, may be safely added about a strange incident connected with part of Laplace's mortal remains, namely his brain. Magendie, his physician, must have held an autopsy and removed in the course of it Laplace's brain. No report that I can hear of was ever published about the results of the autopsy. But about fifteen months after Laplace's death, on June 16, 1828, a paper was read by Magendie before the *Académie des Sciences*; it is entitled "Mémoire physiologique sur le cerveau." The theme of this memoir is that the less cerebro-spinal fluid in the brain the greater is the intelligence. In other words the less fluid the more thinking matter. Towards the end of this memoir occurs the following paragraph:

I once found myself under the sad necessity of examining the brain of a man of genius, who died at an advanced age [78], but when still enjoying the fulness of his intellectual faculties. The sum of cerebro-spinal fluid was not more than two ounces, and the cavities of the brain contained at most a dram.

There is no mention of Laplace's name, and although I have searched French literature I can find no further details of either the autopsy or the brain of Laplace. Here the matter might have rested had not Miss M. Tildesley in 1927 brought me a remarkable letter with which she had been entrusted by Miss Helen Hunter Baillie whose name indicates her relationship to Mrs Joanna Baillie and to her two brothers William and John Hunter, famous authoress and famous anatomists.

The letter dated only "Hampstead, Monday, 1834" is from Joanna Baillie to her great-niece Miss Sophy Milligan, and contains the following important paragraph:

My dear Sophy...Dr Somerville told us not long ago a whimsical circumstance regarding the head of La Place the famous French Astronomer. Some Ladies and Gentlemen went one day to the house of Majendie [*sic!*] the great anatomist to see the brains of this Philosopher, which they conjectured must be of a very ample size, and seeing a preparation on the table answering their expectation they were quite delighted. "Ah! see what a superb brain, what organs, what developments! This accounts completely for all the astonishing power of his intellect, etc." Majendie, who was behind them and overheard all, stepped quietly forward and said: "Yes, that is indeed a large brain, but it belonged to a poor idiot, who when alive scarcely knew his right hand from his left. This, Ladies and Gentlemen" (handing to them a preparation of a remarkably small brain), "this is the brain of Laplace." Dr Somerville was told this anecdote by Majendie himself......Your Affectionate Aunt, J. Baillie.

"Dr Somerville" can scarcely be other than the physician, fellow of the Royal Society, and husband of Mary Somerville, the learned lady who studied Newton's *Principia* in the original, was the friend and correspondent of Laplace, and paraphrased in her *Mechanism of the Heavens* his *Mécanique céleste*. There is accordingly no doubt that Magendie was in possession of Laplace's brain 6 or 7 years after his death, and that this is the brain to which he referred in his "Mémoire physiologique sur le cerveau," probably written in the year of Laplace's death.

I have tried in vain to ascertain in Paris what became of Magendie's collections when he died in 1855. Probably they were sold like his books. Such is the second chapter in the history of Laplace's brain. I now turn to the third stage in this history. I published Joanna Baillie's letter in *Nature* asking if any one knew what had become of Laplace's brain. I received a strange answer. Let me digress for a moment. I well remember as a boy, perhaps I was 10 to 12 years old, a mysterious Museum near the foot of Regent Street, I think near Glasshouse Street, an Anatomical Museum; if I remember rightly, it had a skeleton or the model of a tailed man in the window. Such things had a certain fascination for me, but the mystery of the place was increased by a strict injunction from my parents never to go inside. Under the circumstances, I think, most boys would certainly have gone inside. I, as fortune would have it, did not. I don't think it was because I was a good boy, because I was not; but rather because I was a coward and, although curious, had not the courage to face the contents of that Museum. A different type of parent and a different boy led to the discovery I am about to communicate to you. I received a letter from Mr A. B. Bence-Jones dated June 16, 1927 from 11 King's Bench Walk, Temple:

Dear Sir,

The Brain of Laplace.

I am much interested in your letter printed in *Nature*, 16 April 1927 on this subject, but I have nothing to suggest except very indirect evidence. As a boy I recall a visit with my Father (who died in 1873) to Kahn's or Kühn's Museum in Glasshouse Street, Regent Street, and my attention was directed to a glass vessel said to contain the brain of Laplace, but I cannot recall any mention of Magendie's name.

I do not know if any record or catalogue of this Anatomical Museum exists. Probably some one at the London Museum, Lancaster House, S.W. 1 would know, and I regret that I cannot make inquiry there, just now.

I need only add that my Father was Dr Henry Bence-Jones, F.R.S., and was Secretary of the R. Institution and author of Faraday's *Life and Letters*.

I am, dear Sir, Yours faithfully,

A. B. Bence-Jones.

In a second letter Mr Bence-Jones says that he must insist on Kahn's Museum being in Glasshouse Street: "My one visit to Glasshouse Street impressed me much and in days not later than the seventies, I often observed the place, and shuddered at my recollection of its nature. It was revolting to a boy."

Now Magendie died in 1855. Professor C. Richet kindly tells me that Magendie's

books were sold, and that he has at the present time certain of them. It seems probable therefore that his collections with their preparations were also sold. Now how did Laplace's brain come into Kahn's Museum? Joseph Kahn was a doctor of medicine of the University of Vienna—he set up an anatomical museum and at first appears to have moved the collection up and down England, exhibiting both in Newcastle and London. He started apparently in this country about 1851, and catalogues of the contents of his museum were published in London, 1851, Newcastle-upon-Tyne, 1852, London, 1853 and later dates. *No* catalogue appears to have been published after the date 1855 or 1856 at which we may suppose Magendie's collections to have been purchased. I see no reason to believe that originally Kahn's Museum may not have been what it professed to be—a museum for the study of anatomy—but its proprietor soon found that the shillings rolled in from an inquisitive lay public, and accordingly Dr Kahn started introducing monstrosities of all types approaching near to those of the showman at village fairs. Kahn's residence at one time was 17 Harley Street and one may suppose he then desired to build up a consultant practice. The Museum was at 315 Oxford Street in 1851, in Coventry Street, Leicester Square in 1856, in 1864 at 3 Tichborne Street, Haymarket, where apparently Joseph Kahn resided. In the following years 1865, 1866 it goes on under George Kahn, M.D. at the same address. Probably George was a son of Joseph. In the *Directory* for 1867 there is no occupant given for the house No. 3 Tichborne Street. The Kahns and their museum seem to be wanting in the later *Directories*. The ultimate source of this disappearance was probably due to the medical journal—the *Lancet*—which on June 3, 1865 (p. 600) published an article inveighing against "anatomical museums" and calling for measures to be taken for their suppression. The matter attracted considerable public attention, but nothing appears to have been done till the police at the instance of the Society for the Suppression of Vice in March 1873 seized Kahn's anatomical models, and on application by the Solicitor of the Society these models were destroyed at the Marlborough Street Police Court on Dec. 18, 1873. The Solicitor for the Society said that the museum was closed and the models not seized by the police packed up to be sent abroad. Kahn himself appears to have been tried on January 2, 1874, but I am unaware of what was the result.

It seems highly improbable that the police would remove exhibits like the brain of Laplace, which under no circumstances could be interpreted as conducive to immorality, and there must have been many similar exhibits. Assuming as seems highly probable that Kahn purchased Magendie's preparations then the appearance of Laplace's brain in Glasshouse Street becomes explicable. Granted this, it would be of great interest to ascertain the present locus of the remainder of Kahn's collection. Probably finding London no longer profitable Kahn retreated to his native land or to Germany, taking with him his unseized models, his tailed men, and the still unstudied brain of Laplace. I have not hitherto succeeded in tracing Kahn or his collections. Dr G. M. Morant kindly communicated to me a curious point; he told me that when he was in Munich a few years back advertisements were posted about the streets, announcing the arrival and sojourn for a time in Munich of a

great show—an anatomical museum. I wonder if this was Kahn's original collection, and if so whether it still contains the brain of Laplace—the brain of possibly the greatest mathematician of the ages travelling about the continent in a showman's van!

Imperial Caesar, dead and turned to clay,
May stop a hole to keep the wind away.

The rest of Laplace's mortal remains were buried at Paris in the Père Lachaise cemetery. There they remained for 61 years until 1888, when they were exhumed in fulfilment of the desire of his son General de Laplace (who died in 1874 at the age of 84) and taken to the family estate of Saint Julien de Mailloc, a small hamlet between Lisieux and Orbec in Calvados. On the bye road to these places is a Greek temple with a bronze urn containing the heart of Laplace, and inscriptions commemorating the birth and death of Laplace and the dates of publication of his chief works*. Other members of the family are buried in this temple. The monument from Père Lachaise was given by the Laplace family at the same time to the commune of Beaumont-en-Auge, where it was re-erected in the cemetery.

In June 1871 the Laplace sanctuary at Arcueil which had escaped the Prussians was raided by a band of ruffians from the Mouffetard district. The manuscripts of the great mathematician were thrown into the river Bièvre, from which that of the *Mécanique céleste* was subsequently fished out. The library which was rich in rare books, souvenirs and works of art was looted and devastated (see *Nature*, Vol. IV. p. 108), a sorry ending to Madame Laplace's piety†! Probably owing to this occurrence the remaining personal relics and papers of Laplace were transferred to Saint Julien de Mailloc, but the family chateau at de Mailloc was completely destroyed by a fire on Dec. 11, 1925, and thus perished the whole of the *Laplaciana*. I must confess that I feel personally some pleasure in seeing and handling the relics of the great men of our earth. It may be a silly morbid pleasure akin to the veneration some practise for the bones of saints. Still I could have wished the *Laplaciana* preserved at Arcueil, as one might have hoped that the *Newtoniana* could have been collected at Woolsthorpe, like the *Galtoniana* in this building‡. The books, papers and personal relics of Laplace were destroyed by fire; portions of the library of Newton, which would have told us so much of the writers who had helped to form his mind, and which had been for two centuries preserved unknown to the world, were privately sold only the year before last, and the books with the book plate "Philosophemur" and with Newton's autographs and notes appeared unexpectedly in half-a-dozen different booksellers' catalogues to be dispersed over the world into the collections of the wealthy, who had money to buy them; the remainder is now on sale in the hands of a London bookseller. Perhaps on the whole the history of science would profit, if we still had some of the mediaeval veneration, if not for

* *Nature*, April 2, 1927.

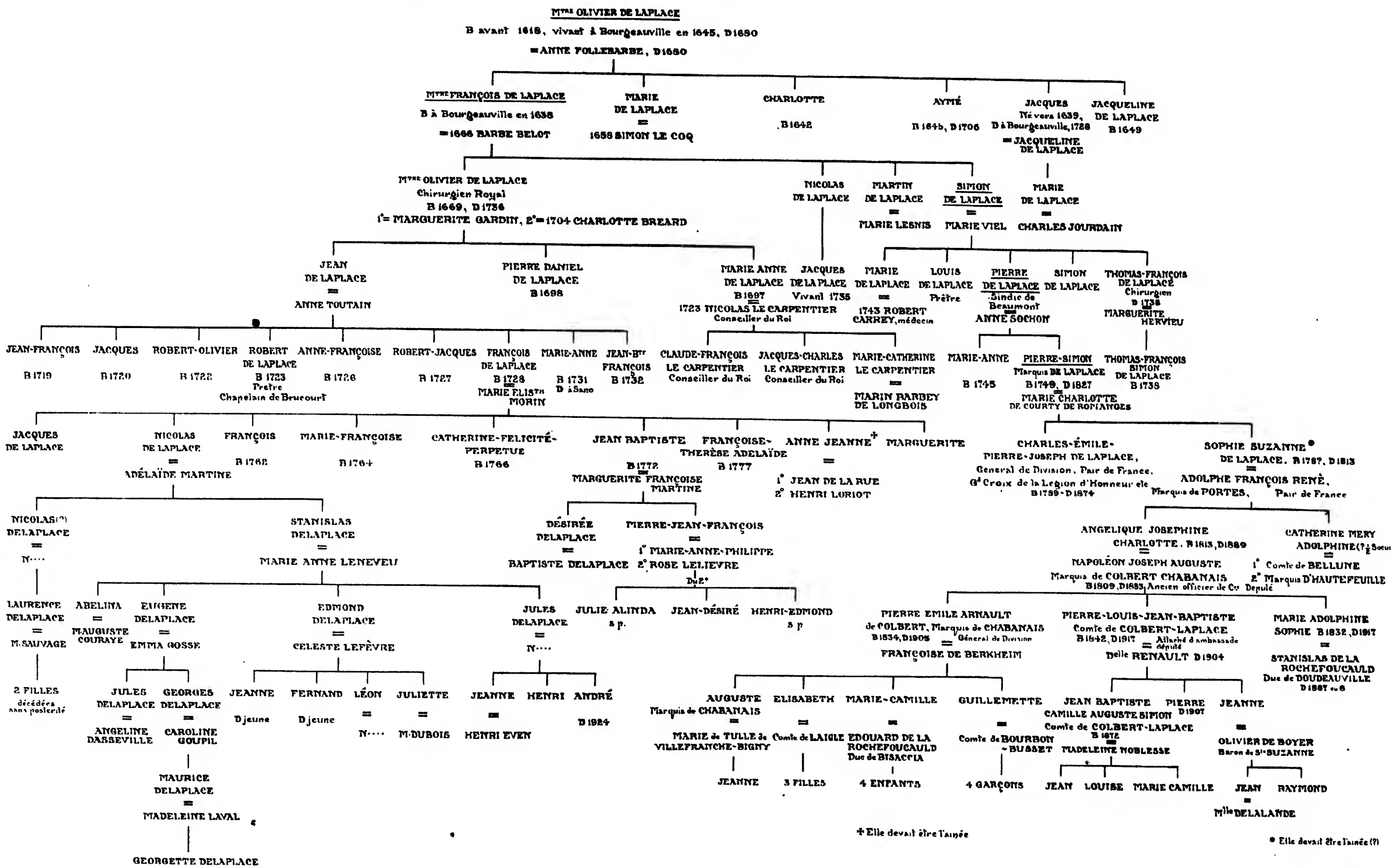
† This has reference to Madame Laplace's proposal to sell Arcueil (discussed in an unpublished part of these lectures) in order to provide funds for the republication of her husband's works, which were out of print and very scarce. The sale was prevented by Arago's *Report* to the French Government, which then undertook to issue a national edition of Laplace's works.

‡ The lectures were delivered in the Galton Laboratory of the University of London.

the bones of saints, at least for the papers and books which are the tools of genius. We know so little of the life, we know so little of the methods of research of Laplace, that a complete destruction of all his papers and relics, before any real life of him has been written, is indeed a lamentable loss.

I have told you all that I have been able to gather of this great Frenchman's life and character. The man who in the first quarter of the nineteenth century appeared as a giant among the intellects of that day, who to our own generation still stands out as one of the greatest mathematicians of all ages, lacks up to the present a critical biographer, who will give him true characterisation as man and as scientist. What I have put before you, however inadequately, may perhaps suffice to warn you against accepting too readily the statements—unauthenticated by documentary evidence—of minor writers on the history of mathematics. The history of science, after becoming an academic study, seems to have dropped (in the modern spirit) scholarly investigation for the methods of journalism.

GÉNÉALOGIE DE LA FAMILLE DE LAPLACE



✝ Elle devait être l'aînée

● Elle devait être l'aînée (?)

To face p. 217

Corrigendum, Biometrika, Vol. XXI, p. 160.

KARL PEARSON and C. H. USHER: "Albinism in Dogs."

In footnote under heading No. 3, line 2 *for* "Darkly pigmented epithelial layers of retina" *read* "Darkly pigmented epithelial layers of iris."

LES ORIGINES DE LAPLACE: SA GÉNÉALOGIE,—SES ÉTUDES.

PAR L'ABBÉ G. A. SIMON.

LES notices sur l'illustre mathématicien Laplace fourmillent d'erreurs, en ce qui concerne ses origines et ses débuts. On affirme qu'il appartenait à une famille pauvre, qu'il fut formé aux sciences mathématiques par son oncle Louis, qui était prêtre, qu'il fut élève de l'École militaire de Beaumont-en-Auge, etc. Autant d'assertions inexactes, que l'on va s'efforcer de rectifier à l'aide des documents.

§ I. GÉNÉALOGIE DE LA FAMILLE DE LAPLACE.

Nous avons dressé cette généalogie à l'aide des registres conservés dans les mairies de Bourgeauville, Criqueville, Angerville, Grangues, Beaumont-en-Auge et Dozulé. Nos renseignements sont donc puisés aux sources et nous fournissent des éléments certains sur les hérédités du savant et le milieu où il a grandi.

Le premier ascendant certain de Laplace est *Olivier de Laplace*, qui vivait à Bourgeauville* en 1645, et qui appartenait certainement à une famille notable, car nous le voyons, en cette même année, 1645, assister à Bourgeauville, au mariage de Pierre Lambert, écuyer, sieur de Saint-Mars, avec Angélique de Montgommery.

D'où venaient ces de Laplace? Je l'ignore. Ils n'appartenaient pas précisément à la noblesse, mais à cette aristocratie terrienne qui faisait presque figure de noblesse dans nos paroisses rurales, et s'alliait souvent avec elle. Il me semble assez vraisemblable que les de Laplace de Bourgeauville étaient de même origine que les de Laplace de Rouen, qui fournirent, dès le XVI^{e} siècle, des Conseillers au Parlement de Rouen. J'ai remarqué, au cours de recherches déjà longues, sur les familles du Pays d'Auge, que nombre de ces familles se retrouvent à Rouen. Bourgeauville, membre de l'Élection de Pont-l'Évêque, appartenait à la Généralité de Rouen, et les rapports de commerce aussi bien que les rapports administratifs étaient fréquents entre Rouen et le Pays d'Auge.

La branche des Laplace de Rouen, la plus anciennement connue, portait: *d'azur à 3 molettes d'or*. Une autre branche rouennaise, celle des Laplace, sieurs de Fumechon, portait: *d'azur à la molette* (alias *à l'étoile*) *d'or surmontée d'un lambel d'or*—ou encore: *de même avec l'étoile d'argent*, ou bien: *d'azur à 3 trèfles d'or*. Comme dans beaucoup de familles bourgeoises, les armoiries ne sont pas très fixes. Nous ne savons pas d'ailleurs si les de Laplace de Bourgeauville avaient conservé le souvenir d'armoiries de famille.

Dans les actes le nom est écrit: de Laplace, de la Place, Delaplace. Cette dernière forme est plus rare avant la Révolution. C'est cependant celle qui a subsisté dans les branches collatérales de la famille†.

* Calvados, canton de Dozulé, arrondissement de Pont-l'Évêque.

† Nous avons adopté, pour plus d'unité, la forme "de Laplace" pour les degrés antérieurs à la Révolution.

I. Olivier de Laplace* avait épousé Anne *Follebarbe,* qui appartenait à une vieille famille du Pays d'Auge, toujours représentée. Ils moururent l'un et l'autre en 1680, à Bourgeauville. Olivier de Laplace est toujours qualifié "Maître," ce qui suppose une situation notable. De ce mariage naquirent au moins six enfants:

1° *Mtre François de Laplace,* qui suit.

2° *Jacques de Laplace,* né vers 1639, décédé à Bourgeauville, le 8 octobre 1728, à l'âge de 90 ans. Il avait épousé sa cousine: Jacqueline de Laplace, dont il eut au moins une fille: Marie, mariée en 1705 à Charles *Jourdain,* dont elle était veuve en 1730.

3° *Marie de Laplace,* mariée en 1658 à Simon *Le Coq.*

4° *Charlotte de Laplace,* baptisée à Bourgeauville le 26 janvier 1642.

5° *Aymé de Laplace,* baptisé à Bourgeauville le 15 novembre 1645, et décédé en 1706.

6° *Jacqueline de Laplace,* baptisée à Bourgeauville le 14 février 1649.

II. *François de Laplace,* qualifié "Maître," comme son père, fut baptisé à Bourgeauville le 5 avril 1638. Il y épousa, en 1666, demoiselle Barbe *Belot.* Il dut mourir à Beaumont-en-Auge† fort âgé, car nous l'y trouvons en 1731.

De ce mariage naquirent au moins quatre enfants:

1° *Mtre Olivier de Laplace,* qui suit.

2° *Nicolas de Laplace,* né vers 1685, décédé à Beaumont le 16 août 1735, et inhumé en présence de son fils: *Jacques de Laplace.*

3° *Martin de Laplace,* marié à Bourgeauville en 1720 à Marie *Lesnis,* fille de feu François Lesnis et de Jeanne *Le Pecq.* Les Lesnis sont une ancienne famille du Pays d'Auge, qui s'est alliée notamment aux Chéron, sieurs du Fresney, et les Le Pecq ont donné naissance, à la fin du XVIIIe siècle, à Louis Le Pecq, sieur de la Clôture, célèbre médecin, anobli par Louis XVI.

4° *Simon de Laplace,* dont la branche sera indiquée dans une section spéciale.

(A) *Première branche.*

III. Maître *Olivier de Laplace,* chirurgien royal, né vers 1669, et décédé à Bourgeauville en avril 1736, âgé de 67 ans 18 jours. Il fut inhumé "dans l'église," privilège réservé aux seigneurs et aux notables. Il épousa: 1° Marguerite *Gardin,* 2° Charlotte *Bréard,* fille de Jacques Bréard et d'Anne Le Petit. Ce dernier mariage eut lieu à Bourgeauville le 12 mai 1704. Les Gardin paraissent originaires de S. Étienne-la-Thillaye‡. Ils étaient alliés aux Le Cordier et aux Isabel, riches familles de la région§. Les Bréard étaient également bien posés.

* Pour plus de clarté, nous avons écrit en majuscules les noms des ancêtres directs du savant.

† Canton et arrondissement de Pont-l'Évêque (Calvados).

‡ Canton et arrondissement de Pont-l'Evêque.

§ H. Le Court: *Généalogie de la famille Le Cordier, seigneurs de Maloisel,* p. 52.

Mtre Olivier de Laplace laissa de son premier mariage :

1° *Jean de Laplace*, qui suit.

2° *Marie-Anne de Laplace*, baptisée à Bourgeauville le 26 décembre 1697. Elle épousa, en cette même paroisse, le 27 septembre 1723, Maître Nicolas *Le Carpentier*, originaire de Criqueville*, fils de Jacques Le Carpentier et de Catherine Bouet, qui devint Conseiller du Roi. Ils eurent pour enfants : (*a*) Claude-François Le Carpentier, Conseiller du Roi, Maître Particulier des Eaux et Forêts, au baillage d'Auge, marié en 1767 à Marie-Anne-Catherine Cambremer, de la famille des *Cambremer de Croismare*. (*b*) Jacques-Charles Le Carpentier, Conseiller du Roi, seigneur et patron de Putot†, lieutenant en l'Élection de Pont-l'Évêque, marié en 1758 à Françoise-Catherine de la Taille. (*c*) Marie-Catherine Le Carpentier, mariée en octobre 1754 à Marin Barbey, bourgeois de Caen, de la famille des *Barbey de Longbois*.

3° *Pierre-Daniel de Laplace*, baptisé à Bourgeauville le 13 novembre 1698.

IV. *Jean de Laplace* épousa Anne *Toutain*. Ils habitaient Bourgeauville. Ils eurent au moins neuf enfants :

1° *Jean-François de Laplace*, baptisé à Bourgeauville le 11 avril 1719.

2° *Jacques de Laplace*, baptisé à Bourgeauville le 9 novembre 1720.

3° *Robert-Olivier de Laplace*, baptisé à Bourgeauville le 14 mars 1722.

4° *Robert de Laplace*, baptisé à Bourgeauville le 17 août 1723. Il devint prêtre et chapelain de Brucourt.

5° *Anne-Françoise de Laplace*, née en 1726.

6° *Robert-Jacques de Laplace*, baptisé à Bourgeauville le 14 mars 1727.

7° *François de Laplace*, qui suit.

8° *Marie-Anne de Laplace*, baptisée à Bourgeauville le 17 mars 1731, décédée le 17 juin 1735.

9° *Jean-Baptiste-François de Laplace*, baptisé à Bourgeauville le 27 août 1732.

V. *François de Laplace* se fixa à Grangues‡, puis à Criqueville§, où il fut inhumé le 18 février 1777. Il avait épousé Marie-Élisabeth *Morin*, dont il eut au moins neuf enfants :

1° *Anne-Jeanne de Laplace*, née à Grangues vers 1751, décédée en 1809 à Angerville‖, mariée successivement à Jean *de la Rue* et à Henri *Loriot*.

2° *Jacques de Laplace*, présent avec ses frères à l'inhumation de son beau-frère Jean de la Rue, à Criqueville, le 20 octobre 1779.

* Canton de Dozulé, arrondissement de Pont-l'Évêque.

† Canton de Dozulé, arrondissement de Pont-l'Évêque.

‡ Canton de Dozulé, arrondissement de Pont-l'Évêque.

§ Canton de Dozulé, arrondissement de Pont-l'Evêque.

‖ Canton de Dozulé, arrondissement de Pont-l'Évêque.

3° *Nicolas de Laplace*, qui suit.

4° *François de Laplace*, baptisé à Criqueville le 3 septembre 1762.

5° *Marie-Françoise de Laplace*, baptisée à Criqueville en 1764.

6° *Catherine-Félicité-Perpétue de Laplace*, baptisée à Criqueville le 26 novembre 1766.

7° *Jean-Baptiste de Laplace*, baptisé à Criqueville le 29 décembre 1772. Il s'établit à Grangues, et épousa *Marguerite-Françoise Martine*. Il est qualifié "propriétaire." Il eut pour enfants: (*a*) *Désirée de Laplace*, née à Grangues, mariée à Baptiste Delaplace son parent, dont j'ignore l'origine. (*b*) *Pierre-Jean François-Hyppolite de Laplace* qui épousa en premières noces: *Marie-Anne-Élisabeth Philippe*, décédée le 5 novembre 1816; et en secondes noces, à Dozulé, le 24 novembre 1824, *Rose-Julie Lelièvre*, née à Gerrots*, fille de *Pierre Lelièvre* et de *Marie-Anne Prentout*†. De cette seconde union naquirent trois enfants: *Julie-Alinda*, *Jean-Désiré-Eugène*, *Henri-Edmond*. La première et le dernier moururent jeunes. J'ignore la destinée du second, né à Dozulé en 1830.

8° *Marguerite de Laplace*, citée comme témoin dans divers actes.

9° *Françoise-Thérèse-Adélaïde de Laplace*, fille posthume, baptisée à Criqueville le 17 mai 1777.

VI. *Nicolas de Laplace* (alias *Delaplace*), établi à Grangues, cité en divers actes avec ses frères, épousa Adélaïde *Martine*, dont il eut deux fils:

1° *Nicolas* (?) *Delaplace*, connu dans la famille sous le nom de "l'aîné Delaplace," décédé à Goustranville‡. Il n'eut qu'une fille: *Laurence Delaplace*, mariée à M. *Sauvage*, dont deux filles décédées sans postérité.

2° *Stanislas Delaplace*, qui suit.

VII. *Stanislas-Casimir Delaplace*, né à Grangues le 10 mars 1796 et décédé à Dozulé le 15 novembre 1875. Il avait épousé Marie-Anne-Véronique *Leneveu*, fille de Jacques-François-Pierre Leneveu et de Marie-Anne-Félicité Leroy, décédée à Dozulé le 15 décembre 1882, dont:

1° *Abélina Delaplace*, née à Criqueville, mariée à *Auguste Couraye*, dont postérité§.

2° *Eugène Delaplace*, qui suit.

3° *Edmond-Constant Delaplace*, domicilié à Angerville, marié à Céleste-Ernestine Lefèvre, dont: (*a*) *Jeanne*, morte jeune; (*b*) *Fernand*, mort jeune; (*c*) *Léon*, domicilié à Beaumont; (*d*) *Juliette*, mariée à M. Dubois.

4° *Jules Delaplace*, marié à N., dont: (*a*) *Jeanne*, mariée à Henri Éven (postérité); (*b*) *Henri*; (*c*) *André*, mort à 17 ans, à Pont-l'Évêque, le 1er avril 1924.

* Canton de Cambremer, arrondissement de Pont-l'Evêque.

† La famille Prentout est ancienne au Pays d'Auge. L'un de ses représentants, M. Henri Prentout, est aujourd'hui professeur d'Histoire de Normandie, à la Faculté des Lettres de Caen.

‡ Canton de Dozulé, arrondissement de Pont-l'Évêque.

§ Cette postérité est représentée par Mme Aimable Moulin et par M. Henri Couraye, maire de Dozulé.

VIII. *Eugène-Étienne-Casimir Delaplace,* né à Criqueville, marié à Louise-Emma *Gosse,* dont :

1° *Jules Delaplace,* marié à Angéline *Dasseville,* dont un fils, mort jeune.

2° *Georges Delaplace,* qui suit.

IX. *Georges Delaplace,* maire de Leaupartie*, délégué cantonal de Cambremer, officier du mérite agricole, né à Troarn, marié à Montreuil†, le 22 mai 1883, à Caroline-Adélaïde *Goupil,* fille d'Adolphe Goupil et d'Euphrasie *Martin des Fontaines.* M. G. Delaplace habite actuellement le manoir de Leaupartie. Il a un fils, qui suit :

X. *Maurice Delaplace,* né à Leaupartie le 21 juillet 1884, brigadier d'artillerie en 1915—1919, Croix de Guerre, marié à Montreuil, le 9 janvier 1925, à Madeleine *Laval,* fille de Pierre Laval et de Mme, née Soulier, dont :

1° *Georgette Delaplace,* née à Leaupartie le 18 juin 1926.

(B) *Seconde branche.*

III. *SIMON DE LAPLACE,* fils de François de Laplace et de Barbe Belot, indiqués plus haut, s'établit à Beaumont avant 1743. En 1738, nous le trouvons à Bourgeauville, parrain de Thomas-François, son neveu, et le 22 décembre 1732 il est parrain à Criqueville de sa petite nièce Marie-Catherine Le Carpentier, fille de Nicolas, Conseiller du Roi, et d'Anne de Laplace. Il avait épousé Marie *Viel,* dont :

1° *Marie de Laplace,* mariée à Beaumont, le 1er juillet 1743, à Maître Robert *Carrey,* médecin à Lisieux, paroisse St Germain. Les Carrey sont une très ancienne famille Lexovienne qui a donné des notaires royaux, des avocats, des médecins. Le manoir Carrey est aujourd'hui l'une des plus curieuses vieilles maisons de la ville.

2° *Louis de Laplace,* prêtre, chapelain de Criqueville, dont nous aurons l'occasion de parler.

3° *PIERRE DE LAPLACE,* qui suit.

4° *Simon de Laplace,* cité avec ses frères au mariage de Robert Carrey et de Marie de Laplace.

5° *Thomas-François de Laplace,* chirurgien, décédé à Bourgeauville, à l'âge de 27 ans, en 1738, marié à Marguerite Hervieu, dont : *Thomas-François,* baptisé à Bourgeauville, le 24 avril 1738.

6° Probablement : *Geneviève,* mariée avant 1745 à Jacques *Mabou.*

* Canton de Cambremer, arrondissement de Pont-l'Évêque.

† Canton de Cambremer, arrondissement de Pont-l'Évêque.

IV. *Pierre de Laplace*, demeurant à Beaumont, sindic de la paroisse, marié à Tourgéville*, le 6 juillet 1744, à Marie-Anne *Sochon*, fille de feu Louis-Robert Sochon et de Marie-Anne *Le Chevalier*. De ce mariage sont nés:

1° *Pierre-Simon de Laplace*, qui suit.

2° *Marie-Anne de Laplace*, baptisée à Beaumont le 15 juin 1745.

V. *Pierre-Simon*, comte, puis marquis de *Laplace*, né à Beaumont le 23 mars 1749. C'est l'illustre savant auquel seront consacrées les pages qui vont suivre. Marié à Marie-Charlotte de Courty de Romanges (famille de Besançon), il en eut un fils et une fille:

1° *Charles-Émile P. J.*, marquis de *La Place*, né 15 avril 1789, mort 27 octobre 1874, général de division, sénateur, Pair de France, Grand-Croix de la Légion d'honneur, chevalier de St Louis.

2° *Sophie-Suzanne de Laplace*, morte 1813, en suite des couches de sa fille, mariée à Adolphe-François-René, marquis de Portes, Pair de France, décédé à Paris le 22 septembre 1852†. Ils eurent une fille: Angélique-Joséphine-Charlotte de Portes, mariée à Napoléon-Joseph-Auguste, Comte de Colbert-Chabannais, décédé le 1er octobre 1883. Le second fils issu de ce mariage: Pierre-Louis-Jean-Baptiste, Comte de Colbert, releva le nom de Laplace, en vertu d'un décret de 1876. Il épousa en 1882 delle Renault, dont postérité. Adolphe-François-René, marquis de Portes, d'un 2me mariage avec Caroline Hutton, américaine, eut deux filles: 1° Catherine-Méry-Adolphine de Portes, mariée en juillet 1846 à Napoléon-Victor-Eugène, comte de Bellune, décédé en 1852, et en secondes noces à Charles-Eugène-Henry-Joseph-Texier, marquis d'Hautefeuille, et 2° Madame de Montgomery.

§ II. Les Sochon, ancêtres maternels de Laplace.

Les Sochon étaient primitivement originaires de Vauville, canton de Pont-l'Évêque. C'étaient riches cultivateurs, dont la branche la plus connue est celle des Sochon de Lavigne, très proche parente de la mère de Laplace.

Nicolas Sochon habitait Vauville en 1669. Il épousa Charlotte *Congnet*‡, d'une famille de Tourgéville, ce qui sans doute amena cette branche à se fixer à Tourgéville.

L'un des fils de Nicolas, Antoine Sochon, épousa à Tourgéville, le 16 juillet 1669, Marguerite Cofin, dont il eut entre autres enfants: Robert-Louis Sochon.

Robert-Louis était né à Tourgéville et y avait été baptisé le 8 janvier 1687. Il épousa Marie Le Chevalier, dont il eut deux filles.

* Canton et arrondissement de Pont-l'Évêque.

† Pour remployer la dot de sa femme au profit de l'enfant mineure, M. de Portes acheta en 1813, sur le conseil de Laplace et de Napoléon lui-même, le château de Mailloc, qui fut détruit en 1925 par une incendie, avec tous les effets appartenant au grand Laplace.

‡ Les Congnet sont aujourd'hui représentés par M. Congnet, rédacteur au Ministère des Finances, dont la mère était sœur de Mme Georges Delaplace, de Leaupartie.

L'une de ces filles, Marie-Anne, épousa Pierre de Laplace. C'est la mère de notre savant.

L'autre, Anne Sochon, fut mariée, à Glanville*, le 30 mai 1752, à François Cordier, Commis pour le Roi au Grenier et Magasin à sel de Danestal, fils de François-Jacques Le Cordier et de Marie-Anne Gondouin†. Ils eurent pour fils: Louis-François Cordier, directeur de la Compagnie des Indes, puis régent de la banque de France, sous le premier Empire. Il était très lié avec son cousin germain Pierre-Simon de Laplace. Il épousa à Paris, le 15 janvier 1788, Françoise-Jeanne-Élisabeth Duclos, fille de Jean-Baptiste Duclos, avocat au Parlement, et d'Anne-Élisabeth Ménard.

Ils eurent deux filles: 1° Louise-Élisabeth, née à Caen en 1788, mariée en 1812 à Louis-François Marchand, chevalier de la Légion d'honneur. 2° Élise, mariée à Jean-Baptiste-Michel, baron de Trétaigne, dont la postérité existe encore.

§ III. LE MILIEU FAMILIAL.

Nous pouvons, grâce aux données précédentes, avoir quelque idée du milieu où grandit Laplace. Ce n'est pas un milieu vulgaire. Les ancêtres ont été des gens distingués. Sans doute le père ne semble pas avoir poursuivi d'études comme ses deux frères, Louis et Thomas-François, le prêtre et le chirurgien, et comme le grand'oncle, Maître Olivier, chirurgien royal, mais évidemment il a acquis au foyer une certaine distinction, et c'est pourquoi ses compatriotes de Beaumont le choisiront pour sindic.

Au nombre de ses plus proches, Pierre-Simon rencontre, dès son tout jeune-âge, l'abbé Louis de Laplace que les biographes nous montrent comme un mathématicien distingué, et son autre oncle Robert Carrey le médecin, qui appartient à une famille célèbre.

Maître Nicolas Le Carpentier, le cousin germain du père, est Conseiller du Roi; c'est donc un personnage de marque, et ses deux fils, tous les deux Conseillers du Roi, l'un Maître des Eaux et Forêts, l'autre Lieutenant en l'Élection et seigneur de Putôt, sont évidemment des gens instruits, des hommes du monde, dont la fréquentation ne pouvait que contribuer à former l'esprit du petit Laplace.

Du côté maternel, il y a l'oncle Cordier, qui mourra en 1757 et sera inhumé dans la nef de l'église de Danestal. Lui aussi est un homme instruit et distingué.

Le futur savant a donc grandi dans un milieu bien capable de former son esprit et de lui donner le goût de la culture. Il faut donc absolument rejeter la fable du petit indigent, fils d'un pauvre laboureur, élevé grâce à la charité des moines de Beaumont.

* Canton de Pont-l'Évêque.

† La *Généalogie des Le Cordier, seigneurs de Maloisel*, a été écrite par M. Henry Le Court. Les Cordier et les Le Cordier étaient de même famille.

§ IV. Le collège de Beaumont.

La maison de M. de Laplace*, le père, s'élévait près du prieuré de Beaumont, où les moines Bénédictins avaient fondé un collège. L'enfant leur sera bientôt confié. Il n'est donc pas indifférent de faire connaissance avec ce nouveau milieu.

La première idée d'un collège à Beaumont date du début du XVIII^e siècle, alors que le prieuré était encore en commende sous Denis-François Bouthilier de Chavigny. L'un des moines, Dom Julien Aubrée, résolut de s'occuper de l'éducation de quelques enfants, Beaumont se trouvant éloigné de tout collège. Voici ce que dit de lui le savant Dom Martène: "Dieu lui avait donné un talent particulier pour bien enseigner les humanités aux enfants. Étant au monastère de Beaumont-en-Auge, les pères et mères d'alentour lui envoyaient leurs enfants pour apprendre de lui le latin. Il avait un grand soin de les former en même temps à la piété. À lui seul, il enseignait toutes les classes et il forma de très bons écoliers†."

La première ébauche prit forme grâce à la protection du duc d'Orléans, héritier des fondateurs du prieuré. Celui-ci obtint du Roi que le monastère serait remis en règle, c.-à-d. n'aurait plus de prieur commendataire, mais un prieur régulier et que la "mense prieurale," ou portion des revenus réservée au prieur, serait réunie à la mense conventuelle, le prieur-moine n'ayant pas de traitement particulier. La suppression de la mense prieurale eut lieu en 1731. Le collège fut définitivement érigé en 1741‡. Il devait être dirigé par les douze moines du prieuré. Le prieur avait surtout la direction spirituelle. Il y avait un régent s'occupant des études.

Les pensionnaires devaient appartenir uniquement aux paroisses relevant du domaine du duc d'Orléans. Il y avait des internes, payant pension, plus six jeunes gentilshommes, dont la pension était payée par le prince. Il y avait également des externes, dont la pension était gratuite, d'après les statuts mêmes rédigés par la volonté du duc d'Orléans.

Les enfants pouvaient être admis à l'âge de sept ans. On ne les prenait pas après douze ans. Les humanités proprement dites commençaient en cinquième.

La maison était bien réputée, et Dumoulin, qui en 1764 publiait sa *Géographie de la France*, y écrivait: "Les Bénédictins ont un beau Collège à Beaumont§."

Le petit Laplace habitait, comme nous l'avons vu, tout près du prieuré. On nous dit, ce qui est extrêmement vraisemblable, qu'il avait "une intelligence précoce," "des dispositions peu communes à un âge où les enfants commencent

* Note par M. le Comte A. de Colbert-Laplace: La famille Laplace avait habité la terre du Mérisier, que je possède encore, depuis quand y étaient-ils? je l'ignore,—la maison a des cheminées du XIV ou XV siècle. Dans les papiers qui je possédais, et qui sont brûlés, je me rappelle avoir vu qu'ils avaient fieffé à N...cette terre, que Laplace a racheté plus tard (179-), par rachat des rentes que la famille en tirait. Mais je me souviens qu'on m'a dit que Laplace était né au Mérisier. Cette terre est du reste sur la Commune de Beaumont—mais à 2 ou 3 kilomètres du bourg. La maison où est apposée une plaque en face de l'église, est une maison neuve ou relativement. Je ne pense pas que si les Laplace ont habité cette maison, ce soit la même.

† *Vie des justes*, éditée par Dom Heurtebize. Paris, 1926, t. III. p. 84.

‡ Les pièces concernant cette affaire se trouvent dans le *Gallia Christiana*, t. XI, *Instrumenta*.

§ T. II. p. 177.

à peine à aborder les premiers éléments de la lecture" et surtout "une prodigieuse mémoire*." À la maison paternelle, ces heureuses dispositions pouvaient être entretenues surtout par l'oncle Louis, autrement dit: l'Abbé de Laplace. Celui-ci habitait Beaumont, peut-être la maison paternelle, peut-être le prieuré. Il n'était pas moine, mais les religieux, trop peu nombreux, faisaient appel aux professeurs et aux répétiteurs ecclésiastiques ou laïques. Ce qui rend vraisemblable la supposition que l'abbé Louis de Laplace faisait la classe au prieuré, c'est qu'il n'apparaît jamais remplissant une fonction ecclésiastique. On le trouve à Beaumont comme sous-diacre en 1745, comme diacre en 1746. Il fut vraisemblablement ordonné prêtre vers 1747. En 1752, il fut nommé chapelain de Criqueville. La chapelle de Criqueville située "au costé gauche du chœur de l'église paroissiale" était un "bénéfice simple," c.-à-d. sans charge d'âmes et n'obligeant pas à résidence†.

C'est de cet oncle que parle Boisard, lorsqu'il écrit: "Le jeune Laplace...reçut les leçons d'un de ses oncles, prêtre et mathématicien fort instruit‡." Il est probable en effet que l'abbé Louis de Laplace inculqua son goût pour les sciences à son jeune neveu, mais il ne put le pousser très loin, car il mourut en 1759§. L'enfant n'avait alors que dix ans.

Il est probable que le jeune Laplace commença ses études régulières au collège à l'âge de sept ans, qu'il atteignait à la fin de mars 1756. Il y sera entré, à la rentrée d'octobre 1756. Le prieur à cette époque était Dom Joachim Hébert de Bailleul. C'était un homme instruit, qui auparavant, notamment d'après des documents de 1741 et de 1746, avait été régent des études.

Les élèves se destinaient les uns à l'armée, les autres à la robe; d'autres enfin à l'état ecclésiastique. Les premiers portaient un uniforme militaire, les seconds un vêtement bleu-de-roi, à revers et parements d'écarlate et épaulettes d'or. Ils se coiffaient d'un chapeau à plumet blanc. Les pensionnaires ecclésiastiques portaient un habit noir, conforme à leur future profession. Les deux dernières catégories formaient le contingent le plus nombreux. "Nous voyons avec satisfaction, déclarent les Bénédictins de Beaumont, nos élèves remplir les cures voisines et occuper les charges de judicature dans les environs‖."

La famille du petit Laplace le destinait à l'état ecclésiastique¶. Peut-être y avait-il pris goût lui-même au contact de son oncle Louis. L'enfant portait donc l'habit noir, et sans nul doute était externe.

La classe du matin commençait à 7 h. $\frac{3}{4}$ et finissait à dix heures moins un quart. On se rendait alors à l'église pour la messe; on se réunissait ensuite à la salle d'études jusqu'à 11 h. $\frac{1}{2}$, heure du dîner.

L'après-midi, la classe recommençait à 2 h. et durait jusqu'à 4 h. Il y avait alors collation et récréation, puis on travaillait à la salle d'études de 4 h. $\frac{3}{4}$ à 6 h. $\frac{1}{2}$.

* L. Puiseux: *Notices sur Malherbe, Laplace etc.* Caen, Laporte, 1847, p. 84.

† *Archives du Chapitre de Bayeux. Insinuations du diocèse de Lisieux*, Registre XXIII, No. 313.

‡ *Notices biographiques, littéraires et critiques sur les hommes du Calvados.* Caen, Pagny, 1848, p. 177. Voir aussi H. Le Court: *Généalogie de la famille Le Cordier*, p. 45.

§ *Insinuations du dioc. de Lisieux*, Reg. XXVII, No. 212.

‖ *Archives du Calvados:* Collège de Beaumont.

¶ Boisard, *op. cit.* p. 177.

Le dimanche, on assistait d'abord à une messe basse, puis il y avait étude pour les devoirs de classe. Durant cette étude, les élèves se rendaient par groupes près des "garçons de chambre" pour se faire friser, poudrer et ajuster. Ensuite avait lieu la grand'messe, suivie d'une instruction sur l'Épître ou l'Évangile du jour. L'après-midi, on assistait aux Vêpres, après quoi, il y avait récréation, puis étude de 5 h. ½ jusqu'au soir.

Les vacances duraient six semaines et commençaient vers le 8 ou le 10 août. La veille avait lieu la distribution des prix, qui était toujours très solennelle. Quelques jours auparavant, on avait organisé des Exercices publics en présence des notabilités locales et des parents, où l'on posait des "questions d'algèbre avec équation au 1er et au 2e degré" et où l'on faisait des démonstrations sur "la Cosmographie, la Fortification, la Trigonométrie plane, la Balistique et les différentes propriétés de l'ellipse et de la parabole." Des questions étaient également posées sur les Belles Lettres, les auteurs latins etc., car au collège de Beaumont l'étude des langues et du latin était fort en honneur.

Le jeune Laplace avait des aptitudes spéciales pour les mathématiques, mais sa culture littéraire allait de pair. Il avait aussi le goût des beaux arts et en particulier de la musique que l'on enseignait également au collège.

Il y avait alors de 50 à 60 élèves.

Le priorat de Dom Joachim de Bailleul cessa en 1756. L'année suivante, le prieur était Dom Jean-Pierre Le Maistre. Un acte de 1760 nous mentionne comme religieux présents au monastère: Dom François Thères, Dom René du Mesnil, Dom Jean Mériel-Bussy, Dom Jean-Charles Foyard, Dom Louis-Charles Gadeau, Dom Louis-Salomon Girouard, Dom Mathieu Crucifix. Ce dernier n'était que diacre, et il sera plus tard professeur à Tiron*.

Aucun de ces religieux n'a laissé de nom ni dans les lettres ni dans les sciences. Ce devaient être simplement de bons professeurs, le Supérieur Général de la Congrégation de Saint-Maur s'étant engagé à envoyer des professeurs "capables... sages et vertueux" dont "il répondait personnellement."

Laplace quitta le collège à l'âge de seize ans†, donc aux vacances de 1765. Il eut par conséquent pour professeur et "régent d'humanités," Dom Charles-Antoine Blanchard, qui avait été envoyé à Beaumont en 1764‡. Né à Réthel (Ardennes) le 20 janvier 1737, Dom Blanchard avait étudié les humanités à Caen, au collège du Bois, durant sept années. Il était entré ensuite à l'abbaye de Jumièges, où il avait fait sa profession monastique en 1757. En 1759 nous le trouvons étudiant la philosophie et la théologie à l'abbaye de Saint-Étienne de Caen. Il fut ordonné prêtre en septembre 1764, et aussitôt envoyé à Beaumont, vraisemblablement pour la rentrée d'octobre. Depuis lors "on l'employa presque toujours à l'instruction

* *Extrait du Nécrologe de l'abbaye du Bec*, éd. par R. N. Sauvage (Extr. *Bulletin philologique et historique*, 1924). Paris, 1926, p. 8.

† Le Fort: *Le collège et l'école militaire de Beaumont*, dans la *Revue illustrée du Calvados*, mai, 1913, p. 76.

‡ Abbé Porée: *Lettres de quelques bénédictins.* Bruges, 1902, p. 5 (Extr. de la *Revue bénédictine*, 1902).

de la jeunesse." C'était "un emploi qui avait toujours eu pour lui des attraits*." Dom Blanchard n'a jamais rien publié, mais il a laissé des *Mémoires historiques sur l'abbaye de Saint-Étienne de Caen*, éditées récemment par M. R. N. Sauvage, au tome XXX du *Bulletin de la Société des Antiquaires de Normandie* (1915). Le nouveau professeur avait 27 ans. Il avait du talent et la passion de l'enseignement. Il put donc avoir une certaine influence sur le goût et la formation littéraire de Laplace †.

Dans les pages qui précèdent nous n'avons rien dit de l'École Militaire de Beaumont dont tous les biographes veulent que Laplace ait été l'élève. C'est que celle-ci ne fut fondée qu'en 1776, dix ans après le départ du jeune étudiant. Il serait peut-être bon de mettre fin à cette erreur tant de fois répétée.

§ V. LAPLACE À CAEN.

Âgé de seize ans, le jeune homme fut envoyé à Caen, au Collège des Arts de l'Université, afin de poursuivre ses études, toujours à titre d'étudiant ecclésiastique. Là, il devait se perfectionner dans les "humanités" et étudier particulièrement la philosophie.

Le Collège des Arts se trouvait "à l'angle de la rue des Grandes-Écoles et de la Cour des Cordeliers‡, tout proche de l'emplacement de l'Université actuelle. Son nom lui venait de ce qu'il avait été fondé par la Faculté des Arts. On en devait suivre deux ans les cours avant de passer en Théologie. Les séminaires d'alors n'étaient pas des maisons d'études, mais s'occupaient uniquement de la préparation spirituelle au sacerdoce. Les étudiants ecclésiastiques suivaient les cours de l'Université, lorsqu'ils en avaient le moyen. Les autres pouvaient étudier près du prêtre de leur paroisse, quitte à passer ensuite des examens. La fortune paternelle permettait à Laplace de prendre pension à Caen et d'y suivre les cours.

Nous sommes en 1766. Le recteur de l'Université est alors Jean-Jacques-François Godard, prêtre, licencié-ès-droits, professeur royal d'éloquence et proviseur du Collège du Mont. C'est un lettré, auteur de quelques poésies et de tragédies d'un caractère scolaire, destinées à être jouées par les élèves, lors des séances littéraires. En fin de mars 1767, il sera remplacé par M. Levêque, dont on ne sait à peu près rien et qui ne fit que passer, car en avril de la même année le rectorat était aux mains de M. Jacques Lentaigne, docteur en théologie, curé de Saint-Sauveur, théologien fougueux, qui résigna sa fonction le 29 septembre et fut remplacé par Jean-Baptiste-Alexandre Hardouin, licencié-ès-droits, proviseur du

* Porée: *L'Abbaye du Bec et ses écoles*. Évreux, 1892, p. 100, et Sauvage: *L'Abbaye de S. Étienne de Caen sous la règle de S. Maur*, p. IX.

† Je n'ai pas trouvé de programme d'études se rapportant aux années passées par Laplace à Beaumont, mais on a publié dans le *Bulletin de la Société d'histoire de Normandie*, t. VII, le programme des Exercices des années 1770—1773 (p. 862 ss.). Ils doivent être sensiblement les mêmes qu'au temps de Laplace.

‡ F. Vaultier: *Histoire de la ville de Caen*. Mancel, 1843, p. 162, et C. Pouthas: *Les collèges de Caen au XVIII^e siècle*, p. 29.

Collège des Arts*. Laplace fut donc particulièrement en rapport avec ce dernier, puisqu'il était chargé du collège dont il suivait les cours. Le professeur de philosophie était M. Christophe Gadbled, dont nous parlerons plus loin.

Laplace, en dehors de l'Université, fut certainement en rapport avec M. Lentaigne, celui-ci, comme nous l'avons vu, étant curé de Saint-Sauveur. Cette église s'élevait tout près des bâtiments du Collège des Arts. C'est un bel édifice gothique malheureusement déparé par un porche du XVIIIe siècle, et plus malheureusement encore transformé en halles aux grains.

Néanmoins d'après une anecdote rapportée par Puiseux†, la paroisse du petit abbé aurait été Notre-Dame-de-Froide-Rue, située à proximité de l'Université, quoique un peu plus loin que Saint-Sauveur‡. Voici cette anecdote: "C'était sous l'Empire, alors que Laplace était revêtu de la haute dignité de chancelier du Sénat. Il parcourait la ville avec notre vénérable M. Lair: passant devant l'église Notre-Dame (aujourd'hui Saint-Sauveur), il lui prit fantaisie d'y entrer. Le curé faisait en ce moment une instruction sur le catéchisme à de petits enfants, et l'un d'eux, rebelle à la leçon sans doute, avait été mis en pénitence. Laplace, qui n'était pas connu, s'avança vers l'ecclésiastique: Monsieur le curé, lui dit-il, j'ai porté autrefois le surplis dans votre église; au nom de ce souvenir qui m'est cher, je vous demande la grâce de ce petit garçon. La grâce fut accordée." Laplace avait donc pris le petit collet et s'associait au clergé paroissial.

Comment fut-il amené à renoncer aux études théologiques pour se consacrer aux sciences? Voici ce que nous rapporte encore M. Puiseux§: "Un jour des livres de hautes mathématiques tombent entre ses mains; il se jette avec ardeur, avec passion sur cet aliment nouveau pour lui et vers lequel pourtant l'entraîne une impérieuse sympathie. De ce jour, sa vocation est décidée: il s'abandonne sans réserve à l'impulsion de son génie; Achille a trouvé ses armes." Le jeune homme fut-il influencé et guidé dans le milieu même de l'Université? Évidemment oui, et l'on cite deux hommes qui paraissent n'avoir pas été étrangers, bien au contraire, à sa vocation scientifique. Nous en avons nommé un, M. Gadbled; l'autre est M. Le Canu.

Christophe Gadbled était né à Saint-Martin-le-Bouillant, près de Villedieu (Manche), le 29 novembre 1732‖. Il était à la fois philosophe et mathématicien. Dans des *Conclusiones philosophicæ*, de 1762, où il figure comme arbitre, il est qualifié: "prêtre, bachelier en la Faculté de Théologie de Paris, Professeur de Philosophie au Collège des Arts de la très célèbre Université de Caen, et membre de la royale Académie des Lettres¶." Dans la *France littéraire* de 1769**, il est

* Eug. Châtel: *Liste des Recteurs de l'Université de Caen*. Caen, Leblanc-Hardel, 1882, p. 47.

† *Op. cit.* p. 36 et p. 62.

‡ Cette même paroisse a pris, depuis la Révolution, le nom de Saint-Sauveur, l'ancien Saint-Sauveur ayant été supprimé.

§ *Op. cit.* p. 36.

‖ Le *Manuel de bibliographie* de Frère porte la date de 1734. Nous devons la rectification à M. R. N. Sauvage, archiviste du Calvados.

¶ Placard imprimé, Arch. Calv. Série D, Université, Conclusions.

** T. I. p. 74.

cité parmi les membres de l'Académie de Caen, avec cette mention: "M. Gadbled, professeur royal de Mathématiques." Enfin dans une Thèse de Mathématiques soutenue en 1772 par Louis-Marin Lancelin, il est indiqué comme président, avec cette mention : "Présidera M. Christophe Gadbled, prêtre...professeur de Philosophie en l'Université de Caen..., de Mathématiques et d'Hydrographie*."

M. Gadbled publia en 1779 un ouvrage intitulé: *Exposé de quelques-unes des vérités rigoureusement démontrées par les Géomètres et rejetées par l'auteur du Compendium de Physique imprimé à Caen en 1775*†. L'auteur du *Compendium de Physique* se nommait M. Adam. Le titre indique qu'à cette époque il y avait des polémiques entre géomètres et physiciens. En cette même année 1779, M. Gadbled publia: *Exercice sur la théorie de la Navigation*, Caen, in 4°. Il mourut à Caen le 11 octobre 1782.

Pierre Le Canu avait été d'abord médecin, mais comme son confrère M. Gadbled il s'adonna, sans toutefois abandonner la médecine, aux mathématiques et à la philosophie, et il enseigna ces trois sciences.

Dans un acte public de 1773, il figure comme "professeur de Médecine et de Philosophie au Collège du Mont de l'Université de Caen‡."

La Bibliothèque de Caen conserve l'exposé d'un *Exercice sur le Calcul infinitésimal*, imprimé à Caen, chez Poisson, 1788, in 8°, 19 p., se terminant par ces mots: "Répondra M. Pierre-Jacques-Guillaume Lair, de Caen. Présidera M. Pierre Le Canu, Professeur Émérite et Royal honoraire de Médecine en l'Université de Caen....Lecteur du Roi et son Professeur de Mathématiques au Collège Royal de Normandie§."

On a de Le Canu un: *Compte-Rendu des maladies qui ont régné pendant l'année 1781, sur les côtes de la Normandie depuis la rivière de Dives jusqu'au Vey.* Ce mémoire fut cité avec éloge par la Société Royale de Médecine, dans sa séance publique du 27 août 1782.

Laplace suivit les cours de ces deux professeurs, qui furent pour lui "plus que des maîtres, des amis," et il fit sous leur direction "des progrès rapides dans le domaine des sciences exactes||."

Il ne semble pas que le jeune savant ait pris le degré de Maître-ès-Arts. Il est certain qu'il n'entra pas dans la cléricature et ne reçut jamais la tonsure.

M. Puiseux¶ nous dit qu'ayant quitté le Collège des Arts, Laplace fut un instant précepteur dans une des branches de la famille d'Héricy. Il s'agit probablement de Philippe-Jacques, marquis d'Héricy, brigadier des armées du Roi, qui avait alors des enfants d'une dizaine d'années, et possédait un hôtel à Caen.

M. Puiseux nous dit encore** qu'avant son départ pour Paris, en 1768, alors

* Bibliothèque municipale de Caen. Imprimé en 4°, 16 pp.

† En 8°. Amsterdam (Caen), 1779.

‡ Archives du Calvados, D, Université, 362.

§ Bibliothèque de Caen. Le Collège royal de Normandie avait remplacé en 1786 l'ancien Collège des Arts.

|| Puiseux, *op. cit.* p. 37. ¶ *O cit.* p. 37. ** *Loc. cit.*

qu'il n'avait environ que 18 ans, Laplace fut "répétiteur" au Collège de Beaumont. Ce ne fut évidemment que pendant fort peu de temps. Peut-être durant quelques mois ou même seulement quelques semaines prêta-t-il le concours de sa science à ses anciens maîtres, trop peu nombreux pour le nombre de leurs élèves. Il faisait en même temps ses préparatifs de départ pour Paris.

À Beaumont, depuis 1767, Dom Le Maître n'était plus prieur. Il avait été remplacé par Dom Jean-Baptiste Marage. Les moines étaient: Dom Alexis Petillon, sous-prieur, Dom Marin Gouges, Dom Louis d'Hée, Dom Augustin Patattier, Dom Jean-Pierre Bride, Dom Henri du Doit, Dom Martin Vatard, Dom Robert Le Guelinel, Dom Charles Blanchard, Dom Pierre Chennebault. Peut-être, entre deux cours de mathématiques, le jeune homme aima-t-il causer un peu littérature avec son ancien maître Dom Blanchard.

Par ailleurs, les relations étaient restées très suivies avec les deux professeurs de Caen, et M. Le Canu, en particulier, approuva chaleureusement le projet de voyage à Paris. Il connaissait quelque peu d'Alembert, et il donna au jeune savant pour l'illustre académicien une lettre de recommandation *.

* Puiseux, *op. cit.* p. 38.

STUDIES IN THE THEORY OF SAMPLING.

By JOSEPH PEPPER, B.A., B.Sc.

1. A GREAT deal of work has been done, both in theory and experiment, on various properties of small samples from a univariate population, with any law of distribution, e.g. the nature of the distribution of the means and variance in samples. With regard to bivariate sampled populations, certain properties of small samples have been found by Professor Pearson, Dr Fisher and recently by Dr Wishart*, but only in the special case when the sampled population is normal. Some of the values obtained by them appear in corollaries to formulae of my paper.

In this paper, I have investigated theoretically the problem of sampling from *any* bivariate population, not necessarily normal or infinite. The method employed is purely algebraical and is an extension to two variates of the methods used by "Student," Dr Church and Dr Neyman†. Although the algebra is often heavy and complicated, yet the method has the advantage of yielding the result in a general form, from which the special cases of univariate, normal or infinite sampled populations may be deduced.

The results obtained may be summarised under the following headings. The notation, which is mostly familiar, is defined in the next section.

(i) The mean values of p_{11}, p_{12}, p_{21}, p_{22}, p_{31}, p_{13} for the general case of any limited sampled population. A special case of these results gives the means of the second, third and fourth moment coefficients in samples from any limited population.

(ii) The standard deviations of p_{11}, p_{12}, p_{21} in samples from any limited population and the corresponding deductions for the second and third moments.

(iii) The standard deviations of p_{31}, p_{13} and p_{22} in samples from any infinite population, giving the standard deviations of the third and fourth moments.

(iv) The third and fourth moment coefficients of the distribution of p_{11} in samples from any infinite population.

(v) The β_1 and β_2 of the distribution of p_{11} in samples from an infinite normal population.

(vi) Correlations between any two of m_x, m_y, σ_x^2, σ_y^2 and p_{11} in samples from any limited population.

(vii) The various deductions of all the above results for the special cases of univariate, normal and infinite sampled populations.

* *Biometrika*, Vol. XVII. p. 176; Vol. X. p. 507; Vol. XX^A. p. 32. See references given in last paper to other writers on this subject.

† *Ibid.* Vol. VI. p. 8; Vol. XVII. pp. 79 and 472.

2. The sampled population is bivariate, with variates

$$(X_1, Y_1),\ (X_2, Y_2),\ \dots\ (X_N, Y_N),$$

where we take
$$S(X_s)=0, \quad S(Y_s)=0.$$

The standard deviations of the variates are σ_X, σ_Y and the correlation R. The product moments, P_{ab}, of the sampled population are given by

$$P_{ab}=\frac{1}{N}S(X_s^a Y_s^b),$$

where a, b are positive integers.

We consider samples of n from this population, where any particular sample has variates $(x_1, y_1), (x_2, y_2), \dots (x_n, y_n)$ and the product moment coefficients in the sample are defined by

$$p_{\alpha\beta}=\frac{1}{n}S(x_s-\bar{x})^\alpha (y_s-\bar{y})^\beta,$$

where
$$\bar{x}=\frac{1}{n}S(x_s), \qquad \bar{y}=\frac{1}{n}S(y_s),$$

and α, β are positive integers. In the later work I have replaced $\bar{x}$, $\bar{y}$ by the notation m_x, m_y respectively. The standard deviations of the variates in the sample are denoted as usual by σ_x, σ_y.

I have deduced many results for the case of sampling from a univariate population by putting $X=Y$ and $x=y$ in the bivariate results. I have then put

$$\mu_p=\frac{1}{N}S(X_s^p) \text{ with } \mu_1=0.$$

The standard deviation is σ_X. In the sample, $p_{\alpha\beta}$ reduces to the moment coefficient and I have put

$$m_q=\frac{1}{n}S(x_s-\bar{x})^q.$$

The value of m_2 is the variance in the sample, which is usually written s^2.

I have denoted the mean of an expression by placing a bar over it. It must be observed that in the notation for the various sums I have indicated different values of the variates by giving them different suffixes, e.g. $S(x_s^2 x_t y_u)$ denotes the sum of terms like $x_1^2 x_2 y_3$, so that s, t, u are combinations of 3 different numbers taken from the numbers 1 to n, and there will be $\frac{1}{6}n(n-1)(n-2)$ such combinations. Similarly $S(x_s^2 y_s y_t)$ denotes the sum of terms like $(x_1^2 y_1 y_2 + x_2^2 y_2 y_1)$, and here there will be $\frac{1}{2}n(n-1)$ terms, and so on, for other sums.

3. (*a*) *The Mean and* (*b*) *the Standard Deviation of* p_{11} *in Samples from a Finite Population with any given Distribution.*

(*a*) By definition,

$$\begin{aligned} p_{11} &= \frac{1}{n}S(x_s-\bar{x})(y_s-\bar{y}) \\ &= \frac{n-1}{n^2}S(x_s y_s)-\frac{1}{n^2}S(x_s y_t) \quad \dots\dots\dots\dots\dots\dots\dots(1). \end{aligned}$$

Now we have in repeated samples

$$\text{Mean } S(x_s y_s) = \frac{n}{N} S(X_s Y_s),$$

$$\text{Mean } S(x_s y_t) = \frac{n(n-1)}{N(N-1)} S(X_s Y_t),$$

and in the sampled population

$$S(X_s Y_s) = NP_{11}, \quad S(X_s Y_t) = -NP_{11},$$

so that from equation (1),

$$\overline{p_{11}} = \frac{n-1}{n^2}(nP_{11}) - \frac{1}{n^2}\left\{-\frac{n(n-1)}{N-1}P_{11}\right\}$$

$$= \frac{\left(1-\frac{1}{n}\right)}{\left(1-\frac{1}{N}\right)} P_{11} \quad \text{.......................................(A)*.}$$

When we put $X = Y$ and $x = y$ in (A), we obtain the familiar result

$$\overline{s^2} = \frac{\left(1-\frac{1}{n}\right)}{\left(1-\frac{1}{N}\right)} \mu_2.$$

(*b*) From equation (1)

$$p_{11}^2 = \frac{(n-1)^2}{n^4}\{S(x_s y_s)\}^2 - \frac{2(n-1)}{n^4} S(x_s y_s) S(x_s y_t) + \frac{1}{n^4}\{S(x_s y_t)\}^2$$

$$= \frac{(n-1)^2}{n^4} S(x_s^2 y_s^2) + \frac{1}{n^4} S(x_s^2 y_t^2) + \frac{2S(x_s y_s x_t y_t)}{n^4}\{(n-1)^2+1\}$$

$$- \frac{2(n-1)}{n^4} S(x_s^2 y_s y_t) - \frac{2(n-1)}{n^4} S(x_s x_t y_t^2) + \frac{S(x_s y_s x_t y_u)}{n^4}\{2-2(n-1)\}$$

$$+ \frac{2}{n^4} S(x_s x_t y_u^2) + \frac{2}{n^4} S(x_s^2 y_t y_u) + \frac{4}{n^4} S(x_s x_t y_u y_v) \quad \text{.....................(2).}$$

In finding the means of the above sums, the following relations will be required:

$$S(X_s^2 Y_t^2) = N^2 P_{20} P_{02} - NP_{22},$$

$$2S(X_s Y_s X_t Y_t) = N^2 P_{11}^2 - NP_{22},$$

$$S(X_s^2 Y_s Y_t) = S(X_s X_t Y_t^2) = -NP_{22},$$

$$S(X_s Y_s X_t Y_u) = 2NP_{22} - N^2 P_{11}^2,$$

$$2S(X_s^2 Y_t Y_u) = 2S(X_s X_t Y_u^2) = 2NP_{22} - N^2 P_{20} P_{02},$$

$$4S(X_s X_t Y_u Y_v) = N^2 P_{20} P_{02} + 2N^2 P_{11}^2 - 6NP_{22}.$$

* This result was first shown to me by Professor Pearson, and I reproduce it here for the sake of completeness.

Taking the mean for all samples from equation (2)

$$\overline{p_{11}^2} = \frac{(n-1)^2}{n^4}(nP_{22}) + \frac{2}{n^4}(\overline{n-1^2}+1)\left\{\frac{n(n-1)(NP_{11}^2 - P_{22})}{2(N-1)}\right\}$$
$$+ \frac{1}{n^4}\frac{n(n-1)}{N-1}(NP_{20}P_{02} - P_{22}) + \frac{2}{n^4}(1-\overline{n-1})\frac{n(n-1)(n-2)}{(N-1)(N-2)}(2P_{22} - NP_{11}^2)$$
$$+ \frac{4}{n^4}\frac{n(n-1)(n-2)}{2(N-1)(N-2)}(2P_{22} - NP_{20}P_{02}) + \frac{4(n-1)}{n^4}\cdot\frac{n(n-1)}{N-1}\cdot P_{22}$$
$$+ \frac{4}{n^4}\frac{n(n-1)(n-2)(n-3)}{4(N-1)(N-2)(N-3)}(NP_{20}P_{02} + 2NP_{11}^2 - 6P_{22}).$$

Now using the result for $\overline{p_{11}}$ in equation (A) we find, for the standard deviation of p_{11}, after reduction,

$$\sigma^2_{p_{11}} = \overline{p_{11}^2} - (\overline{p_{11}})^2$$
$$= \frac{N(N-n)(n-1)}{n^3(N-1)(N-2)(N-3)}\left[(N-n-1)P_{20}P_{02} + (Nn-N-n-1)P_{22}\right.$$
$$\left. - \left\{\frac{n(N-2)(N+1)}{N-1} - 2(N-1)\right\}P_{11}^2\right] \quad \text{...(B).}$$

Corollary 1. For samples from an infinite population

$$\sigma^2_{p_{11}} = \frac{n-1}{n^3}[P_{20}P_{02} + (n-1)P_{22} - (n-2)P_{11}^2].$$

If the sampled population is also normal, $P_{22} = (1+2R^2)\sigma_X^2\sigma_Y^2$, and*

$$\sigma^2_{p_{11}} = \left(\frac{n-1}{n^2}\right)(1+R^2)\sigma_X^2\sigma_Y^2.$$

Corollary 2. Putting $X = Y$ and $x = y$, we obtain the result†

$$\sigma^2_{s^2} = \frac{N(N-n)(n-1)}{n^3(N-1)^2(N-2)(N-3)}$$
$$\times[(Nn-N-n-1)(N-1)\mu_4 - (N^2n - 3N^2 + 6N - 3n - 3)\mu_2^2].$$

4. (*a*) *The Mean and* (*b*) *the Standard Deviation of* p_{21} *and* p_{12} *in Samples from a Finite Population with any Distribution.*

(*a*) Proceeding as in the case of p_{11}, we have:

$$p_{21} = S(x_s - \bar{x})^2(y_s - \bar{y})/n$$
$$= \frac{1}{n}[S(x_s^2y_s) - 2\bar{x}S(x_sy_s) + \bar{x}^2S(y_s) - \bar{y}S(x_s^2) + 2\bar{x}\bar{y}S(x_s) - n\bar{x}^2\bar{y}],$$

which gives on substituting $S(x_s) = n\bar{x}$, $S(y_s) = n\bar{y}$,

$$p_{21} = \frac{1}{n}[S(x_s^2y_s) - 2\bar{x}S(x_sy_s) - \bar{y}S(x_s^2) + 2n\bar{x}^2\bar{y}] \quad \text{..............(3).}$$

* See Dr Wishart: "The Generalised Product Moment Distribution in Samples from a Normal Multivariate Population," *loc. cit.* p. 44 (5).

† This agrees with the result obtained by Dr Neyman in *Biometrika*, Vol. xvii. p. 477, equation (52).

As in the case of p_{11}, we have to express p_{21} in terms of sums such as $S(x_s^2 y_t)$, $S(x_s x_t y_u)$ and so on. For this we have:

$$\bar{x}S(x_s y_s) = \frac{1}{n} S(x_s^2 y_s) + \frac{1}{n} S(x_s x_t y_t),$$

$$\bar{y}S(x_s^2) = \frac{1}{n} S(x_s^2 y_s) + \frac{1}{n} S(x_s^2 y_t),$$

$$\bar{x}^2 \bar{y} = \frac{1}{n^3} [S(x_s^2 y_s) + S(x_s^2 y_t) + 2S(x_s x_t y_t) + 2S(x_s x_t y_u)].$$

Putting these values in (3),

$$p_{21} = \frac{1}{n}\left[\left(1 - \frac{3}{n} + \frac{2}{n^2}\right) S(x_s^2 y_s) + \left(\frac{2}{n^2} - \frac{1}{n}\right) S(x_s^2 y_t) + \left(\frac{4}{n^2} - \frac{2}{n}\right) S(x_s x_t y_t) + \frac{4}{n^2} S(x_s x_t y_u)\right] \quad \text{...(4).}$$

To evaluate now the mean of p_{21}, we have:

$$\text{Mean } S(x_s^2 y_s) = n \text{ Mean } (X_s^2 Y_s) = \frac{n}{N} S(X_s^2 Y_s) = nP_{21},$$

$$\text{Mean } S(x_s^2 y_t) = \frac{n(n-1)}{2} \text{ Mean } (X_s^2 Y_t) = \frac{n(n-1)}{N(N-1)} S(X_s^2 Y_t) = -\frac{n(n-1)}{N-1} P_{21},$$

$$\text{Mean } S(x_s x_t y_t) = + \frac{n(n-1)}{N(N-1)} S(X_s X_t Y_t) = -\frac{n(n-1)}{N-1} P_{21},$$

$$\text{Mean } S(x_s x_t y_u) = \frac{n(n-1)(n-2)}{N(N-1)(N-2)} S(X_s X_t Y_u) = \frac{n(n-1)(n-2)}{(N-1)(N-2)} P_{21}.$$

Hence, taking the mean in repeated samples, we get, after reduction,

$$\overline{p_{21}} = P_{21} \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)}{\left(1 - \frac{1}{N}\right)\left(1 - \frac{2}{N}\right)} \quad \text{........................(C).}$$

The value of $\overline{p_{12}}$ is obtained by replacing P_{21} by P_{12} in the above result.

For samples from a normal bivariate distribution $\overline{p_{21}} = 0$, since $P_{21} = 0$.

For the single variate distribution we get from (C)

$$\overline{m_3} = \mu_3 \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)}{\left(1 - \frac{1}{N}\right)\left(1 - \frac{2}{N}\right)},$$

where m_3 is the third moment about the mean in samples of n, and μ_3 is the third moment in the sampled population.

When the sampled population is infinite, we have, making $N \to \infty$ in (C),

$$\overline{p_{21}} = P_{21}\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right).$$

(*b*) For the evaluation of $\sigma^2_{p_{21}}$ we require the value of $\overline{p_{21}^2}$, or since, from equation (4),

$$p_{21} = \frac{1}{n^3}[(n-1)(n-2)S(x_s^2y_s) - (n-2)S(x_s^2y_t) - 2(n-2)S(x_sx_ty_t) + 4S(x_sx_ty_u)],$$

we have to expand the squares and products of these sums into other sums and find the mean values of the latter sums. For this we have:

$$\begin{aligned}
[S(x_s^2y_s)]^2 &= S(x_s^4y_s^2) + 2S(x_s^2y_sx_t^2y_t),\\
[S(x_s^2y_t)]^2 &= S(x_s^4y_t^2) + 2S(x_s^2y_sx_t^2y_t) + 2S(x_s^2x_t^2y_u^2) + 2S(x_s^4y_ty_u)\\
&\quad + 2S(x_s^2y_sx_t^2y_u) + 4S(x_s^2x_t^2y_uy_v),\\
[S(x_sx_ty_t)]^2 &= S(x_s^2x_t^2y_t^2) + 2S(x_s^2y_sx_t^2y_t) + 2S(x_s^2y_sx_ty_tx_u)\\
&\quad + 2S(x_s^2x_ty_tx_uy_u) + 2S(x_s^2y_s^2x_tx_u) + 4S(x_sy_sx_ty_tx_ux_v),\\
[S(x_sx_ty_u)]^2 &= S(x_s^2x_t^2y_u^2) + 2S(x_s^2x_ty_tx_uy_u) + 2S(x_s^2x_t^2y_uy_v)\\
&\quad + 2S(x_s^2x_ty_tx_uy_v) + 2S(x_s^2y_t^2x_ux_v) + 4S(x_sy_sx_ty_tx_ux_v)\\
&\quad + 4S(x_s^2x_tx_uy_vy_w) + 6S(x_sy_sy_tx_ux_vx_w) + 12S(x_sx_tx_ux_vy_wy_z),\\
S(x_s^2y_s)S(x_s^2y_t) &= S(x_s^4y_sy_t) + S(x_s^2y_s^2x_t^2) + S(x_s^2y_sx_t^2y_u),\\
S(x_s^2y_s)S(x_sx_ty_t) &= S(x_s^3y_sx_ty_t) + S(x_s^3y_s^2x_t) + S(x_s^2y_sx_ty_tx_u),\\
S(x_s^2y_s)S(x_sx_ty_u) &= S(x_s^3y_sx_ty_u) + S(x_s^2y_s^2x_tx_u) + S(x_s^2y_sx_tx_uy_v),\\
S(x_s^2y_t)S(x_sx_ty_t) &= S(x_s^3x_ty_t^2) + S(x_s^3y_sx_ty_t) + S(x_s^3y_tx_uy_u) + S(x_s^3y_sx_ty_u)\\
&\quad + 2S(x_s^2x_ty_tx_uy_u) + S(x_s^2x_ty_t^2x_u) + S(x_s^2x_ty_tx_uy_v),\\
S(x_s^2y_t)S(x_sx_ty_u) &= S(x_s^3x_ty_ty_u) + S(x_s^2y_sx_ty_tx_u) + S(x_s^3y_t^2x_u)\\
&\quad + S(x_s^2x_ty_tx_uy_v) + S(x_s^2y_t^2x_ux_v) + 2S(x_s^3x_ty_uy_v)\\
&\quad + S(x_s^2y_sx_tx_uy_v) + 2S(x_s^2y_tx_ux_vy_w),\\
S(x_sx_ty_t)S(x_sx_ty_u) &= S(x_s^2y_sx_t^2y_u) + S(x_s^2y_sx_ty_tx_u) + S(x_s^2x_ty_t^2x_u)\\
&\quad + S(x_s^2y_sx_tx_uy_v) + S(x_sy_s^2x_tx_ux_v) + S(x_s^2x_ty_tx_uy_v)\\
&\quad + S(x_sy_sx_ty_tx_ux_v) + 3S(x_sy_sy_tx_ux_vx_w).
\end{aligned}$$

In the above expressions there are 28 different sums involved, whose mean values are to be found. As before, we can express the corresponding sums in X and Y in terms of the product moments, P_{ab}, of the sampled population. For the sake of brevity, I have not attempted to show the working but merely state the results, as follows:

$$\begin{aligned}
2S(X_s^2Y_sX_t^2Y_t) &= N^2P_{21}^2 - NP_{42},\\
S(X_s^4Y_t^2) &= N^2P_{40}P_{02} - NP_{42},\\
S(X_s^2X_t^2Y_t^2) &= N^2P_{20}P_{22} - NP_{42},\\
2S(X_s^2X_t^2Y_u^2) &= N^3P_{20}^2P_{02} - N^2(P_{40}P_{02} + 2P_{20}P_{22}) + 2NP_{42},\\
2S(X_s^4Y_tY_u) &= -N^2P_{40}P_{02} + 2NP_{42},
\end{aligned}$$

$$S(X_s^4Y_sY_t) = -S(X_s^4Y_s^2) = -NP_{42} = S(X_s^3Y_s^2X_t),$$

$$S(X_s^3Y_sX_t^2Y_u) = -N^2(P_{21}^2+P_{20}P_{22})+2NP_{42},$$

$$4S(X_s^2X_t^2Y_uY_v) = -N^3P_{20}^2P_{02}+N^2(4P_{20}P_{22}+P_{40}P_{02}+2P_{21}^2)-6NP_{42},$$

$$S(X_s^3Y_sX_tY_t) = N^2P_{31}P_{11}-NP_{42},$$

$$S(X_s^2Y_sX_tY_tX_u) = -N^2(P_{21}^2+P_{31}P_{11})+2NP_{42},$$

$$2S(X_s^2X_tY_tX_uY_u) = N^3P_{11}^2P_{20}-N^2(2P_{31}P_{11}+P_{20}P_{22})+2NP_{42},$$

$$2S(X_s^2Y_s^2X_tX_u) = -N^2P_{20}P_{22}+2NP_{42},$$

$$4S(X_sY_sX_tY_tX_uX_v) = -N^3P_{11}^2P_{20}+N^2(2P_{21}^2+4P_{31}P_{11}+P_{20}P_{22})-6NP_{42},$$

$$S(X_s^3X_tY_t^2) = N^2P_{30}P_{12}-NP_{42},$$

$$S(X_s^3X_tY_tY_u) = -N^2(P_{30}P_{12}+P_{31}P_{11})+2NP_{42},$$

$$S(X_s^3Y_sX_tY_u) = -N^2P_{31}P_{11}+2NP_{42},$$

$$S(X_s^2Y_sX_tX_uY_v) = N^2(P_{21}^2+P_{31}P_{11}+\tfrac{1}{2}P_{20}P_{22})-3NP_{42},$$

$$S(X_s^2X_tY_t^2X_u) = -N^2(P_{30}P_{12}+P_{20}P_{22})+2NP_{42},$$

$$S(X_s^2X_tY_tX_uY_v) = -N^3P_{11}^2P_{20}+N^2(P_{21}^2+3P_{31}P_{11}+2P_{20}P_{22}+P_{30}P_{12}) - 6NP_{42},$$

$$S(X_s^3Y_t^2X_u) = -N^2(P_{40}P_{02}+P_{30}P_{12})+2NP_{42},$$

$$2S(X_s^2Y_t^2X_uX_v) = -N^3P_{20}^2P_{02}+N^2(3P_{20}P_{22}+2P_{40}P_{02}+2P_{30}P_{12}) -6NP_{42},$$

$$2S(X_s^3X_tY_uY_v) = N^2(2P_{31}P_{11}+P_{40}P_{02}+2P_{30}P_{12})-6NP_{42},$$

$$6S(X_sX_tY_t^2X_uX_v) = N^2(2P_{30}P_{12}+3P_{20}P_{22})-6NP_{42},$$

$$2S(X_s^2Y_tX_uX_vY_w) = N^3(P_{11}^2P_{20}+\tfrac{1}{2}P_{20}^2P_{02}) - N^2(4P_{31}P_{11}+2P_{21}^2+4P_{20}P_{22}+P_{40}P_{02}+2P_{30}P_{12}) +12NP_{42},$$

$$12S(X_sY_sY_tX_uX_vX_w) = 5N^3P_{11}^2P_{20} - N^2(5P_{20}P_{22}+16P_{31}P_{11}+6P_{21}^2+\tfrac{4}{3}P_{30}P_{12})+22NP_{42} = -24S(X_sX_tX_uX_vY_wY_z).$$

We are now in a position to write down the mean value of p_{21}^2, and from the equation

$$\sigma^2_{p_{21}} = \overline{p_{21}^2} - (\overline{p_{21}})^2,$$

where $\overline{p_{21}}$ is given by (C), we can find $\sigma^2_{p_{21}}$.

After substitution and simplification, I obtained the result in the following form:

$$\sigma^2_{p_{21}} = \frac{N(n-1)(n-2)(N-n)P_{40}P_{02}}{n^5(N-1)(N-2)}\left[(n-2)-\frac{n^2-12n+28}{N-3}-\frac{8(n-3)(n-6)}{(N-3)(N-4)}\right] + \frac{4N(n-1)(n-2)(N-n)P_{30}P_{12}}{n^5(N-1)(N-2)}\left[(n-2)-\frac{(n-2)(n-10)}{N-3} - \frac{16(n-3)(n-5)}{3(N-3)(N-4)}-\frac{8(n-3)(n-4)}{3(N-3)(N-4)(N-5)}\right]$$

$$-\frac{4N(n-1)(n-2)(N-n)P_{31}P_{11}}{n^5(N-1)(N-2)}\left[(n-2)^2+\frac{9n^2-42n+56}{N-3}\right.$$
$$\left.+\frac{8(n-3)(3n-10)}{(N-3)(N-4)}+\frac{32(n-3)(n-4)}{(N-3)(N-4)(N-5)}\right]$$
$$\frac{2N(n-1)(n-2)(N-n)P_{20}P_{22}}{n^5(N-1)(N-2)}\left[(n-2)(n-3)+\frac{2(5n^2-27n+42)}{N-3}\right.$$
$$\left.+\frac{2(n-3)(13n-58)}{(N-3)(N-4)}+\frac{20(n-3)(n-4)}{(N-3)(N-4)(N-5)}\right]$$
$$\frac{N^2(n-1)(n-2)(N-n)P_{20}{}^2P_{02}}{n^5(N-1)(N-2)(N-3)}\left[(n^2-4n+12)+\frac{4(n-3)(n-6)}{N-4}\right]$$
$$\frac{4N^2(n-1)(n-2)(N-n)P_{11}{}^2P_{20}}{n^5(N-1)(N-2)(N-3)}\left[2(n^2-4n+6)+\frac{(n-3)(7n-22)}{N-4}\right.$$
$$\left.+\frac{10(n-3)(n-4)}{(N-4)(N-5)}\right]$$
$$\frac{N(n-1)(n-2)(N-n)P_{21}{}^2}{n^5(N-1)(N-2)}\left[(n-2)(n-6)+\frac{15(n-2)(n-4)}{N-3}\right.$$
$$+\frac{8(n-3)(5n-18)}{(N-3)(N-4)}+\frac{48(n-3)(n-4)}{(N-3)(N-4)(N-5)}+\frac{2n(7n-6)}{(N-1)(N-2)(N-3)}$$
$$\left.+\frac{2n(3n-2)}{(N-1)(N-2)}\right]$$
$$\frac{(n-1)(n-2)(N-n)P_{42}}{n^5(N-1)}\left[(n-1)(n-2)+\frac{6(n-2)(2n-5)}{N-2}\right.$$
$$+\frac{6(13n^2-72n+100)}{(N-2)(N-3)}+\frac{8(n-3)(23n-94)}{(N-2)(N-3)(N-4)}$$
$$\left.+\frac{176(n-3)(n-4)}{(N-2)(N-3)(N-4)(N-5)}\right]\quad\ldots(\mathrm{D}).$$

It is possible that the above expression may be simplified in some other way, but it is in the most convenient form when considering the case of N infinite. It will be seen that $(N-n)$ is a factor throughout, as we should expect.

Corollary 1. When $N\to\infty$

$$\sigma^2_{p_{31}}=\frac{(n-1)(n-2)^2}{n^5}\left[(n-1)P_{42}+4P_{30}P_{12}-(n-6)P_{21}{}^2+P_{40}P_{02}\right.$$
$$\left.-4(n-2)P_{31}P_{11}-2(n-3)P_{20}P_{22}\right]$$
$$+\frac{(n-1)(n-2)}{n^5}\left[(n^2-4n+12)P_{20}{}^2P_{02}+8(n^2-4n+6)P_{11}{}^2P_{20}\right].$$

Corollary 2. When $N\to\infty$ and we take $X_s=Y_s$

$$\sigma^2_{m_3}=\frac{(n-1)(n-2)^2}{n^5}\left[(n-1)\mu_6-3(2n-5)\mu_4\mu_2-(n-10)\mu_3{}^2\right]$$
$$+\frac{3(n-1)(n-2)(3n^2-12n+20)}{n^5}\mu_2{}^3\,*,$$

where m_3 is the third moment coefficient about the mean in samples of n.

* This result may be written more conveniently, in terms of the β's,

$$\sigma^2_{m_3}=\frac{(n-1)(n-2)}{n^5}\left[(n-1)(n-2)(\beta_4-15)-3(n-2)(2n-5)(\beta_2-3)-(n-2)(n-10)\beta_1+6n^2\right]\mu_2{}^3.$$

Corollary 3. When $N \to \infty$ and both variates are normal

$$\sigma^2_{p_{31}} = \frac{2(n-1)(n-2)}{n^3}(1+2R^2)\,\sigma_X^4\sigma_Y^2 = \frac{2(n-1)(n-2)P_{22}P_{20}}{n^3}.$$

If in this we put $R=1$ and $\sigma_X=\sigma_Y$, the case of a single normal variate, we have

$$\sigma^2_{m_3} = \frac{6(n-1)(n-2)}{n^3}\sigma_X^6,$$

which will be the result obtained when we put the normal values, $\mu_6 = 15\sigma_X^6$, $\mu_4 = 3\sigma_X^4$, $\mu_3 = 0$, in Corollary 2.

Corollary 4. The value of $\sigma^2_{m_3}$ when N is finite is obtained by putting $X=Y$ in (D). As I was not able to simplify the result appreciably by collecting the terms in μ_6, $\mu_4\mu_2$, μ_3^2 and μ_2^3, I have not written down the full expression for $\sigma^2_{m_3}$.

5. *The Mean of* p_{31} *in Samples from a Finite Population with any given Distribution.*

$$p_{31} = \frac{1}{n}S(x_s-\bar{x})^3(y_s-\bar{y})$$

$$= \frac{1}{n}S(x_s^3 - 3\bar{x}x_s^2 + 3\bar{x}^2x_s - \bar{x}^3)(y_s-\bar{y})$$

$$= \frac{1}{n}[S(x_s^3y_s) - 3\bar{x}S(x_s^2y_s) + 3\bar{x}^2S(x_sy_s) - \bar{y}S(x_s^3) + 3\bar{x}\bar{y}S(x_s^2) - 3n\bar{x}^3\bar{y}] \ldots (5).$$

Putting $\bar{x} = \frac{1}{n}S(x_s)$, $\bar{y} = \frac{1}{n}S(y_s)$ and multiplying out the above products, we get

$$\bar{x}S(x_s^2y_s) = \frac{1}{n}[S(x_s^3y_s) + S(x_s^2y_sx_t)],$$

$$\bar{x}^2S(x_sy_s) = \frac{1}{n^2}[S(x_s^3y_s) + S(x_s^2x_ty_t) + 2S(x_s^2y_sx_t) + 2S(x_sx_tx_uy_u)],$$

$$\bar{y}S(x_s^3) \;\;= \frac{1}{n}[S(x_s^3y_s) + S(x_s^3y_t)],$$

$$\bar{x}\bar{y}S(x_s^2) \;\;= \frac{1}{n^2}[S(x_s^3y_s) + S(x_s^2x_ty_t) + S(x_s^3y_t) + S(x_s^2y_sx_t) + S(x_s^2x_ty_u)].$$

To evaluate $\bar{x}^3\bar{y}$ I first expanded $\bar{x}^2\bar{y}$ and then multiplied each of the resulting sums by $\bar{x}$; thus

$$\bar{x}^2\bar{y} = \frac{1}{n^3}[S(x_s^2y_s) + S(x_s^2y_t) + 2S(x_sy_sx_t) + 2S(x_sx_ty_u)].$$

We have evaluated $\bar{x}S(x_s^2y_s)$ above, and for the others

$$\bar{x}S(x_s^2y_t) \;\;= \frac{1}{n}[S(x_s^3y_t) + S(x_s^2x_ty_t) + S(x_s^2x_ty_u)],$$

$$\bar{x}S(x_sy_sx_t) = \frac{1}{n}[S(x_s^2x_ty_t) + S(x_s^2y_sx_t) + 2S(x_sx_tx_uy_u)],$$

$$\bar{x}S(x_sx_ty_u) = \frac{1}{n}[S(x_s^2x_ty_u) + S(x_sx_tx_uy_u) + 3S(x_sx_tx_uy_v)].$$

If we now substitute the values of these products in equation (5) and collect terms belonging to the different sums, we obtain

$$p_{31} = \frac{1}{n^4}[(n-1)(n^2-3n+3)\,S(x_s^3y_s) - 3(n^2-3n+3)\,S(x_s^2y_sx_t) + 3(2n-3)S(x_s^2x_ty_t) + 6(n-3)\,S(x_sx_tx_uy_u) - (n^2-3n+3)\,S(x_s^3y_t) + 3(n-3)\,S(x_s^2x_ty_u) - 18S(x_sx_tx_uy_v)] \quad \text{.........................(6).}$$

For the evaluation of $\overline{p_{31}}$ I obtained the relations

$$\text{Mean } S(x_s^3y_s) = \frac{n}{N}(NP_{31}),$$

$$\text{Mean } S(x_s^2y_sx_t) = \frac{n(n-1)}{N(N-1)}(-NP_{31}),$$

$$\text{Mean } S(x_s^2x_ty_t) = \frac{n(n-1)}{N(N-1)}(N^2P_{20}P_{11} - NP_{31}),$$

$$\text{Mean } S(x_sx_tx_uy_u) = \frac{n(n-1)(n-2)}{2N(N-1)(N-2)}(2NP_{31} - N^2P_{20}P_{11}),$$

$$\text{Mean } S(x_s^3y_t) = \frac{n(n-1)}{N(N-1)}(-NP_{31}),$$

$$\text{Mean } S(x_s^2x_ty_u) = \frac{n(n-1)(n-2)}{N(N-1)(N-2)}(2NP_{31} - N^2P_{20}P_{11}),$$

$$\text{Mean } S(x_sx_tx_uy_v) = \frac{n(n-1)(n-2)(n-3)}{N(N-1)(N-2)(N-3)}\left(\frac{N^2P_{20}P_{11} - 2NP_{31}}{2}\right).$$

Substituting these mean values in equation (6) and collecting the terms in P_{31} and $P_{20}P_{11}$, we find

$$\overline{p_{31}} = \alpha P_{31} + 3\beta P_{20}P_{11} \quad \text{............................(E),}$$

where

$$\alpha = \frac{n-1}{n^3}\left[(n^2-3n+3) + \frac{4n^2-18n+21}{N-1} + \frac{12(n-2)(n-3)}{(N-1)(N-2)} + \frac{18(n-2)(n-3)}{(N-1)(N-2)(N-3)}\right],$$

$$\beta = \frac{N(n-1)}{n^3}\left[\frac{2n-3}{N-1} - \frac{2(n-2)(n-3)}{(N-1)(N-2)} - \frac{3(n-2)(n-3)}{(N-1)(N-2)(N-3)}\right] \quad \text{...(7).}$$

The value of $\overline{p_{13}}$ is obtained by substituting P_{13} and P_{02} for P_{31} and P_{20} in (E).

Corollary 1. When $N \to \infty$

$$\overline{p_{31}} = \frac{n-1}{n^3}[(n^2-3n+3)\,P_{31} + 3(2n-3)\,P_{20}P_{11}].$$

Corollary 2. For a normal population $P_{31} = 3R\sigma_X{}^3\sigma_Y$, $P_{11} = R\sigma_X\sigma_Y$, so that

$$\overline{p_{31}} = 3R\sigma_X{}^3\sigma_Y(\alpha+\beta) = P_{31}(\alpha+\beta),$$

where from (7)

$$\alpha+\beta = \frac{n-1}{n^3}\left[n(n-1) + \frac{2(n^2-3n+3)}{N-1} + \frac{5(n-2)(n-3)}{(N-1)(N-2)} + \frac{9(n-2)(n-3)}{(N-1)(N-2)(N-3)}\right].$$

Corollary 3. Putting the normal values of P_{31}, P_{11} in Corollary 1 or making $N \to \infty$ in Corollary 2, we have for samples from an infinite normal population

$$\overline{p_{31}} = \frac{3(n-1)^2}{n^2} R\sigma_X{}^3\sigma_Y = \left(\frac{n-1}{n}\right)^2 P_{31}.$$

Corollary 4. The mean value of the fourth moment coefficient in samples of n from any population is given by

$$\overline{m_4} = \alpha\mu_4 + 3\beta\mu_2{}^2.$$

For the cases of normal or infinite populations we can simply put $R=1$ and $\sigma_X = \sigma_Y$ in the first three corollaries; thus

$$\overline{m_4} = \frac{n-1}{n^3}[(n^2-3n+3)\mu_4 + 3(2n-3)\mu_2{}^2] \text{ for any infinite population,}$$

$$\overline{m_4} = 3\sigma_X{}^4(\alpha+\beta) \text{ for limited normal population,}$$

$$\overline{m_4} = \frac{3(n-1)^2}{n^2}\sigma_X{}^4 \text{ for infinite normal population.}$$

6. *The Mean of p_{22} in Samples from a Finite Population with any given Distribution.*

$$\begin{aligned} p_{22} &= \frac{1}{n} S(x_s - \bar{x})^2 (y_s - \bar{y})^2 \\ &= \frac{1}{n}[S(x_s{}^2y_s{}^2) - 2\bar{x}S(x_sy_s{}^2) - 2\bar{y}S(x_s{}^2y_s) + \bar{x}^2S(y_s{}^2) + \bar{y}^2S(x_s{}^2) \\ &\qquad + 4\bar{x}\bar{y}S(x_sy_s) - 3n\bar{x}^2\bar{y}^2]. \end{aligned}$$

Expanding these products, we get

$$\bar{x}S(x_sy_s{}^2) = \frac{1}{n}[S(x_s{}^2y_s{}^2) + S(x_sy_s{}^2x_t)],$$

$$\bar{y}S(x_s{}^2y_s) = \frac{1}{n}[S(x_s{}^2y_s{}^2) + S(x_s{}^2y_sy_t)],$$

$$\bar{x}^2S(y_s{}^2) = \frac{1}{n^2}[S(x_s{}^2y_s{}^2) + S(x_s{}^2y_t{}^2) + 2S(x_sy_s{}^2x_t) + 2S(x_sx_ty_u{}^2)],$$

$$\bar{y}^2S(x_s{}^2) = \frac{1}{n^2}[S(x_s{}^2y_s{}^2) + S(x_s{}^2y_t{}^2) + 2S(x_s{}^2y_sy_t) + 2S(x_s{}^2y_ty_u)],$$

$$\bar{x}\bar{y}S(x_sy_s) = \frac{1}{n^2}[S(x_s{}^2y_s{}^2) + 2S(x_sy_sx_ty_t) + S(x_s{}^2y_sy_t) + S(x_sy_s{}^2x_t) + S(x_sy_sx_ty_u)],$$

$$\begin{aligned} \bar{x}^2\bar{y}^2 = \frac{1}{n^4}[&S(x_s{}^2y_s{}^2) + S(x_s{}^2y_t{}^2) + 2S(x_s{}^2y_sy_t) + 2S(x_sy_s{}^2x_t) + 2S(x_s{}^2y_ty_u) \\ &+ 2S(x_sx_ty_u{}^2) + 4S(x_sy_sx_ty_t) + 4S(x_sy_sx_ty_u) + 4S(x_sx_ty_uy_v)]. \end{aligned}$$

These equations give, after substitution,

$$\begin{aligned} \overline{p_{22}} = \frac{1}{n^4}[&\alpha(n-1)S(x_s{}^2y_s{}^2) + (2n-3)S(x_s{}^2y_t{}^2) - 2\alpha\{S(x_s{}^2y_sy_t) + S(x_sy_s{}^2x_t)\} \\ &+ 4(2n-3)S(x_sy_sx_ty_t) + 4(n-3)S(x_sy_sx_ty_u) \\ &+ 2(n-3)\{S(x_s{}^2y_ty_u) + S(x_sx_ty_u{}^2)\} - 12S(x_sx_ty_uy_v)] \quad \text{.........(8),} \end{aligned}$$

where $\alpha = n^2 - 3n + 3$, a quantity which often appears in this work.

The above sums are the same as those which occurred in the evaluation of $\sigma^2_{p_{11}}$. Their means have been found. From these values we obtain, after simplification,

$$\overline{p_{22}} = \alpha P_{22} + \beta\,(P_{20}P_{02} + 2P_{11}^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(F)},$$

where α, β are the same coefficients as those which occur in the value of $\overline{p_{31}}$.

Corollary 1. When $N \to \infty$

$$\overline{p_{22}} = \frac{n-1}{n^3}\left[(n^2 - 3n + 3)\,P_{22} + (2n-3)\,(P_{20}P_{02} + 2P_{11}^2)\right].$$

Corollary 2. For a normal population $P_{22} = (1 + 2R^2)\,\sigma_X^2\sigma_Y^2$, and thus:

$$\overline{p_{22}} = P_{22}\,(\alpha + \beta),$$

and for the case of $N \to \infty$

$$\overline{\quad} = \left(\frac{n-1}{n}\right)^2 P_{22}.$$

Since $\overline{p_{22}}$ and $\overline{p_{31}}$ both reduce to the mean value of the fourth moment, $\overline{m_4}$, in the case of a single variate, we shall obtain the same results from (F) as from (E) when we put $X_s = Y_s$.

7. *The Correlation between the Variances in Samples from any Bivariate Distribution.*

For any sample, we have

$$\sigma_x^2 = \frac{1}{n}\,S\,(x_s^2) - \frac{1}{n^2}\,[S\,(x_s)]^2$$

$$= \frac{n-1}{n^2}\,S\,(x_s^2) - \frac{2}{n^2}\,S\,(x_s x_t).$$

Similarly $$\sigma_y^2 = \frac{n-1}{n^2}\,S\,(y_s^2) - \frac{2}{n^2}\,S\,(y_s y_t).$$

Therefore

$$\sigma_x^2\sigma_y^2 = \frac{(n-1)^2}{n^4}\,[S\,(x_s^2y_s^2) + S\,(x_s^2y_t^2)] + \frac{4}{n^4}\,[S\,(x_s y_s x_t y_t) + S\,(x_s y_s x_t y_u) + S\,(x_s x_t y_u y_v)]$$
$$- \frac{2\,(n-1)}{n^4}\,[S\,(x_s^2 y_s y_t) + S\,(x_s y_s^2 x_t) + S\,(x_s x_t y_u^2) + S\,(x_s^2 y_t y_u)] \quad \ldots\ldots(9).$$

Thus we have once more the sums involved in the expression for p_{11}^2, equation (2). Using the mean values there obtained, we get, after reduction,

$$\overline{\sigma_x^2\sigma_y^2} = \frac{(N-n)(n-1)\,P_{22}}{n^3}\left[\frac{n-1}{N-1} + \frac{2\,(2n-3)}{(N-1)(N-2)} + \frac{6\,(n-2)}{(N-1)(N-2)(N-3)}\right]$$
$$+ \frac{2N\,(N-n)(n-1)\,P_{11}^2}{n^3}\left[\frac{1}{(N-1)(N-2)} - \frac{(n-2)}{(N-1)(N-2)(N-3)}\right]$$
$$+ \frac{NP_{20}P_{02}}{n^3}\left[(n-1)^2\frac{n-1}{N-1} + \frac{(n-1)(n-2)(n-3)}{(N-1)(N-2)(N-3)} + 2(n-1)\frac{(n-1)(n-2)}{(N-1)(N-2)}\right].$$

Now $$\overline{\sigma_x^2} = \frac{N\,(n-1)}{n\,(N-1)}\,P_{20}, \quad \overline{\sigma_y^2} = \frac{N\,(n-1)}{n\,(N-1)}\,P_{02},$$

so that
$$\overline{\sigma_x^2}\,\overline{\sigma_y^2} = \frac{N^2(n-1)^2}{n^2(N-1)^2} P_{20} P_{02},$$
and

$$\overline{\sigma_x^2 \sigma_y^2} - \overline{\sigma_x^2}\,\overline{\sigma_y^2}$$
$$= \frac{(N-n)(n-1)}{n^3} \left[P_{22} \left\{ \frac{n-1}{N-1} + \frac{2(2n-3)}{(N-1)(N-2)} + \frac{6(n-2)}{(N-1)(N-2)(N-3)} \right\} + 2NP_{11}^2 \left\{ \frac{1}{(N-1)(N-2)} - \frac{n-2}{(N-1)(N-2)(N-3)} \right\} - \frac{NP_{20}P_{02}}{(N-1)^2(N-2)(N-3)} \{N(N-2)(n-1)-(n+1)\} \right].$$

We shall find the correlation between σ_x^2 and σ_y^2 from the relation
$$r_{\sigma_x^2, \sigma_y^2} = \frac{1}{\sigma_{\sigma_x^2}\sigma_{\sigma_y^2}} \left(\overline{\sigma_x^2 \sigma_y^2} - \overline{\sigma_x^2}\,\overline{\sigma_y^2}\right) \quad \text{.....................(10).}$$

Thus we now require the standard deviations of the variances, viz. $\sigma_{\sigma_x^2}$, $\sigma_{\sigma_y^2}$. This is given by Corollary (2) to the result (B) and the standard deviations may be written in the form
$$\sigma^2_{\sigma_x^2} = \frac{N(N-n)(n-1)P_{20}^2}{n^3(N-1)^2(N-2)(N-3)} (A\beta_{2,X} - B),$$
$$\sigma^2_{\sigma_y^2} = \frac{N(N-n)(n-1)P_{02}^2}{n^3(N-1)^2(N-2)(N-3)} (A\beta_{2,Y} - B),$$
where
$$A = (N-1)(Nn - N - n - 1),$$
$$B = N^2 n - 3N^2 + 6N - 3n - 3,$$
and
$$\beta_{2,X} = \frac{P_{40}}{P_{20}^2}, \quad \beta_{2,Y} = \frac{P_{04}}{P_{02}^2}.$$
Therefore
$$\sigma_{\sigma_x^2}\sigma_{\sigma_y^2} = \frac{N(N-n)(n-1)P_{20}P_{02}}{n^3(N-1)^2(N-2)(N-3)} \sqrt{(A\beta_{2,X} - B)(A\beta_{2,Y} - B)}.$$

Hence, using the above equation (10) for the correlation, this may be written in the form
$$r_{\sigma_x^2, \sigma_y^2} = \frac{AP_{22} + 2(N-1)(N-n-1)P_{11}^2 - P_{20}P_{02}\{N(N-2)(n-1) - (n+1)\}}{P_{20}P_{02}\sqrt{(A\beta_{2,X} - B)(A\beta_{2,Y} - B)}} \quad \text{......(G).}$$

Corollary 1. When $N \to \infty$
$$r_{\sigma_x^2, \sigma_y^2} = \frac{(n-1)(P_{22} - P_{20}P_{02}) + 2P_{11}^2}{P_{20}P_{02}\sqrt{\{(n-1)\beta_{2,X} - (n-3)\}\{(n-1)\beta_{2,Y} - (n-3)\}}}.$$

Corollary 2. For samples from an infinite normal population we must put
$$P_{22} = (1 + 2R^2)\sigma_X^2 \sigma_Y^2, \quad \beta_{2,X} = \beta_{2,Y} = 3,$$
and we obtain the simple result
$$r_{\sigma_x^2, \sigma_y^2} = R^2$$
independent of the size of the sample, a result already familiar*.

* See Wishart: *loc. cit.* p. 43.

8. *The Correlation between σ_x^2 and p_{11} in Samples from a Finite Population with any given Distribution.*

We have the equations

$$x = \frac{\qquad}{n^2} S(x_s^2) - \frac{2}{n^2} S(x_s x_t),$$

$$p_{11} = \frac{n-1}{n^2} S(x_s y_s) - \frac{1}{n^2} S(x_s y_t),$$

$$\sigma_x^2 p_{11} = \frac{(n-1)^2}{n^4} S(x_s^3 y_s) + \frac{n^2-2n+3}{n^4} S(x_s^2 x_t y_t) - \frac{3(n-1)}{n^4} S(x_s^2 y_s x_t)$$

$$-\frac{2(n-3)}{n^4} S(x_s y_s x_t x_u) - \frac{(n-1)}{n^4} S(x_s^3 y_t) - \frac{(n-3)}{n^4} S(x_s^2 x_t y_u)$$

$$+\frac{6}{n^4} S(x_s x_t x_u y_v) \quad \text{.........(11).}$$

These sums are the same as occur in the expression for p_{31}, and their means have been found. Hence, using these values, and the following means, previously reached,

$$\overline{\sigma_x^2} = \frac{N(n-1)}{n(N-1)} P_{20}, \quad \overline{p_{11}} = \frac{N(n-1)}{n(N-1)} P_{11},$$

we obtain after simplification

$$\overline{\sigma_x^2 p_{11}} - \overline{\sigma_x^2}\,\overline{p_{11}} = \frac{N(N-n)(n-1)}{n^3(N-1)^2(N-2)(N-3)} (AP_{31} - BP_{20}P_{11}),$$

where A and B are the expressions in the previous section.

Now to find the correlation between σ_x^2 and p_{11} we need the following standard deviations:

$$\sigma^2_{p_{11}} = \frac{N(N-n)(n-1)}{n^3(N-1)^2(N-2)(N-3)} [AP_{22} + (N-1)(N-n-1)P_{20}P_{02}$$
$$- \{n(N-2)(N+1) - 2(N-1)^2\} P_{11}^2],$$

$$\sigma^2_{\sigma_x^2} = \frac{N(N-n)(n-1)P_{20}^2}{n^3(N-1)^2(N-2)(N-3)} (A\beta_{2,x} - B),$$

giving the result

$$r_{\sigma_x^2, p_{11}} = \frac{AP_{31} - BP_{20}P_{11}}{P_{20}\sqrt{A\beta_{2,x} - B}\sqrt{AP_{22} + (N-1)(N-n-1)P_{20}P_{02} - CP_{11}^2}} \quad \text{...(H),}$$

where $\quad C = n(N-2)(N+1) - 2(N-1)^2.$

Corollary 1. When $N \to \infty$

$$r_{\sigma_x^2, p_{11}} = \frac{(n-1)P_{31} - (n-3)P_{20}P_{11}}{P_{20}\sqrt{(n-1)\beta_{2,x} - (n-3)}\sqrt{(n-1)P_{22} + P_{20}P_{02} - (n-2)P_{11}^2}}.$$

Corollary 2. For samples from an infinite normal population, where

$$P_{31}=3R\sigma_X{}^3\sigma_Y,\quad P_{22}=(1+2R^2)\,\sigma_X{}^2\sigma_Y{}^2,\quad \beta_{2,X}=3,$$

$$r_{\sigma_x{}^2,p_{11}}=\sqrt{\frac{2R^2}{1+R^2}},$$

once again independent of the size of the sample, a result already known*.

9. *The Correlation of* (*a*) m_x *and* m_y, (*b*) $\sigma_x{}^2$ *and* m_y, (*c*) m_x *and* p_{11}, *in Samples from a Finite Population with any given Distribution.*

(*a*) We have:

$$\overline{m_x m_y}=\frac{1}{n^2}\overline{S(x_s)\,S(y_s)}$$

$$=\frac{1}{n^2}\overline{S(x_s y_s)+S(x_s y_t)}$$

$$=\frac{1}{n^2}\left[nP_{11}-\frac{n(n-1)}{N(N-1)}P_{11}\right]$$

$$=\frac{(N-n)}{n(N-1)}P_{11}.$$

We also find:

$$\overline{m_x}=\overline{m_y}=0,$$

$$\sigma^2_{m_x}=\frac{N-n}{n(N-1)}P_{20},\quad \sigma^2_{m_y}=\frac{N-n}{n(N-1)}P_{02}.$$

Hence

$$r_{m_x,m_y}=\frac{P_{11}(N-n)}{n(N-1)}\Big/\frac{(N-n)}{n(N-1)}\sqrt{P_{20}P_{02}}=\frac{P_{11}}{\sigma_X\sigma_Y}=R_{X,Y}\quad\text{.........(J).}$$

It will be observed that this result is true for any sampled population, not necessarily infinite or normal, and it is independent of n, the size of the sample.

(*b*) We have:

$$\sigma_x{}^2=\left(\frac{n-1}{n^2}\right)S(x_s{}^2)-\frac{2}{n^2}S(x_s x_t),\quad m_y=\frac{1}{n}S(y_s),$$

$$\sigma_x{}^2 m_y=\frac{n-1}{n^3}\left[S(x_s{}^2y_s)+S(x_s{}^2y_t)\right]-\frac{2}{n^3}\left[S(x_s y_s x_t)+S(x_s x_t y_u)\right]\quad\text{...(12).}$$

If we refer back to Section (4) on $\overline{p_{21}}$, we shall find the mean values of the above sums written down. These give

$$\overline{\sigma_x{}^2 m_y}=\frac{N(n-1)(N-n)P_{21}}{n^2(N-1)(N-2)}.$$

Now $\overline{\sigma_x{}^2}\,\overline{m_y}=0$, since $\overline{m_y}=0$,

and

$$\sigma^2_{\sigma_x{}^2}=\frac{N(N-n)(n-1)P_{20}{}^2}{n^3(N-1)^2(N-2)(N-3)}(A\beta_{2,X}-B),\quad \sigma^2_{m_y}=\frac{(N-n)P_{02}}{n(N-1)},$$

giving

$$r_{\sigma_x{}^2,m_y}=\frac{P_{21}}{(N-2)P_{20}}\sqrt{\frac{(n-1)N(N-1)(N-2)(N-3)}{P_{02}(A\beta_{2,X}-B)}}\quad\text{......(K)†.}$$

* See Wishart: *loc. cit.* p. 43.

† It will be observed that both results (K) and (L), when we put $X=Y$, reduce to the result obtained by Dr Neyman for the correlation between the mean of a sample and its variance, given in equation (67), *Biometrika*, Vol. XVII. p. 479.

Corollary. When $N \to \infty$

$$r_{\sigma_x^2, m_y} = \frac{P_{21}}{P_{20}\sqrt{P_{02}}}\sqrt{\frac{n-1}{(n-1)\beta_{2,X}-(n-3)}}.$$

Since for a normal population $P_{21}=0$, $r_{\sigma_x^2, m_y}$ will be zero in this case.

(*c*) We have: $p_{11} = \frac{n-1}{n^2} S(x_s y_s) - \frac{1}{n^2} S(x_s y_t)$, $\quad m_x = \frac{1}{n} S(x_s)$,

$$m_x p_{11} = \frac{n-1}{n^3} S(x_s^2 y_s) - \frac{1}{n^3} S(x_s^2 y_t) + \frac{n-2}{n^3} S(x_s x_t y_t) - \frac{2}{n^3} S(x_s x_t x_u) \quad \ldots (13).$$

Again using the mean values of the sums found in p_{21} we get

$$\overline{m_x p_{11}} = \frac{N(n-1)(N-n)P_{21}}{n^2(N-1)(N-2)},$$

the same result as for $\overline{\sigma_x^2 m_y}$.

Hence, remembering that $\overline{m_x p_{11}}=0$ and using the values of $\sigma^2_{m_x}$ and $\sigma^2_{p_{11}}$ as before, we obtain the correlation

$$r_{m_x, p_{11}} = \frac{P_{21}}{(N-2)\sqrt{P_{20}}}\sqrt{\frac{(n-1)N(N-1)(N-2)(N-3)}{AP_{22}+(N-1)(N-n-1)P_{20}P_{02}-CP_{11}^2}} \quad \text{(L)*}.$$

Corollary. When $N \to \infty$

$$r_{m_x, p_{11}} = \frac{P_{21}}{\sqrt{P_{20}}}\sqrt{\frac{(n-1)}{(n-1)P_{22}+P_{20}P_{02}-(n-2)P_{11}^2}}.$$

10. *The Standard Deviation of* (*a*) $m_x - m_y$, (*b*) $m_x m_y$.

(*a*) $m_x - m_y = \frac{1}{n}[S(x_s) - S(y_s)]$,

$$(m_x - m_y)^2 = \frac{1}{n^2}[S(x_s^2) + 2S(x_s x_t) + S(y_s^2) + 2S(y_s y_t) - 2S(x_s y_s) - 2S(x_s y_t)],$$

$$\overline{(m_x - m_y)^2} = \frac{1}{n^2}\left[\frac{n}{N} NP_{20} - \frac{n(n-1)}{N(N-1)} NP_{20} + \frac{n}{N} NP_{02} - \frac{n(n-1)}{N(N-1)} NP_{02} \right.$$
$$\left. - \frac{2n}{N} NP_{11} + \frac{2n(n-1)}{N(N-1)} NP_{11}\right]$$
$$= \frac{N-n}{n(N-1)}[P_{20}+P_{02}-2P_{11}].$$

Since $\overline{m_x - m_y} = 0$, we have

$$\sigma^2_{m_x - m_y} = \frac{N-n}{n(N-1)}(\sigma_X^2 + \sigma_Y^2 - 2R\sigma_X\sigma_Y).$$

For $N \to \infty$ this agrees with the familiar result

$$\sigma^2_{m_x - m_y} = \left(\frac{\sigma_X}{\sqrt{n}}\right)^2 + \left(\frac{\sigma_Y}{\sqrt{n}}\right)^2 - 2\left(\frac{\sigma_X}{\sqrt{n}}\right)\left(\frac{\sigma_Y}{\sqrt{n}}\right)R.$$

* See footnote †, p. 245.

$$(b) \qquad m_x^2 = \frac{1}{n^2}[S(x_s^2) + 2S(x_s x_t)], \qquad m_y^2 = \frac{1}{n^2}[S(y_s^2) + 2S(y_s y_t)],$$

$$m_x^2 m_y^2 = \frac{1}{n^4}[S(x_s^2 y_s^2) + S(x_s^2 y_t^2) + 2S(x_s^2 y_s y_t) + 2S(x_s y_s^2 x_t) + 2S(x_s^2 y_t y_u) + 2S(x_s x_t y_u^2) + 4S(x_s y_s x_t y_t) + 4S(x_s y_s x_t y_u) + 4S(x_s x_t y_u y_v)] \ldots (14).$$

Using the values of the means of these sums, previously found in § 3, it will be found, after reduction,

$$\overline{m_x^2 m_y^2} = \frac{(N-n)P_{22}}{n^3}\left[\frac{1}{N-1} - \frac{6(n-1)(N-n-1)}{(N-1)(N-2)(N-3)}\right] + \frac{N(N-n)(n-1)(N-n-1)(P_{20}P_{02} + 2P_{11}^2)}{n^3(N-1)(N-2)(N-3)}.$$

From previous work, we also have

$$\overline{m_x m_y} = \frac{(N-n)}{n(N-1)} P_{11}.$$

Hence

$$\sigma^2_{m_x m_y} = \overline{m_x^2 m_y^2} - (\overline{m_x m_y})^2$$
$$= \frac{(N-n)P_{22}}{n^3}\left[\frac{1}{N-1} - \frac{6(n-1)(N-n-1)}{(N-1)(N-2)(N-3)}\right] + \frac{N(N-n)(n-1)(N-n-1)P_{20}P_{02}}{n^3(N-1)(N-2)(N-3)} + \frac{(N-n)P_{11}^2}{n^3}\left[\frac{2N(n-1)(N-n-1)}{(N-1)(N-2)(N-3)} - \frac{n(N-n)}{(N-1)^2}\right]^*.$$

11. *The Standard Deviations of (a) p_{31} and p_{13}, (b) p_{22}, in Samples from an Infinite Population with any given Distribution.*

(*a*) I had worked out this case for a limited sampled population but owing to the great length of the final result which I obtained in an unsimplified form I decided for the purposes of this paper to confine myself to the case when the sampled population is infinite. Even here the result is rather long, but it reduces considerably when the population follows the normal law.

We have already expanded p_{31} in sums of x_s and y_s. For the standard deviation of p_{31}, we must square this expression and find the mean values of the resulting sums. As we are taking N infinite, many of these sums will vanish when their mean values are taken, owing to the fact that $S(X_s)$ and $S(Y_s)$ vanish in the sampled population. Therefore, in the following expansions which are needed for p_{31}^2, I have omitted these sums. Thus:

$$[S(x_s^3 y_s)]^2 = S(x_s^6 y_s^2) + 2S(x_s^3 y_s x_t^3 y_t),$$

$$[S(x_s^2 y_s x_t)]^2 = S(x_s^4 y_s^2 x_t^2) + 2S(x_s^3 y_s x_t^3 y_t) + 2S(x_s^2 y_s x_t^2 y_t x_u^2) + \text{etc.},$$

$$[S(x_s^2 x_t y_t)]^2 = S(x_s^4 x_t^2 y_t^2) + 2S(x_s^3 y_s x_t^3 y_t) + 2S(x_s^4 x_t y_t x_u y_u) + 2S(x_s^3 y_s x_t y_t x_u^2) + 2S(x_s^2 y_s^2 x_t^2 x_u^2) + 4S(x_s y_s x_t y_t x_u^2 x_v^2),$$

* This will be found to agree with Dr Neyman's formula (59) in *Biometrika*, Vol. XVII. p. 478, on putting $X = Y$.

$$[S(x_s y_s x_t x_u)]^2 = S(x_s^2 y_s^2 x_t^2 x_u^2) + 2S(x_s^2 y_s x_t^2 y_t x_u^2) + 2S(x_s y_s x_t y_t x_u^2 x_v^2) + \text{etc.},$$

$$[S(x_s^3 y_t)]^2 = S(x_s^6 y_t^2) + 2S(x_s^3 y_s x_t^3 y_t) + 2S(x_s^3 x_t^3 y_u^2) + \text{etc.},$$

$$[S(x_s^2 x_t y_u)]^2 = S(x_s^4 x_t^2 y_u^2) + 2S(x_s^3 x_t^2 y_t x_u y_u) + 2S(x_s^4 x_t y_t x_u y_u) + 2S(x_s^3 x_t^3 y_u^2) + 2S(x_s^2 y_s x_t^2 y_t x_u^2) + 6S(x_s^2 x_t^2 x_u^2 y_v^2) + 4S(x_s^2 x_t^2 x_u y_u x_v y_v) + \text{etc.},$$

$$[S(x_s x_t x_u y_v)]^2 = S(x_s^2 x_t^2 x_u^2 y_v^2) + 2S(x_s^2 x_t^2 x_u y_u x_v y_v) + \text{etc.},$$

$$S(x_s^3 y_s)\, S(x_s^2 y_s x_t) = S(x_s^4 y_s x_t^2 y_t) + \text{etc.},$$

$$S(x_s^3 y_s)\, S(x_s^2 x_t y_t) = S(x_s^5 y_s x_t y_t) + S(x_s^4 y_s^2 x_t^2) + S(x_s^3 y_s x_t y_t x_u^2),$$

$$S(x_s^3 y_s)\, S(x_s^3 y_t) = S(x_s^3 y_s^2 x_t^3) + \text{etc.},$$

$$S(x_s^2 y_s x_t)\, S(x_s^2 x_t y_t) = S(x_s^4 y_s x_t^2 y_t) + S(x_s^3 y_s^2 x_t^3) + S(x_s^3 x_t^2 y_t x_u y_u) + 2S(x_s^2 y_s x_t^2 y_t x_u^2) + \text{etc.},$$

$$S(x_s^2 y_s x_t) S(x_s y_s x_t x_u) = S(x_s^3 y_s x_t y_t x_u^2) + \text{etc.},$$

$$S(x_s^2 y_s x_t)\, S(x_s^3 y_t) = S(x_s^5 y_s x_t y_t) + S(x_s^4 x_t^2 y_t^2) + S(x_s^3 x_t^2 y_t x_u y_u) + \text{etc.},$$

$$S(x_s^2 y_s x_t)\, S(x_s^2 x_t y_u) = S(x_s^3 y_s x_t y_t x_u^2) + 2S(x_s^2 y_s^2 x_t^2 x_u^2) + \text{etc.},$$

$$S(x_s^2 x_t y_t) S(x_s y_s x_t x_u) = S(x_s^3 x_t^2 y_t x_u y_u) + \text{etc.},$$

$$S(x_s^2 x_t y_t)\, S(x_s^3 y_t) = S(x_s^5 x_t y_t^2) + S(x_s^4 y_s x_t^2 y_t) + S(x_s^3 x_t^2 y_t x_u y_u) + S(x_s^3 x_t y_t^2 x_u^2) + \text{etc.},$$

$$S(x_s^2 x_t y_t) S(x_s^2 x_t y_u) = S(x_s^3 x_t y_t^2 x_u^2) + 2S(x_s^2 y_s x_t^2 y_t x_u^2) + \text{etc.},$$

$$S(x_s y_s x_t x_u)\, S(x_s^3 y_t) = 2S(x_s^4 x_t y_t x_u y_u) + \text{etc.},$$

$$S(x_s y_s x_t x_u) S(x_s^2 x_t y_u) = S(x_s^3 y_s x_t y_t x_u^2) + S(x_s^3 x_t y_t^2 x_u^2) + S(x_s^3 x_t^2 y_t x_u y_u) + 4S(x_s y_s x_t y_t x_u^2 x_v^2) + \text{etc.},$$

$$S(x_s^3 y_t)\, S(x_s^2 x_t y_u) = S(x_s^4 x_t^2 y_u^2) + S(x_s^3 y_s x_t y_t x_u^2) + \text{etc.}$$

To find the mean values of these sums, it is necessary to express the corresponding sums in X and Y in terms of the product moments of the sampled population. However, it is only necessary in each case to consider the term with the highest degree in N as those of lower degree vanish when N is infinite.

For example,

$$\begin{aligned}
\text{Mean } S(x_s^3 y_s x_t^3 y_t) &= \lim_{N\to\infty} \frac{n(n-1)}{N(N-1)} S(X_s^3 Y_s X_t^3 Y_t) \\
&= \lim_{N\to\infty} \frac{n(n-1)}{N(N-1)} \frac{N^2 P_{31}^2 - N P_{62}}{2} \\
&= \tfrac{1}{2} n(n-1) P_{31}^2,
\end{aligned}$$

$$\text{Mean } S(x_s^4 y_s^2 x_t^2) = n(n-1) P_{42} P_{20},$$

$$\text{Mean } S(x_s^3 y_s x_t y_t x_u^2) = n(n-1)(n-2) P_{31} P_{20} P_{11},$$

$$\text{Mean } S(x_s^2 x_t^2 x_u y_u x_v y_v) = \tfrac{1}{4} n(n-1)(n-2)(n-3) P_{20}^2 P_{11}^2,$$

and similarly the means of the other sums may be written down and hence the

value of $\overline{p_{31}^2}$. We have already found the mean of p_{31} in (E) and when N is infinite

$$(\overline{p_{31}})^2 = \frac{(n-1)^2}{n^6}\left[\alpha^2 P_{31}^2 + 9(2n-3)^2 P_{20}^2 P_{11}^2 + 6\alpha(2n-3) P_{31} P_{20} P_{11}\right],$$

where, as before, $\alpha = n^2 - 3n + 3$.

Hence, from the equation

$$\sigma^2_{p_{31}} = \overline{p_{31}^2} - (\overline{p_{31}})^2,$$

it will be found, after some reduction, that

$$\begin{aligned}
n^7\sigma^2_{p_{31}} = {} & \alpha^2(n-1)^2(P_{62} - P_{31}^2 - 6P_{41}P_{21} - 2P_{32}P_{30}) \\
& + \alpha^2(n-1)\{P_{60}P_{02} + 9P_{42}P_{20} + 6P_{51}P_{11} + 10P_{31}^2 + 6P_{40}P_{22} \\
& + 6(n-2)P_{30}P_{21}P_{11} + (n-2)P_{30}^2P_{02} + 9(n-2)P_{21}^2P_{20}\} \\
& + 9(n-1)(2n-3)^2[P_{40}P_{22} + P_{31}^2 + (n-2)\{P_{11}^2P_{40} \\
& + 2P_{31}P_{20}P_{11} + P_{20}^2P_{22} + (n-3)P_{20}^2P_{11}^2\}] \\
& + 9(n-1)(n-2)(n-3)^2\{6P_{30}P_{21}P_{11} + 4P_{30}P_{20}P_{12} + 5P_{21}^2P_{20} \\
& + P_{30}^2P_{02} + P_{40}P_{02}P_{20} + 4P_{31}P_{20}P_{11} + P_{11}^2P_{40} + 2P_{20}^2P_{22} \\
& + (n-3)(P_{20}^3P_{02} + 7P_{20}^2P_{11}^2)\} \\
& + 6\alpha(n-1)^2(2n-3)(P_{51}P_{11} + P_{42}P_{20} - 2P_{31}P_{20}P_{11}) \\
& - 6\alpha(n-1)(2n-3)\{P_{50}P_{12} + 4P_{41}P_{21} + 3P_{32}P_{30} \\
& + (n-2)(4P_{30}P_{21}P_{11} + P_{30}P_{20}P_{12} + 3P_{21}^2P_{20})\} \\
& - 6\alpha(n-1)(n-2)(n-3)(10P_{31}P_{20}P_{11} + P_{40}P_{02}P_{20} + 2P_{11}^2P_{40} + 3P_{20}^2P_{22}) \\
& + 18(n-1)(n-2)(n-3)(2n-3)(P_{30}P_{20}P_{12} + 2P_{30}P_{21}P_{11} + P_{21}^2P_{20}) \\
& + 54(n-1)(n-2)(n-3)(P_{20}^3P_{02} + 3P_{20}^2P_{11}^2) \\
& - 9n(n-1)^2(2n-3)^2P_{20}^2P_{11}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\mathrm{M}).
\end{aligned}$$

Corollary 1. For a normal population, the product moments above have the values:

$$\begin{array}{lll}
P_{62} = 15(1+6R^2)\sigma_X^6\sigma_Y^2, & P_{31} = 3R\sigma_X^3\sigma_Y, & P_{60} = 15\sigma_X^6, \\
P_{42} = 3(1+4R^2)\sigma_X^4\sigma_Y^2, & P_{51} = 15R\sigma_X^5\sigma_Y, & P_{22} = (1+2R^2)\sigma_X^2\sigma_Y^2, \\
P_{40} = 3\sigma_X^4, & P_{41} = P_{21} = P_{12} = P_{32} = P_{30} = 0. &
\end{array}$$

On substitution of these values, $\sigma^2_{p_{31}}$ reduces to the value

$$\sigma^2_{p_{31}} = \frac{3(n-1)\sigma_X^6\sigma_Y^2}{n^4}[(5n^2 - 12n + 9) + 3R^2(9n^2 - 20n + 13)].$$

Corollary 2. Put $X = Y$ in the result, thus giving the standard deviation of the fourth moment in samples of n. I have written the result in terms of the β's of the sampled population, thus:

$$\begin{aligned}
\frac{n^7\sigma^2_{m_4}}{\sigma^8} = {} & \alpha^2(n-1)^2[(\beta_6 - 105) - 8\beta_3 - (\beta_2 - 3)^2 - 6(\beta_2 - 3)] \\
& + 16\alpha^2(n-1)[(\beta_4 - 15) + (\beta_2 - 3)^2 + 6(\beta_2 - 3) + (n-2)\beta_1] \\
& + 18(n-1)(2n-3)^2[(\beta_2 - 3)^2 + 2(n+1)(\beta_2 - 3)]
\end{aligned}$$

$$+72(n-1)(n-2)(n-3)^2[(\beta_2-3)+2\beta_1]$$
$$+12\alpha(n-1)^2(2n-3)[(\beta_4-15)-(\beta_2-3)]$$
$$-48\alpha(n-1)(2n-3)[\beta_3+(n-2)\beta_1]-96\alpha(n-1)(n-2)(n-3)(\beta_2-3)$$
$$+72(n-1)(n-2)(n-3)(2n-3)\beta_1+24n^3(n-1)(4n^2-9n+6)\ \ldots(15),$$

where $$\alpha = n^2-3n+3.$$

Corollary 3. When the sampled population is normal all the terms, except the last, vanish in the above result giving, in this case,

$$\sigma^2_{m_4} = 24\left(\frac{n-1}{n^4}\right)(4n^2-9n+6)\sigma^8$$
$$= 24\frac{(n-1)^3}{n^4}\left\{4-\frac{1}{n-1}+\frac{1}{(n-1)^2}\right\}\sigma^8 \quad \ldots\ldots\ldots\ldots\ldots(16).$$

(*b*) Referring back to equation (8) giving the value of p_{22}, we shall require the following results in the expansion of $p_{22}{}^2$:

$$[S(x_s^2y_s^2)]^2 = S(x_s^4y_s^4)+2S(x_s^2y_s^2x_t^2y_t^2),$$
$$[S(x_s^2y_t^2)]^2 = S(x_s^4y_t^4)+2S(x_s^2y_s^2x_t^2y_t^2)+2S(x_s^4y_t^2y_u^2)$$
$$+2S(x_s^2y_s^2x_t^2y_u^2)+2S(x_s^2x_t^2y_u^4)+4S(x_s^2x_t^2y_u^2y_v^2),$$
$$[S(x_s^2y_sy_t)]^2 = S(x_s^4y_s^2y_t^2)+2S(x_s^2y_s^2x_t^2y_t^2)+2S(x_s^2y_sx_t^2y_ty_u^2)+\text{etc.},$$
$$[S(x_sy_sx_ty_t)]^2 = S(x_s^2y_s^2x_t^2y_t^2)+2S(x_s^2y_s^2x_ty_tx_uy_u)+6S(x_sy_sx_ty_tx_uy_ux_vy_v),$$
$$[S(x_sy_sx_ty_u)]^2 = S(x_s^2y_s^2x_t^2y_u^2)+2S(x_s^2y_s^2x_ty_tx_uy_u)+2S(x_s^2y_sx_ty_t^2x_uy_u)$$
$$+2S(x_s^2y_t^2x_uy_ux_vy_v)+2S(x_s^2y_sx_t^2y_ty_u^2)$$
$$+2S(x_sy_s^2x_ty_t^2x_u^2)+24S(x_sy_sx_ty_tx_uy_ux_vy_v)+\text{etc.},$$
$$[S(x_s^2y_ty_u)]^2 = S(x_s^4y_t^2y_u^2)+2S(x_s^2y_sx_t^2y_ty_u^2)+\text{etc.},$$
$$[S(x_sx_ty_uy_v)]^2 = S(x_s^2x_t^2y_u^2y_v^2)+2S(x_s^2y_t^2x_uy_ux_vy_v)$$
$$+6S(x_sy_sx_ty_tx_uy_ux_vy_v)+\text{etc.},$$
$$S(x_s^2y_s^2)S(x_s^2y_t^2) = S(x_s^4y_s^2y_t^2)+S(x_s^2y_s^4x_t^2)+S(x_s^2y_s^2x_t^2y_u^2),$$
$$S(x_s^2y_s^2)S(x_s^2y_sy_t) = S(x_s^2y_s^3x_t^2y_t)+\text{etc.},$$
$$S(x_s^2y_s^2)S(x_sy_sx_ty_t) = S(x_s^3y_s^3x_ty_t)+S(x_s^2y_s^2x_ty_tx_uy_u),$$
$$S(x_s^2y_t^2)S(x_s^2y_sy_t) = S(x_s^4y_sy_t^3)+S(x_s^2y_s^3x_t^2y_t)+2S(x_s^2y_sx_t^2y_ty_u^2)$$
$$+S(x_s^2y_sx_t^2y_u^3)+\text{etc.},$$
$$S(x_s^2y_t^2)S(x_sy_sx_ty_t) = S(x_s^3y_sx_ty_t^3)+S(x_s^3y_sx_ty_ty_u^2)+S(x_sy_s^3x_ty_tx_u^2)$$
$$+S(x_s^2y_t^2x_uy_ux_vy_v),$$
$$S(x_s^2y_t^2)S(x_sy_sx_ty_u) = S(x_s^3y_t^3x_uy_u)+S(x_s^2y_sx_ty_t^2x_uy_u)+\text{etc.},$$
$$S(x_s^2y_t^2)S(x_s^2y_ty_u) = S(x_s^2y_sx_t^2y_u^3)+\text{etc.},$$
$$S(x_sy_sx_ty_t)S(x_sy_sx_ty_u) = S(x_s^2y_sx_ty_t^2x_uy_u)+\text{etc.},$$
$$S(x_sy_sx_ty_t)S(x_s^2y_ty_u) = S(x_s^2x_ty_t^2x_uy_u^2)+\text{etc.},$$
$$S(x_s^2y_sy_t)S(x_sy_sx_ty_t) = S(x_s^3y_s^2x_ty_t^2)+S(x_s^2y_sx_ty_t^2x_uy_u)+\text{etc.},$$

$$S(x_s^2y_sy_t)\,S(x_sy_sx_ty_u)=S(x_s^3y_sx_ty_ty_u^2)+2S(x_s^2y_s^2x_ty_tx_uy_u)+\text{etc.},$$
$$S(x_s^2y_sy_t)\,S(x_s^2y_ty_u)=S(x_s^2y_s^2x_t^2y_u^2)+\text{etc.},$$
$$S(x_s^2y_sy_t)\,S(x_sx_ty_u^2)=S(x_s^3y_sx_ty_ty_u^2)+\text{etc.},$$
$$S(x_s^2y_ty_u)\,S(x_sx_ty_u^2)=S(x_s^3y_t^3x_uy_u)+S(x_s^2y_s^2x_ty_tx_uy_u)$$
$$+S(x_s^2y_t^2x_uy_ux_vy_v)+\text{etc.},$$
$$S(x_s^2y_sy_t)\,S(x_sx_ty_t^2)=S(x_s^3y_sx_ty_t^3)+S(x_s^3y_s^3x_ty_t)+S(x_s^2y_sx_ty_t^3x_uy_u)+\text{etc.},\quad\ldots\ldots(17).$$

The mean values of these sums may be written down as in the case (a). Thus, in repeated samples,

$$\text{Mean } 2S(x_s^2y_s^2x_t^2y_t^2)=n(n-1)P_{22}^2,$$
$$\text{Mean } S(x_s^4y_t^4)=n(n-1)P_{40}P_{04},$$
$$\text{Mean } 2S(x_s^4y_t^2y_u^2)=n(n-1)(n-2)P_{40}P_{02}^2,$$
$$\text{Mean } 4S(x_s^2x_t^2y_u^2y_v^2)=n(n-1)(n-2)(n-3)P_{20}^2P_{02}^2,$$
$$\text{Mean } 24S(x_sy_sx_ty_tx_uy_ux_vy_v)=n(n-1)(n-2)(n-3)P_{11}^4,$$

and so on.

Corollary 1 to the result (F) gives the value of $\overline{p_{22}}$ when N is infinite, so that

$$(\overline{p_{22}})^2=\frac{(n-1)^2}{n^6}[\alpha^2P_{22}^2+(2n-3)^2(P_{20}P_{02}+2P_{11}^2)^2+2\alpha(2n-3)P_{22}(P_{20}P_{02}+2P_{11}^2)],$$

where $\alpha=n^2-3n+3.$

Hence, from the equation

$$\sigma^2_{p_{22}}=\overline{p_{22}^2}-(\overline{p_{22}})^2,$$

we obtain, after simplification,

$$\begin{aligned}
n^7\sigma^2_{p_{22}}=\;&\alpha^2(n-1)^2[-a+d-4(i+i')]\\
&+(n-1)(2n-3)^2[9a+16(n-2)b-8(2n-3)c+2(n-2)g-8(2n-3)h\\
&\qquad-2(2n-3)k+8m+(n-2)(q+q')+t+8(n-2)(u+u')]\\
&+4\alpha^2(n-1)[2a+(n-2)(e+e')+2(n-2)f+2j+(l+l')+2m]\\
&+4(n-1)(n-2)(n-3)^2[5b+4(n-3)c+5(e+e')+16f+4g+13(n-3)h\\
&\qquad+(n-3)k+4(p+p')+\tfrac{1}{2}(q+q')+2r+4(u+u')]\\
&+36(n-1)(n-2)(n-3)[c+4h+k]\\
&+2\alpha(n-1)^2(2n-3)[-4b-2g+4j+(l+l')]\\
&-4\alpha(n-1)(2n-3)[(n-2)(e+e')+8(n-2)f+5(i+i')\\
&\qquad+(n-2)(p+p')+(s+s')]\\
&+4(n-1)(n-2)(n-3)(2n-3)[2(e+e')+10f+(p+p')+2r]\\
&-8\alpha(n-1)(n-2)(n-3)[4b+2g+3(u+u')]\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{N}),
\end{aligned}$$

where $\alpha=n^2-3n+3$, and a, b, c, etc., have the same values as in equation (21). This results from the fact that the same sums occur in the expressions for p_{22}^2 and p_{11}^4.

Corollary. For a normal sampled population, the above product moments have the values stated on pp. 249 and 254 giving, on reduction,

$$p_{22} = \frac{4(n-1)^3\sigma_X{}^4\sigma_Y{}^4}{n^4}\left[(5R^4+17R^2+2) - \frac{R^4+4R^2+1}{n-1} + \frac{R^4+4R^2+1}{(n-1)^2}\right].$$

12. *The Third and Fourth Moment Coefficients of the Distribution of p_{11} in Samples from an Infinite Population.*

The third moment coefficient of p_{11} may be written in the form

$$\begin{aligned} {}_{p_{11}}M_3 &= \text{Mean value of } (p_{11} - \overline{p_{11}})^3 \\ &= \overline{p_{11}{}^3} - (\overline{p_{11}})^3 - 3\sigma^2_{p_{11}}\overline{p_{11}} \quad \text{......................(18).} \end{aligned}$$

We have already evaluated $\overline{p_{11}}$ and $\sigma^2_{p_{11}}$, so that it remains to find $\overline{p_{11}{}^3}$. This I have done by multiplying the sums in equation (2) by

$$p_{11} = \frac{n-1}{n^2}S(x_sy_s) - \frac{1}{n^2}S(x_sy_t),$$

and finding the mean values of the resulting sums. These were as follows:

$$\begin{aligned}
S(x_sy_s)\,S(x_s{}^2y_s{}^2) &= S(x_s{}^3y_s{}^3) + S(x_s{}^2y_s{}^2x_ty_t),\\
S(x_sy_s)\,S(x_s{}^2y_t{}^2) &= S(x_s{}^3y_sy_t{}^2) + S(x_s{}^2x_ty_t{}^3) + S(x_s{}^2y_t{}^2x_uy_u),\\
S(x_sy_s)\,S(x_sy_sx_ty_t) &= S(x_s{}^2y_s{}^2x_ty_t) + 3S(x_sy_sx_ty_tx_uy_u),\\
S(x_sy_s)\,S(x_s{}^2y_sy_t) &= S(x_s{}^2y_sx_ty_t{}^2) + \text{etc.},\\
S(x_sy_s)\,S(x_sx_ty_t{}^2) &= S(x_s{}^2y_sx_ty_t{}^2) + \text{etc.},\\
S(x_sy_t)\,S(x_s{}^2y_t{}^2) &= S(x_s{}^3y_t{}^3) + S(x_s{}^2y_sx_ty_t{}^2) + \text{etc.},\\
S(x_sy_t)\,S(x_sy_sx_ty_t) &= S(x_s{}^2y_sx_ty_t{}^2) + \text{etc.},\\
S(x_sy_t)\,S(x_s{}^2y_sy_t) &= S(x_s{}^3y_sy_t{}^2) + S(x_s{}^2y_s{}^2x_ty_t) + \text{etc.},\\
S(x_sy_t)\,S(x_sx_ty_t{}^2) &= S(x_sy_s{}^3x_t{}^2) + S(x_s{}^2y_s{}^2x_ty_t) + \text{etc.}
\end{aligned}$$

I have again omitted the sums which vanish when their means are taken. Now we have

$$\begin{aligned}
\text{Mean } S(x_s{}^3y_s{}^3) &= nP_{33},\\
\text{Mean } S(x_s{}^2y_s{}^2x_ty_t) &= n(n-1)P_{22}P_{11},\\
\text{Mean } S(x_s{}^3y_sy_t{}^2) &= n(n-1)P_{31}P_{02},\\
\text{Mean } S(x_s{}^2x_ty_t{}^3) &= n(n-1)P_{13}P_{20},\\
\text{Mean } S(x_s{}^2y_t{}^2x_uy_u) &= n(n-1)(n-2)P_{20}P_{02}P_{11},\\
\text{Mean } S(x_sy_sx_ty_tx_uy_u) &= \tfrac{1}{6}n(n-1)(n-2)(n-3)P_{11}{}^3,\\
\text{Mean } S(x_s{}^2y_sx_ty_t{}^2) &= n(n-1)P_{21}P_{12},\\
\text{Mean } S(x_s{}^3y_t{}^3) &= n(n-1)P_{30}P_{03}.
\end{aligned}$$

Hence, using the values, when N is infinite, of

$$\overline{p_{11}} = P_{11}\frac{\left(1-\frac{1}{n}\right)}{\left(1-\frac{1}{N}\right)},$$

and $$\sigma^2_{p_{11}} = \frac{N(N-n)(n-1)}{n^3(N-1)(N-2)(N-3)}\left[(N-n-1)P_{20}P_{02}+(Nn-N-n-1)P_{22}\right.$$
$$\left.-\left\{\frac{n(N-2)(N+1)}{N-1}-2(N-1)\right\}P_{11}^2\right],$$

we may write, from equation (18),

$$\begin{aligned}n^5({}_{p_{11}}M_3) = {} & (n-1)^3[b+(n-1)c]+(n-1)^2[d+e+(n-2)f]\\
& +a(n-1)^2[2c+(n-2)g]-4(n-1)^3h-(n-1)(i+h)-2a(n-1)h\\
& +2(n-1)^2(2c+d+e)-4(n-1)(n-2)f+2(n-1)(n-2)^2(f+g)\\
& -n^2(n-1)^3g-3n(n-1)^2[f+(n-1)c-(n-2)g],\end{aligned}$$

where
$$\begin{array}{lll} b=P_{33}, & c=P_{11}P_{22}, & d=P_{02}P_{31},\\ e=P_{20}P_{13}, & f=P_{11}P_{20}P_{02}, & g=P_{11}^3,\\ h=P_{21}P_{12}, & i=P_{30}P_{03}, & a=(n-1)^2+1.\end{array}$$

I simplified the above by collecting terms in n^3, n^2, n and the constant (the terms in n^5 and n^4 vanishing) and finally obtained

$$\begin{aligned}{}_{p_{11}}M_3 = {} & \frac{1}{n^2}(P_{33}-3P_{22}P_{11}+2P_{11}^3-6P_{21}P_{12})\\
& -\frac{1}{n^3}(3P_{33}-15P_{22}P_{11}-3P_{31}P_{02}-3P_{13}P_{20}+12P_{11}P_{20}P_{02}+14P_{11}^3-18P_{21}P_{12})\\
& +\frac{1}{n^4}(3P_{33}-21P_{22}P_{11}-6P_{31}P_{02}-6P_{13}P_{20}+30P_{11}P_{20}P_{02}+24P_{11}^3-21P_{21}P_{12}-P_{30}P_{03})\\
& -\frac{1}{n^5}(P_{33}-9P_{22}P_{11}-3P_{31}P_{02}-3P_{13}P_{20}+18P_{11}P_{20}P_{02}+12P_{11}^3-9P_{21}P_{12}-P_{30}P_{03})\end{aligned}$$
......(19)*.

The case of the fourth moment coefficient, although somewhat longer to work out, is simpler than the previous case, owing to the fact that the expressions involved are symmetrical and that the sums occurring in p_{22}^2 are the same as those used to find p_{11}^4, so that their mean values have been found. We now write

$${}_{p_{11}}M_4 = \overline{p_{11}^4} - 4\overline{p_{11}}\,({}_{p_{11}}M_3) - 6\,(\overline{p_{11}})^2\,\sigma^2_{p_{11}} - (\overline{p_{11}})^4 \quad \text{............(20)},$$

so that we have to evaluate $\overline{p_{11}^4}$, as the other terms are now known.

The value of p_{11}^2 in equation (2) has now to be squared, and the values of the squares and products of the sums are given in equation (17); their mean values are also known. Thus ${}_{p_{11}}M_4$ may be written down from equation (20). To shorten the writing I have put the result in the form

$$\begin{aligned}\frac{n^7({}_{p_{11}}M_4)}{n-1} = {} & 3n^4(a-2b+c)\\
& +n^3\{-12a+36b-21c+d+12(e+e')+72f\\
& +6g-6h-12(i+i')-4j\}\end{aligned}$$

* This agrees with the result of Dr Church in *Biometrika*, Vol. XVII. p. 82, giving the third moment coefficient of the distribution of variance in samples from an infinite population, when we put $X_s = Y_s$ in the above equation.

$$+ n^2 \{30a - 150b + 108c - 3d - 60(e+e') - 288f - 48g + 162h + 36(i+i')$$
$$+ 24j + 9k + 6(l+l') + 12m - 48(u+u') - 12(p+p')\}$$
$$+ n\{-36a + 252b - 216c + 3d + 108(e+e') + 468f + 108g - 504h$$
$$- 48(i+i') - 36j - 45k - 12(l+l') - 24m + 132(u+u') + 48(p+p')$$
$$+ 3(q+q') + 20r - 4s\}$$
$$+ \{18a - 144b + 144c - d - 72(e+e') - 288f - 72g + 432h + 24(i+i')$$
$$+ 16j + 54k + 6(l+l') + 16m - 96(u+u') - 48(p+p')$$
$$- 6(q+q') - 32r + 4s + t\} \quad \ldots\ldots\ldots\ldots(21)^*,$$

where $a = P_{22}^2$, $b = P_{11}^2 P_{22}$, $c = P_{11}^4$, $d = P_{44}$,
$e = P_{21}^2 P_{02}$, $f = P_{21} P_{12} P_{11}$, $g = P_{22} P_{20} P_{02}$, $h = P_{11}^2 P_{20} P_{02}$,
$i = P_{21} P_{23}$, $j = P_{33} P_{11}$, $k = P_{20}^2 P_{02}^2$, $l = P_{42} P_{02}$,
$m = P_{31} P_{13}$, $u = P_{31} P_{11} P_{02}$, $p = P_{30} P_{02} P_{12}$, $q = P_{20}^2 P_{04}$,
$r = P_{30} P_{03} P_{11}$, $s = P_{41} P_{03}$, $t = P_{40} P_{04}$.

The dashed letters denote the functions obtained by interchanging X and Y; so that $e' = P_{12}^2 P_{20}$, $i' = P_{12} P_{32}$, and so on.

13. *The β_1 and β_2 of the Distribution of p_{11} in Samples from an Infinite Normal Population* †.

The results (A) and (B) and equations (19) and (21) give the first four moment coefficients of the distribution of p_{11} in samples from a population not necessarily normal.

Make $N \to \infty$ in (A) and (B) and substitute the normal values of the product moments in the sampled population, which are as follows:

$P_{22} = (1 + 2R^2)\,\sigma_X^2 \sigma_Y^2$, $P_{33} = 3R(3 + 2R^2)\,\sigma_X^3 \sigma_Y^3$, $P_{31} = 3R\sigma_X^3 \sigma_Y$,
$P_{44} = 3(3 + 24R^2 + 8R^4)\,\sigma_X^4 \sigma_Y^4$, $P_{42} = 3(1 + 4R^2)\,\sigma_X^4 \sigma_Y^2$, $P_{40} = 3\sigma_X^4$.

The remaining moments, P_{ab}, in which $a + b$ is odd, vanish in this case.

The four moments may now be written:

$$_{p_{11}}M_1 = \left(\frac{n-1}{n}\right) R\sigma_X\sigma_Y,$$

$$_{p_{11}}M_2 = \left(\frac{n-1}{n^2}\right)(1 + R^2)\,\sigma_X^2\sigma_Y^2,$$

$$_{p_{11}}M_3 = 2\left(\frac{n-1}{n^3}\right)(3R + R^3)\,\sigma_X^3\sigma_Y^3,$$

$$_{p_{11}}M_4 = 3\left(\frac{n-1}{n^4}\right)\{n(1+R^2)^2 + (1 + 10R^2 + R^4)\}\,\sigma_X^4\sigma_Y^4 \quad \ldots\ldots\ldots(22).$$

* **Transforming the result above to the case of one variate, it will be found to agree with the value of $_2M_4$ given by Dr Church in *Biometrika*, Vol. XVII. p. 83.**

† **See Wishart, *op. cit.* p. 42.**

CURVE OF β_1 FOR THE FREQUENCY DISTRIBUTION OF n_{11} IN SAMPLES OF n FROM AN INFINITE BIVARIATE NORMAL POPULATION OF CORRELATION R.

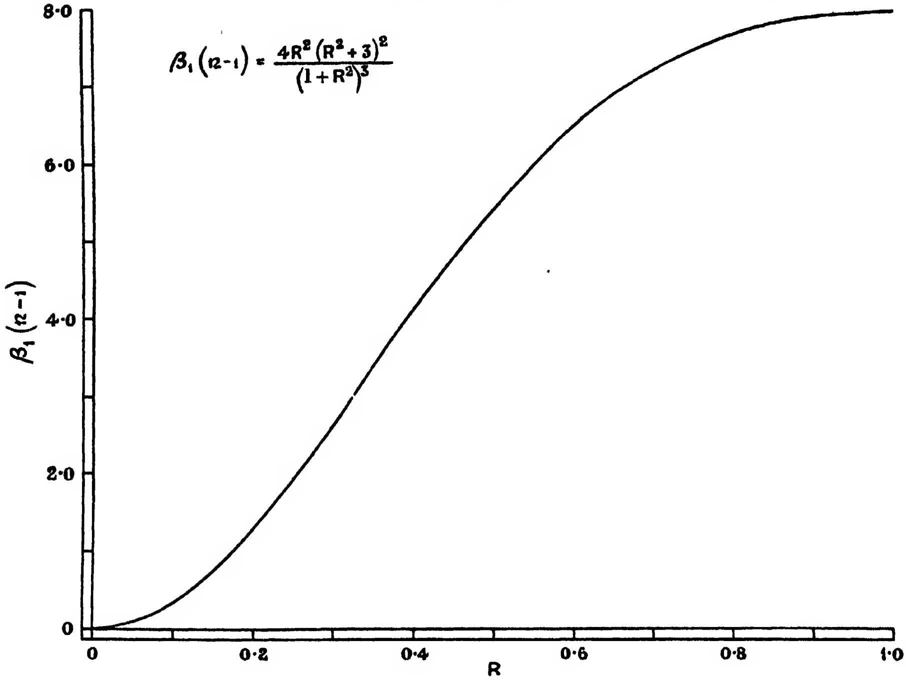


FIG I

CURVE OF β_2 FOR THE FREQUENCY DISTRIBUTION OF n_{11} IN SAMPLES OF n FROM AN INFINITE BIVARIATE NORMAL POPULATION OF CORRELATION R.

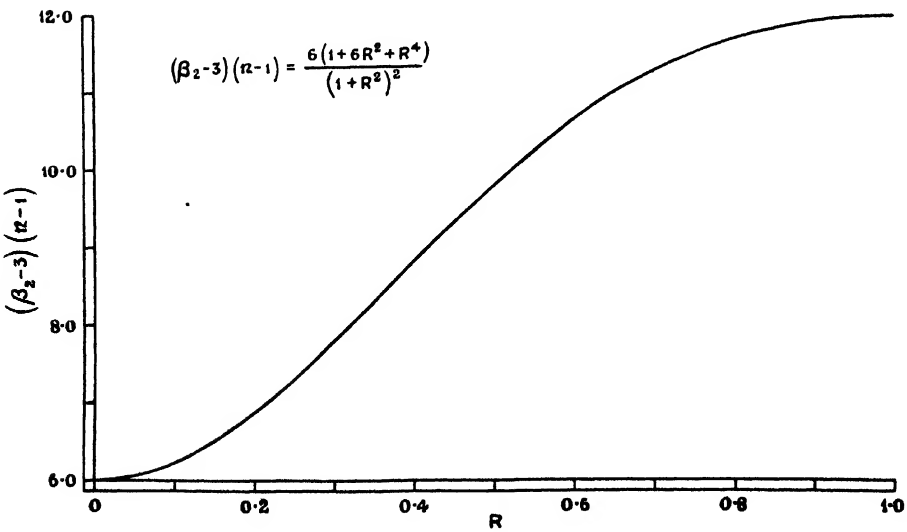
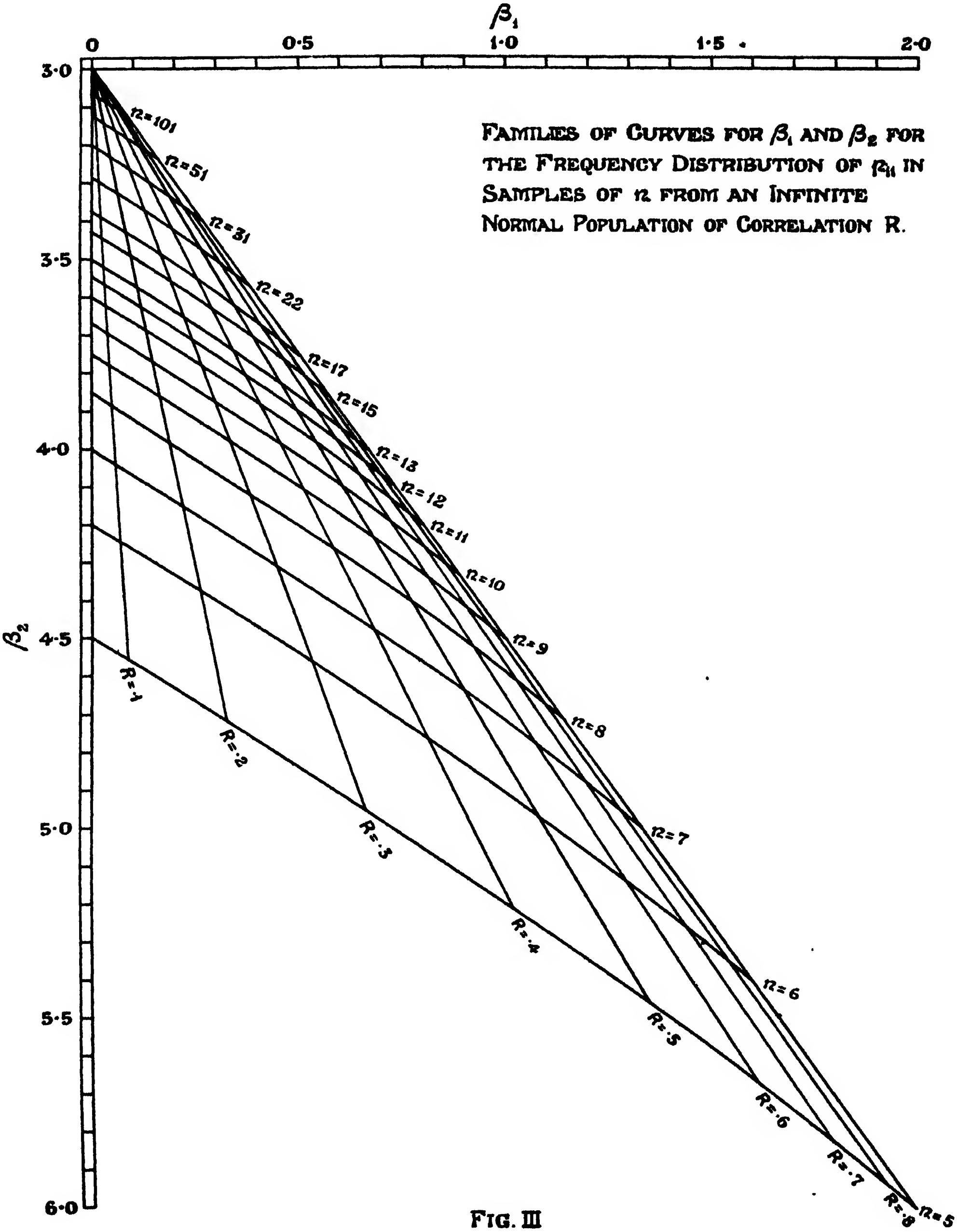


FIG. II



β_1
0
0·5
1·0
1·5
2·0
3·0
3·5
4·0
4·5
5·0
5·5
6·0
β_2
FAMILIES OF CURVES FOR β_1 AND β_2 FOR THE FREQUENCY DISTRIBUTION OF r_{11} IN SAMPLES OF n FROM AN INFINITE NORMAL POPULATION OF CORRELATION R.
n=101
n=51
n=31
n=22
n=17
n=15
n=13
n=12
n=11
n=10
n=9
n=8
n=7
n=6
n=5
R=·1
R=·2
R=·3
R=·4
R=·5
R=·6
R=·7
R=·8

FIG. III

Hence
$$\left.\begin{aligned}\beta_1 &= \frac{M_3^2}{M_2^3} = \frac{4R^2(R^2+3)^2}{(n-1)(R^2+1)^3},\\ \beta_2 &= \frac{M_4}{M_2^2} = 3 + \frac{6(1+6R^2+R^4)}{(n-1)(R^2+1)^2}\end{aligned}\right\} \quad\text{.(23).}$$

The values of $\beta_1(n-1)$ and $(\beta_2-3)(n-1)$ have been plotted for different values of R and the graphs are shown in Figures I and II respectively. For a given value of R, β_1 and (β_2-3) are found by dividing the ordinates by $(n-1)$.

The result of eliminating $(n-1)$ from equations (23) gives

$$3\beta_1(1+R^2)(1+6R^2+R^4) - 2\beta_2 R^2(3+R^2)^2 + 6R^2(3+R^2)^2 = 0 \quad \text{...(24).}$$

Thus, for any given R, the point (β_1, β_2) lies on a straight line passing through the Gaussian point $\beta_1 = 0$, $\beta_2 = 3$.

When $R=0$, the line becomes $\beta_1 = 0$, and when $R=1$, (β_1, β_2) satisfies the condition for the Pearson Type III curve, viz. $2\beta_2 - 3\beta_1 - 6 = 0$.

From Equations (23) it is seen that as $n \to \infty$, $\beta_1 \to 0$ and $\beta_2 \to 3$, the normal values.

When R is eliminated from Equations (23), we obtain

$$(\phi-2)^3 + (2\theta - 3\phi + 2)^2 = 0 \quad \text{.......................(25),}$$

where
$$\theta = \left(\frac{n-1}{4}\right)\beta_1, \quad \phi = \left(\frac{n-1}{6}\right)(\beta_2 - 3).$$

Thus, for any given size n of the sample, (β_1, β_2) lies on the cubic curve (25).

The system of straight lines (24) and the system of cubics (25) are shown in Figure III, which enables the values of (β_1, β_2) to be read off from given values of R and n. The rapid approach to normality as n increases is seen from this figure, and for high values of the correlation R, (β_1, β_2) lies very close to the Type III line, $2\beta_2 - 3\beta_1 - 6 = 0$.

The main Pearson Types covered by the points in Figure III are IV and VI, and Types III and VII are obtained for the special cases of $R=1$ and 0 respectively. For the particular case of $n=3$ and $R=1$ Equations (23) give $\beta_1 = 4$, $\beta_2 = 9$, that is: the Exponential Point in the (β_1, β_2) plane.

14. *Summary.* Finally, to give some indication of the nature of the formulae provided in this paper, I have summarised them in the manner shown below.

I. General values for *any distribution* and for a *limited sampled population* have been found for:

(*a*) ***Mean values*** of $p_{11}, p_{12}, p_{21}, p_{22}, p_{31}, p_{13}$ and consequently of m_2, m_3, m_4.

(*b*) ***Standard Deviations*** of p_{11}, p_{12}, p_{21} and consequently of m_2 and m_3.

(*c*) ***Correlations*** between $m_x, m_y, \sigma_x^2, \sigma_y^2, p_{11}$.

Of the above the *univariate* results for $\overline{m_2}, \overline{m_3}, \overline{m_4}, \sigma^2_{m_2}$ and r_{m_x, σ_x^2} are easy to compute in any given case, while $\sigma^2_{m_3}$ (see Corollary 4, p. 239), although rather long, does not present an insuperable amount of arithmetic labour to use in practice.

The bivariate results in this general case have a theoretical interest and may be used, for example, in the determination of certain bivariate distributions.

II. General values for *any distribution* and an *infinite sampled population* have been found for:

(*a*) The standard deviations of p_{31}, p_{13}, p_{22} and consequently of m_4.

The result for $\sigma^2_{m_4}$ (p. 249) has been given in terms of the β's of the sampled population and is quite easy to compute.

(*b*) The third and fourth moment coefficients of p_{11} and consequently of m_2 (or s^2). These results, already obtained by Dr Church, are quite easy to compute.

(*c*) The univariate results in I. These are correspondingly easier to compute when $N \to \infty$.

III. Values for an *infinite normal sampled population* have been found for:

(*a*) All the results given in I and II. These reduce to very simple forms, the *univariate* results involving only the size of the sample (n) and the standard deviation (σ) of the sampled population, and the *bivariate* results involving n, the correlation R of the sampled population and σ_X, σ_Y the standard deviations of X and Y. These results are all easy to compute.

(*b*) The β_1 and β_2 of the distribution of p_{11}. These are simple expressions in n and R (Equations (23)) and have been computed for different values of n and R, illustrated in Figures I, II and III.

In conclusion, I wish to thank Professor Karl Pearson for suggesting the subject of this paper to me, and for his invaluable advice and kindly encouragement throughout its progress; also Miss Ida McLearn for her preparation of the diagrams.

THE DISTRIBUTION OF FREQUENCY CONSTANTS IN SMALL SAMPLES FROM NON-NORMAL SYMMETRICAL AND SKEW POPULATIONS.

2nd Paper: The Distribution of "Student's" z.

By EGON S. PEARSON, D.Sc., ASSISTED BY N. K. ADYANTHĀYA, B.Sc.
AND OTHERS.

CONTENTS.

1. THE USE OF "STUDENT'S" z-TEST WITH POPULATIONS NOT NORMAL.

One of the most important problems with which the mathematical statistician is faced is that of bringing his theoretical structures into some degree of correspondence with the situations of practical experience. This is no doubt hardest when the samples are small, for here he will often be faced with two difficulties. In the first place his populations may not be completely stable; the sample when drawn may be a random one, but owing to difficulties in control or to some changing time factor he cannot be sure that it will be quite the same population with which he will be concerned in further work And then, even if he is sure of the stability of his population, it will generally be impossible for him to obtain any certain estimate of its exact form. For purposes of inference he may calculate from the sample one or more statistical measures, but the first difficulty will make him hesitate to lay too much stress on the exact value of the figures found on entering his probability table, while the second may make him wonder whether there is after all any appropriate table in existence.

The questions of stability and randomness of sampling can only be dealt with in each problem as it arises, but though the statistician may be prepared to accept these conditions as approximately true, he is still faced with the second problem. "The majority of tests dealing with small samples," he may say, "have only been worked out for the case in which the population distributions are normal. I do not know whether my distribution is normal, although from my general experience in the past I do not think that it is likely to be excessively skew or leptokurtic. How sensitive are the 'normal theory' tests to changes in population form? May I use some with less hesitation than others?"

In the present paper an attempt will be made to answer this question as far as it concerns some of the tests connected with "Student's" Type VII distribution of z, the two fundamental tests considered being those dealing with the mean of a single sample and the difference between the means of two samples. It may be well to illustrate the problem by taking a concrete example. A commercial firm, let us suppose, is considering whether to introduce a new method of production, which may be of advantage perhaps either through a saving of time or because it seems likely to lead to an improvement in the quality of the article produced. A series of experiments is carried out in which some variable quantity x is carefully observed under both methods. As a result two samples are available, one of n_1 values of x with a mean $\bar{x}_1$ and standard deviation s_1, the other of n_2 with $\bar{x}_2$ and s_2. In this case the most useful answer that statistical analysis could give would perhaps be as follows: "The exact difference that would be found to hold in the long run between the average values of x arising from the two methods cannot of course be determined, but the odds are k to 1 that this difference lies between d_1 and d_2." With such an answer as this before it the firm could decide whether the innovation showed an improvement of sufficient significance to be commercially profitable, or if the question remained in doubt whether it seemed worth the expense of undertaking further experiments in order to narrow down these limits, d_1 and d_2. But unfortunately an answer in so precise a form cannot be given without serious assumptions which at once destroy the precision.

If the samples are small we may use R. A. Fisher's two-sample z-test* and calculate

$$z = \frac{\bar{x}_1 - \bar{x}_2 - d}{\sqrt{n_1 s_1^2 + n_2 s_2^2}} \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}.$$

Then choosing a suitable value of α, such as ·01, and entering "Student's" tables, we may find the values of d_1 and d_2 corresponding to $z = \pm z_\alpha$ for which $P_z = 2\alpha$, where

$$P_z = 2\int_z^\infty c_0 (1 + z^2)^{-\frac{n_1+n_2-1}{2}} dz \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}.$$

On the assumption that the two distributions of x are normal and have the same variance, we can then say that if the difference between the population mean values of x were (1) as low as d_1, or (2) as high as d_2, then the chance would be α of obtaining in pairs of random samples of n_1 and n_2: (1) a positive deviation of z, or (2) a negative deviation of z as great or greater than that observed. But it is not possible to speak in any exact sense of the odds being $1 - 2\alpha$ to 2α that the difference in means lies between d_1 and d_2. Such a use of the inverse probability would involve an assumption regarding the *à priori* probability distributions of the

* *Metron*, 1925, Vol. v. No. 3, p. 7; *Statistical Methods for Research Workers*, 1928, p. 107.

The relation of this and the single sample z-test to the criterion of likelihood was discussed by Neyman and Pearson in *Biometrika*, Vol. xx^A^. pp. 190 and 207. The symbol z will be used throughout this paper. "Student's" later tables in *Metron*, Vol. v. are entered with $t = z\sqrt{n' - 1}$, and the t-notation is that used by Fisher. In equation (2) c_0 has the value $\Gamma(\frac{1}{2}(n_1+n_2-1))/\{\Gamma(\frac{1}{2}(n_1+n_2-2))\sqrt{\pi}\}$.

population means and standard deviations; and further in the case where it is not even certain that these populations are exactly normal a more complex *à priori* assumption still must be introduced, so that any approach to an exact solution in terms of inverse probability becomes impossible. The difficulty and one method of treating it was discussed briefly by "Sophister" in the last number of this Journal* in dealing with the distribution of z found on sampling from a skew population. With the fuller results now available it will be possible to analyse the situation a little more in detail than he was able to do last year.

It is true that it may be more helpful for the practical worker to look at his problem from the inverse point of view, and to obtain some measure of the odds for or against the population parameters lying within certain limits. But a little reflection suggests that unless he is prepared to grapple with *à priori* probability, his justification in the use of any such rough and ready guide must depend on the validity of employing the probability tables of the z-distribution in dealing with the following questions:

(*a*) There is a sample of n individuals measured for a certain character. We wish to test the probability of the hypothesis that this sample has been drawn from a population whose mean is at a distance $m = \bar{x} - a$ from the sample mean $\bar{x}$.

(*b*) There are two samples of n_1 and n_2, and on the assumption that they come from populations with the same variance, we wish to test the hypothesis that the means in these populations differ by d† (d of course will often be zero).

In discussing the various experimental results we shall therefore be concerned chiefly with the adequacy of the "normal theory" in testing these two fundamental hypotheses. There are many cases in which the problem presents itself in almost exactly one or other of these forms, but the reader should find no difficulty in interpreting the tables given in any manner which seems more applicable to the particular form of problem with which he is concerned.

In practice an hypothesis will be accepted or rejected with a varying degree of confidence; no precise line between acceptance or rejection can be drawn. Yet some light is thrown upon the problem if it is supposed for the moment that this vague edge of uncertainty can be given precision, the statistician being compelled to make a definite decision one way or the other; he will reject the hypothesis when $P_z \leqslant 2\alpha$‡ and accept it when $P_z > 2\alpha$, where the value of α used will depend upon the nature of his problem. If this be so errors in judgment cannot be avoided, and it is seen that they will be of two kinds:

(1) The hypothesis is rejected when it is in fact true.

(2) It is accepted when it is false.

* *Biometrika*, Vol. xx^A. p. 421.

† The question of finding the probable limits d_1 and d_2 in the commercial problem suggested above seems really to consist at bottom in testing the second of these hypotheses for varying values of d. We can then get a good appreciation of these limits without attempting to assign numerical odds to the chance that the difference in population mean lies between them.

‡ In the case of two samples P_z is as in equation (2) above. For the single sample hypothesis

$$P_z = 2\int_z^\infty c_0 (1+z^2)^{-\frac{n}{2}} dz, \text{ where } c_0 = \Gamma(\tfrac{1}{2}n)/\{\Gamma(\tfrac{1}{2}(n-1))\sqrt{\pi}\}.$$

It is impossible to estimate the relative proportions in which these two types of error occur, as it will depend upon the kind of problems to which the statistician applies his tests, but we can analyse separately each type.

If the populations sampled are all of a form such that z follows "Student's" distribution, then the source of error (1) may be completely controlled. In the long run such errors will be committed in $100 \times 2\alpha\,\%$ of the cases in which the hypothesis tested was really true, and the statistician may choose α according to the risk he is prepared to take of making this form of error of judgment. From this point of view, $|z|$* is as good a criterion as z. Suppose that $\pm z_\alpha$ are the deviations

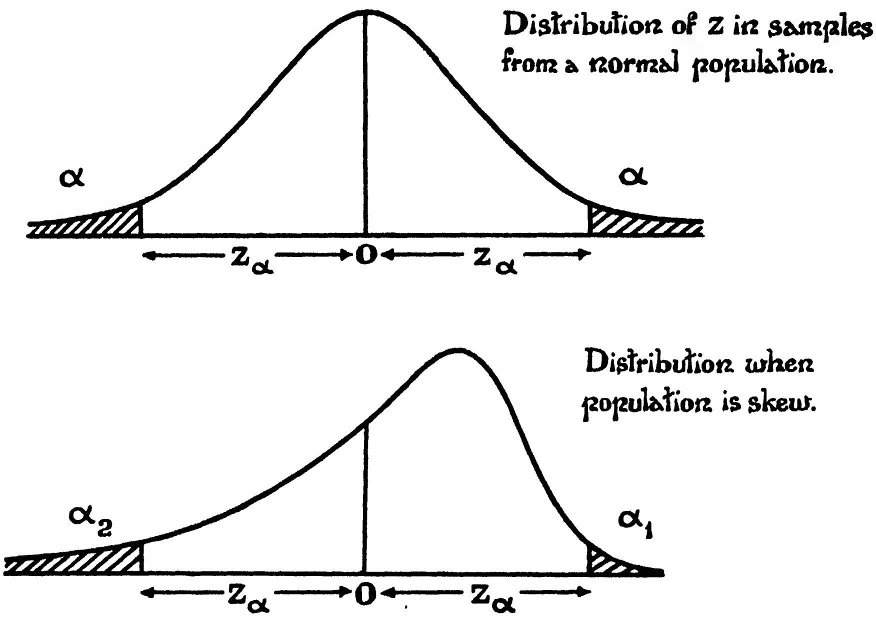


Fig. 1.

corresponding to tail areas of α when "Student's" tables are entered, but that in fact, as the population is not normal, the distribution of z in repeated samples follows a certain skew curve. The tail areas beyond $\pm z_\alpha$ are now α_1 and α_2 which are not equal. But if we know that $\alpha_1 + \alpha_2 = 2\alpha$ (or very nearly so) and this for a wide range of values of α†, then our control of the first source of error will be as good in sampling from the non-normal population as from the normal one.

We shall therefore first consider below how far $|z|$ follows "Student's" distribution in samples from a variety of non-normal populations‡. We may note here, incidentally, that any other statistical constant, z', for which the sampling

* The expression $|z|$ indicates that the numerical value of z is to be given a positive sign.

† Say between $\alpha = \cdot 100$ and $\cdot 005$.

‡ Previous experimental work in this direction has been carried out by Shewart and Winters, *Journal of the American Statistical Association*, Vol. XXIII. pp. 144—58; Neyman and Pearson, *Biometrika*, Vol. XX^A. pp. 197—207, using Church's sampling data; "Sophister," *Biometrika*, Vol. XX^A. pp. 408—21, and Rider in the present volume, pp. 124—43.

distribution of $|z'|$ is as invariable for changing populations as $|z|$, will be of equal value as a criterion *in so far as the control of source of error* (1) *is concerned.*

When, however, we consider the second type of error, the position is somewhat different. It will not generally in practice be of serious consequence if we accept the hypothesis tested when in fact the mean of the sampled population differs by some small quantity, τ, from the supposed value a; nor in the case of two samples, if the population means differ by $d+\tau$ instead of by d. This cannot be avoided, but we should like to have some appreciation of the rapidity with which the untrue hypotheses are rejected as τ, or rather the ratio of τ to the population standard deviation, increases. If we accept the hypothesis when $P_z > 2\alpha$, are we likely to be doing so when really the true population mean is at a distance of σ or perhaps even 2σ from its supposed position? Or in the second test, when the means of the two populations really differ by $d+\sigma$ or even $d+2\sigma$? We are concerned now with what may be termed the sensitivity of the test in the rejection of false hypotheses, and this will depend upon (a) the size of the sample, (b) the form of the population sampled, and (c) the sign of τ. It has been pointed out that in testing any given hypothesis there will be an indefinite number of criteria which will ensure the control of the first source of error, but it seems probable that for each type of population there will be one of these which is more efficient than any other in controlling the second error. This point will be examined in more detail below in connection with the experimental results, and the sensitivity of "Student's" z and of the ratio z' = sample centre/(half sample range)* will be compared for samples of 5 and 10 from a variety of populations.

2. THE POPULATIONS SAMPLED.

No experimental programme could possibly cover all the populations likely to be met in common experience, but a variety of types of frequency form have been represented by taking samplings from Pearson-curves of the following nature:

TABLE I.

Population Curve	β_1 and β_2 of grouped distribution	Samples	S.D. of population in terms of grouping unit
Type II	0, 2·50	1000 of 2	63·25
		500 of 5, 500 of 10	6·32
		500 of 20	10·54
Type VII	0, 4·12	1000 of 2	56·67
		500 of 5, 500 of 10, 500 of 20	5·67
Type VII	0, 7·07	1000 of 2	64·48
		1000 of 5, 500 of 10, 500 of 20	6·45
Type III	·20, 3·30	1000 of 2	50·00
		1000 of 5, 500 of 10	5·00
		500 of 20	6·67
Type III	·50, 3·73	1000 of 2	50·00
		1000 of 5, 500 of 10, 1000 of 20	5·00

* The use of the "centre" or mid-point between extreme observations in the sample as an estimate of the population mean was discussed in *Biometrika*, Vol. xx^A. pp. 212 and 358.

The sampling was carried out with the help of Tippett's Random Numbers*. Certain results obtained from the sampling of the three symmetrical populations have already been published†. The samples of 5 and 20 from the skewer of the two Type III populations are those obtained by "Sophister"‡, who has kindly placed his data unreservedly at our disposal. To these we have added samples of 2 and of 10, and have taken a fresh Type III population to fill in the gap between his population and the normal population.

In drawing samples of 2 from a grouped population distribution, both individual values will occasionally fall in the same group so that the value of z becomes indeterminate. And even if the values fall into groups, one or two units apart, considerable uncertainty must exist as to the true value of z if it be supposed that the population distribution is really continuous. By taking a very fine grouping, i.e. 50 or more groups to the population standard deviation, this difficulty was reduced to a minimum. Only about 10 cases occurred in the 5000 samples of 2 in which both individuals fell in the same group; these cases were discarded and fresh samples taken, and it was assumed that no serious systematic error would arise in other cases if z were calculated on the assumption that the variates had mid-group values.

TABLE II (a).

Frequencies of z in 1000 *Samples of* 2.

$\lvert z \rvert$ greater than		Populations Sampled						
		$\beta_1=0{\cdot}00$ $\beta_2=1{\cdot}80$	$\beta_1=0{\cdot}00$ $\beta_2=2{\cdot}50$	$\beta_1=0{\cdot}00$ $\beta_2=3{\cdot}00$	$\beta_1=0{\cdot}00$ $\beta_2=4{\cdot}12$	$\beta_1=0{\cdot}00$ $\beta_2=7{\cdot}07$	$\beta_1=0{\cdot}20$ $\beta_2=3{\cdot}30$	$\beta_1=0{\cdot}50$ $\beta_2=3{\cdot}73$
0·0		*1000·0*	1000	*1000·0*	1000	1000	1000	1000
0·5		*666·7*	682·5	*704·8*	716·5	723·5	731·5	699
1·0		*500·0*	487	*500·0*	512	526	509·5	487·5
1·5		*400·0*	373	*374·4*	374·5	372·5	376·5	355
2·0		*333·3*	308·5	*295·2*	296·5	272	290·5	280·5
2·5		*285·7*	249·5	*242·2*	233·5	204	226	231·5
3·0		*250·0*	222	*204·8*	199·5	173	188	196
3·5		*222·2*	193	*177·2*	170·5	147	152·5	168·5
4·0		*200·0*	178	*156·0*	156	131	134	150
4·5		*181·8*	162	*139·4*	147	113	119·5	126
5·0		*166·7*	145	*125·6*	132	109	106·5	116
6·0		*142·8*	119·5	*105·2*	112	97	84	101
7·0		*125·0*	99	*90·4*	100	88	74·5	88·5
8·0		*111·1*	90·5	*79·2*	87·5	76	67·5	78·5
9·0		*100·0*	80	*70·4*	79	68	64	71
10·0		*90·9*	73·5	*63·4*	69·5	62·5	57	61
15·0		*62·5*	50	*42·4*	42	42	39·5	36
20·0		*47·6*	31·5	*31·8*	30	29·5	28	26·5
Goodness of Fit	$\lvert z \rvert$ P	—	·137	—	·684	·006	·394	·871
	$\lvert z \rvert$ n'	—	17	—	17	17	17	17
	z P	—	—	—	—	—	·242	·022
	z n'	—	—	—	—	—	26	26

* *Tracts for Computers*, No. xv. A fresh sampling was carried out for each of the 20 sets, and the columns of the sampling book and the number scale were frequently altered so as to ensure as far as possible complete independence between the different sets.

† *Biometrika*, Vol. xx^A. pp. 356—60.

‡ *Biometrika*, Vol. xx^A. pp. 389—423.

3. THE SINGLE SAMPLE TEST.

Tables II (*a*), (*b*), (*c*) and (*d*) give the results of the sampling. In the first place they show the number of samples in which $|z|$ lay beyond the limit given in the leading column. The figures in italics are theoretical values, the others

TABLE II (*b*).

Frequencies of z in 500 Samples of 5.

$\lvert z \rvert$ greater than	Populations Sampled						Normal distribution with S.D. $=1/\sqrt{2}$
	$\beta_1=0{\cdot}00$, $\beta_2=2{\cdot}50$	$\beta_1=0{\cdot}00$, $\beta_2=3{\cdot}00$	$\beta_1=0{\cdot}00$, $\beta_2=4{\cdot}12$	$\beta_1=0{\cdot}00$, $\beta_2=7{\cdot}07$	$\beta_1=0{\cdot}20$, $\beta_2=3{\cdot}30$	$\beta_1=0{\cdot}50$, $\beta_2=3{\cdot}73$	
0·0	500	*500·0*	500	500*	500*	500*	*500·0*
0·1	432	*425·6*	436	430	422·5	433	*443·8*
0·2	355	*354·8*	376	371·5	360·5	357·5	*388·6*
0·3	284	*290·4*	296	305·5	302	291·5	*335·7*
0·4	216·5	*234·3*	235	254	246	231·5	*285·8*
0·5	179	*187·0*	186·5	203·5	192	183	*239·8*
0·6	147	*148·2*	146	158·5	153	150	*198·1*
0·7	121	*117·1*	113	123·5	121	117	*161·1*
0·8	95·5	*92·4*	81	88	92	88	*128·9*
0·9	76	*73·1*	65	70	73	71·5	*101·6*
1·0	65·5	*58·1*	50	51	58	60·5	*78.7*
1·1	55	*46·3*	38	39·5	50	50·5	*59·9*
1·2	43	*37·2*	30	30·5	42·5	41	*44·9*
1·3	32	*30·0*	26	24	35	31·5	*33·0*
1·4	26	*24·4*	22	19·5	28·5	23	*23·9*
1·5	22	*20·0*	19	14·5	22	21	*16·9*
1·6	20	*16·4*	18	13	18·5	16·5	*11·8*
1·7	20	*13·6*	14	8	16·5	14·5	*8·1*
1·8	14	*11·4*	13	7·5	14	12·5	*5·5*
1·9	11	*9·6*	8	5	12·5	10	*3·6*
2·0	8·5	*8·1*	6·5	3·5	10	9·5	*2·3*
Goodness of Fit $\lvert z \rvert$ P	·629	—	·634	·057	·678	·407	—
Goodness of Fit $\lvert z \rvert$ n'	15	—	15	18	18	18	—
Goodness of Fit z P	—	—	—	—	·182	<·001	—
Goodness of Fit z n'	—	—	—	—	30	28	—
Mean z	−·0268	*0* S.E. *·0224*†	−·0247	+·0066	−·0244	−·1283	—
σ_z	·7273‡	*·7071*	·6556	·6447	·7235	·7005	—

experimental. Thus to take Table II (*a*), we find among 1000 samples of 2 the following numbers having $|z| > 5{\cdot}0$, that is to say with z outside the limits $-5{\cdot}0$ and $+5{\cdot}0$:

* Figures in these columns reduced from results for 1000 samples.

† Standard error of Mean z for 1000 samples from a normal population. The standard error of σ_z would be theoretically infinite were the sampled population truly normal.

‡ This value of σ_z has been calculated, omitting one very divergent value of z of $-8{\cdot}5$; including it $\sigma_z = {\cdot}8193$.

Rectangular Population, 166·7 (theory)*; Symmetrical Platykurtic Population ($\beta_2 = 2{\cdot}5$), 145; Normal Population, 125·6 (value from "Student's" tables); Symmetrical Leptokurtic Population ($\beta_2 = 4{\cdot}1$), 132; etc.

TABLE II (c).

Frequencies of z in 500 Samples of 10.

$\|z\|$ greater than	Populations Sampled							Normal distribution with S.D. $=1/\sqrt{7}$
	$\beta_1=0{\cdot}00$ $\beta_2=2{\cdot}50$	$\beta_1=0{\cdot}00$ $\beta_2=3{\cdot}00$	$\beta_1=0{\cdot}00$ $\beta_2=4{\cdot}12$	$\beta_1=0{\cdot}00$ $\beta_2=7{\cdot}07$	$\beta_1=0{\cdot}22$ $\beta_2=3{\cdot}16$	$\beta_1=0{\cdot}20$ $\beta_2=3{\cdot}30$	$\beta_1=0{\cdot}50$ $\beta_2=3{\cdot}73$	
·00	500	*500·0*	500	500	500†	500	500	*500·0*
·05	435	*442·1*	446	446		433	438	*447·4*
·10	370	*385·5*	390	388	381·5	380	379	*395·7*
·15	312	*331·6*	349	335		330	332	*345·7*
·20	265	*281·7*	307	294	270	287·5	273	*298·4*
·25	224	*236·2*	269	246		239	245	*254·2*
·30	186	*195·8*	218	209	185	207	200	*213·7*
·35	153	*160·5*	180	168		172	167	*177·2*
·40	121	*130·4*	137·5	138	124	145	139	*145·0*
·45	95	*105·0*	104	108		123	112	*116·9*
·50	78	*83·9*	82	86	83	99·5	96	*92·9*
·55	64	*66·7*	62	71		80	73	*72·8*
·60	51	*52·7*	51	57	44·5	59	60	*56·2*
·65	41	*41·5*	38	41		41	49	*42·7*
·70	35	*32·6*	34	30	31	37	41	*32·1*
·75	29	*25·5*	25	24		33	35	*23·6*
·80	26	*19·9*	19	19	21·5	29	31	*17·1*
·85	20	*15·6*	17	15		26	29	*12·2*
·90	15	*12·2*	11	13	12	19	22	*8·6*
·95	11	*9·6*	8	11		16	19	*6·0*
1·00	9	*7·5*	8	7	7·5	11	11	*4·0*
Goodness of Fit $\|z\|$ P	·867	—	·219	·935	·195	·334	·066	—
Goodness of Fit $\|z\|$ n'	16	—	16	16	11	16	16	—
Goodness of Fit z P	—	—	—	—	·033	·071	·005	—
Goodness of Fit z n'	—	—	—	—	18	26	26	—
Mean z	+·0007	*0* S.E. *·0169*‡	−·0226	+·0208	−·0120	+·0022	−·0556	—
σ_z	·3763	*·3780* S.E. *·0151*‡	·3808	·3815	·3709	·4112	·3992	—

The last columns of Tables II (b), (c) and (d) give the corresponding frequencies obtained on the assumption that the distribution of z is normal, with a standard deviation of $1/\sqrt{n-3}$ or $1/\sqrt{2}$, $1/\sqrt{7}$ and $1/\sqrt{17}$ respectively.

* For samples of two, z is the same as z', or the ratio, centre/($\frac{1}{2}$ range), for which the distribution $y=\frac{1}{2}(1+|z'|)^{-2}$ was given in *Biometrika*, Vol. xx[A]. p. 211.

† Figures in this column reduced from results for 1000 samples.

‡ Standard errors of Mean z and σ_z for 500 samples from a normal population.

In the lower part of Tables II (a), (b), (c) and (d) are given:

(1) The result of applying the (P, χ^2) test for Goodness of Fit, the theoretical distribution being in each case that of the Type VII z-curve of "normal theory." In the test for $|z|$, corresponding positive and negative values of z have been combined, or the z-curve was doubled over about $z=0$; in that for z the positive

TABLE II (*d*).

Frequencies of z in 500 Samples of 20.

$\lvert z \rvert$ greater than		Populations Sampled						Normal distribution with S.D. $=1/\sqrt{17}$
		$\beta_1=0{\cdot}00$ $\beta_2=2{\cdot}50$	$\beta_1=0{\cdot}00$ $\beta_2=3{\cdot}00$	$\beta_1=0{\cdot}00$ $\beta_2=4{\cdot}12$	$\beta_1=0{\cdot}00$ $\beta_2=7{\cdot}07$	$\beta_1=0{\cdot}20$ $\beta_2=3{\cdot}30$	$\beta_1=0{\cdot}50$ $\beta_2=3{\cdot}73$	
·00		500	*500·0*	500	500	500	500*	*500·0*
·05		415	*414·9*	402	409	412	413	*418·3*
·10		333	*333·9*	324	317	323	332·5	*340·0*
·15		253	*260·5*	251	244	240	261	*268·1*
·20		182	*197·1*	196	176	194	198·5	*204·8*
·25		136	*144·7*	147	125	145	150	*151·4*
·30		101	*103·3*	103	84	98	105	*108·0*
·35		69	*71·8*	80	54	65	74·5	*74·5*
·40		46	*48·7*	51	38	52	57	*49·6*
·45		35	*32·3*	32	21	36	37·5	*31·8*
·50		26	*21·1*	20	11	25	26	*19·6*
·55		20	*13·5*	15	6	12	16·5	*11·7*
·60		13	*8·5*	10	1	11	10·5	*6·7*
·65		7	*5·3*	3	1	7	7	*3·7*
·70		6	*3·3*	1	1	5	6	*2·0*
Goodness of Fit	$\lvert z \rvert$ P	·535	—	·618	·487	·172	·797	—
	$\lvert z \rvert$ n'	11	—	11	11	11	13	—
	z P	—	—	—	—	·022	·049	—
	z n'	—	—	—	—	20	20	—
Mean z		+·0006	*0* S.E. *·0108*†	−·0056	+·0135	−·0217	−·0223	—
σ_z		·2439	*·2425* S.E. *·0084*†	·2409	·2187	·2436	·2494	—

and negative values were kept separate. n' is the number of groups used in applying the test.

(2) For samples of 5, 10 and 20, Mean z and σ_z are given‡. The results have been represented graphically in Figures 2 (a), (b), (c) and (d). The continuous

* Figures in this column reduced from results for 1000 samples.

† Standard errors of Mean z and σ_z for 500 samples from a normal population.

‡ Here and throughout this paper the standard error of a standard deviation has been taken as $\frac{1}{2}\sigma\sqrt{\frac{\beta_2-1}{N}}$, where σ and β_2 are the constants of the theoretical distribution of the variable, and N is the number of samples upon which the value of the standard deviation has been based.

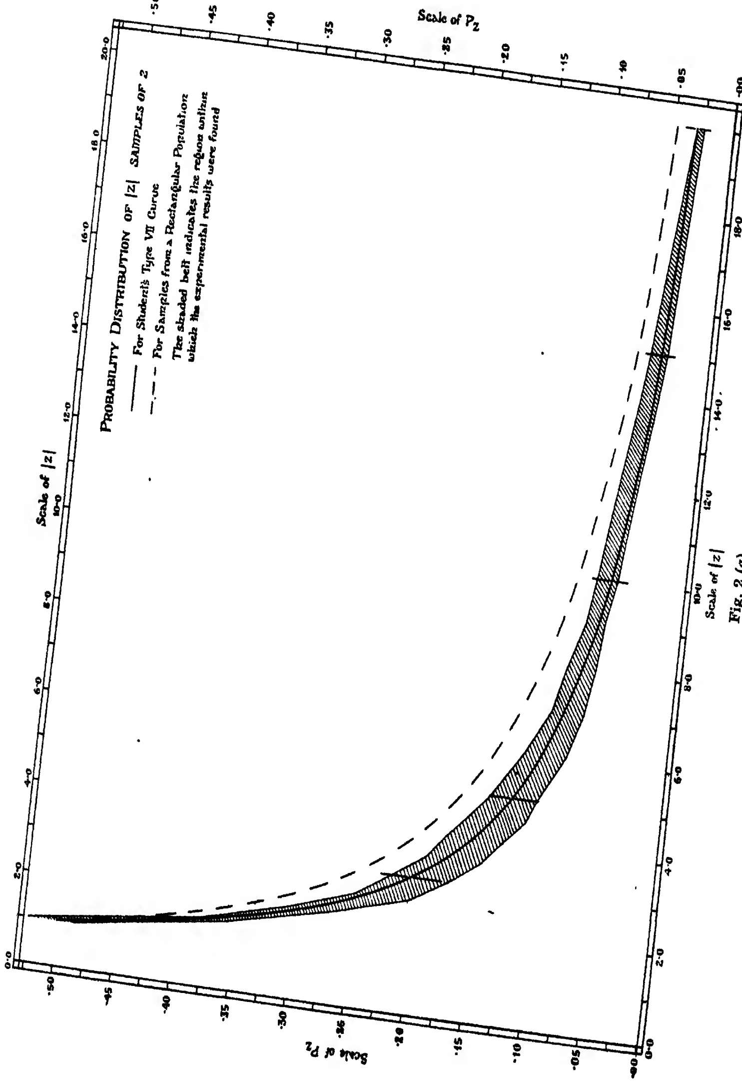

PROBABILITY DISTRIBUTION OF |z| SAMPLES OF 2
For Student's Type VII Curve
For Samples from a Rectangular Population
The shaded belt indicates the region within which the experimental results were found
Scale of |z|
Scale of P_z

Fig. 2 (*a*).

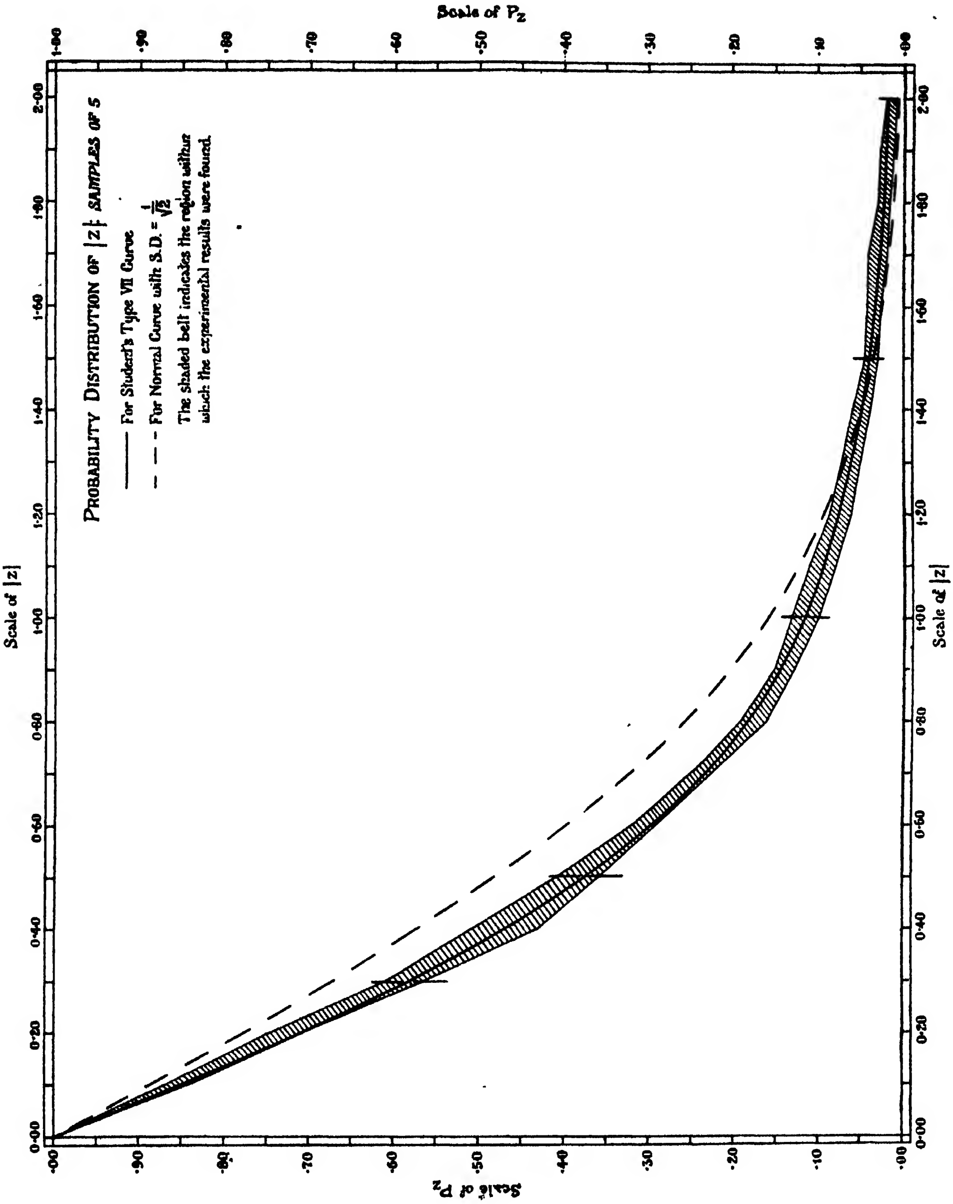

Scale of P_z
Scale of |z|
PROBABILITY DISTRIBUTION OF |z|: SAMPLES OF 5
—— For Student's Type VII Curve
– – – For Normal Curve with S.D. = $\frac{1}{\sqrt{2}}$
The shaded belt indicates the region within which the experimental results were found.

Fig. 2 (b).

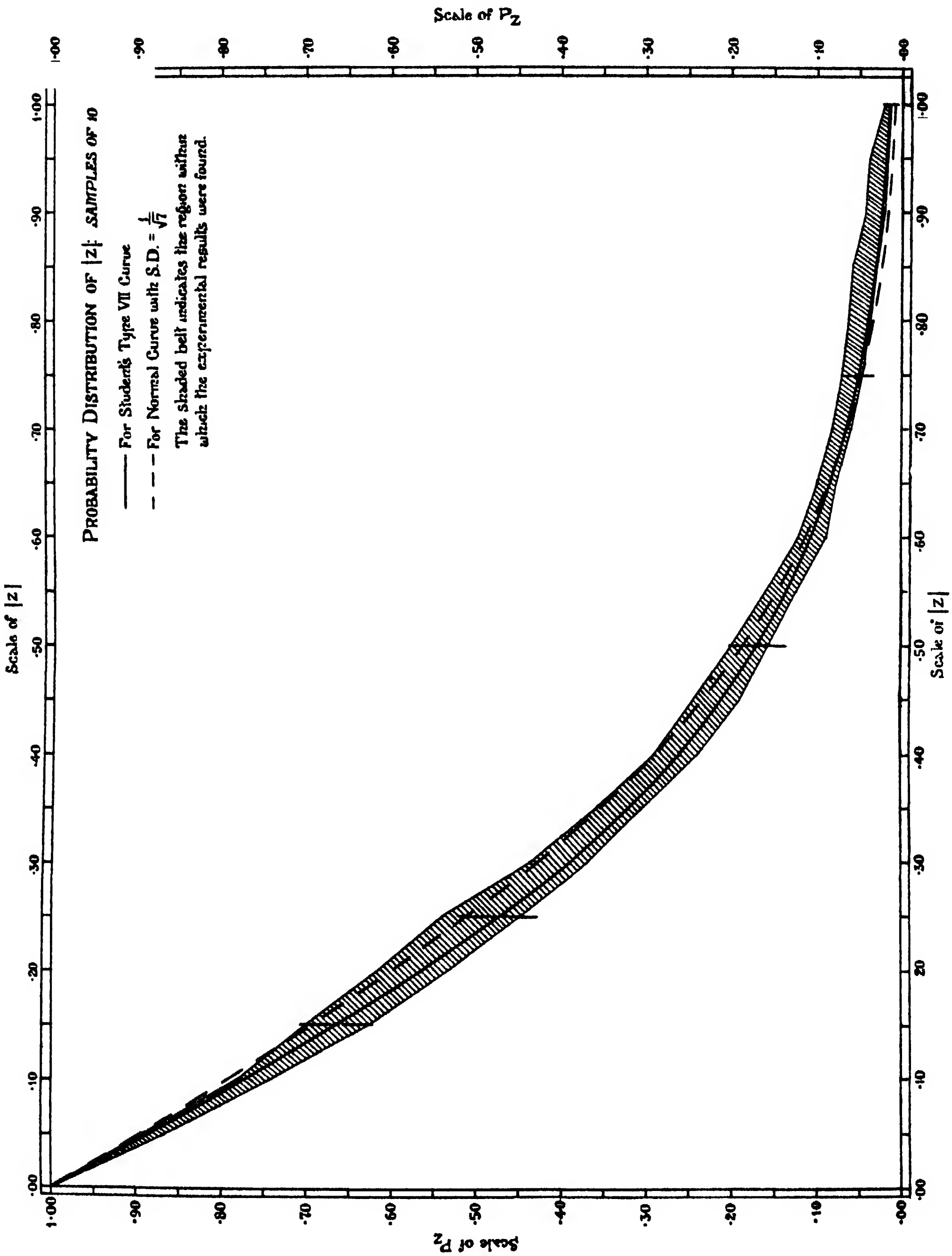

PROBABILITY DISTRIBUTION OF |z|: SAMPLES OF 10
For Student's Type VII Curve
For Normal Curve with S.D. = 1/√7
The shaded belt indicates the region within which the experimental results were found.
Scale of P_Z
Scale of |z|
·00
·10
·20
·30
·40
·50
·60
·70
·80
·90
1·00

Fig. 2 (c).

curve shows the change in the P_z of "normal theory" as $|z|$ increases, and the upper and lower limits of the shaded belts represent the highest and lowest observed frequencies of the corresponding row of Table II when divided by 500 or 1000 as the case may be. To give some indication of the sampling variation that might be expected to arise at different points on the z-scale if the true distribution

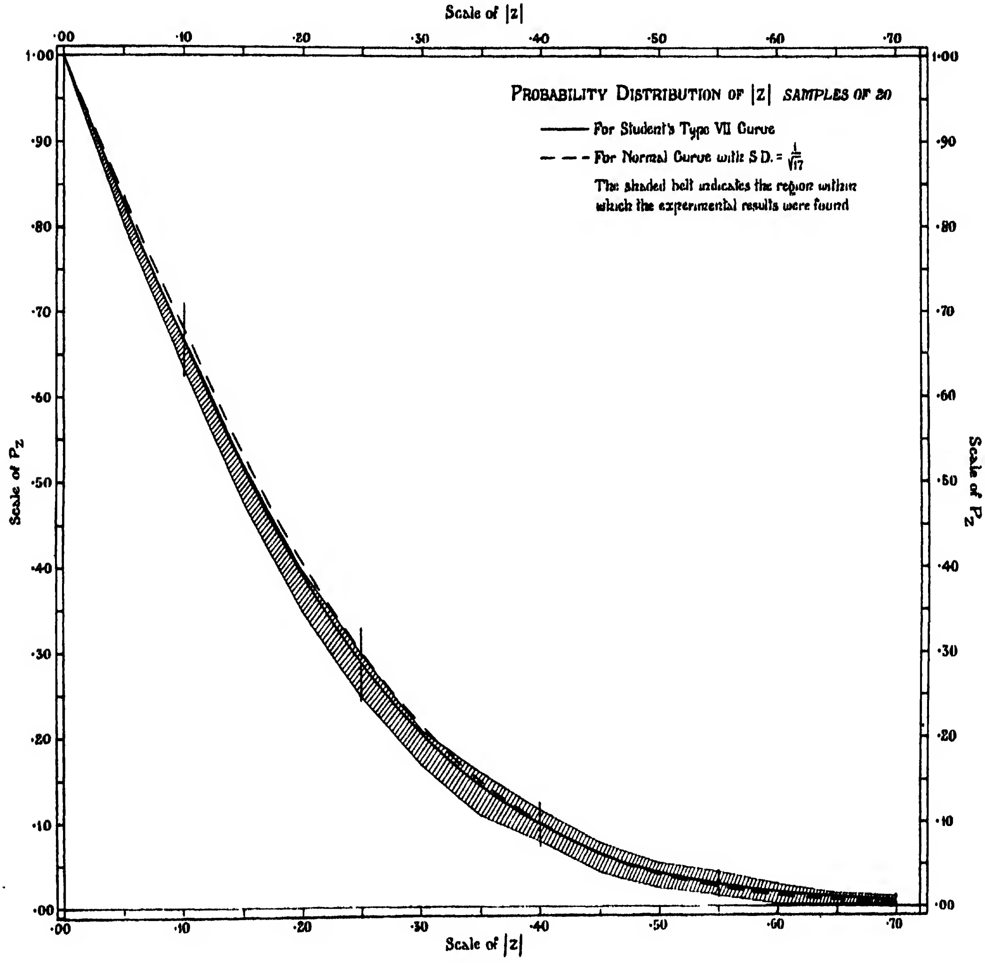


Fig. 2 (*d*).

followed "Student's" curve, lengths equal to twice the standard error of these reduced frequencies have been plotted on either side of the continuous curve*. But of course the systematic manner in which the frequencies differ in certain cases from "normal theory," as shown in the tables, makes it clear that the width of the shaded belt has at any rate sometimes a real significance. Owing to the

* If P_z is as defined in the footnote to p. 261 above, then the standard error of the proportion of N samples in which $|z|$ is greater than a certain value is $\sqrt{P_z(1-P_z)/N}$.

inevitable sampling fluctuations too much stress cannot be laid on any single difference taken alone, but in combination the results support one another. We shall consider them briefly in order.

Samples of 2.

Taking the symmetrical populations, the progressive change in the frequencies corresponding to a given value of $|z|$ as we pass from the rectangular population to the most leptokurtic population ($\beta_2 = 7{\cdot}07$) is very marked. The changes are less clear in passing from the normal population down the Type III line; this is the result of doubling over a skew z-distribution, but for reasons given above we shall be content in the present paper with considering the distribution of $|z|$ only*.

Samples of 5.

The table and diagram show that a normal curve with $\sigma_z = 1/\sqrt{2}$ provides a very poor approximation to "Student's" curve. For the symmetrical populations, σ_z diminishes steadily as the population β_2 increases, and the tail frequencies in the 1000 samples of 5 from population ($\beta_2 = 7{\cdot}07$) are quite clearly less than those expected on normal theory. The correspondence for the other four populations is really very good. The distribution of z (not doubled over) is, however, very skew in the case of the population ($\beta_1 = {\cdot}50$, $\beta_2 = 3{\cdot}73$). This is "Sophister's" case and has been fully discussed by him.

Samples of 10.

Figure 2 (c) shows that the line representing the tail area of the normal curve with $\sigma_z = 1/\sqrt{7}$, although still differing rather widely from the line representing "Student's" curve, now falls largely within the shaded belt. For $|z| > {\cdot}6$ the difference between the two curves is not great. For the samples from the three symmetrical curves, σ_z lies very close to the "normal theory" value, and the correspondence in frequencies is good. For samples from population ($\beta_1 = 0$, $\beta_2 = 7{\cdot}07$) there is curiously no evidence of the shortage of frequency in the tail which appears for samples of 2, 5 and 20. For the two Type III populations there is an excess of high values of $|z|$ which shows itself in the upper limit of the belt in the diagram. This tendency is not at all evident in the 1000 samples from Church's population ($\beta_1 = {\cdot}22$, $\beta_2 = 3{\cdot}16$), but great caution must be exercised in drawing conclusions from apparent differences in these cumulative frequency distributions. If we take the distribution of $|z|$, (a) for the 500 samples from the Type III population ($\beta_1 = {\cdot}20$, $\beta_2 = 3{\cdot}30$), and (b) for the 1000 samples from Church's population ($\beta_1 = {\cdot}22$, $\beta_2 = 3{\cdot}16$), and apply the (P, χ^2) difference test, we obtain for 11 groups a P of ${\cdot}592$; that is to say the observed differences which look large in the columns of Table II (c) are not inconsistent with a common theoretical law of distribution for $|z|$.

* That the distribution of z for samples from the skewest population is also skew, is shown by the drop in value of P from ${\cdot}871$ (test for $|z|$) to ${\cdot}022$ (test for z) in the goodness of fit tests.

The distribution of z (not doubled over) for samples from "Sophister's" population ($\beta_1 = \cdot 50$, $\beta_2 = 3\cdot 73$) is definitely negatively skew.

Samples of 20.

Figure 2 (*d*) suggests that we have now reached a size of sample where the normal curve with standard deviation $1/\sqrt{n-3}$ represents the z-distribution very well. The observed distributions of $|z|$ are quite closely represented by "normal theory" in all cases except that of the extreme leptokurtic population. Here there is again a shortage of high values of $|z|$, although judged by the test of goodness of fit this difference is not exceptional. For the Type III populations the distributions of z are again skew.

A completely satisfactory analysis of the position will only be possible when the theoretical distribution of z in samples from any non-normal population has been found. But in the meantime these results enable a good appreciation to be formed of the extent of variation from "normal theory" that may be expected in sampling from a fairly wide variety of populations*. They suggest that within this range there will not in practice be a danger of any serious loss of control of the source of error (1), if $|z|$ be assumed to follow "Student's" law. The least satisfactory agreement occurs among the samples from the very leptokurtic population ($\beta_2 = 7\cdot 1$). Taken together, we find that the 21 tests of goodness of fit for $|z|$ give a mean value of P of ·463; even if the variations were all due to chance we should only expect a value of ·500.

In Table III a comparison is made at about the level $P_z = \cdot 04$ of the chances, theoretical and observed, of obtaining $|z|$ greater than the values indicated in the

TABLE III.

Comparative values of P_z near ·04.

		Population						Normal distribution with
n	$\lvert z\rvert$	$\beta_1=0\cdot 00$ $\beta_2=2\cdot 50$	$\beta_1=0\cdot 00$ $\beta_2=3\cdot 00$	$\beta_1=0\cdot 00$ $\beta_2=4\cdot 12$	$\beta_1=0\cdot 00$ $\beta_2=7\cdot 07$	$\beta_1=0\cdot 20$ $\beta_2=3\cdot 30$	$\beta_1=0\cdot 50$ $\beta_2=3\cdot 73$	S.D. $=1/\sqrt{n-3}$
2	15·0	·050	*·042*	·042	·042	·039	·036	—
5	1·5	·044	*·040*	·038	·029	·044	·042	*·034*
10	0·8	·052	*·040*	·038	·038	·058	·062	*·034*
20	0·5	·052	*·042*	·040	·022	·050	·052	*·039*

2nd column. The observed frequencies have been divided by 500 or 1000 according to the number of samples, and the figures are therefore subject to sampling errors. But even if they represented the true values of P_z in sampling from the corresponding populations, the differences between them and the "normal theory"

* Certain incomplete results suggest that the population skewness cannot be increased much further without beginning to modify the distribution of $|z|$ appreciably.

values, as shown in the 4th column, are hardly large enough to lead to any serious errors in inference.

With characteristic intuition "Student" anticipated the adequacy of his test in sampling from symmetrical leptokurtic systems more than twenty years ago in his original paper*; the idea of the "doubling over" in the case of skew populations lies also to his credit.

4. The Two Sample Test.

We may now examine the adequacy of Fisher's two sample z-test in controlling the source of error (1) when sampling from non-normal populations. If two independent samples of n_1 and n_2 are drawn from the same normal population, then

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{n_1 s_1^2 + n_2 s_2^2}} \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

is distributed according to the law

$$y = \text{constant} \times (1 + z^2)^{-\frac{n_1+n_2-1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(4).$$

Equation (4) is the distribution of z in the single sample problem with $n_1 + n_2 - 1$ written for n. It does not, however, necessarily follow that when sampling from a non-normal population the distribution of $z = m/s$ for $n = 14$, let us say, will be the same as that of the z of (3) when $n_1 = 5$, $n_2 = 10$. This fact is illustrated in the case of sampling from the leptokurtic population ($\beta_1 = 0$, $\beta_2 = 7{\cdot}07$); there is here a considerable positive correlation between the values of m and s in a sample. Large deviations in mean tend to be associated with large deviations in standard deviation, and as a result the preceding tables have suggested that the ratio, z, is slightly less variable than on "normal theory." But if we combine the samples of 5 and 10, taking $n_1 = 5$, $n_2 = 10$, and calculate the z of (3), the most variable term in the numerator is x_1, the mean of the smaller sample, while the most important term in the denominator is $10 s_2^2$, which is quite uncorrelated with $\bar{x}_1$. There is not compensation, therefore, as in the previous case, and as a result the z is somewhat more variable than that of "normal theory."

Tables IV (*a*), (*b*) and (*c*) show the result of pairing together samples of (*a*) 5 and 10, (*b*) 5 and 20, and (*c*) 10 and 20, from the three populations (0, 2·5), (0, 7·07) and (·50, 3·73). Results for the other two populations sampled are not yet available†. The tables show the observed and theoretical frequencies lying beyond certain values, not of $|z|$, but of a multiple of $|z|$ (as shown at the head of the leading column), which was a simpler ratio to obtain in the computation. The values of Mean z and σ_z are, however, given below, as well as the results of testing the doubled-over distribution for goodness of fit. The skewness and goodness of fit of the undoubled-over z-distributions have not yet been examined. The final columns of each table contain the frequencies found from a normal curve with standard deviation equal to $1/\sqrt{n_1 + n_2 - 4}$; even for samples of 5 and 10

* *Biometrika*, Vol. vi. p. 19.

† See Addendum, p. 285.

these frequencies never differ very widely from those of the true "normal theory" z-curve. The sampling results as they stand do not suggest that the distribution of $|z|$ varies in any simple way as the sampled population changes. But this could hardly be expected owing to the complex structure of the ratio, in which $\bar{x}_1$ is correlated with s_1^2, and $\bar{x}_2$ with s_2^2, but with no cross correlation. It seems justifiable,

TABLE IV (a).

Distribution of z in Pairs of Samples of 5 and 10.

Frequencies in 500 pairs.

$\sqrt{1\cdot5}\,\vert z\vert$ greater than	Populations Sampled				Normal distribution with S.D. $=1/\sqrt{11}$
	$\beta_1=0\cdot00$ $\beta_2=2\cdot50$	$\beta_1=0\cdot00$ $\beta_2=3\cdot00$	$\beta_1=0\cdot00$ $\beta_2=7\cdot07$	$\beta_1=0\cdot50$ $\beta_2=3\cdot73$	
·00	500	*500·0*	500	500	*500·0*
·05	447	*442·6*	459	443	*446·1*
·10	402	*386·6*	399	390	*393·3*
·15	352	*333·0*	341	337	*342·3*
·20	297	*283·0*	295	277	*294·0*
·25	235	*237·4*	256	232	*249·2*
·30	196	*196·4*	222	197	*208·3*
·35	157	*160·8*	184	156	*171·6*
·40	131	*130·0*	153	130	*139·4*
·45	106	*104·0*	121	99	*111·5*
·50	86	*82·5*	102	85·5	*87·9*
·55	67	*64·7*	86	72	*68·2*
·60	44	*50·4*	67	53	*52·1*
·65	34	*39·0*	61	44	*39·2*
·70	23	*30·0*	49	33	*29·0*
·75	15	*22·9*	35	23	*21·1*
·80	12	*17·4*	23	18	*15·1*
·85	9	*13·2*	19	14	*10·7*
·90	9	*10·0*	18	10	*7·4*
·95	7	*7·5*	10	9	*5·0*
1·00	7	*5·7*	7	6	*3·4*
Goodness of Fit P	·161	—	·034	·591	—
Goodness of Fit n'	16	—	16	16	—
Mean z	− ·0116	*0* S.E. *·0135*	+ ·0141	− ·0071	—
σ_z	·3019	*·3015* S.E. *·0110*	·3317	·3024	—

however, to conclude, after examining the tables, that the practical worker will be led to make no very serious error of judgment if he refers the value of z to "Student's" tables (or even to the normal tables with $\sigma_z=1/\sqrt{n_1+n_2-4}$) when examining the difference between the means of pairs of small samples, taken from moderately skew, leptokurtic or platykurtic populations. Possibly the position might be less satisfactory if n_1 and n_2 were below the values of 5 and 10.

The average of the nine values of P found in the goodness of fit tests is now ·332. The standard errors given for Mean z and σ_z are for 500 samples from a normal population.

TABLE IV (b).

Distribution of z in Pairs of Samples of 5 *and* 20.

Frequencies in 500 pairs.

$\frac{1}{2}\sqrt{5}\,\lvert z\rvert$ greater than	Populations Sampled				Normal distribution with S.D. $=1/\sqrt{21}$
	$\beta_1=0{\cdot}00$, $\beta_2=2{\cdot}50$	$\beta_1=0{\cdot}00$, $\beta_2=3{\cdot}00$	$\beta_1=0{\cdot}00$, $\beta_2=7{\cdot}07$	$\beta_1=0{\cdot}50$, $\beta_2=3{\cdot}78$	
·00	500	*500·0*	500	500	*500·0*
·05	411	*416·0*	412	417·5	*418·8*
·10	332	*336·0*	340	340·5	*340·9*
·15	255	*263·2*	274	260	*269·3*
·20	198	*199·9*	224	183·5	*206·2*
·25	143	*147·3*	162	139	*152·8*
·30	106	*105·4*	123	101	*109·4*
·35	64	*73·4*	82	72·5	*75·7*
·40	49	*49·8*	53	47	*50·6*
·45	33	*33·0*	36	30·5	*32·6*
·50	23	*21·4*	23	18	*20·2*
·55	13	*13·6*	16	8	*12·1*
·60	7	*8·5*	10	4	*7·0*
·65	6	*5·2*	4	1·5	*3·9*
·70	2	*3·2*	3	0·5	*2·1*
·75	2	*1·9*	3	—	*1·1*
·80	2	*1·1*	2	—	*0·5*
Goodness of Fit P	·610	—	·397	·085	—
Goodness of Fit n'	11	—	11	13	—
Mean z	− ·0094	*0* S.E. *·0098*	+ ·0099	− ·0058	—
σ_z	·2175	*·2182* S.E. *·0074*	·2280	·2088	—

5. Examination of the Second Type of Error.

Suppose that on finding a value of z such that $P_z > 2\alpha$ (say, > ·10 perhaps), it is decided to accept the hypothesis that the mean of the sampled population has a value b. How often is this likely to occur when in fact the true population mean lies at a instead of b? In such a case "Student's" tables will have been entered with $\zeta = (\bar{x} - b)/s$ instead of with $z = (\bar{x} - a)/s$, and the error in judgment will arise because the test is not sensitive enough to detect this fact. What we require is to have, for different values of $(a - b)$, some appreciation of the chance that $-z_a < \zeta < +z_a$*, for the smaller the chance the more effective is the control of this source of error.

* z_a being, as above, the value of z giving $P_z = 2\alpha$.

The position may be explored with the aid of the experimental sampling results. We have fixed in the first place on two different values of α, ·05 and ·01, which backward interpolation in "Student's" tables shows to correspond to deviations (z_α) of 1·066 and 1·873 for $n=5$, and of ·611 and ·941 for $n=10$. We have then chosen out randomly for each of the five sampled populations, and also for a normal population*, 100 of our samples and have given in succession to $(a-b)$ the values

TABLE IV (c).

Distribution of z in Pairs of Samples of 10 *and* 20.

Frequencies in 500 pairs.

$\sqrt{1\cdot5}\,\lvert z\rvert$ greater than	Populations Sampled				Normal distribution with S.D. $=1/\sqrt{26}$
	$\beta_1=0\cdot00$, $\beta_2=2\cdot50$	$\beta_1=0\cdot00$, $\beta_2=3\cdot00$	$\beta_1=0\cdot00$, $\beta_2=7\cdot07$	$\beta_1=0\cdot50$, $\beta_2=3\cdot73$	
·00	500	*500·0*	500	500	*500·0*
·05	411	*415·3*	410	416	*417·5*
·10	310	*334·5*	329	330	*338·6*
·15	239	*261·1*	258	273	*266·2*
·20	174	*197·4*	196	216	*202·5*
·25	124	*144·6*	144	155	*149·0*
·30	97	*102·7*	105	119	*105·8*
·35	75	*70·8*	76	82	*72·5*
·40	51	*47·4*	55	59	*47·9*
·45	32	*30·9*	31	34	*30·5*
·50	25	*19·7*	19	19	*18·7*
·55	16	*12·2*	12	8	*11·0*
·60	12	*7·4*	8	3	*6·2*
·65	8	*4·4*	5	3	*3·4*
·70	4	*2·6*	4	—	*1·8*
·75	1	*1·5*	2	—	*0·9*
Goodness of Fit P	·104	—	·917	·092	—
Goodness of Fit n'	11	—	11	11	—
Mean z	·0000	*0* S.E. *·0088*	− ·0036	− ·0085	—
σ_z	·1947	*·1961* S.E. *·0066*	·1972	·1997	—

$\sigma/\sqrt{n}$, $2\sigma/\sqrt{n}$, $3\sigma/\sqrt{n}$, ..., etc. ($n=5$ and 10), where σ is the standard deviation of the population sampled. $(a-b)$ has then been added in each case to the observed deviation in the sample mean and the result divided by s, the corresponding sample standard deviation, to give ζ. In samples from a normal population, if $(a-b)$ were zero, the percentage of values of $\zeta\,(=z)$, which should lie in the long run within the limits $\pm z_\alpha$, should be 90 for $\alpha=\cdot05$ and 98 for $\alpha=\cdot01$. For the non-normal

* One hundred random samples of 5 and 10 were specially drawn from a normal population for this purpose.

population, the results discussed in Section (3) above suggest that these percentages will also be fairly nearly approached. As $(a-b)$ is increased from zero the number of values of ζ found between these limits will decrease and will represent the percentage of false hypotheses accepted. The situation can be explained most clearly by turning to Table V.

This table shows the percentage of samples for which $-z_\alpha < \zeta < +z_\alpha$ when different multiples of $\theta = \sigma/\sqrt{n}$ have been added to or subtracted from the sample mean. Suppose for instance that we have a sample of 10 from the population ($\beta_1 = 0$, $\beta_2 = 7{\cdot}07$) and wish to test the hypothesis that the population mean lies at b, and decide to accept it if $|z| = |(\bar{x} - b)|/s < z_\alpha$. Then the experimental results suggest that

(1) if the true population mean were to lie at $a = b \pm 2\sigma/\sqrt{10} = b \pm {\cdot}63\sigma$ instead of at b, we should in repeated sampling accept 38 % of these false hypotheses if we took $\alpha = {\cdot}05$ as the critical level, and 67 % if we took $\alpha = {\cdot}01$;

(2) if the true population mean were to lie at $a = b \pm 4\sigma/\sqrt{10} = b \pm 1{\cdot}26\sigma$ we should accept in repeated sampling only 2 % of these false hypotheses in taking $\alpha = {\cdot}05$, and 9 % with $\alpha = {\cdot}01$.

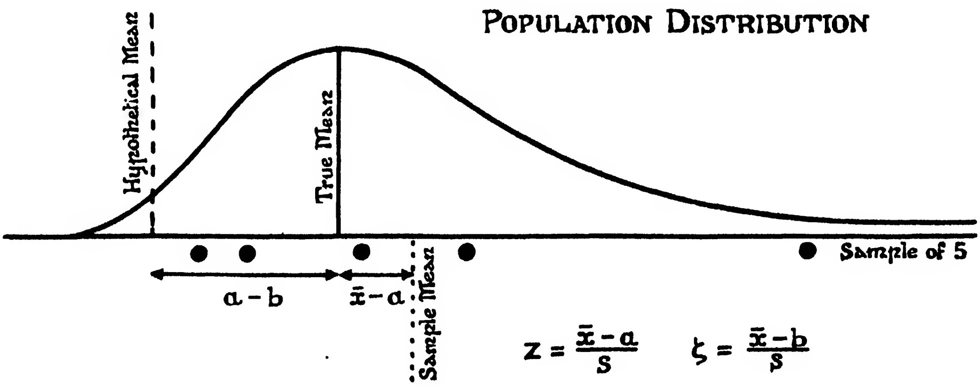


Fig. 3.

The table therefore shows the sensitiveness of "Student's" test in rejecting false hypotheses when applied to samples from various populations. We may comment on the results briefly as follows:

(*a*) For symmetrical populations the test will be equally sensitive whether $(a-b)$ be positive or negative. Multiples of $\sigma/\sqrt{n}$ were therefore only *added* to the observed samples*.

(*b*) There is extremely little difference in the degree of sensitiveness among the samples from the four symmetrical populations, the percentages being of course subject to sampling errors.

(*c*) The Type III populations were both positively skew, and the position is represented diagrammatically in Figure 3. An examination of Table V shows that

* For convenience in comparison with the results for the skew populations the figures for the normal population have been repeated on the negative side.

TABLE V.

z-Test. Table showing Percentage of False Hypotheses accepted when the True Population Mean lies at Increasing Distances $(a-b)$ from its Supposed Position. $(\theta = \sigma/\sqrt{n}.)$

Values of $(a-b)$.

	$-11, 10, 9\theta$	-8θ	-7θ	-6θ	-5θ	-4θ	-3θ	-2θ	$-\theta$	Population β_1	Population β_2	$+\theta$	$+2\theta$	$+3\theta$	$+4\theta$	$+5\theta$	$+6\theta$	$+7\theta$	$+8, 9, 10\theta$
Samples of 5										0	2·50	75	57	28	7	2			
$\alpha =$ ·05						7	24	48	70	0	3·00	70	48	24	7				
$z_\alpha = 1·066$										0	4·12	73	49	23	6	1			
										0	7·07	71	42	20	7	2	1		
			1	1	4	7	16	38	67	·20	3·30	84	54	14	3				
					4	12	27	45	73	·50	3·73	84	50	21	3				
Samples of 5										0	2·50	93	80	65	47	22	10	4	1
$\alpha =$ ·01			1	9	20	39	56	76	86	0	3·00	86	76	56	39	20	9	1	
$z_\alpha = 1·873$										0	4·12	94	78	61	45	18	10	4	1
										0	7·07	86	73	49	35	17	8	2	2, 1, 1
	1, 1, 1	3	4	7	12	26	55	73	81	·20	3·30	96	88	63	39	11	4		
	1	2	6	14	24	38	53	75	89	·50	3·73	99	88	66	33	12	6	1	
Samples of 10										0	2·50	74	39	13	3				
$\alpha =$ ·05						1	10	44	69	0	3·00	69	44	10	1				
$z_\alpha =$ ·611										0	4·12	74	42	21	4				
										0	7·07	68	38	9	2				
						3	18	46	84	·20	3·30	72	31	9	1				
					1	8	14	42	66	·50	3·73	80	45	8	1				
Samples of 10										0	2·50	92	71	41	11	3			
$\alpha =$ ·01					3	12	40	67	89	0	3·00	89	67	40	12	3			
$z_\alpha =$ ·941										0	4·12	93	73	47	22	4			
										0	7·07	88	67	34	9	2			
				2	4	18	48	82	95	·20	3·30	93	67	31	10	1	1		
			1	2	10	15	40	62	85	·50	3·73	95	77	46	9				

for these two populations the test is quicker in rejecting false hypotheses when the true population mean is, as shown in the figure, to the right of its supposed position than when it is in the other direction.

(*d*) For values of $(a-b)$ with the same sign as the population skewness the test appears to be slightly more sensitive in rejection than in the normal case. But for the opposite sign the position is distinctly less favourable when the populations are skew. In other words when dealing with skew populations there is more danger of failing to detect a faulty hypothesis when the long tail of the true population distribution points towards the position of the supposed mean than when the steep tail does. In certain problems the direction of the skewness, if not its exact magnitude, may be clear; in such cases we shall know that the chance of error is less completely controlled in one direction than in the other.

(*e*) The control of what has been termed the first source of error is as good for small samples as for large, provided that the population is such that $|z|$ follows approximately "Student's" law. It is in dealing with the second source of error that small samples are at a disadvantage. Suppose for example we are dealing with normal populations and on obtaining a sample $(\bar{x}, s)$ decide to accept the hypothesis that the population mean lies at b whenever $\zeta=(\bar{x}-b)/s < z_a$ or $P_z > 2\alpha = \cdot 10$, say. Then a rough interpolation in Table V suggests that for samples of 5 we may be accepting the hypothesis in as many as about 42 % of cases where the true population mean differs from b by as much as the population standard deviation; while in samples of 10 this will happen only in about 9 % of such cases*. For samples of 20 the risk would be almost negligible. There is nothing new in this except perhaps the method of approach; it is the old tale that no conceivable method of statistical analysis will enable differences below a certain limit to be detected from the evidence of a single small sample.

6. An Alternative Test.

In a recent paper† it was shown that in sampling from a rectangular population the appropriate criterion to use in testing a hypothesis regarding the position of the mean, a, was not z but the ratio $z'=(G-a)/\frac{1}{2}R$, where

u and v are the highest and lowest values of the variable in the sample,

G is the sample "centre," $=\frac{1}{2}(u+v)$,

R is the sample range, $=u-v$.

The theoretical distribution of z' in samples of n from this population was obtained, and it was suggested that perhaps it might be of wider application, just as "Student's" z-distribution has been found to be adequate for populations differing considerably from the normal. Further analysis, however, soon showed that the

* For $n=5$, $\alpha=\cdot 05$ we have interpolated roughly between the columns $a-b=2\theta=2\sigma/\sqrt{5}=\cdot 894\sigma$ and $a-b=3\theta=3\sigma/\sqrt{5}=1\cdot 342\sigma$, i.e. between the percentages 48 and 24. For $n=10$ we interpolate between $a-b=3\sigma/\sqrt{10}$ and $a-b=4\sigma/\sqrt{10}$.

† *Biometrika*, Vol. xx^A. p. 212.

"rectangular theory" z'-distribution would not be appropriate for samples from the populations of common statistical experience. That this is so is suggested at once by an examination of the values of $\sigma_{z'}$ found from the sampling experiments and given in Table VI. The whole form of the curve also changes.

TABLE VI.

Comparison of Observed Distribution of z' with Empirical "Normal Theory."

Populations.

n		$\beta_1=0\cdot00$ $\beta_2=1\cdot80$	0·00 2·50	0·00 3·00	0·00 4·12	0·20 3·30
5	Goodness of Fit P	—	·472	—	·558	·175
	Goodness of Fit n'	—	12	—	12	15
	Mean z'	0	− ·0103	0	− ·0224	+ ·0274
	S.E.*	—	·0242	—	·0242	·0171
	$\sigma_{z'}$	*·5773*†	·5515	·5418	·5150	·5651
	S.E.*	—	·0309	—	·0309	·0218
10	Goodness of Fit P	—	·003	—	·063	·007
	Goodness of Fit n'	—	13	—	13	13
	Mean z'	0	+ ·0160	0	+ ·0034	+ ·0737
	S.E.*	—	·0132	—	·0132	·0132
	$\sigma_{z'}$	*·1890*†	·2629	·2947	·3169	·3246
	S.E.*	—	·0103	—	·0103	·0103

It seemed, however, worth undertaking the following research:

(*a*) Find experimentally the distribution of z' in samples of 5 and of 10 from a normal population, and by fitting the data with curves obtain empirically "normal theory" z'-curves.

(*b*) Test the adequacy of these curves to represent the distribution of z' in the samples from the three neighbouring non-normal populations, with β_1 and β_2: (0, 2·5), (0, 4·1), (0·2, 3·3). That is to say examine the adequacy of these distributions in controlling the error (1).

(*c*) As in the case of z, examine the sensitiveness of the z'-test in rejecting false hypotheses (control of error (2)).

(*d*) Make a comparison of the sensitiveness of the z- and z'-tests for samples of 5 and 10 from the same populations.

* Standard errors for samples of 500 or 1000 if the distribution law of z' were of the empirical "normal theory" form.

† These are theoretical values obtained from equation (xliii), *Biometrika*, Vol. xx^A. p. 211.

Let us take these steps in order:

(*a*) Mr L. H. C. Tippett very kindly placed at our disposal the 1000 samples of 5 and of 10 from a normal population which he had used in his work on the Distribution of Range*. He also undertook some preliminary computation. The distribution of z' must clearly be symmetrical; the following values were obtained by using the 2nd and 4th moment coefficients about $z' = 0$.

$$n = 5 \quad \sigma_{z'} = \cdot 5418 \quad \beta_2 = 7\cdot 5225,$$

$$n = 10 \quad \sigma_{z'} = \cdot 2947 \quad \beta_2 = 3\cdot 4342.$$

Type VII curves were fitted to the observations and gave on applying tests for goodness of fit, for $n = 5$, $P = \cdot 715$; and for $n = 10$, $P = \cdot 491$. These curves were taken to represent the standard z'-curves of "normal theory," and the chance of exceeding any given value of z' could be obtained by interpolating in "Student's" Tables of t (*Metron*, Vol. v. No. 3, p. 26).

(*b*) The two curves were then doubled over and fitted to the observed distributions of $|z'|$ for the three non-normal populations with the result shown in Table VI. The fits appear quite reasonable for samples of 5, but are no longer satisfactory when $n = 10$. That is to say it would appear that the "normal theory" z'-curves will only represent the distribution of $|z'|$ from moderately non-normal populations in very small samples. It did not seem worth while attempting the fitting in the more extreme cases of sampling from the populations with β_1 and β_2 (0·00, 7·07) and (0·50, 3·73). The table shows how, for symmetrical populations, $\sigma_{z'}$ decreases with β_2 for $n = 5$ and increases for $n = 10$. For samples from the skew Type III population the distributions of z' are negatively skew, and Mean z', at any rate for $n = 10$, differs quite significantly from zero. In dealing of course with a skew population the mean value in repeated samples of G, the "centre," is no longer at the population mean but at a point which changes as n is increased.

(*c*) The sensitiveness of the test in the control of error (2) was examined in precisely the same manner as for the z-test. The error arises because on taking $P_{z'} = 2\alpha$ as the limiting probability†, we find $-z_\alpha' < \zeta' < +z_\alpha'$, where $\zeta' = (G - b)/\frac{1}{2}R$ has been calculated instead of $z' = (G - a)/\frac{1}{2}R$, the supposed population mean being at b, the true one at a. Table VII gives the observed results based as before on 100 samples in each case. The following appear its most important features:

(1) For the symmetrical populations the test becomes less sensitive the more leptokurtic the population. This is not connected with the change in $\sigma_{z'}$, which, as we have seen, takes place in opposite directions for $n = 5$ and 10, but arises because the z' criterion becomes less and less efficient in controlling error (2) as we move away from the rectangular population for which it is theoretically most suitable.

* *Biometrika*, Vol. XVII. pp. 364—87.

† $P_{z'} = 2\int_{z'}^{\infty} f(z')\, dz'$. Using the empirical distribution referred to above it was found that

for $\alpha = \cdot 05$, $z_\alpha' = \cdot 852$ when $n = 5$ and $z_\alpha' = \cdot 482$ when $n = 10$,

for $\alpha = \cdot 01$, $z_\alpha' = 1\cdot 404$ when $n = 5$ and $z_\alpha' = \cdot 710$ when $n = 10$.

TABLE VII.

z'-Test. Table showing Percentage of False Hypotheses accepted when the True Population Mean lies at Increasing Distances $(a-b)$ from its Supposed Position. $(\theta = \sigma/\sqrt{n}.)$

Values of $(a-b)$.

	-10θ	-9θ	-8θ	-7θ	-6θ	-5θ	-4θ	-3θ	-2θ	$-\theta$	Population β_1	Population β_2	$+\theta$	$+2\theta$	$+3\theta$	$+4\theta$	$+5\theta$	$+6\theta$	$+7\theta$	$+8\theta$	$+9\theta$
Samples of 5											0	2·50	77	60	28	11	1				
						7	14	29	52	77	0	3·00	77	52	29	14	7				
$\alpha =$ ·05											0	4·12	82	54	35	15	7	4			
$z_\alpha' =$ ·852					1	5	13	28	63	82	0·2	3·30	72	51	21	3					
Samples of 5											0	2·50	91	82	67	42	26	9	3		
			4	7	15	23	39	61	78	87	0	3·00	87	78	61	39	23	15	7	4	
$\alpha =$ ·01											0	4·12	96	79	59	45	27	15	7	4	2
$z_\alpha' = 1$·404			2	5	13	22	42	67	81	92	0·2	3·30	92	77	62	35	17	2			
Samples of 10											0	2·50	80	56	29	9	1				
						7	13	29	59	81	0	3·00	81	59	29	13	7				
$\alpha =$ ·05											0	4·12	75	57	33	17	6	3	1		
$z_\alpha' =$ ·482			1	1	4	9	20	44	66	80	0·2	3·30	67	46	25	6	1				
Samples of 10											0	2·50	93	82	58	31	15	3			
				2	7	15	36	60	79	95	0	3·00	95	79	60	36	15	7	2		
$\alpha =$ ·01											0	4·12	92	76	57	37	20	7	4	2	1
$z_\alpha' =$ ·710	1	1	2	6	12	24	45	67	80	91	0·2	3·30	91	74	50	27	5	2			

(2) In sampling from the skew population, the control is slightly better than in the normal case when $(a-b)$ is positive, but worse when this difference has a negative sign. Exactly the same effect was observed in the case of z.

(*d*) We shall conclude with a comparison between z and z'. For the second type of error, this can be obtained by comparing Tables V and VII, but for convenience the results for the case $\alpha = \cdot 05$ have been placed together in Table VIII. The figures give the percentages of false hypotheses accepted for increasing values of $(a-b)$. It will be seen that for samples of 5 the z-test is not very much more sensitive than the z'-test, but that when n has increased to 10 the former has a very marked advantage. The difference is least for the platykurtic population.

TABLE VIII.

Comparing the Efficiency of the z- and z′-Tests in Rejecting False Hypotheses. $(\theta = \sigma/\sqrt{n}.)$

	$n=5$										$n=10$									
β_1 / β_2	0·0 / 2·5		0·0 / 3·0		0·0 / 4·1		0·2 / 3·3				0·0 / 2·5		0·0 / 3·0		0·0 / 4·1		0·2 / 3·3			
							Positive θ		Negative θ								Positive θ		Negative θ	
$a-b$	z	z'	z	z'	z	z'	z	z'	z	z'	z	z'	z	z'	z	z'	z	z'	z	z'
θ	*75*	77	*70*	77	*73*	82	*84*	72	*67*	82	*74*	80	*69*	81	*74*	75	*72*	67	*84*	80
2θ	*57*	60	*48*	52	*49*	54	*54*	51	*38*	63	*39*	56	*44*	59	*42*	57	*31*	46	*46*	66
3θ	*28*	28	*24*	29	*23*	35	*14*	21	*16*	28	*13*	29	*10*	29	*21*	33	*9*	25	*18*	44
4θ	*7*	11	*7*	14	*6*	15	*3*	3	*7*	13	*3*	9	*1*	13	*4*	17	*1*	6	*3*	20
5θ	*2*	1	—	7	*1*	7	—	—	*4*	5	—	1	—	7	—	6	—	1	—	9
6θ	—	—	—	—	—	4	—	—	*1*	1	—	—	—	—	—	3	—	—	—	4
7θ	—	—	—	—	—	—	—	—	*1*	—	—	—	—	—	—	1	—	—	—	1
8θ	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1

The figures for z have been printed in italics to aid the eye in comparison.

We have available, therefore, the distribution of two criteria in samples from a normal population, one (of z) known exactly and the other (of z') found empirically. That of the former, which is in theory ideal at the normal point, has been shown to be still applicable for a very considerable variety of population forms. But the distribution of z', while providing complete control of the first type of error at this point, begins to lose this control much more quickly than does that of z as the population form is modified. And further the z' criterion is less sensitive than the other in the detection of false hypotheses regarding the position of the population mean. Were the populations of experience clustered round the rectangular point the situation would almost certainly be reversed.

Owing to the simplicity in calculation it seems, however, possible that in problems where the population is known to be approximately normal, the criterion

$$z' = (u + v - 2a)/(u - v)$$

may be of value in providing a rapid method of testing the validity of a hypothesis

regarding a population mean from a knowledge of the two extreme individuals in the sample only. This would be in cases where n is not greater than, say, 7 or even 10; for $n=2$ of course $z=z'$. It is therefore hoped to provide shortly brief tables of the empirical "normal theory" probability integral of z'.

Elsewhere in statistical theory there may well be cases of two criteria, for both of which the frequency distributions in sampling from a given population are known. In the case of z and z' the method of likelihood expresses in simple logical form the reason for the choice of z at the normal point and of z' at the rectangular point. This method should be applicable in other cases, and its value in picking out the right criterion is supported by the conclusions of this paper which have been reached by a quite different process of argument. In any case, however, the sensitiveness of the tests to changes in population form could not have been gauged except by the present form of experiment or by surmounting certain stubborn obstacles in the mathematical theory of sampling.

In conclusion, it is necessary to emphasise the extent to which this paper is a result of co-operation. The labour of sampling and computation is far too great to have been undertaken by a single individual. Mr N. K. Adyanthāya has been entirely responsible for this and other work on the symmetrical population with $\beta_2=4\cdot1$. As has been stated above, the results for samples of 5 and 20 from the skewest of the populations have been taken from "Sophister's" paper, and acknowledgments have also been made to Dr A. E. R. Church and Mr L. H. C. Tippett. Far the greater part of the remaining computing has been courageously undertaken by Mrs L. J. Comrie, while other computers have been Miss Marie H. Anderson, Mr A. B. Thomson and Mr Ernest Martin. To Mr A. E. Stone we are indebted for some 11,000 samples, and the diagrams are the work of Miss Ida McLearn. To all these the chief author is exceedingly grateful.

ADDENDUM: *Distribution of z in 500 Pairs of Samples from Population $\beta_1=0\cdot20$, $\beta_2=3\cdot30$.*

Samples of 5 and 10			Samples of 5 and 20			Samples of 10 and 20		
$\sqrt{1\cdot5}\,\lvert z\rvert$ greater than	Observation	Normal Theory	$\frac{1}{2}\sqrt{5}\,\lvert z\rvert$ greater than	Observation	Normal Theory	$\sqrt{1\cdot5}\,\lvert z\rvert$ greater than	Observation	Normal Theory
·00	500	500·0	·00	500	500·0	·00	500	500·0
·15	361	333·0	·10	340	336·0	·10	342	334·5
·30	204	196·4	·20	200	199·9	·20	203	197·4
·40	135	130·0	·30	121	105·4	·30	119	102·7
·50	88	82·5	·40	59	49·8	·40	64	47·4
·60	52	50·4	·50	23	21·4	·50	25	19·7
·70	29	30·0	·60	9	8·5	·55	15	12·2
·80	14	17·4	·65	6	5·2	·60	8	7·4
·90	6	10·0	·70	3	3·2	·65	6	4·4
1·00	5	5·7	·75	2	1·9	·70	5	2·6
P	·337		P	·441		P	·185	
n'	16		n'	12		n'	11	
σ_z	·3023	·3015	σ_z	·2204	·2182	σ_z	·1983	·1961

The above results correspond, in somewhat abbreviated form, to those of Tables IV (*a*), (*b*) and (*c*) above. The values of σ_z show very close agreement with "normal theory," and the frequencies do not appear to differ seriously.

For the population $\beta_1 = 0{\cdot}00$, $\beta_2 = 4{\cdot}12$ the result for samples of 5 and 10 alone is available. Testing goodness of fit to "normal theory" it is found that $P = {\cdot}350$, while $\sigma_z = {\cdot}3111$ against the normal value of ·3015. The distribution of z is somewhat too variable, but not as much so as in the case of samples from the extremely leptokurtic population ($\beta_2 = 7{\cdot}07$).

SAMPLING WHEN THE PARENT POPULATION IS OF PEARSON'S TYPE III.

By CECIL CALVERT CRAIG, Ph.D.

Introduction. It is an immediate extension of my thesis* to apply the methods there developed in a more detailed way in the study of sampling in cases in which the parent distribution is skew. One of the most useful and important of the skew frequency functions is the Type III of Pearson. If its equation is written in the form

$$f(z)=\frac{b^b e^{-b}}{a^b\Gamma(b)}(a+z)^{b-1}e^{-bz/a} \quad \text{.........................(1)},$$

in which $z=\dfrac{x-m_x}{\sigma_x}$, $a=\dfrac{2}{\alpha_{3\cdot x}}$, and $b=a^2$ $(\alpha_{3:x}=\sqrt{\beta_1})$,

the semi-invariants, λ_r, of this frequency function follow the simple law†

$$\lambda_r=\frac{(r-1)!\,a^r}{b^{r-1}}=\frac{(r-1)!}{a^{r-2}}, \qquad r \geqslant 2 \quad \text{....................(2)}.$$

($\lambda_0=1$ and $\lambda_1=0$ since (1) is written in standard units with the origin at the mean.) This simple, explicit expression for any semi-invariant invites an application of the method of semi-invariants to the case in which the parent distribution is of this type.

The three parameters of the Type III distribution are the mean, m_x, the standard deviation. σ_x, and the skewness, $\alpha_{3:x}$. (The measure of skewness is in this case just twice that given by Pearson.) The problem is to find the semi-invariants of the frequency distributions of these three characteristics as found from samples of N, each taken from an infinite parent population which is distributed according to the Type III law.

More explicitly let the infinite parent be given by means of the semi-invariants of (2). Let infinitely many random samples of N each be taken and the mean, the standard deviation, and the skewness be calculated for each sample. To find the semi-invariants, $\lambda_{r:m_x}$, $\lambda_{r:\sigma_x}=d_r$, and $\lambda_{r:\alpha_3:x}=b_r$, of the frequency distributions of the sample means, sample standard deviations, and sample skewnesses thus obtained.

Section I. The Frequency Distribution of Sample Means. In the case of sample means it has already been shown that they also form a Type III distribution if the parent does‡. I believe however that it will be interesting to show how neatly the same conclusion is reached using semi-invariants.

* *Metron*, Vol. VII. pp. 3—75.

† Steffensen, J. F., *Matematisk Iagttagelseslære*, G. E. C. Gads, Copenhagen, 1923, p. 60. Also the development of Section I follows exactly the same lines.

‡ Church, A. E. R., "Means and Squared Standard Deviations of Small Samples from any Population." *Biometrika*, Vol. XVIII. (1926), pp. 335—338. Also, Irwin, J. O., "On the Frequency Distribution of the Means of Samples, etc." *Biometrika*, Vol. XIX. (1927), pp. 228, 229.

If from an infinite population distributed according to a frequency law which has $\lambda_1, \lambda_2, \ldots \lambda_r, \ldots$ for its semi-invariants, infinitely many random samples of N be taken, it is a property of semi-invariants that the distribution of means, m_x, of these samples, has semi-invariants, $\lambda_{r.m_x}$, given by*

$$\lambda_{r.m_x} = \frac{\lambda_r}{N^{r-1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3).$$

If the parent distribution be of Type III then

$$\lambda_{r:m_x} = \frac{(r-1)!\,a^r}{(Nb)^{r-1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4).$$

Comparison with (1) and (2) suggests that in this case

$$F(y) = \frac{(Nb)^{Nb} e^{-Nb}}{a^{Nb}\Gamma(Nb)} (a+y)^{Nb-1} e^{-\frac{Nb}{a}y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5),$$

in which $y = m_x$. It is only necessary to verify this by finding the semi-invariants of (5). This is done by equating the coefficients of like powers of t in

$$e^{\theta_1 t + \frac{1}{2!}\theta_2 t^2 + \frac{1}{3!}\theta_3 t^3 + \ldots} = \frac{(Nb)^{Nb} e^{-Nb}}{a^{Nb}\Gamma(Nb)} \int_{-a}^{\infty} (a+y)^{Nb-1} e^{-\frac{Nb}{a}} e^{yt}\, dy \quad \ldots\ldots (6).$$

The minimum value of z in (1) is $-a$ and this will also be the minimum value for y. Put $a + y = \omega$ in the right member of (6) and it becomes

$$\frac{(Nb)^{Nb} e^{-Nb}}{a^{Nb}\Gamma(Nb)} \int_0^{\infty} \omega^{Nb-1} e^{-\frac{Nb}{a}(\omega - a)} e^{-(a-\omega)t}\, d\omega = \frac{(Nb)^{Nb} e^{-at}}{a^{Nb}\Gamma(Nb)} \int_0^{\infty} \omega^{Nb-1} e^{-\omega\left(\frac{Nb}{a} - t\right)}\, d\omega$$

$$\frac{\left(Nb\right)^{Nb}}{\left(\frac{Nb}{a} - t\right)^{Nb}} \qquad \frac{e^{-at}}{\left(1 - \frac{at}{Nb}\right)^{Nb}}.$$

Then taking the logarithms of both sides,

$$\theta_1 t + \frac{1}{2!}\theta_2 t^2 + \frac{1}{3!}\theta_3 t^3 + \ldots = -at - Nb \log\left(1 - \frac{at}{Nb}\right)$$

$$= \frac{(at)^2}{2Nb} + \frac{(at)^3}{3(Nb)^2} + \quad \ldots\ldots\ldots\ldots\ldots\ldots (7),$$

or

$$\theta_r = \lambda_{r:m_x} = \frac{(r-1)!\,a^r}{(Nb)^{r-1}}, \qquad r \geqslant 2,$$

as anticipated. Note that the coefficient of t^0 on both sides of (7) is zero; that is, the constant term in (5) has already been so chosen that the total frequency is unity.

The final step is to write (5) in its own standard units. The mean is already at the origin since $\lambda_{1:m_x} = 0$. Also

$$\left.\begin{aligned} \sqrt{\lambda_{2:m_x}} &= \sqrt{\frac{a^2}{bN}} = \frac{1}{\sqrt{N}} \\ \text{and} \qquad \alpha_{3:m_x} &= \frac{\lambda_{3:m_x}}{\sigma^3_{m_x}} = \frac{2a^3}{N^2 b^2} N^{\frac{3}{2}} = \frac{2}{a\sqrt{N}} = \frac{\alpha_{3:x}}{\sqrt{N}} \end{aligned}\right\} \quad (8).$$

* Thiele, T. N., *The Theory of Observations*, C. and E. Layton, London, 1903, p. 42.

If now I write
$$z = \frac{y}{\sigma_y} = y\sqrt{N},$$
$$\frac{2}{\alpha_{3:m_x}} = a\sqrt{N},$$
and
$$B = A^2 = Nb,$$
(5) reduces to
$$F(y) = \frac{N^{\frac{1}{2}} B^B e^{-B}}{A^B \Gamma(B)} (A+z)^{B-1} e^{-\frac{B}{A}z}$$
or
$$\frac{F(y)\sigma_y}{1} = F(z) = \frac{B^B e^{-B}}{A^B \Gamma(B)} (A+z)^{B-1} e^{-\frac{B}{A}z} \quad \text{.................(9)},$$
which is the desired form.

Knowing that the distribution of sample means is of Type III, in practice formula (4) is the one that will be used. For example, if for the parent $\alpha_3 = 0{\cdot}5$, then $a = 4$ and $b = 16$. If I choose $N = 100$,
$$\lambda_{1:m_x} = a = 4,$$
$$\lambda_{2:m_x} = \frac{a^2}{Nb} = \frac{1}{N} = \sigma^2_{m_x},$$
$$\sigma_{m_x} = 0{\cdot}1,$$
$$\lambda_{3\cdot m_x} = \frac{2a^3}{(Nb)^2} = 0{\cdot}00005,$$
$$\alpha_{3:m_x} = \frac{\lambda_{3:m_x}}{\sigma^3_{m_x}} = 0{\cdot}05.$$
These are the characteristics of the Type III distribution of means of samples of 100 taken from an infinite parent population which is distributed according to the Type III law.

In general terms, this Type III distribution of sample means has the same mean as the parent distribution, and a standard deviation and a skewness, α_3, equal to those of the parent respectively divided by $\sqrt{N}$, where N is the size of each sample.

Section II. The Semi-invariants of the Distributions of Sample Standard Deviations and α_3's.

In my thesis I used the following method for approximating the values of the semi-invariants of the distribution of standard deviations in samples of N taken at random from an infinite parent population, the frequency law for which is given by the semi-invariants, $\lambda_1, \lambda_2, \ldots \lambda_r, \ldots$. The desired semi-invariants, $d_1, d_2, \ldots d_r, \ldots$, are defined by
$$e^{d_1 t + \frac{1}{2!} d_2 t^2 + \frac{1}{3!} d_3 t^3 + \cdots} = \int_{-\infty}^{\infty} d\sigma_x f_1(\sigma_x) e^{\sigma_x t} = \int_{-\infty}^{\infty} d\nu_2 f_2(\nu_2) e^{\sqrt{\nu_2}\, t} \quad \text{......(10)},$$
in which $f_1(\sigma_x)$ is the frequency function for σ_x due to sampling and $f_2(\nu_2)$ is the

same for ν_2. ($\nu_2 = \sigma_x^2$.) The second form on the right arises by virtue of a well-known property of semi-invariants. I write

$$\nu_2 = \lambda_2 + \epsilon,$$

and then

$$m_\epsilon = -\frac{1}{N}\lambda_2,$$

so that

$$\sqrt{\nu_2} = \sqrt{\lambda_2}\left(1 + \frac{\epsilon}{\lambda_2}\right)^{\frac{1}{2}}.$$

On substitution in (10), I had

$$e^{d_1 t + \frac{1}{2!} d_2 t^2 + \frac{1}{3!} d_3 t^3 + \dots} = \int_{-\infty}^{\infty} d\nu_2 f_2(\nu_2)\, e^{\lambda_2^{\frac{1}{2}}\left(1 + \frac{\epsilon}{\lambda_2}\right)^{\frac{1}{2}} t} \quad \text{..............(11).}$$

Expansion of the exponential on the right gave

$$\left.\begin{aligned}
d_1 &= \lambda_2^{\frac{1}{2}}\left(1 + \tfrac{1}{2}\frac{\bar{\nu}_1}{\lambda_2} - \tfrac{1}{8}\frac{\bar{\nu}_2}{\lambda_2^2} + \tfrac{1}{16}\frac{\bar{\nu}_3}{\lambda_2^3} - \tfrac{5}{128}\frac{\bar{\nu}_4}{\lambda_2^4} + \dots\right) \\
d_2 &= \lambda_2\left(1 + \frac{\bar{\nu}_1}{\lambda_2}\right) - d_1^2 \\
d_3 &= \lambda_2^{\frac{3}{2}}\left(1 + \tfrac{3}{2}\frac{\bar{\nu}_1}{\lambda_2} + \tfrac{3}{8}\frac{\bar{\nu}_2}{\lambda_2^2} - \tfrac{1}{16}\frac{\bar{\nu}_3}{\lambda_2^3} + \tfrac{3}{128}\frac{\bar{\nu}_4}{\lambda_2^4} + \dots\right) - 3d_1 d_2 - d_1^3 \\
d_4 &= \lambda_2^2(1 + 2\bar{\nu}_1 + \bar{\nu}_2) - 4d_1 d_3 - 3d_2^2 - 6d_2 d_1^2 - d_1^4
\end{aligned}\right\} \dots (12),$$

in which $\bar{\nu}_r$ is the rth moment about the mean of the samples' ν_2's. For the $\bar{\nu}_r$'s were substituted their values in terms of the semi-invariants $S_r(\nu_2)$ of ν_2 due to sampling. In the results g_r was written for $S_r(\nu_2)/\lambda_2^r$. Also $S_1(\nu_2)$ is of order -1 in N and $S_r(\nu_2)$ is of order $-(r-1)$ in N for $r > 1$. Below are given the results when all terms of order -3 and higher in N are retained:

$$\left.\begin{aligned}
d_1 &= \lambda_2^{\frac{1}{2}}\big(1 + \tfrac{1}{2}g_1 - \tfrac{1}{8}g_2 - \tfrac{1}{8}g_1^2 + \tfrac{1}{16}g_3 + \tfrac{3}{16}g_2 g_1 + \tfrac{1}{16}g_1^3 - \tfrac{5}{128}g_4 - \tfrac{5}{32}g_3 g_1 \\
&\qquad - \tfrac{15}{128}g_2^2 - \tfrac{15}{64}g_2 g_1^2 + \tfrac{35}{128}g_3 g_2 + \tfrac{105}{256}g_2^2 g_1 - \tfrac{315}{1024}g_2^3\big) \\
d_2 &= \frac{\lambda_2}{4}\left(g_2 - \tfrac{1}{2}g_3 - g_2 g_1 + \tfrac{5}{16}g_4 + g_3 g_1 + \tfrac{7}{8}g_2^2 + g_2 g_1^2 - \tfrac{17}{8}g_3 g_2 - \tfrac{21}{8}g_2^2 g_1 + \tfrac{75}{32}g_2^3\right) \\
d_3 &= \frac{\lambda_2^{\frac{3}{2}}}{8}\left(g_3 - \tfrac{3}{4}g_4 - \tfrac{3}{2}g_3 g_1 - \tfrac{3}{2}g_2^2 + \tfrac{39}{8}g_3 g_2 + \tfrac{15}{4}g_2^2 g_1 - \tfrac{83}{16}g_2^3\right) \\
d_4 &= \frac{\lambda_2^2}{16}\left(g_4 - 6g_3 g_2 + 6g_2^3\right)
\end{aligned}\right\} \dots (13).$$

For further details of these calculations and of similar ones in the case of α_3 consult my thesis.

For α_3 essentially the same device was used. The semi-invariants, $b_1, b_2, b_3, \dots$, are defined by

$$e^{b_1 t + \frac{1}{2!} b_2 t^2 + \frac{1}{3!} b_3 t^3 + \dots} = \int_{-\infty}^{\infty} d\alpha_3\, \phi(\alpha_3)\, e^{\alpha_3 t},$$

in which $\phi(\alpha_3)$ is the frequency function of the α_3's due to sampling. Using this, I wrote

$$\nu_2 = \lambda_2 + \epsilon_1,$$

as before, and

$$\nu_3 = \lambda_3 + \epsilon_2,$$

in which λ_3 is the third semi-invariant (or third moment about the mean) of the parent. Then I rewrote the above,

$$e^{b_1 t+\frac{1}{2!}b_2t^2+\frac{1}{3!}b_3t^3+\dots}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d\nu_2\,d\nu_3\,\theta(\nu_2,\nu_3)\,e^{\frac{(\lambda_3+e_3)}{\lambda_2^{\frac{3}{2}}}\left(1+\frac{e_1}{\lambda_2}\right)^{-\frac{3}{2}}t} \quad\dots\dots(14),$$

as in (10), in which $\theta(\nu_2, \nu_3)$ is the correlation function of ν_2 and ν_3 due to sampling. The exponential term on the right was expanded, and I had

$$\left.\begin{aligned}
b_1&=\frac{\lambda_3}{\lambda_2^{\frac{3}{2}}}\left(1-\tfrac{3}{2}\frac{\bar\nu_{10}}{\lambda_2}+\tfrac{15}{8}\frac{\bar\nu_{20}}{\lambda_2^2}-\tfrac{35}{16}\frac{\bar\nu_{30}}{\lambda_2^3}+\dots\right)\\
&\qquad+\frac{1}{\lambda_2^{\frac{3}{2}}}\left(\bar\nu_{01}-\tfrac{3}{2}\frac{\bar\nu_{11}}{\lambda_2}+\tfrac{15}{8}\frac{\bar\nu_{21}}{\lambda_2^2}-\tfrac{35}{16}\frac{\bar\nu_{31}}{\lambda_2^3}+\dots\right)\\
b_2&=\frac{\lambda_3^2}{\lambda_2^3}\left(1-3\frac{\bar\nu_{10}}{\lambda_2}+6\frac{\bar\nu_{20}}{\lambda_2^2}-10\frac{\bar\nu_{30}}{\lambda_2^3}+\dots\right)\\
&\qquad+\frac{2\lambda_3}{\lambda_2^3}\left(\bar\nu_{01}-3\frac{\bar\nu_{11}}{\lambda_2}+6\frac{\bar\nu_{21}}{\lambda_2^2}-10\frac{\bar\nu_{31}}{\lambda_2^3}+\dots\right)\\
&\qquad+\frac{1}{\lambda_2^3}\left(\bar\nu_{02}-3\frac{\bar\nu_{12}}{\lambda_2}+6\frac{\bar\nu_{22}}{\lambda_2^2}-10\frac{\bar\nu_{32}}{\lambda_2^3}+\dots\right)-b_1^2\\
b_3&=\frac{1}{\lambda_2^{\frac{9}{2}}}\sum_{i=0}^{3}{}_3C_i\lambda_3^{3-i}\left(\bar\nu_{0i}-\tfrac{9}{2}\frac{\bar\nu_{1i}}{\lambda_2}+\tfrac{99}{8}\frac{\bar\nu_{2i}}{\lambda_2^2}-\tfrac{429}{16}\frac{\bar\nu_{3i}}{\lambda_2^3}+\dots\right)-3b_1b_2-b_1^3\\
b_4&=\frac{1}{\lambda_2^6}\sum_{i=0}^{4}{}_4C_i\lambda_3^{4-i}\left(\bar\nu_{0i}-6\frac{\bar\nu_{1i}}{\lambda_2}+21\frac{\bar\nu_{2i}}{\lambda_2^2}-56\frac{\bar\nu_{3i}}{\lambda_2^3}+\dots\right)\\
&\qquad-4b_1b_3-3b_2^2-6b_2b_1^2-b_1^4
\end{aligned}\right\}\dots(15),$$

in which the $\bar\nu_{rs}$'s are moments of the correlation function $\theta(\nu_2, \nu_3)$. $(\bar\nu_{00}=1.)$

For the $\bar\nu_{rs}$'s their values in terms of $S_{ij}(\nu_2, \nu_3)$'s the semi-invariants of $\theta(\nu_2, \nu_3)$ were substituted in the above, and in the results g_{kl} written for $\dfrac{S_{kl}(\nu_2, \nu_3)}{\lambda_2^{\frac{2k+3l}{2}}}$. In my thesis I have the final expressions which include all terms of order -3 and higher in N. Here, for reasons to be discussed later in this paper, I will only reproduce all terms of order -2 and higher in N. These are:

$$\left.\begin{aligned}
b_1&=a_3\left(1-\tfrac{3}{2}g_{10}+\tfrac{15}{8}g_{20}+\tfrac{15}{8}g_{10}^2-\tfrac{35}{16}g_{30}-\tfrac{105}{16}g_{20}g_{10}+\tfrac{945}{128}g_{20}^2\right)\\
&\quad+\left(g_{01}-\tfrac{3}{2}g_{11}-\tfrac{3}{2}g_{10}g_{01}+\tfrac{15}{8}g_{21}+\tfrac{15}{8}g_{20}g_{01}+\tfrac{15}{4}g_{11}g_{10}-\tfrac{105}{16}g_{20}g_{11}\right)\\
b_2&=a_3^2\left(\tfrac{9}{4}g_{20}-\tfrac{45}{8}g_{30}-\tfrac{45}{4}g_{20}g_{10}+\tfrac{855}{32}g_{20}^2\right)\\
&\quad+2a_3\left(-\tfrac{3}{2}g_{11}+\tfrac{33}{8}g_{21}+\tfrac{9}{4}g_{20}g_{01}+6g_{11}g_{10}-\tfrac{165}{8}g_{20}g_{11}\right)\\
&\quad+\left(g_{02}-3g_{12}-3g_{02}g_{10}-3g_{11}g_{01}+\tfrac{39}{4}g_{11}^2+6g_{20}g_{02}\right)\\
b_3&=a_3^3\left(-\tfrac{27}{8}g_{30}+\tfrac{405}{16}g_{20}^2\right)+3a_3^2\left(\tfrac{9}{4}g_{21}-18g_{20}g_{11}\right)\\
&\quad+3a_3\left(-\tfrac{3}{2}g_{12}+\tfrac{9}{2}g_{20}g_{02}+\tfrac{33}{4}g_{11}^2\right)+\left(g_{03}-9g_{02}g_{11}\right)
\end{aligned}\right\}\dots(16);$$

b_4 is of order -3 in N.

The series in (12) and (15) give rise to questions of convergence. In my thesis I imposed a sufficient condition on the sampling which has the effect in practice of ensuring convergence. This restriction was that the frequency distribution of sample ν_2's be of limited variation. This got rid of the difficulties on the point of convergence in (12) and (15). Then, strictly speaking, the $\bar\nu$'s in (12) and (15) and

the S's which are substituted for them in obtaining (13) and (16) should be the moments and semi-invariants of the frequency distribution of sample ν_2's and of the correlation surface of sample ν_2's and ν_3's respectively in which in both cases the range of sample ν_2's is restricted to an arbitrary interval about their mean. But the moments and the semi-invariants actually used are those on which no such limitation has been made. It was assumed for moments and semi-invariants of low order that the limited range is still large enough in practice for the error made to be negligible. But since the order of contact of the tail of a Type III distribution is not so high as it is in the case of the tails of a normal distribution, it appeared that in the case of a Type III parent this assumption had better be more carefully studied. My digression for this purpose grew to such proportions that it seemed better to give it a separate existence as a study of the semi-invariants and moments of incomplete frequency distributions both normal and of Type III. In that investigation, which I hope shortly to publish, it is found that the assumption is valid for semi-invariants of ν_2 of order as high as four for skewness of the parent not exceeding unity. Also this other paper makes it possible to answer another question which arises in the present connection, namely, concerning the systematic error which is introduced when in the sampling formulae arrived at in this present investigation, values of semi-invariants and moments determined from a finite and necessarily incomplete sample distribution are substituted in place of those of the parent. It must be well known that in the case of the Type III parent, at least, the differences so caused are by no means negligible; it appears from my work that these discrepancies alone render practically valueless the inclusion of semi-invariants or moments of any high order in such formulae as I give below.

The calculation of the d's to order -3 in N and of the b's to order -2 in N is carried out without the use of semi-invariants of order higher than four for either ν_2 or ν_2 and ν_3 together and with the use of only the first eight semi-invariants of the parent distribution. Into the formulae (13) and (16) the values of the g's as found from my thesis and expressed in terms of α_3 by means of (2) were inserted and after reduction I found:

To order -3 in N,

$$\left.\begin{aligned}
d_1 &= 1 - \frac{1}{N}\left(\tfrac{3}{16}\alpha_3^2 + \tfrac{3}{4}\right) + \frac{1}{N^2}\left(\tfrac{105}{512}\alpha_3^4 + \tfrac{49}{64}\alpha_3^2 - \tfrac{7}{32}\right) \\
&\qquad - \frac{1}{N^3}\left(\tfrac{8505}{8192}\alpha_3^6 + \tfrac{9675}{2048}\alpha_3^4 + \tfrac{1179}{512}\alpha_3^2 + \tfrac{9}{128}\right) \\
d_2 &= \frac{1}{4N}\left(\tfrac{3}{2}\alpha_3^2 + 2\right) - \frac{1}{4N^2}\left(\tfrac{57}{32}\alpha_3^4 + \tfrac{49}{4}\alpha_3^2 + \tfrac{1}{2}\right) \\
&\qquad + \frac{1}{4N^3}\left(\tfrac{2205}{256}\alpha_3^6 + \tfrac{2571}{64}\alpha_3^4 + \tfrac{363}{16}\alpha_3^2 - \tfrac{3}{4}\right) \\
d_3 &= \frac{1}{8N^2}\left(\tfrac{33}{8}\alpha_3^4 + 13\alpha_3^2 + 2\right) - \frac{1}{8N^3}\left(\tfrac{2781}{128}\alpha_3^6 + \tfrac{3303}{32}\alpha_3^4 + \tfrac{519}{8}\alpha_3^2 - \tfrac{3}{2}\right) \\
d_4 &= \frac{3\alpha_3^2}{16N^3}\left(\tfrac{21}{2}\alpha_3^4 + 47\alpha_3^2 + 28\right)
\end{aligned}\right\}\quad \ldots(17).$$

To order -2 in N,

$$\left.\begin{aligned} b_1 &= \alpha_3\left[1-\frac{1}{N}\left(\tfrac{27}{16}\alpha_3^2+\tfrac{27}{4}\right)+\frac{1}{N^2}\left(\tfrac{6585}{512}\alpha_3^4+\tfrac{3697}{64}\alpha_3^2+\tfrac{809}{32}\right)\right] \\ b_2 &= \frac{1}{N}\left[\left(\tfrac{15}{8}\alpha_3^4+9\alpha_3^2+6\right)-\frac{1}{N}\left(\tfrac{8073}{128}\alpha_3^6+\tfrac{5409}{16}\alpha_3^4+\tfrac{2781}{8}\alpha_3^2+36\right)\right] \\ b_3 &= \frac{\alpha_3}{N^2}\left(\tfrac{4509}{64}\alpha_3^6+\tfrac{3587}{8}\alpha_3^4+\tfrac{2001}{4}\alpha_3^2+216\right) \end{aligned}\right\} \quad \ldots(18).$$

I was considerably perturbed over the apparent danger there was of b_2 becoming negative or quite small for small values of N. It is possible to calculate the b's to order -3 in N using the semi-invariants of ν_2 and ν_3 to not more than the fourth order and using only the first twelve semi-invariants of the parent distribution. I spent literally months performing the necessary preliminary computations and I did complete b_1 and b_2 to order -3 in N before deciding it after all to be labour in vain. In the first place the results involve α_3 to powers as high as the tenth and the standard error of α_3 is of the order of $\sqrt{6/N}$. For small values of N it is quite useless to include high powers of α_3 in the values of the b's. And for small values of N also the value of α_3 obtained from the sample is shown by my other investigation to be nearly valueless for use in values of b's which include high powers of α_3 as determined from the parent.

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NOTE ON DR CRAIG'S PAPER.

By EGON S. PEARSON, D.Sc.

THE formulae (17) and (18) of the preceding paper contain approximate expressions for the semi-invariants and moment coefficients of the standard deviation and of $\sqrt{\beta_1}$ in samples from a Pearson Type III curve. The results are of considerable interest, but as it is important from the practical point of view to appreciate how far the expressions are convergent and how far they are modified by changes in sample size and population form, it seemed desirable to examine them numerically. With this suggestion Dr Craig has readily concurred, but he is of course in no way responsible for the conclusions which I have drawn. I have taken sizes of sample with $N = 5, 10, 20, 50, 100, 250, 500$ and 1000 (the last two only in considering the semi-invariants of $\sqrt{\beta_1}$), and examined the position for samples from five populations with increasing skewness, namely $\beta_1 = 0$, $0{\cdot}2$, $0{\cdot}5$, $1{\cdot}0$ and $1{\cdot}5$ corresponding to $\alpha_3 = \sqrt{\beta_1} = 0$, $\cdot 4472$, $\cdot 7071$, $1{\cdot}0000$ and $1{\cdot}2247$ in Dr Craig's notation. Table I contains the numerical values of the successive terms of the expansion (17), and Table II of (18). It will be remembered that the population standard deviation is taken as unity.

To summarise and compare these results I have formed Tables III, IV, V and VI. It is not of course possible to fix any exact point at which the expansions (17) and (18) become inadequate; this will depend partly on the purpose to which the results are turned. But I have assigned a rough scale of (? ?) and (?) to borderline cases. Any reader who is not satisfied with this classification can form his own directly from the Tables I and II. The expressions for d_4 (4th semi-invariant of the standard deviation) and for b_3 (3rd semi-invariant of $\sqrt{\beta_1}$) contain only one term, and therefore the marks noted against the tabled values of the B_2 of s and the B_1 of $\sqrt{\beta_1}$ are very arbitrary. It will be clearest to discuss the latter tables in detail separately.

The Mean of the Standard Deviation ($d_1 = \text{Mean } s$). Table III.

Table I shows that the convergence is good except for very small samples from the skewer population. For the normal population the values obtained from (17) agree exactly to four decimal places with those obtained from "Student's" curve and given in *Biometrika*, Vol. x. p. 529. The results show how with increasing N, the Mean s converges somewhat more slowly on the population σ as the variation deviates from normality, but the difference is not great.

The Standard Error of s ($\sqrt{d_2} = \sigma_s$). Table IV.

I have compared Craig's value from (17) with two other approximations, namely

$$\text{Approximation A,}\quad \sigma_s = \frac{1}{2\sigma}\sigma_{s^2} = \frac{\sigma}{2}\frac{N-1}{N^{3/2}}\sqrt{\beta_2 - 3 + \frac{2N}{N-1}}$$

$$\text{Approximation B,}\quad \sigma_s = \frac{\sigma}{2}\sqrt{\frac{\beta_2 - 1}{N}}.$$

TABLE I.

Distribution Constants of the Standard Deviation, s, in Samples of N.

N	d_1 = Mean s	$d_2 = \sigma_s^2$	$d_3 = \mu_3(s)$
	(1) Normal Population with $\beta_1 = \alpha_3^2 = 0$		
5	1 − ·150 000 − ·008 750 − ·000 562	·100 000 − ·005 000 − ·001 500	·010 000 + ·001 500
10	1 − 75 000 − 2 187 − 70	50 000 − 1 250 − 187	2 500 + 187
20	1 − 37 500 − 547 − 9	25 000 − 312 − 23	625 + 23
50	1 − 15 000 − 87 − 1	10 000 − 50 − 1	100 + 1
100	1 − 7 500 − 22	5 000 − 12	25
250	1 − 3 000 − 3	2 000 − 2	4
	(2) Population with $\beta_1 = \alpha_3^2 = 0{\cdot}2$		
5	1 − ·157 500 − ·002 297 − ·005 825	·115 000 − ·020 212 + ·010 927	·023 825 − ·015 778
10	1 − 78 750 − 574 − 728	57 500 − 5 053 + 1 366	5 956 − 1 972
20	1 − 39 375 − 144 − 91	28 750 − 1 263 + 171	1 489 − 247
50	1 − 15 750 − 23 − 6	11 500 − 202 + 11	238 − 16
100	1 − 7 875 − 6 − 1	5 750 − 51 + 1	60 − 2
250	1 − 3 150 − 1	2 300 − 8	9 (5) − 0 (1)
	(3) Population with $\beta_1 = \alpha_3^2 = 0{\cdot}5$		
5	1 − ·168 750 + ·008 613 − ·020 260	·137 500 − ·045 703 + ·043 427	·047 656 − ·059 458
10	1 − 84 375 + 2 153 − 2 532	68 750 − 11 426 + 5 428	11 914 − 7 432
20	1 − 42 187 + 538 − 317	34 375 − 2 856 + 679	2 979 − 929
50	1 − 16 875 + 86 − 20	13 750 − 457 + 43	477 − 59
100	1 − 8 437 + 22 − 3	6 875 − 114 + 5	119 − 7
250	1 − 3 375 + 3	2 750 − 18	19 (1) − 0 (5)
	(4) Population with $\beta_1 = \alpha_3^2 = 1{\cdot}0$		
5	1 − ·187 500 + ·030 078 − ·065 083	·175 000 − ·095 312 + ·141 445	·095 625 − ·188 320
10	1 − 93 750 + 7 520 − 8 135	87 500 − 23 828 + 17 681	23 906 − 23 540
20	1 − 46 875 + 1 880 − 1 017	43 750 − 5 957 + 2 210	5 977 − 2 943
50	1 − 18 750 + 301 − 65	17 500 − 953 + 141	956 − 188
100	1 − 9 375 + 75 − 8	8 750 − 238 + 18	239 − 24
250	1 − 3 750 + 12 − 1	3 500 − 38 + 1	38 (2) − 1 (5)
	(5) Population with $\beta_1 = \alpha_3^2 = 1{\cdot}5$		
5	1 − ·206 250 + ·055 645 − ·141 261	·212 500 − ·153 828 + ·305 476	·153 906 − ·401 382
10	1 − ·103 125 + 13 911 − 17 658	·106 250 − 38 457 + 38 184	38 477 − 50 173
20	1 − 51 562 + 3 478 − 2 207	53 125 − 9 614 + 4 773	9 619 − 6 272
50	1 − 20 625 + 556 − 141	21 250 − 1 538 + 305	1 539 − 401
100	1 − 10 312 + 139 − 18	10 625 − 385 + 38	385 − 50
250	1 − 4 125 + 22 − 1	4 250 − 62 + 2	61 (6) − 3 (2)

TABLE II.

Distribution Constants of $a_3=\sqrt{\beta_1}$ in Samples of N.

Size of Sample	(1) Normal Population with $\beta_1=a_3^2=0$		(2) Population with $\beta_1=a_3^2=0\cdot2$		(3) Population with $\beta_1=a_3^2=0\cdot5$	
	$b_1=\text{Mean}\sqrt{\beta_1}$	$b_2=\sigma^2_{\sqrt{\beta_1}}$	$\dfrac{b_1}{a_3}=\dfrac{\text{Mean}\sqrt{\beta_1}}{\text{Population}\sqrt{\beta_1}}$	$b_2=\sigma^2_{\sqrt{\beta_1}}$	$\dfrac{b_1}{a_3}=\dfrac{\text{Mean}\sqrt{\beta_1}}{\text{Population}\sqrt{\beta_1}}$	$b_2=\sigma^2_{\sqrt{\beta_1}}$
5	0	1·200 000 − 1·440 000	1 − 1·417 500 + 1·493 953	1·575 000 − 4·782 082	1 − 1·518 750 + 2·295 176	2·193 750 − 12 088 477
10	0	·600 000 − 360 000	1 − ·708 750 + ·373 488	·787 500 − 1·195 521	1 − ·759 375 + ·573 794	1·096 875 − 3·022 119
20	0	·300 000 − 90 000	1 − ·354 375 + 93 372	·393 750 − ·298 880	1 − ·379 687 + ·143 448	·548 437 − ·755 530
50	0	·120 000 − 14 400	1 − ·141 750 + 14 940	·157 500 − 47 821	1 − ·151 875 + 22 952	·219 375 − ·120 885
100	0	60 000 − 3 600	1 − 70 875 + 3 735	78 750 − 11 955	1 − 75 937 + 5 738	·109 687 − 30 221
250	0	24 000 − 576	1 − 28 350 + 598	31 500 − 1 913	1 − 30 375 + 918	43 875 − 4 835
500	0	12 000 − 144	1 − 14 175 + 149	15 750 − 478	1 − 15 187 + 230	21 937 − 1 209
1000	0	6 000 − 36	1 − 7 078 + 37	7 875 − 120	1 − 7 594 + 57	10 969 − 302

Size of Sample	(4) Population with $\beta_1=a_3^2=1\cdot0$		(5) Population with $\beta_1=a_3^2=1\cdot5$	
	$\dfrac{b_1}{a_3}=\dfrac{\text{Mean}\sqrt{\beta_1}}{\text{Population}\sqrt{\beta_1}}$	$b_2=\sigma^2_{\sqrt{\beta_1}}$	$\dfrac{b_1}{a_3}=\dfrac{\text{Mean}\sqrt{\beta_1}}{\text{Population}\sqrt{\beta_1}}$	$b_2=\sigma^2_{\sqrt{\beta_1}}$
5	1 − 1·687 500 + 3·836 328	—	—	—
10	1 − ·843 750 + ·959 082	—	1 − ·928 125 + 1·408 677	—
20	1 − ·421 875 + ·239 771	·843 750 − 1·961 895	1 − ·464 062 + ·352 169	—
50	1 − ·168 750 + 38 363	·337 500 − ·313 903	1 − ·185 625 + 56 347	·474 375 − ·612 376
100	1 − 84 375 + 9 591	·168 750 − 78 476	1 − 92 812 + 14 087	·237 187 − ·153 094
250	1 − 33 750 + 1 535	67 500 − 12 556	1 − 37 125 + ·2 254	94 875 − 24 495
500	1 − 16 875 + 384	33 750 − 3 139	1 − 18 562 + 563	47 437 − 6 124
1000	1 − 8 437 + 96	16 875 − 785	1 − 9 281 + 141	23 719 − 1 531

TABLE III.

Mean of Standard Deviation (d_1 = Mean s).

Population

Size of Sample	$\beta_1=0$		$\beta_1=0{\cdot}2$	$\beta_1=0{\cdot}5$	$\beta_1=1{\cdot}0$	$\beta_1=1{\cdot}5$
	Craig (17)	*Biometrika* Vol. x. p. 529	Craig	Craig	Craig	Craig
5	·840 688	·8407	·8344	·8196 (?)	·7775 (??)	—
10	·922 743	·9227	·9199	·9152 (?)	·9056 (?)	·8931 (??)
20	·961 944	·9619	·9604	·9580	·9540	·9497 (?)
50	·984 912	·9849	·9842	·9832	·9815	·9798
100	·992 478	·9925	·9921	·9916	·9907	·9898
250	·996 997	—	·9968	·9966	·9963	·9959

It will be seen that for the normal population, the values from (17) almost agree throughout to 4 decimal places with those given in *Biometrika*, Vol. x. For very small samples the expansion to 3 terms in (17) becomes inadequate as the population becomes skew, but there is no reason to suppose that the Approximation A is any more satisfactory in these cases. It will be noted however that as soon as the run of the terms in Table I suggests that the values from (17) are satisfactory, these values agree closely enough for most practical purposes with those of Approximation A.

This is a result of some interest and shows the value of the latter expression as an approximation to the standard error of the standard deviation in non-normal material. For samples of more than 50 the very simple expression, Approximation B, provides good values.

In *Biometrika*, Vol. XII. pp. 276—277, K. Pearson has given general expressions for both Mean s and σ_s for samples from any population in terms of the first six moment coefficients of that population. On making use of the appropriate relations between the moment coefficients of Type III curves, it will be found that his equations (R) and (W) correspond exactly with Craig's expressions for d_1 and d_2 in (17) as far as the terms in $1/N^2$. That is to say Craig has provided an additional term in these two expressions; the point at which this term can be neglected may be seen by examining Table I.

TABLE IV.

Standard Error of Standard Deviation. ($\sqrt{d_2} = \sigma_s$.)

Population

Size of Sample	$\beta_1 = 0$				$\beta_1 = 0 \cdot 2$			$\beta_1 = 0 \cdot 5$		
	Craig (17)	*Biomet.* Vol. x.	Approx. A	Approx. B	Craig	Approx. A	Approx. B	Craig	Approx A	Approx. B
5	·3058 (?)	·3052	·2828	·3162	·3251 (?)	·2993	·3391	—	·3225	·3708
10	·2204	·2203	·2121	·2236	·2320 (?)	2260	·2398	·2505 (??)	·2453	·2622
20	·1571	·1570	·1541	·1581	·1663	·1647	·1696	·1794	·1795	·1854
50	·0997	·0997	·0990	·1000	·1063	·1060	·1072	·1155	·1158	·1173
100	·0706	·0706	·0704	·0707	·0755	·0754	·0758	·0823	·0824	·0829
250	·0447	—	·0446	·0447	·0478	·0478	·0480	·0523	·0523	·0524

Size of Sample	$\beta_1 = 1 \cdot 0$			$\beta_1 = 1 \cdot 5$		
	Craig	Approx. A	Approx. B	Craig	Approx. A	Approx. B
5	—	·3578	·4183	—	·3899	·4610
10	—	·2745	·2958	—	·3009	·3260
20	·2000 (??)	·2017	·2092	·2197 (?)	·2217	·2305
50	·1292	·1304	·1323	·1415	·1435	·1458
100	·0924	·0929	·0935	·1014	·1023	·1031
250	·0588	·0590	·0592	·0647	·0650	·0652

TABLE V.

The Coefficients B_1 and B_2 of the Sampling Distribution of the Standard Deviation.

$B_1 = d_3^2/d_2^3$

Size of Sample	Population $\beta_1 = 0$ Craig (17)	Population $\beta_1 = 0$ *Biomet.* Vol. x.	$\beta_1 = 0{\cdot}2$ Craig	$\beta_1 = 0{\cdot}5$ Craig	$\beta_1 = 1{\cdot}0$ Craig	$\beta_1 = 1{\cdot}5$ Craig
5	·1618	·1646	—	—	—	—
10	·0631	·0634	·102 (??)	—	—	—
20	·0280	·0281	·073 (?)	·126 (??)	—	—
50	·0104	·0105	·034	·074 (?)	·127 (??)	·161 (??)
100	·0050	·0051	·017	·040	·075 (?)	·103 (?)
250	·0020	—	·007	·017	·033	·046

$B_2 = 3 + d_4/d_2^2$

Size of Sample	Population $\beta_1 = 0$ Craig (17)	Population $\beta_1 = 0$ *Biomet.* Vol. x	$\beta_1 = 0{\cdot}2$ Craig	$\beta_1 = 0{\cdot}5$ Craig	$\beta_1 = 1{\cdot}0$ Craig	$\beta_1 = 1{\cdot}5$ Craig
5	3	3·0593	—	—	—	—
10	3	3·0106	3·49 (??)	—	—	—
20	3	3·0022	3·23 (?)	3·61 (??)	—	—
50	3	3·0003	3·089	3·23 (?)	3·46 (??)	3·69 (??)
100	3	3·0000	3·044	3·111	3·22 (?)	3·33 (?)
250	3	—	3·017	3·044	3·086	3·125

The Coefficients B_1 and B_2 for the Standard Deviation. Table V.

$$(B_1 = d_3^2/d_2^3,\ B_2 = 3 + d_4/d_2^2).$$

For the normal population the values of B_1 agree closely with those given in *Biometrika*, Vol. x., but as the single term in d_4 contains the factor α_3 which is zero in this case, the values for B_2 obtained from (17) are all equal to 3. The convergence of the expression for d_3 is not satisfactory except for large samples, and therefore the values of B_1 can hardly be treated as accurate until we have reached a stage where for most purposes they could be taken as zero. There is no check on the degree of convergence of d_4, so that no great reliance must be placed on the differences between B_2 and 3. The value of the formulae seems therefore to lie in giving a rough appreciation of how soon the distribution of standard deviations may be taken as normal.

The Mean $\sqrt{\beta_1}$ (b_1). Table VI.

For samples from a normal or any symmetrical population this is zero. The quantity given in the first section of Table VI is the ratio of the Mean Sample $\sqrt{\beta_1}$ to the Population $\sqrt{\beta_1}$ or b_1/α_3 in Craig's notation; it tends to unity as N increases but not as quickly as the corresponding ratio for Mean s shown in Table III. For a given N the ratio changes only very slowly with increasing skewness.

The Standard Error of $\sqrt{\beta_1}$ ($\sigma_{\sqrt{\beta_1}} = \sqrt{b_2}$). Table VI.

An expression for the standard error of β_1 has long been used*; it is however only the first order term in an expansion and vanishes if the population be symmetrical. For this reason, when dealing with normal populations, the standard error of $\sqrt{\beta_1}$ has been employed; this to the first order is $\sqrt{6/N}$. Formula (18) provides in addition a second order term, namely for a normal population it gives

$$\sigma_{\sqrt{\beta_1}} = \sqrt{\frac{6}{N}\left(1 - \frac{6}{N}\right)}.$$

The relative magnitude of these two terms for different values of N will be seen in Table II. The formula is also valuable in giving the standard error of $\sqrt{\beta_1}$ for skew Type III populations, provided the population be large enough.

The Coefficient B_1 for the distribution of $\sqrt{\beta_1}$ ($B_1 = b_3^2/b_2^3$). Table VI.

As the expression for b_3 in (18) contains only one term, there is no means of judging at what point it becomes adequate, but clearly the convergence will be less satisfactory than for b_2. The question-marks added to the figures in the third section of Table VI have therefore been somewhat arbitrarily assigned. It seems probable however that there is considerably greater skewness in the distribution of $\sqrt{\beta_1}$ than in that of s.

Dr Craig has referred at the end of his paper to the inaccuracy that would be involved by inserting into the formulae for the semi-invariants high powers of α_3

* *Phil. Trans.* A, Vol. 198, 1902, p. 278. Numerical values are given in *Tables for Statisticians and Biometricians.*

TABLE VI.

Frequency Constants for Sampling Distribution of $\sqrt{\beta_1}$.

	(Mean sample $\sqrt{\beta_1}$)/(Population $\sqrt{\beta_1}$) $= b_1/a_2$				Standard Error of $\sqrt{\beta_1} = \sigma_{\sqrt{\beta_1}} = \sqrt{b_2}$				
Size of Sample	$\beta_1=0{\cdot}2$	$\beta_1=0{\cdot}5$	$\beta_1=1{\cdot}0$	$\beta_1=1{\cdot}5$	Normal Population	$\beta_1=0{\cdot}2$	$\beta_1=0{\cdot}5$	$\beta_1=1{\cdot}0$	$\beta_1=1{\cdot}5$
20	·7390 (??)	—	—	—	·46 (??)	—	—	—	—
50	·8732 (?)	·8711 (?)	·8696 (??)	·8707 (??)	·32 (?)	·33 (??)	—	—	—
100	·9329	·9298	·9252 (?)	·9213 (?)	·237	·26 (?)	·28 (??)	—	—
250	·9722	·9705	·9678	·9651	·153	·172	·20 (?)	·23 (??)	·27 (??)
500	·9860	·9850	·9835	·9820	·109	·124	·144	·175	·20 (?)
1000	·9929	·9925	·9917	·9909	·077	·088	·103	·127	·149

	$B_1 = b_3^2/b_2^3$				
Size of Sample	Normal Population	$\beta_1=0{\cdot}2$	$\beta_1=0{\cdot}5$	$\beta_1=1{\cdot}0$	$\beta_1=1{\cdot}5$
20	0	—	—	—	—
50	0	—	—	—	—
100	0	·89 (??)	—	—	—
250	0	·26 (?)	·94 (??)	—	—
500	0	·119	·39 (?)	1·06 (?)	2·00 (?)
1000	0	·057	·180	·456	·81 (?)

determined from the *sample*. In practice however the difficulty is perhaps not so serious as he believes, for an exact appreciation of the higher moments of the population is not really necessary. The value of this work of his and of similar research lies mainly in the light thrown on the manner in which the distribution of frequency constants in samples varies with changes in population form. Standard errors are associated with a population and not a sample, and it will be found that the answer to many statistical problems must be obtained not by assigning a standard error to the sample constants, but by considering the following question. Is it or is it not likely that the observed sample could have been drawn from a population of a certain specified form? If, for example, we believe that a sample with a given $\sqrt{\beta_1}$ comes from *some* Type III population, we do not need to assign a standard error to this $\sqrt{\beta_1}$, but rather to find out two limiting population parameters α_3 and α_3', the one above and the other below $\sqrt{\beta_1}$, such that

$$\alpha_3 - \sqrt{\beta_1} = k\sigma_{\sqrt{\beta_1}} \text{ and } \sqrt{\beta_1} - \alpha_3' = k'\sigma'_{\sqrt{\beta_1}},$$

the two standard errors being associated with α_3 and α_3' respectively*. That is to say we insert into the formula for b_2 of (18) not a sample value of $\alpha_3 = \sqrt{\beta_1}$, but certain hypothetical population values.

* The values given to k and k' will depend on the probability limit which is chosen, and on the degree of skewness in the sampling distribution of $\sqrt{\beta_1}$. Certainly this latter is difficult to ascertain, and we may often have to be content with putting $k = k' = 3$, say.

ON RACIAL DIFFERENCES IN STATURE LONG BONE REGRESSION FORMULAE, WITH SPECIAL REFERENCE TO STATURE RECONSTRUCTION FORMULAE FOR THE CHINESE.

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CONTENTS.

1. *Introduction.*

The first attempt to obtain a prediction of stature from measurements of the long bones on the basis of the correlational calculus was made by Pearson in 1898†. The particular problem that Pearson set himself to solve at that time was that of reconstructing the stature of prehistoric races. The only data then available for calculating the necessary coefficients of correlation between human stature and long bones, and between the various long bones themselves, were those provided by Rollet in his work *De la Mensuration des Os longs des Membres*, Lyons, 1889, and it was from the means, standard deviations, and correlation coefficients of this French material that Pearson's original stature regression formulae were derived. The question of applying formulae of one local race to another was carefully considered. Pearson in the course of his discussion of this question in the original paper states that "the extension of the stature regression formulae from one local race—say, modern French—to other races—say, palaeolithic man—must be made with very great caution," nevertheless such extension was deemed theoretically permissible on the assumptions (1) that stature represents an indirectly selected racial character, and (2) that whereas regression formulae in general might be expected to change from one race to another yet certain of these, viz. regression formulae of indirectly on directly selected characters, should not change; while in the case of others, where mere size is the chief factor involved, the differences encountered should be only of the second or third order of small quantities‡. These conclusions seemed to find practical justification in the trial application of

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† "On the Reconstruction of the Stature of Prehistoric Races," *Phil. Trans.* A, Vol. 192, pp. 169—244.

‡ *Loc. cit.* p. 177.

the regression formulae based on the French data to the reconstruction of stature from the long bones in the case of such a widely separated racial group as the Aino.

The accumulation of comparable osteometric data in the case of a second and distinct contemporaneous racial group, the Chinese, affords an opportunity of testing directly the validity of applying regression formulae derived from one branch of the human race to another. Without anticipating in too great a degree the conclusions arrived at below, it may be stated that the results of such a test do not sustain the early confidence in the general applicability of the particular regression formulae in question to all racial groups. While such a conclusion may call for a general reconsideration of the premises upon which the original assumptions were made, a more immediate implication lies in the desirability of working out specific regression formulae for different racial groups as soon as the necessary data become available*. Such racially specific formulae should not only better serve the practical end of more reliable predictions in the case of the groups for which they have been derived but should also, in their differences, provide a means of studying the direction and degree of organic differentiation among the racial groups thus studied.

The present paper presents stature regression formulae specifically derived from Chinese data and compares these with Pearson's original stature regression formulae based on Rollet's French data. Use is made also of certain available constants for Aino and Naqada skeletons for a slightly wider comparison of racial variations in general. The writer gratefully acknowledges his personal indebtedness to Professor Pearson for his stimulating interest in the problem under consideration, and for his generous permission to make use of certain preliminary notes made by him on the same subject.

2. *Data and Treatment.*

The data necessary for the calculation of stature regression formulae in the case of the Chinese are to be found in the osteometric records based on the collection of Chinese osteological material in the Department of Anatomy of the Peiping Union Medical College of Peiping. The actual measurements were made by Drs Gerhard von Bonin and M. T. P'an. The writer assumes full responsibility for the calculation of the various constants, correlation coefficients and regression formulae based upon these measurements. Lack of adequate female skeletal material confines the discussion to males alone, and the material is further restricted to representatives of the North China population.

* In this connection Professor Pearson states that he prepared many years ago a schedule for taking cadaver and long bone lengths in the post-mortem room, and sent it to a number of anatomists. It produced nothing at all in England. In Strasburg Gustav Schwalbe promised aid, and shortly before the Great War measurements had been made on 80 male and some 40 female subjects. Schwalbe died during the war, and Professor Pearson has been unable to find anyone who knows what has become of the material. The data which were in England in 1913 or 1914 were returned to Schwalbe as he thought he would be able ultimately to complete 100 cadavers of each sex. The rediscovery and reduction of these valuable data would be of great importance.

All measurements on the Chinese material were taken on well-macerated bones, with cartilage removed and after the lapse of ample time to ensure thorough drying. Maximum length measurements only are used, those of the tibia were made without the spine but including the malleolus. To render the Chinese formulae comparable in all respects to Pearson's formulae for the French the measurements of the right bones only are used. Rigid rejection and selection in the interests of assured normality still further restricted the Chinese material finally used to forty-eight skeletons. The constants for the French, Aino and Naqada tabled together with those for the Chinese are taken in the first two instances from the original paper by Pearson (*loc. cit.*) and in the last from the report of Warren on the Naqada race*.

The theory of regression undoubtedly affords the most reliable method of determining the best prediction of stature from given measurement of the long bones. Thus if $\bar{X}_0$, $\bar{x}_n$ be the mean values of the stature and the length of one of the long bones respectively as calculated from a sample of the population in question, likewise σ_0, σ_n their standard deviations and r_{0n} the coefficient of correlation between them, then the most probable stature X_0 to be predicted from a particular bone length x_n is given by the simple regression formula

$$X_0 - \bar{X}_0 = r_{0n}\frac{\sigma_0}{\sigma_n}(x_n - \bar{x}_n) \quad \text{(i)},$$

which may be transformed into

$$X_0 = \left(\bar{X}_0 - r_{0n}\frac{\sigma_0}{\sigma_n}\bar{x}_n\right) + r_{0n}\frac{\sigma_0}{\sigma_n}x_n$$
$$= c_1 + c_2 x_n,$$

where c_1 and c_2 are constants specifically derived from the sampled values of the two variates under consideration. The probable error of this determination is $\cdot 67449\sigma_0\sqrt{1 - r_{0n}^2}$ if the stature prediction is made for a single individual, or this amount divided by $\sqrt{n}$ if the x_n used in the determination represents the mean of measurements on n individuals, as would be the case if the prediction of the most probable stature of a group or race is desired and the mean value of measurements on n individuals is available for the prediction.

For the prediction of stature on the basis of given measurements of more than one bone, recourse must be had to the multiple regression formula. The most convenient working form of this formula is that in which the multiple regression coefficients are expressed in the form of determinants, viz.

$$X_0 - \bar{X}_0 = -\frac{\sigma_0}{\Delta_{00}}\left\{\frac{\Delta_{01}}{\sigma_1}(x_1 - \bar{x}_1) + \frac{\Delta_{02}}{\sigma_2}(x_2 - \bar{x}_2) + \ldots + \frac{\Delta_{0n}}{\sigma_n}(x_n - \bar{x}_n)\right\} \quad \text{(ii)},$$

where $x_1 \ldots x_n$ represent the respective long bones, and $\Delta_{00} \ldots \Delta_{0n}$ the minors of the determinant

$$\Delta = \begin{vmatrix} 1 & r_{01} & r_{02} & \ldots & r_{0n} \\ r_{10} & 1 & r_{12} & \ldots & r_{1n} \\ r_{20} & r_{21} & 1 & \ldots & r_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ r_{n0} & r_{n1} & r_{n2} & \ldots & 1 \end{vmatrix}$$

* *Phil. Trans.* B, Vol. 89, pp. 135—228.

in which $r_{01} \ldots r_{0n}$ represent the various correlations between stature and the different long bones, and $r_{12} \ldots r_{n-1n}$ the correlations between the bones themselves.

Substitution of the values of the variable factors concerned in formula (ii) transforms the equation into the form

$$X_0 = c_1 + c_2 x_1 + c_3 x_2 \ldots c_{n+1} x_n,$$

where the stature is again expressed in terms of a linear function of the various long bone measurements. Formula (ii) has a probable error of $\cdot 67449 \sigma_0 \sqrt{\frac{\Delta}{\Delta_{00}}}$, with a similar provision for reduction by $\sqrt{n}$ in the case of the determination being based upon measurements of more than one individual.

3. *Comparisons of Data involved in Stature Regression Formulae.*

It is quite obvious from a consideration of their derivation as shown above, that the constants c_1 and c_2 in the final regression formulae are functions of the respective racial means, standard deviations and coefficients of correlation. We will commence therefore with a comparison of these variates in the case of the French and Chinese (Table I). Flanking these values for the French and Chinese are those also of the Naqada and Aino, providing a wider basis of comparison of the general interracial differences in the characters concerned.

The first thing to attract attention in the above table is the significant difference in cadaver length in the case of the French and Chinese. Lest the figure for the latter be suspected in error it may be mentioned that all the cadavers in the series in question are of Northern Chinese, whose mean living stature has been adequately determined as 168·830 cms.* In regard to the bone lengths of the French and Chinese only the humerus shows a difference that proves statistically significant, yet the consistently smaller value for the Chinese limb bones is particularly worthy of attention in the light of the reverse total body length difference noted above. The Naqada and Aino on different ends of the size scale show very marked differences in limb bone lengths. When the four races are grouped together as representing Mediterranean and Oriental racial stocks respectively a racial differentiation on the basis of shorter limb segment lengths in the case of the latter seems to be clearly indicated.

As regards variability the table shows that the Chinese are both absolutely (cf. standard deviations) and relatively (cf. coefficients of variation) significantly less variable than the French. While the French and Naqada variabilities are not individually significantly different, the ancient Naqada considered as a whole show

* There are well-demonstrated total size differences between the Chinese of different regions of China—those of Central and South China having mean statures of 165·1 and 163·0 cms. respectively as compared with the considerably higher stature of 168·8 cms. of the North. (This latter figure is based on unpublished data on eleven hundred individuals.) Cf. P. H. Stevenson, "Collected Anthropometric data on the Chinese," *China Medical Journal*, 1925, Vol. xxxix. pp. 855—898.

greater variability than the French. The Aino tends to agree in the main with the French and Naqada in being somewhat more variable than the Chinese. We may summarise the figures shown in this table by saying that the limb bones of the Naqada and French races are longer than those of the Chinese and Aino, and the Aino agrees with the two former races in exhibiting a greater variability than the Chinese. The possible influence of these differences on the regression formulae will be discussed later.

TABLE I.

Stature (Cadaver) and Dry Long Bone Measurements.

(Absolute measurements in cms.)

	Naqada[(a)]	French[(b)]	Chinese	Aino[(b)]
Means:				
Stature	—	166·260 ± 0·525	168·923 ± 0·528	157·900[(c)]
Femur	45·930 ± 0·170	[(d)] 44·578 ± 0·226	43·975 ± 0·174	40·770 ± 0·193
Tibia	37·970 ± 0·137	36·336 ± 0·172	36·248 ± 0·149	33·895 ± 0·183
Humerus	32·618 ± 0·146	32·600 ± 0·147	31·073 ± 0·115	29·502 ± 0·135
Radius	25·697 ± 0·127	24·174 ± 0·112	23·779 ± 0·096	22·913 ± 0·121
Standard Deviations:				
Stature	—	5·502 ± 0·371	5·424 ± 0·373	—
Femur	2·519 ± 0·134	2·372 ± 0·160	1·788 ± 0·123	1·898 ± 0·136
Tibia	1·877 ± 0·097	1·799 ± 0·121	1·535 ± 0·106	1·668 ± 0·129
Humerus	1·701 ± 0·103	1·538 ± 0·104	1·182 ± 0·081	1·343 ± 0·095
Radius	1·290 ± 0·090	1·170 ± 0·079	0·987 ± 0·068	1·117 ± 0·086
Coefficients of Variation:				
Stature	—	3·309 ± 0·223	3·211 ± 0·221	—
Femur	5·484 ± 0·293	5·425 ± 0·354	4·066 ± 0·280	4·655 ± 0·336
Tibia	4·943 ± 0·256	4·888 ± 0·330	4·234 ± 0·291	4·921 ± 0·381
Humerus	5·216 ± 0·317	4·659 ± 0·314	3·802 ± 0·262	4·552 ± 0·322
Radius	5·021 ± 0·350	4·796 ± 0·323	4·150 ± 0·286	4·875 ± 0·375

(*a*) Taken from Warren, *Phil. Trans.* B, Vol. 189, 1897, pp. 135—191.

(*b*) Taken from Pearson and Lee, *R. S. Proc.* Vol. 61, 1897, p. 347 and *Phil. Trans.* A, Vol. 192, pp. 169 *et seq.*

(*c*) This measurement of the Aino represents the cadaver length as estimated on the basis of a living stature of 156·7 cms. plus 1·2 cms., which is a proportional estimate of the amount figured by Pearson (*loc. cit.* p. 191) as the difference between living stature and corpse length. [The living stature given here is for the Yezo and Sachalin Aino, *not* for the Shikotan Aino.—Ed.]

(*d*) The mean lengths given here for the French long bones are not based on the data given originally by Rollet but represent the mean lengths found by Pearson after the subtractions calculated by him (*loc. cit.* p. 195) as necessary to reduce the lengths of freshly autopsied bones with cartilages to the corresponding lengths of dry bones without cartilage and free of animal matter. All the measurements of long bones in this table are therefore strictly comparable.

TABLE II.

Stature (Cadaver) and Long Bone Correlations (Males)[a].

Race		Stature	Femur	Tibia	Humerus	Radius
Stature	Naqada	—	—	—	—	—
	French	1	·8105 ± ·0327	·7769 ± ·0378	·8091 ± ·0329	·6956 ± ·0492
	Chinese	1	·8036 ± ·0344	·8563 ± ·0260	·6128 ± ·0608	·6801 ± ·0523
	Aino	—	—	—	—	—
Femur	Naqada	—	1	·9164 ± ·0115	·8416 ± ·0248	·8465 ± ·0295
	French	·8105 ± ·0327	1	·8058 ± ·0335	·8421 ± ·0277	·7439 ± ·0426
	Chinese	·8036 ± ·0344	1	·8904 ± ·0202	·7377 ± ·0443	·6696 ± ·0536
	Aino	—	1	·8266 ± ·0338	·8584 ± ·0274	·7891 ± ·0418
Tibia	Naqada	—	·9164 ± ·0115	1	·8497 ± ·0218	·8505 ± ·0247
	French	·7769 ± ·0378	·8058 ± ·0335	1	·8601 ± ·0248	·7804 ± ·0373
	Chinese	·8563 ± ·0260	·8904 ± ·0202	1	·6866 ± ·0514	·7642 ± ·0405
	Aino	—	·8266 ± ·0338	1	·7447 ± ·0481	·8655 ± ·0286
Humerus	Naqada	—	·8416 ± ·0248	·8497 ± ·0218	1	·8232 ± ·0308
	French	·8091 ± ·0329	·8421 ± ·0277	·8601 ± ·0248	1	·8451 ± ·0273
	Chinese	·6128 ± ·0608	·7377 ± ·0443	·6866 ± ·0514	1	·6715 ± ·0534
	Aino	—	·8584 ± ·0274	·7447 ± ·0481	1	·7763 ± ·0429
Radius	Naqada	—	·8465 ± ·0295	·8505 ± ·0247	·8232 ± ·0308	1
	French	·6956 ± ·0492	·7439 ± ·0426	·7804 ± ·0373	·8451 ± ·0273	1
	Chinese	·6801 ± ·0523	·6696 ± ·0536	·7642 ± ·0405	·6715 ± ·0534	1
	Aino	—	·7891 ± ·0418	·8655 ± ·0286	·7763 ± ·0429	1

(*a*) French, Aino and Naqada values taken from Pearson, Lee and Warren as cited under Table I.

We turn next to the correlations; these are given in Table II. It is to be noted at the outset that owing to the smallness of the members in the various series* the probable errors of the coefficients run high notwithstanding the high values of the correlations. Care must be exercised therefore in attributing significance to the differences observed. Of the correlations between stature and various long bones in the French and Chinese, only in the case of that between the stature and humerus do we find a difference approaching statistical significance, the difference here being 2·9 times its probable error. A peculiar fact to be noted in connection with the stature long bone correlations in these two races is that whereas in the Chinese it is the distal limb segments that show the highest correlation with stature, the reverse is true in the case of the French.

* The number of individuals figuring in the above correlations are Chinese 48, French 50, Aino varying from 32 to 39, Naqada varying from 24 to 38.

Turning to the correlations between the various long bones themselves we note that with the exception of that between femur and tibia all the Chinese correlations are smaller than the French. In addition to its low correlation with stature noted above the Chinese humerus seems to be significantly low in its correlations with other long bones also. Glancing at the correlations on the part of the Naqada and Aino groups we note that the Aino, except in the single case of the correlation between the humerus and tibia, is in closer agreement with the French and Naqada than with the Chinese. These correlation differences are not great however, and until confirmed (or otherwise) by data on other allied races should not have too great importance attached to them.

The remaining constants necessary to complete the formulae are the combined lengths of femur plus tibia and humerus plus radius. These are:

Femur plus Tibia	Mean	S. D.	Correlation
Chinese	80·223	3·231	·8515
French	80·848	3·979	·8384
Humerus plus Radius			
Chinese	54·852	1·984	·7032
French	56·738	2·536	·7973

Having noted the racial differences in size, variability and correlations in the Chinese and French statures and long bones we are now ready to study the resultant differences in the respective regression formulae derived from them. Substituting in the generalised regression formulae (i), (ii) the appropriate racial means, standard deviations and correlation coefficients, we obtain the two series of regression formulae tabled together in Table III. The first constant in the French formulae has been recalculated to provide in each case the cadaver length (rather than the living stature) prediction*. The two sets of formulae are therefore strictly comparable in all respects.

There is no need to pause over the individual differences in the corresponding constants in these regression formulae, such differences being expected in the nature of the case. Glancing at the probable errors it is to be noted that while in

* The French formulae as given here differ slightly therefore from either of the two series derived by Pearson. His first series (*loc. cit.* Table VII) were "Formulae for the Reconstruction of the Stature as Corpse, the Maximum Lengths of the F., H., R. and T. without Spine being measured with the Cartilage on and in a Humid State"; his second (Table XIV), "Living Stature from Dead Long Bones." The French formulae as recalculated for this paper are for Cadaver Length from Dead (Dry) Long Bones, the same as in the case of the Chinese formulae. [The "recalculation" should consist in adding 1·26 cms. to Pearson's formulae for stature prediction (*loc. cit.* pp. 187 and 191). I do not understand how Dr Stevenson reaches a fourth decimal place in the constant terms, Pearson having only three, or why his values of the constant terms are far from agreeing with Pearson's constants plus 1·26 cms. ED.]

the case of the French the best single bone prediction is to be had from the femur (*a*), in the case of the Chinese the best single bone prediction is to be had from the tibia (*c*). Further, whereas in the French the combination of the tibia with the femur, either by adding the two lengths together (*e*) or by using a bivariate formula (*f*), materially improves the prediction, in the Chinese no appreciable gain in accuracy is to be obtained through combining any other bone with the tibia. In the French the excellent prediction on the basis of the bivariate formula (*i*) is hardly improved upon by using all the bones (*k*). In the Chinese the best prediction of all is based on all the bones (*k*).

TABLE III.

Prediction Equations for Cadaver Lengths from Dry Long Bones.

	From Chinese Data			From French Data	
(*a*)	$C = 61{\cdot}7207 + 2{\cdot}4378\,F$	$\pm 2{\cdot}1756$	(*a*)	$C = 82{\cdot}5661 + 1{\cdot}880\,F$	$\pm 2{\cdot}174$
(*b*)	$= 81{\cdot}5115 + 2{\cdot}8131\,H$	$\pm 2{\cdot}8903$	(*b*)	$= 71{\cdot}9156 + 2{\cdot}894\,H$	$\pm 2{\cdot}181$
(*c*)	$= 59{\cdot}2256 + 3{\cdot}0263\,T$	$\pm 1{\cdot}8916$	(*c*)	$= 79{\cdot}9257 + 2{\cdot}376\,T$	$\pm 2{\cdot}337$
(*d*)	$= 80{\cdot}0276 + 3{\cdot}7384\,R$	$\pm 2{\cdot}6791$	(*d*)	$= 87{\cdot}1868 + 3{\cdot}271\,R$	$\pm 2{\cdot}666$
(*e*)	$= 54{\cdot}2522 + 1{\cdot}4294\,(F+T)$	$\pm 1{\cdot}9214$	(*e*)	$= 72{\cdot}5572 + 1{\cdot}159\,(F+T)$	$\pm 2{\cdot}023$
(*f*)	$= 55{\cdot}3865 + 0{\cdot}6024\,F + 2{\cdot}4014\,T$	$\pm 1{\cdot}8605$	(*f*)	$= 72{\cdot}7052 + 1{\cdot}220\,F + 1{\cdot}080\,T$	$\pm 2{\cdot}030$
(*g*)	$= 63{\cdot}4865 + 1{\cdot}9222\,(H+R)$	$\pm 2{\cdot}6529$	(*g*)	$= 68{\cdot}1033 + 1{\cdot}730\,(H+R)$	$\pm 2{\cdot}240$
(*h*)	$= 64{\cdot}4272 + 1{\cdot}3052\,H + 2{\cdot}6889\,R$	$\pm 2{\cdot}5691$	(*h*)	$= 71{\cdot}2767 + 2{\cdot}769\,H + 0{\cdot}195\,R$	$\pm 2{\cdot}179$
(*i*)	$= 59{\cdot}7828 + 2{\cdot}3397\,F + 0{\cdot}2012\,H$	$\pm 2{\cdot}1747$	(*i*)	$= 69{\cdot}6483 + 1{\cdot}030\,F + 1{\cdot}557\,H$	$\pm 1{\cdot}962$
(*j*)	$= 57{\cdot}1954 + 2{\cdot}8594\,T + 0{\cdot}3398\,R$	$\pm 1{\cdot}8838$	(*j*)	* $= 73{\cdot}7659 + 1{\cdot}831\,T + 1{\cdot}074\,R$	$\pm 2{\cdot}276$
(*k*)	$= 52{\cdot}2596 + 0{\cdot}6640\,F + 2{\cdot}2065\,T - 0{\cdot}1008\,H + 0{\cdot}4464\,R$	$\pm 1{\cdot}8201$	(*k*)	$= 68{\cdot}3990 + 0{\cdot}913\,F + 0{\cdot}600\,T + 1{\cdot}225\,H - 0{\cdot}187\,R$	$\pm 1{\cdot}961$

4. *Application of Formulae of one Race to another.*

We pass now to the question of applying formulae specifically derived from data of one race to prediction in the case of a second. The first test of such application is found in Table IV. The probable errors of the predictions herein tabulated are for 48 and 50 individuals respectively. It is to be seen at a glance that the average Chinese stature prediction from French formulae is over 4 cms. too small, and that of French stature from Chinese formulae about the same amount too large. These differences are of an order of more than seven times their own probable errors, and are quite obviously too great to justify the application of French formulae to Chinese or vice versa. In fact such a difference indicates a statistical improbability of the order of several millions to one that the formulae of one of these two races will provide a satisfactory prediction of the stature of an individual belonging to the other.

It will be of interest to review briefly at this point the various racial factors entering into the derivation of these formulae, with the purpose in mind of determining which are responsible for the failure of the formulae of one of the two races in question to provide suitable predictions in the case of the other.

[* This formula seems to me erroneous; it has been worked out for cadaver length and humid bones with cartilages attached. I think the constant term should be 74·864. ED.]

In this connection we must take note of differences in absolute size, absolute and relative variabilities, correlations between the factors involved, and the racial differences in stature-limb proportions. We will briefly discuss these points, as far as our data permits, in the reverse order of that just named.

TABLE IV.

Trial Reconstruction of Stature (Cadaver Length).

Formula	Chinese from French	French from Chinese
True Value	168·923 ± ·528	166·260 ± ·525
(*a*)	165·239 ± ·314	170·247 ± ·307
(*b*)	161·841 ± ·318	173·218 ± ·408
(*c*)	166·051 ± ·307	169·190 ± ·267
(*d*)	164·968 ± ·384	170·400 ± ·378
(*e*)	165·536 ± ·291	169·825 ± ·271
(*f*)	165·502 ± ·293	169·461 ± ·263
(*g*)	162·997 ± ·323	172·617 ± ·375
(*h*)	161·955 ± ·314	171·978 ± ·363
(*i*)	163·323 ± ·283	170·501 ± ·307
(*j*)	165·675 ± ·328*	169·309 ± ·266
(*k*)	163·915 ± ·283	169·500 ± ·257
Mean (*a*)—(*k*)	164·273 ± ·312	170·568 ± ·315

(*a*) *Influence of Differences in Stature-Limb Proportions.* With shorter limb bones the Northern Chinese are nevertheless taller than the French. The question immediately arises as to the possibility of judging in advance as to the applicability (or otherwise) of formulae of one race to a second on the basis of similarity or dissimilarity of the sitting height-stature index†. At first sight this suggestion seems reasonable on the grounds that the sitting height-stature index, by definitely indicating the total lower limb-stature proportion, might be expected to provide an indication of stature-limb proportions in general. The natural assumption would be that the greater the difference between two races in the sitting height-stature index the less likely would the formulae of either race prove applicable to the other.

To test this assumption it is necessary to have recourse to stature reconstruction data on a third race, as well as the sitting height-stature indices of all three races under consideration. The racial variables given for the Aino prove useful for this

[* This value should, I think, be 166·773, raising the mean to 164·383. Ed.]

† Professor Pearson in commenting on the proportionately shorter limb bones of the Chinese says: "Their vertebral columns must be relatively longer, and accordingly their index: 100 sitting height/standing height, should differ very sensibly from that of the French. Before applying our French reconstruction formulae to a second race, it would certainly be wise, where it is possible, to test whether the above index is approximately the same for the two races."

purpose. Table V presents the parallel series of Aino stature reconstructions on the bases of the French and Chinese formulae respectively, and Table VI contains the figures necessary for an estimate of the influence of sitting height-stature index on the applicability of the Chinese and French regression formulae to each of the three races. The sitting height-stature indices for the French and Aino are taken from Martin*, that for the Chinese from the writer's unpublished data on over one thousand Chinese.

TABLE V.

Aino Stature Predictions.

Formulae	French	Chinese
(*a*)	159·214	161·110
(*b*)	157·294	164·503
(*c*)	160·460	161·802
(*d*)	162·135	165·685
(*e*)	159·094	160·978
(*f*)	159·051	161·342
(*g*)	158·781	164·239
(*h*)	157·436	164·544
(*i*)	157·576	161·108
(*j*)	160·436	161·901
(*k*)	157·814	161·375
Mean (*a*)—(*k*)	159·026	162·599

TABLE VI.

Influence of Sitting Height-Stature Index.

Race	Sitting Height-Stature Index	Actual Stature†	Predicted Statures from			
			Ch. Formula	Diff.	Fr. Formula	Diff.
French	51·9	166·260	170·568	4·308	—	—
Chinese	53·9	168·923	—	—	164·273	4·650
Aino	54·8‡	157·900	162·599	4·699	159·026	1·126

The results shown in Table VI are interesting and not a little surprising. The sitting height-stature index of the Aino, as is well known, is very high. This race in fact represents one extreme in trunk-limb proportion. The index for the Chinese is much nearer the Aino than either of these is to the French. On the

* Rudolf Martin, *Lehrbuch der Anthropologie*, 2nd edition, Vol. I. p. 339. Jena, Gustav Fischer, 1928.

† Cadaver length.

[‡ I do not see why the Shikotan Aino should be cited by Martin, nor used by Dr Stevenson; Koganei, who took the measurements, considers this value exceptional, and gives for the Yezo Aino 52·4 and for his general Aino mean 52·7, which place the Aino nearer to the French than to the Chinese. ED.]

assumption stated above the Chinese formulae should give a much better prediction of Aino stature than the French formulae. The fact is, however, that the prediction of Aino stature by use of the French formulae, in spite of the wide divergence in sitting height index, is very much better than that given by the Chinese.

It may be objected with reason that the index in question is concerned only with relative proportions of the lower limbs to the stature. Using the formulae based on lower limb segments alone might therefore be expected to give a better Aino stature prediction from the Chinese than from the French formulae. But Chinese formulae (*a*), (*c*), (*e*) and (*f*), in which the lower limb segments alone are involved, give an average prediction of Aino stature of 161·308 as against 159·454 cms. given by the same formulae of the French; the actual stature being 157·90 cms. Clearly we must conclude that the sitting height-stature formula fails singularly as a criterion of the applicability of racial formula from one race to another.

TABLE VII.

Comparison of Separate Limb Segment-Stature Ratios.

	Femur	Humerus	Tibia	Radius
French ...	·2678	·1961	·2185	·1454
Chinese ...	·2603	·1839	·2146	·1468
Aino ...	·2582	·1868	·2147	·1451

But there are other much more critical limb proportion tests than the one just used. Table VII for instance gives a comparison of the separate limb segment to stature ratios in the three races under consideration. Here it is to be noted that with the single exception of the radius the Aino and Chinese are again nearer to each other than either is to the French. Excluding the formulae containing the radius should therefore, if limb proportions play an important rôle in determining the applicability of formulae of one race to another, provide a better prediction of Aino stature from the Chinese than from the French formulae. The average Aino statures predicted from formulae without the radius, however, are from the French formulae 158·781 cms., and from the Chinese 161·807 cms. The French formulae still give the better prediction.

The results just noted suggest a still further step in limb proportion analysis. Table VIII gives a comparison of the stature and individual limb segment ratios of the Aino and Chinese, each with respect to the French. The total size differences between the two races obscure the comparison in the first two columns, but when the Aino ratios are adjusted to what they would be were the statures of the two races the same—i.e. multiplying each by the factor necessary to equalise the statures, as is done in the last column of the table—then the relative ratios of

the Aino and Chinese limb segments to the corresponding French limb segments are indicated clearly. The results are the same as those noted above, namely, except in the case of the radius, the Aino and Chinese limb segments bear practically the same proportions to the French. The Chinese should, on this basis, have as good prediction values from the French formulae as the Aino.

TABLE VIII.

Stature and Limb Segment Ratios of Aino and Chinese respectively to French.

	Aino	Chinese	Aino (a)*
Stature ...	·9497	1·0160	1·0160
Femur ...	·9158	·9878	·9797
Humerus ...	·9050	·9532	·9681
Tibia ...	·9329	·9976	·9980
Radius ...	·9478	·9837	1·1040

The evidence just presented leads us to the conclusion therefore that racial similarities or differences in limb segment and stature proportions, or limb segment proportions *per se*, are not the chief factors concerned in the question of the satisfactory application of stature reconstruction formulae from one race to another.

(*b*) *Influence of Size, Variability and Correlation.* It is not difficult to show that the values of the different constants in the various regression formulae depend upon the means, standard deviations and correlation coefficients of the racial characters involved. The following form of the regression equation—

$$S = \left(M_S - r_{SB}\frac{\sigma_S}{\sigma_B}M_B\right) + r_{SB}\frac{\sigma_S}{\sigma_B}B$$
$$= c_1 + c_2 B,$$

where S and B represent the stature and one of the long bones respectively, and M_S and M_B, σ_S and σ_B, and r_{SB} their means, standard deviations and correlation coefficients—expresses clearly this intimate relation between the ultimate values of the constants c_1 and c_2 and the various racial variables. Thus the first constant (c_1) is seen to be a function of all five variables, while the second (c_2) is the regression coefficient itself $\left(r_{SB}\frac{\sigma_S}{\sigma_B}\right)$. Although it is possible to formulate a mathematical proof of the influence of single or associated deviations in the case of each of these racial variables on the derived constants, yet the empirical results given in Table IX will suffice to demonstrate the general character of the resultant changes brought about in the constants by the indicated changes in the respective variables.

* Adjusted to Chinese stature equivalents.

TABLE IX.

Influence of Variations in Racial Size, Variability and Correlations on Regression Constants.

A. VARIATIONS IN SIZE.

	Stature Mean	Stature S.D.	Long Bone Mean	Long Bone S.D.	Correlation	Equation Constants c_1	Equation Constants c_2
(a) Of stature.							
a	160·00	5·00	45·00	2·00	·80	70·00	2·00
a	165·00	„	„	„	„	75·00	„
a	170·00	„	„	„	„	80·00	„
(b) Of long bone.							
b	165·00	„	43·64	„	„	77·72	„
b	„	„	45·00	„	„	75·00	„
b	„	„	46·36	„	„	72·28	„
(c) Of both stature and long bone, in same direction.							
c	160·00	„	43·64	„	„	72·72	„
c	165·00	„	45·00	„	„	75·00	„
c	170·00	„	46·36	„	„	77·28	„
(d) Of both stature and long bone, in opposite directions.							
d	160·00	„	46·36	„	„	67·28	„
d	165·00	„	45·00	„	„	75·00	„
d	170·00	„	43·64	„	„	82·72	„

B. VARIATIONS IN VARIABILITY.

	Stature Mean	Stature S.D.	Long Bone Mean	Long Bone S.D.	Correlation	c_1	c_2
(a) Of stature.							
a	165·00	4·00	45·00	2·00	·80	93·00	1·60
a	„	5·00	„	„	„	75·00	2·00
a	„	6·00	„	„	„	57·00	2·40
(b) Of long bone.							
b	„	5·00	„	1·60	„	52·50	2·50
b	„	„	„	2·00	„	75·00	2·00
b	„	„	„	2·40	„	90·00	1·67
(c) Of both stature and long bone, in same direction.							
c	„	4·00	„	1·60	„	75·00	2·00
c	„	5·00	„	2·00	„	75·00	2·00
c	„	6·00	„	2·40	„	75·00	2·00
(d) Of both stature and long bone, in opposite directions.							
d	„	4·00	„	2·40	„	105·00	1·33
d	„	5·00	„	2·00	„	75·00	2·00
d	„	6·00	„	1·60	„	30·00	3·00

C. VARIATIONS IN CORRELATIONS.

	Stature Mean	Stature S.D.	Long Bone Mean	Long Bone S.D.	Correlation	c_1	c_2
a	165·00	5·00	45·00	2·00	·70	86·25	1·75
a	„	„	„	„	·80	75·00	2·00
a	„	„	„	„	·90	63·75	2·25

If we start with a set of hypothetical racial values—e.g. for stature, let the mean be 165·00 cms., S.D. 5·00; for the long bone (femur), mean 45·00, S.D. 2·00; correlation $r_{SB} = \cdot 80$; the regression equation being $S = 75{\cdot}00 + 2{\cdot}00\,F$—then Table IX shows in the last two columns the changes that occur in the values of the two constants of the regression equation incident upon the various changes in the racial variables indicated in the sub-headings of the various sections of the table. The centre row of each section repeats the hypothetical values just given; the line above this centre row shows the effect of a decrease in the variable concerned, the row below the effect of an increase. Variations in size and variability in the case of the femur are proportional to the variations arbitrarily chosen for the stature. The various ranges of variations chosen for illustration are all of them perfectly normal and likely to occur in the light of the racial values given in Tables I and II and experience with other racial data.

A glance at the results given in the last two columns of Table IX (p. 315) shows several things. In the first place, as is easily understood from a knowledge of the derivation of the constants, variations in size (cf. Section A of the table) affect only the first constant. Any variation in the first variable alone, for instance, results in a change in c_1 corresponding both in amount and in direction, while a variation in the second variable alone produces a change in the opposite direction in the constant and of an amount determined by the regression coefficient $\left(r_{SB}\frac{\sigma_S}{\sigma_B}\right)$. It is in the case of differences in the sizes of both variables and in opposite directions, cf. A (d), that the greatest change occurs in the constant. It is in this latter category, it so happens, that the stature and long bone differences fall in the case of our French and Chinese material.

Differences in variability (cf. Section B of the table) affect both constants except when the differences are exactly proportional and in the same direction. Furthermore, the resultant changes in the first constant are considerably larger than those resulting from differences in size only. The balancing effect of proportional differences in both variables and in the same direction is also to be noted, cf. B (c), although this fact is easily explainable by the particular relation these variables bear to each other $\left(\frac{\sigma_S}{\sigma_B}\right)$ in the derivation of the constants. With respect to those of the French the variabilities of our Chinese material may be considered as falling into category B (b) of the table.

The effect of variations in correlation are shown in Section C of the table. Both constants are affected, and always in the same direction.

Although there is nothing in the above table that a knowledge of the theory of regression would not have anticipated, yet the results tabulated therein make possible a very simple visualisation of the relative influence of common racial variations in size, variability and correlation on the two regression formula constants. In actual practice, however, matters are rarely as simple as indicated in the table, owing to the fact that we usually meet with various combinations of

differences in all three of the variables at once. Considering the human family at large it must be realised that the possibilities of such combinations are practically infinite. Judgment in advance of the applicability of regression formulae of one race to a second, except in rare cases of practical agreement in respect to all factors concerned, would seem to be most difficult, if not impossible. Fortuitous combinations of differences in size, variabilities and correlations in the case of widely separated races may occasionally provide an equally fortuitous combination of regression equation constants that may yield otherwise quite unexplainable results in regard to applicability of the regression formulae of one to the other of the two races. It is suggested here, though certainly not proven, that such a fortuitous combination of circumstances, especially in regard to variability and correlations, is operative in the case of the French and the Aino. The urgent need of similar regression formulae for a much wider range of racial groups is vital to the problem in hand.

5. *Conclusions.*

A series of Chinese stature and long bone regression formulae, based on associated measurements of the cadaver length and dry long bone lengths of forty-eight Chinese male skeletons, is presented herewith (Table X). These formulae, together with the racial variables involved in their derivation, are compared with a similar series of regression formulae derived by Pearson from French data.

The validity of applying regression formulae of one racial group to a second is tested by the trial application of each of these two sets of formulae to stature predictions in the case of the opposite race. An analysis of the factors underlying the resulting failure of the formulae of one race to give satisfactory prediction results for the second is then attempted. Through the application of each of these two sets of formulae to a third race, the Aino, the influence of such factors as (*a*) stature-limb proportions and individual limb segment proportions and (*b*) variations in racial size, variabilities and correlations on the respective prediction results is noted. Differences or similarities in stature-limb proportions or the intersegmental proportions of the various limb elements seem to play a minor rôle in determining the applicability of racial formulae of one race to another, although this phase of the subject requires much more study than it is possible to give to it at present on account of the lack of suitable data. The influence of common racial differences in size, variability and correlation, especially of the second of these factors, is noted in the case of a tabulated résumé of the variations observed in regression constants incident upon variations in the underlying racial factors just mentioned.

Lastly, the urgent need of additional data in the form of similar series of regression formulae based on comparable material for other races is strongly emphasised.

TABLE X.

Chinese Statures (Cadaver Lengths) and Right Long Bone Measurements.

Cad. No.	Stature	Femur	Humerus	Tibia	Radius
20	159·2	41·7	30·4	34·3	22·3
77	159·6	43·3	30·6	35·1	23·9
94	159·7	41·9	29·6	35·0	22·8
88	160·0	41·7	29·8	33·2	22·5
14	161·0	41·1	28·5	34·2	22·7
81	161·5	43·0	32·2	34·7	22·7
84	163·1	44·0	30·9	34·3	22·3
23	163·7	41·3	29·8	33·6	22·5
90	164·7	43·2	30·7	35·9	23·8
30	165·0	43·8	31·0	34·6	23·4
36	165·0	43·8	30·9	35·0	23·1
49	165·0	43·5	30·6	36·4	24·4
83	165·4	39·9	30·2	34·4	22·8
74	165·6	43·4	31·6	36·2	23·0
76	165·7	44·1	30·5	35·4	23·4
28	165·7	43·4	29·5	36·5	23·1
109	165·9	42·5	30·2	35·3	24·1
106	166·7	42·6	31·8	36·2	24·6
46	167·1	43·4	30·0	35·3	23·0
99	168·0	42·5	30·5	35·1	22·5
34	168·0	43·8	31·4	35·5	22·9
79	168·2	41·7	31·7	35·1	24·8
95	168·2	44·2	31·4	37·2	24·1
37	169·0	42·3	28·3	35·6	22·2
56	169·0	44·2	31·6	36·5	24·0
92	169·1	43·1	31·1	35·0	22·3
48	169·7	42·8	29·9	34·9	23·2
18	170·0	44·3	30·4	36·2	25·3
73	170·1	45·1	31·5	37·2	24·4
54	170·3	43·5	31·1	37·1	24·0
110	170·5	45·5	33·2	37·5	24·3
22	170·5	46·7	32·6	38·4	24·5
29	170·5	42·6	30·6	35·4	24·6
31	171·0	46·0	30·9	37·8	24·1
17	172·0	44·8	31·6	37·4	24·5
114	172·2	44·8	30·4	37·5	23·7
80	172·5	44·7	31·0	36·2	23·3
100	173·5	44·4	31·1	36·4	23·5
53	173·7	45·2	31·6	37·4	25·0
35	174·0	45·4	31·3	37·2	24·2
72	174·1	45·4	31·0	37·0	24·2
69	174·7	44·9	32·6	38·3	24·5
75	175·8	46·8	33·0	38·6	25·9
97	176·5	45·1	30·2	37·9	23·7
21	176·5	45·4	32·3	37·8	25·7
102	177·1	47·2	33·2	39·0	25·3
101	180·0	48·0	32·7	39·1	24·5
40	184·0	48·8	34·5	40·0	25·8
Mean	168·9229	43·9750	31·0729	36·2479	23·7792
S.D.	5·4243	1·7881	1·1816	1·5348	0·9868
C. of V.	3·2111	4·0662	3·8024	4·2343	4·1499

NOTE. I think there should be some hesitation in accepting all Dr Stevenson's conclusions. I am prepared to admit that better results for the regression formulae will be obtained by applying the formula peculiar to a race itself than by applying a formula arising from a second race. Yet the results of Table IV seem more divergent than I should consider possible, and become more remarkable when we notice how well the French formulae reconstruct Aino stature. Dr Stevenson tells us that the stature of the Northern Chinese (measured on 1100 individuals) was 168·830 cms. I should expect the cadaver length therefore to be about 170·090 cms. The cadaver length of the 48 subjects was 168·923, corresponding roughly to a stature length of 167·7. The 1·1 cms. difference in stature between the general population of Northern China and the post-mortem room population may be possible; such a population is usually not a random sample from a general population. On the whole, however, it would be desirable to discover whether the cadaver length was measured by Rollet and by the Peiping anatomists in the same manner. If this really were so, then it must follow as far as I can see that it is the correlation of the vertebral column with the stature which is affecting the differentiation in the racial results. The sitting height index would be a sign of racial differences in the vertebral column, and I suggested it might possibly be a measure of whether a racial formula could reasonably be applied from one race to a second. To this Dr Stevenson gives the reply that whereas the French formulae give Aino stature with fair approximation, the Chinese formulae do not, although the Aino and Chinese sitting height indices are fairly close to each other and divergent from the French. To this the answer must be that we do not know adequately the sitting height index of the Aino, whose long bones were measured. If they were Shikotan Aino with a sitting height index of 58·4, then their average living stature was 157·9 cms. (not the 156·6 of the Yezo and Sachalin Aino), corresponding to 159·2 cms. cadaver length, which is astonishingly close to the value 159·0 of the French prediction.

To surmount this difficulty Dr Stevenson introduces his Table IX, in which he gives arbitrary values to his stature, long bone lengths, standard deviations and correlations and calculates the new regression coefficients; he infers that because these quantities are not the same for two different races, it is due to the actual means, standard deviations and correlations not being the same. As I have said Dr Stevenson gives *arbitrary* values to these quantities and shows as a result that the regression coefficients will be modified. Let us look at this a little more closely.

Intraracially there exists a high correlation between stature and long bones, also between their standard deviations and correlation coefficients; there is also correlation between individual standard deviations and between individual correlation coefficients. If the distribution be non-normal there may be correlation even between the means themselves and these other constants. It would therefore be quite impossible *within the race* to take a group from the population with the sort of changes denoted by those in Dr Stevenson's Table IX, for the odds against their coexistence would be in most cases excessive.

Now let us turn to *interracial* data, that is to say to tables based on racial means, from which the standard deviations, correlations and regression equations of racial means are derived. These interracial constants are determinable, and have been determined in a certain number of cases. In the matter of stature and long bones, the correlations are likely to be high, the race with short average stature will have short average long bones and the tall races will have greater average long bones. Further the interracial standard deviations will usually be smaller than the intraracial standard deviations. Means have not such a wide range of distribution as individuals—the individuals belonging to a race stretch out over a wide range which may cover the whole range of interracial means.

It is accordingly not possible for Dr Stevenson to give *arbitrary* changes to his means, standard deviations and correlations; all these quantities in the interracial tables will be correlated and they cannot be measured by the scale of intraracial standard deviations, and treated as possible systems of change for other races. I feel fairly confident that a race with stature 160·0 cms. and femur length 46·4 cms., and one with 170·0 cms. of stature and 43·6 cms. of femur, are so improbable that Dr Stevenson will search the world for them in vain. The fact that interracially stature and femur have standard deviations of order 5·0 and 2·2 cms. respectively does not justify us in attributing changes of this order in combination to racial means.

One further point, let us suppose that we were in possession of a multiple regression equation for the group of men from whom by a process of selection we believe all races of mankind to have sprung. Let it be

$$\tilde{s} = c_0 + c_1 x_1 + c_2 x_2 + \ldots + c_n x_n,$$

where $\tilde{s}$ is the probable stature of the group in this race which has skeletal parts represented by $x_1, x_2, \ldots x_n$. Then if these skeletal parts include *all* those which have been *directly* selected in the course of evolution, the regression equation for stature would remain the same in all races, although the means, standard deviations and correlations might change in a great variety of ways*. The regression coefficients would be unaltered by the selection; in other words they have a stability far higher than that of means, standard deviations and correlations. If these regresssion coefficients change it must be because some other skeletal parts $x_{n+1}, x_{n+2}, \ldots x_m$ not included in our formula, but highly correlated with stature, have been selected. The divergence therefore between the stature-prediction formulae for the French and Chinese must be due to one or more skeletal parts, which are highly correlated with stature, having been omitted from the formulae. If we consider the parts of the skeleton not taken into consideration, and which suggest selection, we naturally turn to the vertebral column as the most important. Of course the pelvic and cranial heights might present appreciable correlations, but the first subject for study seems to me the vertebral column. At

* Pearson, "On the Influence of Natural Selection on the Variability and Correlation of Organs." *Phil. Trans.* A, Vol. 200, 1902, p. 21.

present nobody knows the correlations between individual vertebrae, nor the correlation between any individual vertebra and the total length of the column. It is quite possible that it might not be needful to use all the vertebrae, but that the correlation of stature with the height of one or two vertebrae might be nearly as efficient as measuring the whole series. The investigation would be well worth while making, if the Chinese material extends to measurements on the vertebral column.

Unfortunately Dr Stevenson being in Peiping I cannot talk matters over with him and I am uncertain whether his conception of bones "free of animal matter" quite coincides with the view I had in my memoir of 1898. Still I do not think that the corrections he has made in this respect would materially alter the difficulty that my formulae while giving good results for the Aino give bad ones for the Chinese, but on the other hand his formulae for the Chinese give bad results for both French and Aino. Some light might possibly be thrown on our difficulties could we ascertain from hospital data the true relationship of living stature to corpse length for the Chinese. It is a case where far more data and far more research, especially as to the part played by the vertebral column, are requisite.

K. P.

MEASUREMENTS OF MACEDONIAN MEN.

BY MARGARET M. HASLUCK, B.A., AND G. M. MORANT, D.Sc.

I. *Introduction, by M. M. Hasluck.*

THE measurements analysed in the following paper were made in 1921—3 in what Professor Ripley once called* the "practically unworked" field of South-West Macedonia. Its extreme boundaries may be given as the River Vardar (the ancient Axios) on the East, the Pindus mountains on the West, Mount Olympus and the Thessalo-Macedonian frontier on the South, and Mount Kaimakchalan and the Graeco-Serbian frontier on the North. The whole region was transferred from Turkish to Greek sovereignty after the Balkan wars of 1912—3, and it has an area of some seventy square miles.

Measurements were made among six different groups of people, who have been called respectively Greeks, Vlachs, Christian Bulgars, Mohammedan Bulgars, Turks, and Greek-speaking Mohammedans. These names divide the groups into Christians and Mohammedans according to their religion, and into Greeks, Vlachs, Bulgars, and Turks according to their language. The Rumanian patois spoken by the Vlachs (i.e. Wallachians) is meant by the Vlach language.

The linguistic touchstone which gave these names was the language spoken at home by the women. The additional language or languages spoken by some women and many men were ignored as adventitious accomplishments. The three Christian groups belong to the Eastern or Orthodox church. The vast majority of the Mohammedans are Sunnis, only half the small group of Greek-speaking Mohammedans and a very few Turks being Shiahs (of the Bektashi Order of dervishes). The Greek-speaking Mohammedans and the Mohammedan Bulgars are respectively Greeks† and Bulgars‡ who abandoned Christianity for Mohammedanism at least two hundred years ago§. From the linguistic point of view the six groups are thus only four.

The Turks descend from Asiatic Turks who came to Macedonia from Asia Minor a little before A.D. 1400. Some of these early Turks were cavalry in the service of Sultan Murad I‖ (r. 1360—1389), the Turkish conqueror of Macedonia, and of his son Sultan Bayezid I¶ (r. 1389—1402), and others were colonists imported by Sultan Bayezid with their wives and children to keep down the conquered natives**.

* *Races of Europe*, London, 1900, p. 422.

† Wace and Thompson, *Nomads of the Balkans*, London, 1914, pp. 29—30.

‡ Colonel W. M. Leake, *Travels in Northern Greece*, London, 1835, III. p. 270.

§ The evidence will be set out in detail in the book which the observer hopes to write on the folklore of the Greeks, Turks, and Albanians in the Western Balkans.

‖ Chalcondyles, *De Rebus Turcicis*, II. p. 52 B and C—D. Further details will be found in the observer's book.

¶ *Ibid.* II. p. 52 B.

** *Ibid.* II. pp. 31 B, 53 A.

MAP OF SOUTH-WEST MACEDONIA IN 1923.

To face p. 323

The Bulgars, coming from the Volga to the Danube and then sweeping south and west across the Balkan peninsula, made themselves masters of Macedonia about A.D. 850*. The Slavs with whom they fused† had come there towards the close of the sixth century‡. The Vlachs are not mentioned until just before A.D. 600§, but their Latin language shows that they are older‖. The Greeks are at least as old as the Vlachs. An Albanian strain of recent origin is traceable among the Greek-speaking Mohammedans¶, and possibly a Pecheneg (Russian) strain dating from A.D. 1091 among the Mohammedan Bulgars of Karajova (Moglena)**.

Intermarriage between any two of the six groups is so rare in normal times as to be biologically negligible††. The Christian women reputed to have been carried off to Turkish harems have not been seen in the harems by impartial witnesses and seem to have existed mainly in the imaginations of propagandists eager to inflame Christian Europe against the Turkish Government. The Christian Macedonians denied the existence of such women. In times of disturbance on the other hand, after an abortive or an actual rebellion or after a war of conquest, intermarriage may have occurred on a biologically important scale. For instance, all the Turkish cavalry of the early sultans can hardly have brought their wives with them like the colonists of Sultan Bayezid‡‡, and they may safely be presumed to have taken native women to wife. The same is probably true of the conquering Slavs and Bulgars.

Since the Mohammedan Bulgars and the Greek-speaking Mohammedans are only islands in a surrounding sea of Christian Bulgars and Greeks, the six groups are to be regarded as only four from the geographical point of view. The Christian and Mohammedan members of the Greek and Bulgar groups may live side by side, even in the same village, but the four linguistic groups live each in its separate district. The Bulgars live in the Vardar valley and along the Graeco-Serbian frontier, that is to say, on the eastern and northern fringes of the area investigated. The Turks occupy the central region, the fertile plateau that stretches from Sorovich past the large village of Kayalar to the towns of Kozani and Servia. The Greeks live on, or west of, the plateau of Anaselitza, which lies south-west of the Turkish area and is walled off from it by Mounts Sinatziko and Burunos. The Vlachs live in high-lying, sub-Alpine villages among the Pindus mountains, on the slopes of Mounts Sinatziko and Vermion, and on the Gumenje Balkan of Karajova, for they are shepherds and merchants in contradistinction to the others who are all agriculturists.

The accompanying map portrays this distribution of the village population and attempts also to suggest the relative numbers of the different groups. As much of

* G. Weigand, *Ethnographie von Makedonien*, Leipzig, 1924, p. 16.

† *Ibid.* p. 15.

‡ *Ibid.* p. 10.

§ Wace and Thompson, p. 256.

‖ *Ibid.* p. 272: cf. Weigand, p. 11.

¶ The observer will give details in her future book.

** Zonaras, *Annales*, XVIII. p. 23. Weigand perhaps exaggerates the strength of this Pecheneg strain: cf. his *Aromunen* (Leipzig, 1895), I. p. 250, and his *Ethnographie*, p. 22, with his *Ethnographie*, pp. 39, 56.

†† The case will be fully set out in the observer's book.

‡‡ Chalcondyles, II. p. 31 B.

Macedonia is very mountainous or is infested with scrub, an erroneous idea of the population is given by maps that only shade or colour large tracts of country and do not mark each village distinctively. Two caveats must be entered, however. In the first place, the villages differ considerably in size in proportion to the fertility of the soil or, in the case of the Vlachs, the range of pasturage. No reliable statistics exist, but possible averages for the Vlach, Turkish, Mohammedan Bulgar, Greek-speaking Mohammedan, Christian Bulgar, and Greek villages respectively may be given, though with some reserve, as 1800, 500, 350, 300, 280, and 270 souls. In the second place, the map only claims to represent the population as it was in 1923 and the immediately preceding years, and it takes no note of such linguistic changes as Dr Weigand* and Messrs Wace and Thompson† witnessed, or of such political changes as the wholesale substitution in 1924 of Greek refugees from Asia Minor for the Mohammedans under the Convention appended to the Treaty of Lausanne for the Exchange of Populations between Greece and Turkey.

The measurements were all made on villagers. The population of the towns, especially the Mohammedan population, was too mixed in origin to be biologically valuable. The Mohammedan Bulgars measured lived in the villages of Rudina, Polyan, Kosturyan, Kapinyan, Prabodicha, Subotsko, and Fushtan on the very fertile plain of Karajova (Moglena) in the North-East. The Christian Bulgars came from fifty-six scattered villages. To evade political difficulties most of their measurements were made, by the kindness of the Greek authorities, on young army recruits who were serving in Athens. The Turks came from the villages of Chukur Anbar, Shahinlar, Dedeler, Kalbujalar, Sarihanlar, Yenikeui, Ak Bunar, Koja Ahmedli, Kuchuk Ahmedli, Islamli, Sinekli, and Hasankeui, all of which lie immediately west of, or south of, Kozani. The Greek-speaking Mohammedans came from all the villages of their group. The Greeks measured came from thirty-seven different villages in the extreme West. The chief were Bogatsko near Hrupista, Yerania beside Shatista, Konstantziko and Zhupan in the Pindus, and Dovrunista south of Lapsista. The Vlachs measured came from the Pindus villages of Samarina, Smixi, Avdela, and Perivoli.

In each of the Greek, Greek-speaking Mohammedan, Turkish, and Vlach groups two hundred heads were measured. One hundred Christian Bulgars and one hundred Mohammedan Bulgars were measured, but the records of forty-five of the latter were lost by an unfortunate accident. Only men were measured, because the thickness of the women's hair made the measurements of their heads unreliable. The men measured were between eighteen and fifty years of age, and generally between eighteen and thirty.

The measurements made were stature (without shoes), cranial circumference (through the glabella), head (glabello-occipital) length, head breadth (greatest transverse breadth of the cranium), face (nasio-mental) height, face (bizygomatic) breadth, nose height (distance between nasion and subnasal point), and nose width (without pressure). The stature was measured with the sectional height standard

* *Aromunen*, I. p. 249. † *Nomads*, pp. 30—31.

recommended by the Anthropological Institute of Great Britain. With the exception of the nose width the measurements taken were made with moderate pressure. The colouring was recorded by eye only and not by comparison with tinted wools or other substances. The adjectives employed explain themselves with the exception of "medium" as applied to eyes. The indeterminate shade usually called hazel in England is meant. The moustache was usually lighter than the hair and was often bleached.

II. *Reduction of M. M. Hasluck's Material, by G. M. Morant.*

Table I gives the mean measurements of the six groups of Macedonian men. They are arranged in order of the cephalic indices and it will be seen that there is no other measurement which furnishes the same order. Not only is that so, but there is actually no single pair of measurements which arrange the types in the same way. Such a state of affairs might still be found if the means for the total populations were available, but the apparent lack of any high inter-racial correlations between the measurements is probably due in part to the fact that statistically small samples are being dealt with. It is clear that no reliable conclusions can be deduced from the comparison of single characters.

TABLE I.

Mean Measurements of Groups of Macedonian Men.

	Turks	Greeks	Greek-speaking Mohammedans	Christian Bulgars	Vlachs	Mohammedan Bulgars
No. of Individuals	200	200	200	100	200	55
Stature ...	1679·2 ± 2·91	1672·6 ± 2·76	1675·9 ± 2·62	1679·2 ± 3·84	1686·5 ± 3·04	1668·5 ± 4·78
Head Length ...	180·93 ± ·31	183·29 ± ·30	181·67 ± ·29	183·24 ± ·46	187·96 ± ·34	186·78 ± ·64
Head Breadth	157·65 ± ·27	157·33 ± ·27	153·57 ± ·27	152·10 ± ·36	155·81 ± ·28	150·24 ± ·65
Horizontal Circumference	543·79 ± ·69	546·30 ± ·70	538·11 ± ·69	539·93 ± ·93	550·17 ± ·79	532·91 ± 1·28
Facial Height	124·22 ± ·31	121·28 ± ·36	121·84 ± ·31	117·14 ± ·42	119·66 ± ·30	120·79 ± ·47
Facial Breadth	142·28 ± ·25	140·58 ± ·26	135·21 ± ·30	137·08 ± ·33	137·48 ± ·34	136·48 ± ·48
Nasal Height ...	53·95 ± ·22	53·26 ± ·21	53·06 ± ·20	50·85 ± ·25	52·90 ± ·20	52·24 ± ·36
Nasal Breadth	35·65 ± ·15	35·79 ± .17	34·83 ± ·14	33·25 ± ·19	35·26 ± ·17	36·29 ± ·35
Cephalic Index	87·20 ± ·17	85·92 ± ·19	84·64 ± ·17	83·26 ± ·27	82·98 ± ·17	80·53 ± ·37
Facial Index ...	87·40 ± ·23	86·20 ± ·27	90·31 ± ·27	85·47 ± ·29	87·31 ± ·25	88·47 ± ·47
Nasal Index ...	67·20 ± ·39	68·39 ± ·41	67·05 ± ·41	65·80 ± ·40	68·00 ± ·39	69·95 ± ·91

Standard deviations for all 11, and coefficients of variation for the 8 absolute measurements, are given in Table II. A number of significant differences in variability may be noted, but the differences between the extreme standard deviations only exceed 3·5 times their probable errors in the case of the facial height and breadth, the nasal height and the facial and nasal indices. All the measurements of the face show more significant differences in variability than do the measurements of the brain-

TABLE II.

Constants of Variation for Series of Macedonian Men.

	Christian Bulgars	Turks	Greek-speaking Mohammedans	Mohammedan Bulgars	Greeks	Vlachs	Mean Standard Deviations (from weighted σ^2)
No. of Individuals	100	200	200	55	200	200	
	Standard Deviations						
Stature	56·98 ± 3·19	60·98 ± 2·06	55·02 ± 1·86	52·58 ± 3·38	57·88 ± 1·95	63·76 ± 2·15	58·86
Head Length ...	6·88 ± ·33	6·60 ± ·22	6·16 ± ·21	7·00 ± ·45	6·38 ± ·22	7·03 ± ·24	6·61
Head Breadth ...	5·38 ± ·26	5·65 ± ·19	5·76 ± ·19	7·14 ± ·46	5·67 ± ·19	5·94 ± ·20	5·81
Horizontal Circumference ...	13·82 ± ·66	14·45 ± ·49	14·54 ± ·49	14·12 ± ·91	14·78 ± ·50	16·50 ± ·56	14·91
Facial Height ...	6·16 ± ·29	6·59 ± ·22	6·51 ± ·22	5·15 ± ·33	7·52 ± ·25	6·32 ± ·21	6·61
Facial Breadth ...	4·85 ± ·23	5·21 ± ·18	6·19 ± ·21	5·28 ± ·34	5·51 ± ·19	7·05 ± ·24	5·88
Nasal Height ...	3·69 ± ·18	4·60 ± ·16	4·23 ± ·14	3·94 ± ·25	4·50 ± ·15	4·15 ± ·14	4·28
Nasal Breadth ...	2·87 ± ·14	3·08 ± ·10	3·01 ± ·10	3·80 ± ·24	3·56 ± ·12	3·57 ± ·12	3·30
Cephalic Index ...	4·01 ± ·19	3·50 ± ·12	3·66 ± ·12	4·05 ± ·26	3·97 ± ·13	3·55 ± ·12	3·73
Facial Index ...	4·32 ± ·21	4·72 ± ·16	5·60 ± ·19	5·18 ± ·33	5·67 ± ·19	5·32 ± ·18	5·23
Nasal Index ...	5·88 ± ·28	8·11 ± ·27	8·56 ± ·29	10·03 ± ·65	8·66 ± ·29	8·21 ± 28	8·27
Mean of σ^2/Mean σ^2	·830	·947	·976	1·057	1·062	1·087	—
	Coefficients of Variation						
Stature	3·39 ± ·16	3·63 ± · 12	3·28 ± ·11	3·15 ± ·20	3·46 ± ·12	3·78 ± ·13	—
Head Length ...	3·75 ± ·18	3·65 ± · 12	3·39 ± ·11	3·75 ± ·24	3·48 ± ·12	3·74 ± ·13	—
Head Breadth ...	3·54 ± ·17	3·58 ± · 12	3·75 ± ·13	4·75 ± ·31	3·60 ± ·12	3·81 ± ·13	—
Horizontal Circumference ...	2·56 ± ·12	2·66 ± ·09	2·70 ± ·09	2·65 ± ·17	2·71 ± ·09	3·00 ± ·10	—
Facial Height ...	5·26 ± ·25	5·31 ± ·18	5·34 ± ·18	4·26 ± ·27	6·20 ± ·21	5·38 ± ·18	—
Facial Breadth ...	3·54 ± ·17	3·66 ± ·12	4·58 ± ·15	3·87 ± ·25	3·92 ± ·13	5·13 ± ·17	—
Nasal Height ...	7·26 ± ·35	8·53 ± ·29	7·97 ± ·27	7·54 ± ·49	8·45 ± ·29	7·84 ± ·27	—
Nasal Breadth ...	8·63 ± ·41	8·64 ± ·29	8·64 ± ·29	10·47 ± ·68	9·95 ± ·34	10·12 ± ·34	—

box. The Christian Bulgars have the lowest constants for head breadth, horizontal circumference, facial and nasal breadths, nasal height and facial and nasal indices. The Vlachs are more variable than the other series for stature, horizontal circumference and facial breadth. The standard deviations in the right-hand column are average ones found by weighting the squared standard deviations of the series with the number of individuals they contain. A measure of the relative variability of the series based on all the characters was obtained by averaging for the 11 measurements the squared serial σ's divided by these mean σ's squared. The Christian Bulgars are found to be appreciably less variable than any other population represented.

The coefficients of racial likeness between the six groups are given in Table III* All the available 11 characters have been used for this purpose and it is known that the intra-racial correlations between some pairs are greater than 0·5. The theoretical condition that the measurements used should be uncorrelated, or, at any rate, lowly correlated with one another, is thus not fulfilled, but if a selection were made the number of characters would be too small. As the series contain different numbers of individuals, the coefficients were reduced to the values they would have if each sample in the comparison consisted of 100. Direct comparison may be made between these adjusted values. The populations dealt with live in adjoining regions and in close contact, but they can be distinguished easily. In spite of differences in language and religion, the connection between the Greeks and Turks is the only intimate one. The inter-relationships of the types suggested by the coefficients can be seen more readily in Fig. 1. The Turks and Mohammedan Bulgars are most widely separated, but they are connected through the other four populations. The lack of any close connection between the Christian and Mohammedan Bulgars is surprising

TABLE III.

Coefficients of Racial Likeness between Series of Macedonian Men†.

	Turks (200)	Greeks (200)	Greek-speaking Mohammedans (200)	Christian Bulgars (100)	Vlachs (200)	Mohammedan Bulgars (55)
			Crude Coefficients			
Turks ... ... (200)	—	5·39	28·85	32·57	35·75	29·92
Greeks ... ... (200)	5·39	—	22·20	19·29	14·86	21·21
Greek-speaking Mohammedans (200)	28·85	22·20	—	12·67	22·41	10·95
Christian Bulgars ... ... (100)	32·57	19·29	12·67	—	13·56	11·97
Vlachs ... ... (200)	35·75	14·86	22·41	13·56	—	12·35
Mohammedan Bulgars ... (55)	29·92	21·21	10·95	11·97	12·35	—
			Coefficients reduced to $n_1 n_2/(n_1+n_2)=50$			
Turks	—	2·69	14·43	24·43	17·87	34·68
Greeks	2·69	—	11·10	14·47	7·43	24·58
Greek-speaking Mohammedans ...	14·43	11·10	—	9·50	11·21	12·69
Christian Bulgars	24·43	14·47	9·50	—	10·17	16·87
Vlachs	17·87	7·43	11·21	10·17	—	14·31
Mohammedan Bulgars	34·68	24·58	12·69	16·87	14·31	—

* With the usual notation the form of the coefficient used was:

$$S\left(\frac{1}{M}\frac{n_s n'_s}{n_s + n'_s} \times \frac{(m_s - m'_s)^2}{\sigma_s^2}\right) - 1 \pm \cdot 67449\sqrt{\frac{2}{M}\left(1-\frac{1}{M}\right)}.$$

The average standard deviations given in the right-hand column of Table II were used for this purpose.

† All the coefficients are based on the 11 characters given in Table I. The probable errors are ±·27.

and it will be seen that the Turks resemble the Christian Greeks very much more closely than they do the Greek-speaking Mohammedans. Several very distinct racial types are evidently represented in Macedonia and more abundant material from that region, and from neighbouring districts, would be needed to unravel their blood relationships. The possession of a common religion or of a common language gives no indication whatever of descent. The coefficients of racial likeness have been calculated between a number of cranial series from South-Eastern Europe *. The values reduced to samples of 100 each are given in Table IV for 6 of these.

TABLE IV.

Coefficients of Racial Likeness between Male Cranial Series from South-Eastern Europe reduced to $\frac{\bar{n}_1\bar{n}_2}{\bar{n}_1+\bar{n}_2}=50$.

		Slovenes (59·6)†	Rumanians (40·0)	Turks (67·0)	Greeks (89·7)	Serbo-Croats (79·8)	Magyars (Mediaeval) (27·6)
Slovenes (59·6)†	C.R.L. No. of Characters	— —	3·24 ± ·22 17	5·22 ± ·22 17	8·46 ± ·21 20	8·20 ± ·21 20	16·51 ± ·20 21
Rumanians (40·0)	C.R.L. No. of Characters	3·24 ± ·22 17	— —	8·88 ± ·18 27	6·17 ± ·19 24	8·10 ± ·19 24	9·88 ± ·21 19
Turks (67·0)	C.R.L. No. of Characters	5·22 ± ·22 17	8·88 ± ·18 27	— —	3·10 ± ·19 24	6·26 ± ·19 24	5·21 ± ·21 19
Greeks (89·7)	C.R.L. No. of Characters	8·46 ± ·21 20	6·17 ± ·19 24	3·10 ± ·19 24	— —	7·27 ± ·18 27	7·46 ± ·20 22
Serbo-Croats (79 8)	C.R.L. No. of Characters	8·20 ± ·21 20	8·10 ± ·19 24	6·26 ± ·19 24	7·27 ± ·18 27	— —	8·32 ± ·20 22
Magyars (Mediaeval) (27·6)	C.R.L. No. of Characters	16·51 ± ·20 21	9·88 ± ·21 19	5·21 ± ·21 19	7·46 ± ·20 22	8·32 ± ·20 22	— —

The Turkish skulls came from cemeteries in Constantinople. The Greek series was made up by pooling a group of 50 specimens from Europe and 45 from Asia Minor, the coefficient between the two samples being $-0{\cdot}04 \pm {\cdot}18$. The groups of skulls were drawn from regions much further apart than any in Macedonia and greater racial differences would be anticipated. But actually the coefficients in Table IV are of a decidedly lower order than the ones for the living in Table III adjusted so that the sizes of the samples are the same in the two cases. The difference in the

* G. M. Morant, "A Preliminary Classification of European Races based on Cranial Measurements," *Biometrika*, Vol. xx^{B}, 1928, pp. 301—375.

† The numbers in brackets are the mean numbers of skulls ($\bar{n}$) available for the characters used in computing the coefficients.

TABLE V.

Values of a reduced to* $\frac{\bar{n}_1 \bar{n}_2}{\bar{n}_1 + \bar{n}_2} = 50$ *between Series of Macedonian Men.*

	Stature	Nasal Index	Nasal Breadth	Facial Index	Nasal Height	Facial Height	Horizontal Circum-ference	Head Length	Facial Breadth	Head Breadth	Cephalic Index
Turks and Greeks	0·7	1·1	0·1	2·7	1·1	10·7	1·4	6·6	4·5	0·2	6·1
" " Greek-speaking Mohammedans	0·1	0·0	3·3	15·3	2·0	7·3	7·3	0·7	79·5	24·3	24·3
" " Christian Bulgars	0·0	1·1	26·4	6·6	26·2	64·4	3·4	6·1	42·7	45·7	54·5
" " Vlachs	0·7	0·5	0·5	0·0	3·0	25·9	9·2	57·7	36·3	5·1	63·3
" " Mohammedan Bulgars	1·7	5·5	2·0	2·2	4·3	14·7	26·7	39·8	53·1	83·3	161·0
Greeks and Greek-speaking Mohammedans	0·1	1·3	4·6	30·7	0·1	0·3	15·1	2·9	46·0	20·3	6·1
" " Christian Bulgars	0·7	4·3	29·9	0·9	16·3	22·5	9·2	0·0	19·3	40·1	24·3
" " Vlachs	2·8	0·1	1·1	2·2	0·5	3·3	3·4	25·3	15·1	3·3	30·1
" " Mohammedan Bulgars	0·3	1·8	1·1	9·7	9·9	0·3	40·4	14·0	26·5	74·7	104·6
Greek-speaking Mohammedans and Christian Bulgars ...	0·1	8·0	9·3	42·1	13·8	28·3	0·7	2·5	5·7	3·8	6·1
" " Vlachs	1·7	0·7	1·1	16·5	0·1	5·7	32·9	45·4	8·3	7·2	9·2
" " Mohammedan Bulgars	0·8	6·1	10·3	5·9	12·0	1·3	6·1	29·7	2·6	17·1	60·3
Christian Bulgars and Vlachs	0·7	3·0	19·3	5·9	11·5	8·7	23·9	26·3	0·3	20·3	0·3
" " Mohammedan Bulgars	1·7	11·5	42·7	16·4	51·5	17·5	11·0	14·8	0·5	5·3	28·1
Vlachs and Mohammedan Bulgars	4·6	2·8	4·6	2·7	14·4	1·6	67·4	1·7	1·6	46·5	22·4
Mean a's	1·11	3·17	10·43	10·64	11·11	14·16	17·21	18·24	22·81	26·49	40·03

* $a = \frac{n_s n'_s}{n_s + n'_s} \times \frac{(m_s - m'_s)^2}{\sigma}$.

number and choice of characters used may partly account for this state of affairs, but the evidence still suggests forcibly that the coefficient of racial likeness between two racial types is of a much higher order for the living head than for the skull. This may mean that the skeleton is of more fundamental importance than the living body and that it is a more reliable guide to racial constitution. It may be noted that the living Vlachs, Greeks and Turks are related in the same way as the cranial forms of the Rumanians, Greeks and Turks. The Greeks occupy the intermediate position and their very close resemblance to the Turks is brought out by both comparisons.

The significance of the differences between the various characters considered singly is conveniently measured by the α's found in computing the coefficients. Values of α less than 10 may be taken to indicate that the types are undifferentiated by the particular character. The α's reduced to values they would have for samples of 100 each are given in Table V*. As for the skull, there are found to be profound differences between the average contributions which the different characters make to the coefficients of racial likeness. Not a single significant difference is found between the mean statures of the six series. The nasal index is almost as constant and it only serves to distinguish the two Bulgarian types from one another. It is interesting to find these relations since so much importance has been attached to the stature and nasal index in attempting to classify European races: in the case of this small group they are perfectly worthless characters for the purpose. More significant differences are found between the other facial measurements, but the cephalic index and head breadth are of still greater importance. The last two, together with the head length, control the coefficients about as much as the other 8 characters together. The following relations are observed from a comparison of Tables I and V:

(*a*) The stature is constant for the six types.

(*b*) The nasal index of the Mohammedan Bulgars is greater than that of the Christian Bulgars, but no other differences are found for this character. The Christian Bulgars are distinguished from all other types by their small nasal height and nasal breadth.

(*c*) The only marked differences between the facial indices are occasioned by the high value for the Greek-speaking Mohammedans. The facial height is peculiarly large for the Turks and peculiarly small for the Christian Bulgars. The facial breadth makes a distinction between the Turks and Greeks on the one hand, with their high values, and the remaining four series on the other.

(*d*) The small horizontal circumference of the Mohammedan Bulgars leads to the most significant differences for this character, but the Vlachs show an appreciably higher value than the Greek-speaking Mohammedans and Christian Bulgars.

(*e*) For the cephalic index 10 of the total 15 comparisons are significant; for the head breadth there are 9 such and for the head length 8. The arrangement of

* After the sizes of the samples have been adjusted it may not be true to say that an α greater than 10 indicates a significant difference, but that limit may still be adopted conventionally.

the types given by these 3 characters is shown in Fig. 1 and it will be seen that it agrees well with one which would have been suggested by the coefficient of racial likeness. The only disagreement between the two methods is due to the absence

INTER-RELATIONSHIPS OF MACEDONIAN TYPES.

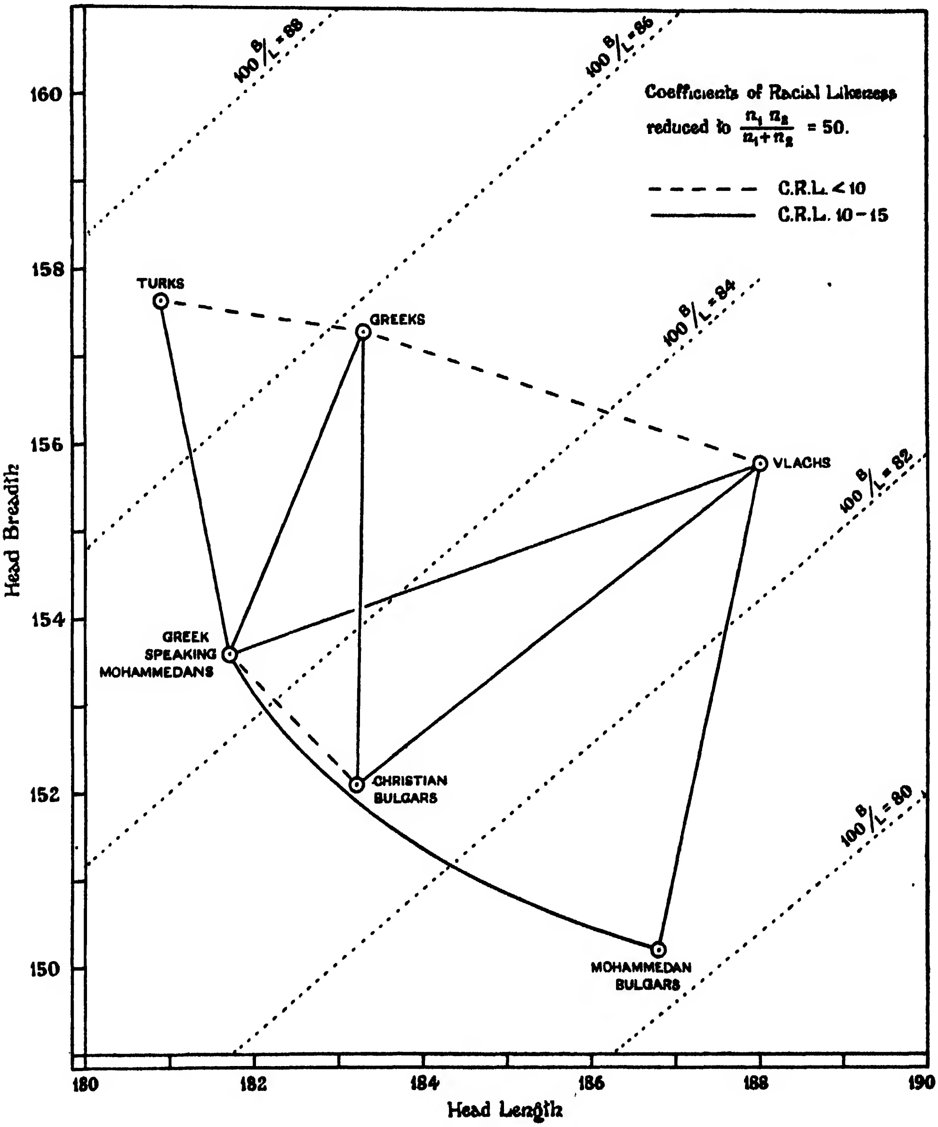


Fig. 1.

of a low coefficient between the Christian and Mohammedan Bulgars. The very significant differences between the nasal measurements of the two clearly connotes a racial difference which is not revealed by the head length, head breadth and

cephalic index. Though the last is quite the most valuable single measurement, it is not able, by itself, to provide a reliable guide to racial relationships.

The relative significance of the average differences between single measurements of racial types has been examined for the skull for 41 series from various parts of Europe as in Table V above. The 10 head measurements there correspond roughly to cranial measurements: the facial height of the head may be supposed homologous to the upper facial height of the skull ($G'H$), the facial breadth to the bizygomatic breadth (J) and the facial index to the upper facial index ($100\ G'H/GB$). The characters are arranged in Table V in order of their mean α's. The arrangement for the 10 corresponding cranial characters is: nasal index, nasal breadth, nasal height, upper facial height, upper facial index, horizontal circumference, facial breadth, skull length, skull breadth and cephalic index *. The two orders are closely similar, so the characters which are most constant and most variable inter-racially appear to be the same whether the group of racial types is represented by measurements of the living from a small area such as Macedonia, or by skull measurements from Europe as a whole.

With regard to the qualitative characters, "Shape of Nose," "Body Build," "Skin Tint," "Hair Colour" and "Eye Colour," the last two classified without standard scales, it was clear on tabulation that very little could be done with them. Such classifications are worth less than measured characters on the living, as the latter are worth less than measured characters on the skull. Tables VI—IX contain the reduced data exhibited as total frequencies and as percentages. Little of real racial value can be deduced from these tables as they stand. For example: taking curliness of hair, Turks and Christian Greeks stand closest, both are significantly different from Greek-speaking Mohammedans and Vlachs, but scarcely from Christian Bulgars. On the other hand, skin tint data do not separate Christian Greeks from Vlachs, but separate both from the groups of Turks, of Greek-speaking Mohammedans and of Christian and Mohammedan Bulgars, all of whom are indistinguishable in skin tint from each other. Skin tint must therefore be determined by a graduated scale.

From skin tint and curliness of hair no definite result seemed to flow and no further regard was paid to these characters. The other characters have sufficient categories just to admit of our applying the method given in *Biometrika*, Vol. VIII. pp. 250—254, to measure the probability that two independent distributions of frequency are really samples of the same population. If this probability be represented by P, Tables X—XIII give the values of this quantity. At first sight it seemed wholly impossible to draw any conclusions from these tables—they contradicted each other in such a serious manner. Finally we took the means for each pair of groups for the four qualitative characters, "Shape of Nose," "Body Build," "Hair Colour" and "Eye Colour,"—thus endeavouring to get an average probability for each pair of races; and treated this as a measure of their racial relationship. From this procedure Table XIV (p. 335) resulted.

* *Biometrika*, Vol. XX^B. 1928, Table XVI facing p. 336.

TABLE VI. *Shape of Nose.*

		Hooked	Straight	Tip-tilted	Low at Root	Wavy
200 Greeks	Frequency Percentage	18 9·0±1·4	165 82·5±1·8	14 7·0±1·2	1 0·5±0·3	2 1·0±0·5
200 Greek-speaking Mohammedans	Frequency Percentage	19 9·5±1·4	157 78·5±1·9	13 6·5±1·2	6 3·0±0·8	5 2·5±0·7
200 Turks	Frequency Percentage	17 8·5±1·3	177 88·5±1·5	4 2·0±0·7	2 1·0±0·5	0 —
200 Vlachs	Frequency Percentage	16 8·0±1·3	165 82·5±1·8	7 3·5±0·9	4 2·0±0·7	8 4·0±0·9
100 Christian Bulgars	Frequency Percentage	8 8·0±1·8	52 52·0±3·4	40 40·0±3·3	0 —	0 —
55 Mohammedan Bulgars	Frequency Percentage	1 1·8±1·2	51 51·0±4·5	3 5·5±2·1	0 —	0 —

TABLE VII. *Body Build and Skin Tint.*

		Body Build			Skin		
		Narrow	Medium	Broad	Dark	Fair	Freckled
200 Greeks	Frequency Percentage	60 30·0±2·2	77 38·5±2·3	63 31·5±2·2	160 80·0±1·9	40 20·0±1·9	3 1·5±0·6
200 Greek-speaking Mohammedans	Frequency Percentage	88 44·0±2·4	59 29·5±2·2	53 26·5±2·1	124 62·0±2·3	76 38·0±2·3	0 —
200 Turks	Frequency Percentage	61 30·5±2·2	83 41·5±2·3	56 28·0±2·1	129 64·5±2·3	71 35·5±2·3	0 —
200 Vlachs	Frequency Percentage	76 38·0±2·3	57 28·5±2·2	67 33·5±2·3	165 82·5±1·8	35 17·5±1·8	5 2·5±0·7
100 Christian Bulgars	Frequency Percentage	44 44·0±3·3	35 35·0±3·2	21 21·0±2·7	65 65·0±3·2	35 35·0±3·2	0 —
55 Mohammedan Bulgars	Frequency Percentage	24 43·6±4·5	14 25·5±4·0	17 30·9±4·2	34 61·8±4·4	21 38·2±4·4	0 —

TABLE VIII. *Colour of Hair.*

		Black	Dark Brown	Brown	Red	Fair	Curly
200 Greeks	Frequency Percentage	28 14·0±1·7	134 67·0±2·2	30 15·0±1·7	1 0·5±0·3	7 3·5±0·9	12 6·0±1·1
200 Greek-speaking Mohammedans	Frequency Percentage	18 9·0±1·4	114 57·0±2·4	42 21·0±1·9	0 —	26 13·0±1·6	23 11·5±1·5
200 Turks	Frequency Percentage	2 1·0±0·5	125 62·5±2·3	56 28·0±2·1	0 —	17 8·5±1·3	9 4·5±1·0
200 Vlachs	Frequency Percentage	73 36·5±2·3	87 43·5±2·4	25 12·5±1·6	0 —	15 7·5±1·3	38 19·0±1·9
100 Christian Bulgars	Frequency Percentage	8 8·0±1·8	47 47·0±3·4	31 31·0±3·1	2 2·0±0·9	12 12·0±2·2	8 8·0±1·8
55 Mohammedan Bulgars	Frequency Percentage	8 14·5±3·3	26 47·3±4·3	18 32·7±4·5	1 1·8±1·2	2 3·6±1·7	0 —

TABLE IX. *Colour of Eyes.*

		Dark	Medium	Light
200 Greeks	Frequency Percentage	110 55·0 ± 2·4	56 28·0 ± 2·1	34 17·0 ± 1·8
200 Greek-speaking Mohammedans	Frequency Percentage	72 36·0 ± 2·3	71 35·5 ± 2·3	57 28·5 ± 2·2
200 Turks	Frequency Percentage	91 45·5 ± 2·4	66 33·0 ± 2·2	43 21·5 ± 1·9
200 Vlachs	Frequency Percentage	95 47·5 ± 2·4	59 29·5 ± 2·2	46 23·0 ± 2·0
100 Christian Bulgars	Frequency Percentage	61 61·0 ± 3·3	14 14·0 ± 2·3	25 25·0 ± 2·9
55 Mohammedan Bulgars	Frequency Percentage	19 34·5 ± 4·3	10 18·2 ± 3·5	26 47·3 ± 4·5

TABLE X. *Shape of Nose* (5 *Groups*). Values of *P*.

	Greeks	Greek-speaking Mohammedans	Turks	Vlachs	Christian Bulgars	Mohammedan Bulgars
Greeks	—	·2767	·0812	·0981	·0000	·3464
Greek-speaking Mohammedans	·2767	—	·0110	·5600	·0000	·1172
Turks	·0812	·0110	—	·0417	·0000*	·1663*
Vlachs	·0981	·5600	·0417	—	·0000	·1606
Christian Bulgars	·0000	·0000	·0000*	·0000	—	·0000†
Mohammedan Bulgars	·3464	·1172	·1663*	·1606	·0000†	—

* For 4 groups. † For 3 groups.

TABLE XI. *Body Build* (3 *Groups*). Values of *P*.

	Greeks	Greek-speaking Mohammedans	Turks	Vlachs	Christian Bulgars	Mohammedan Bulgars
Greeks	—	·0144	·7466	·0825	·0384	·1079
Greek-speaking Mohammedans	·0144	—	·0110	·2886	·4963	·7844
Turks	·7466	·0110	—	·0248	·0673	·0740
Vlachs	·0825	·2886	·0248	—	·0013	·7745
Christian Bulgars	·0384	·4963	·0673	·0013	—	·3053
Mohammedan Bulgars	·1079	·7844	·0740	·7745	·3053	—

TABLE XII. *Hair Colour* (5 *Groups*). Values of P.

	Greeks	Greek-speaking Mohammedans	Turks	Vlachs	Christian Bulgars	Mohammedan Bulgars
Greeks	—	·0000	·0000	·0000	·0001	·0216
Greek-speaking Mohammedans	·0000	—	·0002*	·0000*	·0882	·0771
Turks	·0000	·0002*	—	·0000*	·0015	·0000
Vlachs	·0000	·0000*	·0000*	—	·0000	·0002
Christian Bulgars	·0001	·0882	·0015	·0000	—	·3746
Mohammedan Bulgars	·0216	·0771	·0000	·0002	·3746	—

* For 4 groups.

TABLE XIII. *Eye Colour* (3 *Groups*). Values of P.

	Greeks	Greek-speaking Mohammedans	Turks	Vlachs	Christian Bulgars	Mohammedan Bulgars
Greeks	—	·0004	·1645	·2266	·0169	·0001
Greek-speaking Mohammedans	·0004	—	·1162	·0675	·0000	·0127
Turks	·1645	·1162	—	·7718	·0019	·0006
Vlachs	·2266	·0675	·7718	—	·0110	·0018
Christian Bulgars	·0169	·0000	·0019	·0110	—	·0052
Mohammedan Bulgars	·0001	·0127	·0006	·0018	·0052	—

TABLE XIV.

Probabilities that each pair of Macedonian Groups might have been samples of the same Race.

	Greeks	Greek-speaking Mohammedans	Turks	Vlachs	Christian Bulgars	Mohammedan Bulgars
Greeks	—	·0729	·2481	·1018	·0138	·1190
Greek-speaking Mohammedans	·0729	—	·0346	·2140	·1461	·2478
Turks	·2481	·0346	—	·2096	·0177	·0602
Vlachs	·1018	·2140	·2096	—	·0031	·2343
Christian Bulgars	·0138	·1461	·0177	·0031	—	·1713
Mohammedan Bulgars	·1190	·2478	·0602	·2343	·1713	—

Without laying much stress on this table, we may draw the following results from it:

(i) The Greeks are most like the Turks.

(ii) The Greek-speaking Mohammedans are most like the Mohammedan Bulgars.

(iii) The Turks are most like the Greeks.

(iv) The Vlachs are most like the Mohammedan Bulgars.

(v) The Christian Bulgars are most like the Mohammedan Bulgars.

(vi) The Mohammedan Bulgars are most like the Greek-speaking Mohammedans.

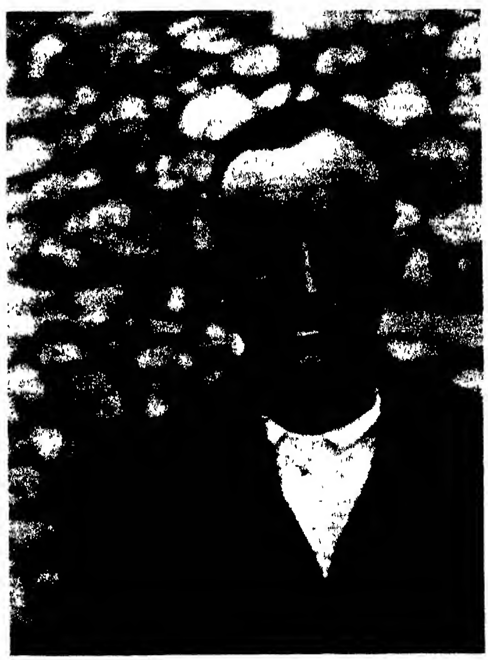
The whole of these results would flow from the first series of crude coefficients in Table III on p. 327. The other values in Table XIV for order of resemblance have considerable correspondence with the same values in Table III. Thus these qualitative characters may be said generally to give such support as lies in them to the relationships deduced from the measured characters first discussed.

In conclusion the observer begs to offer her grateful thanks to the successive royalist and republican governments of Greece for the facilities which they gave her in spite of the political and military difficulties of 1921—3. She thanks Professor R. W. Reid, too, for his kindly help, and the statistician and she could scarcely have compiled this paper without the practical interest of Professor Karl Pearson, while they have also to thank Miss Ida McLearn for the preparation of the figure and map.


Christian Greek (Kozani).


Greek-speaking Mohammedan (Chotil).

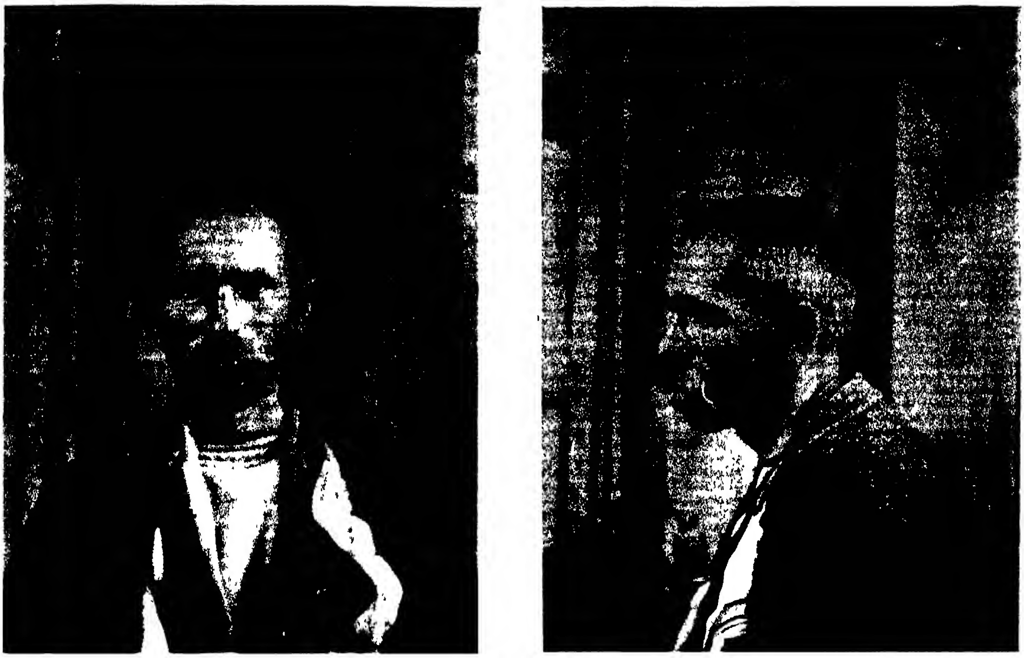
Mohammedan Turk (Hadovo), probably pure Asiatic type.



Mohammedan Turk (Sofular), ancestry probably intermarried with Christians.

Mohammedan Bulgar (Kosturyan in Karajova).

Christian Bulgar of stock type (Pateli).



Christian Vlach of usual dark type (Mejidieh).

Christian Vlach of fair type (Samarina).

SOME NOTES ON SAMPLING TESTS WITH TWO VARIABLES.

By E. S. PEARSON, D.Sc.

CONTENTS.

(1) Introductory.

Suppose that we are considering the distribution of a single variable, x, and that the population sampled is divided into a groups such that in the rth group x is normally distributed with standard deviation σ about a mean $\tilde{x}_r$. In general $\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_a$ are not equal, although in a special case they may be so. A sample of N is now drawn in which n_1 individuals are taken randomly from the first group, n_2 from the second, and so on, where

$$n_1 + n_2 + \ldots + n_a = N \quad \text{.............................(1).}$$

Estimates $X_1, X_2, \ldots X_a$ are made from the sample of the true population group means $\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_a$, and

$$u = \mathop{S}_{r=1}^{a} \left\{ \mathop{S}_{t=1}^{n_r} (x_{rt} - X_r)^2 \right\} \Big/ \sigma^2 \quad \text{..........................(2)}$$

is calculated. Then if the quantities X_r have been obtained in a suitable manner, it can be shown that the distribution of u in repeated samples of N* follows the Type III law

$$f(u)\,du = \text{constant} \times u^{\frac{N-c-2}{2}} e^{-\frac{1}{2}u}\,du \quad \text{....................(3),}$$

where c will depend upon the method of estimation of the X's. For example, if X_r is the mean of the n_r values of x sampled from the rth population group

$$(r = 1, 2, \ldots a),$$

then $c = a$; or if in the population $\tilde{x}_1 = \tilde{x}_2 = \ldots = \tilde{x}_a$, and we take $X_1 = X_2 = \ldots = X_a$ = mean of the N sample values of x, then $c = 1$. It will be noted that (3) gives

$$\text{Mean } u = N - c, \quad \sigma_u = \sqrt{2(N-c)}.$$

* That is to say, samples in which n_r individuals are drawn randomly from the rth population group $(r = 1, 2, \ldots a)$, n_r remaining fixed.

Dr R. A. Fisher has based a number of simple but important statistical tests on the equation (3), which he classes under the heading of "Analysis of Variance*." The expression

$$\underset{r=1}{\overset{a}{S}}\left\{\underset{t=1}{\overset{n_r}{S}}(x_{rt}-X_r)^2\right\}\Big/(N-c)=u\sigma^2/(N-c) \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

he describes as an estimate of the population variance, σ^2, based upon $N-c$ degrees of freedom; its mean value in repeated samples is seen to be σ^2.

The expression u, containing as it does the population σ^2, is not of much direct value if this quantity be unknown, but in a number of problems the appropriate criterion to use is the ratio $\theta=u'/u$, where u' is a quantity similar to the u of (2) but based upon an independent estimate of σ^2, such that

$$f(u')\,du'=\text{constant}\times u'^{\frac{N'-c'-2}{2}}e^{-\frac{1}{2}u'}\,du' \quad \ldots\ldots\ldots\ldots\ldots\ldots(5).$$

θ is now independent of σ^2 and, as Fisher has shown (also it may be easily proved from (3) and (5)), if u and u' are uncorrelated, then in repeated sampling θ is distributed according to the Type VI law

$$f(\theta)\,d\theta=\text{constant}\times\theta^{\frac{N'-c'-2}{2}}(1+\theta)^{-\frac{N+N'-c-c'}{2}}\,d\theta \quad \ldots\ldots\ldots(6).$$

Here the constant term is independent of σ^2. In dealing with this distribution Fisher uses the transformation

$$z=\frac{1}{2}\left\{\log_e\frac{u\sigma^2}{N-c}-\log_e\frac{u'\sigma^2}{N'-c'}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7),$$

that is to say he takes z as half the difference of the natural logarithms of the two estimates of variance. He has given tables showing for different values of $n_1=N-c$ and $n_2=N'-c'$ the value of z, the chance of exceeding which is ·05 and ·01†. It will be noticed that by writing $\zeta=\theta/(1+\theta)$, (6) may be transformed into the Type I distribution

$$f(\zeta)\,d\zeta=\text{constant}\times\zeta^{\frac{N'-c'-2}{2}}(1-\zeta)^{\frac{N-c-2}{2}}\,d\zeta \quad \ldots\ldots\ldots\ldots\ldots(8),$$

whose probability integral depends upon the Incomplete Beta Function.

The various tests based upon the frequency law (6) depend upon the variables being normally distributed. As soon as non-normality is introduced the distribution of θ will be modified in a direction varying with the particular test. Not only may (3) and (5) be no longer applicable, but u and u' will in certain cases be correlated where previously they were independent. In the present paper it is proposed to examine how far deviations from normality are likely to affect one of Fisher's tests, that for the goodness of fit of regression curves. The experimental results used to illustrate this point will also help to throw some light on the distribution of the correlation coefficient in small samples from non-normal populations.

* A description of these tests is given in a paper entitled "On a Distribution yielding the Error-Functions of several well known Statistics," read before the International Mathematical Congress at Toronto in 1924 but only recently published. The methods of application without the mathematical framework are given in Dr Fisher's *Statistical Methods for Research Workers*, 1925 and 1928, pp. 178 *et seq.*

† Table VI, *Statistical Methods for Research Workers*. This z must be distinguished from the original z of "Student's" test which Fisher writes as $t/\sqrt{n}$.

(2) THE APPLICATION OF THE PRINCIPLE OF LIKELIHOOD.

In two recent papers an attempt has been made by Dr J. Neyman and the author to connect together the various tests that are applied in different sampling problems by deducing from a common basis the criterion appropriate in each case*. It will perhaps be of interest to illustrate the use of this method in a further instance. The problem of the goodness of fit of regression curves presents itself commonly in the following form. We have before us a sample, Σ, and wish to know whether it is likely that this has been drawn from a population, Π, for which the curve of regression of y on x, let us say, follows a law

$$Y_x = F(x;\ \alpha_1, \alpha_2, \ldots \alpha_c) \qquad \text{(9)},$$

where F is of given form but the constants α are unspecified. That is to say, we are testing what has been described as a "composite hypothesis"; it would become a "simple hypothesis" only if the values of α were specified in advance. What is the appropriate criterion to use? In the case where $c=1$, and we suppose that in the population Y_x is constant, should we take the correlation ratio η_{y_x}? And when in the population Y_x is supposed to lie on a sloping straight line ($c=2$), should we consider $\eta - r$ or $\eta^2 - r^2$ or even (as one of Blakeman's alternatives†) the ratio of η to r? The general problem in which the array distributions may be of any form would probably be extremely difficult to solve, at any rate for small samples, but the solution in one important case—that in which the arrays of y for constant x are homoscedastic normal curves—can be obtained. And here the principle of likelihood appears to provide a method of finding the appropriate criterion.

Σ is a sample of N in the form of a correlation table, for which the marginal totals, the means, and the standard deviations in the a y-arrays are respectively n_x, $\bar{y}_x$, and s_x ($x = 1, 2, \ldots a$). The set Ω‡ of all possible populations from which Σ may have been drawn is that in which the y-arrays are homoscedastic normal curves, but the regression of y on x as well as the distribution of the x-arrays and of both marginal distributions may be of any form whatever§. The sub-set ω of Ω contains the populations for which the regression of y on x is given by the law (9). As a step in measuring the probability that Σ is a sample from a member of ω we shall find the likelihood of this composite hypothesis. Let Π be a member of Ω for which the standard deviation in the y-arrays is σ, and the proportions in the marginal totals of these arrays are $p_1, p_2, \ldots p_a$, where of course

$$p_1 + p_2 + \ldots + p_a = 1 \qquad \text{(10)}.$$

* "On the Use and Interpretation of Certain Test Criteria for Purposes of Statistical Inference," *Biometrika*, Vol. xx^A. pp. 175—240 and 264—294.

† *Biometrika*, Vol. IV. pp. 332—350.

‡ This terminology was explained in *Biometrika*, Vol. xx^A. pp. 263—265.

§ There is no need for the x-variate to be continuous; in fact, if it be, the distribution in the y-arrays is only likely to be normal if the number of arrays, a, be fairly large. If, for example, dx be the breadth of a y-array and we are dealing with a bivariate normal surface, then the array distributions will only be strictly normal in the limit as dx tends to zero.

Then the chance of drawing from Π a sample

(1) in which n_x individuals come from the xth array ($x = 1, 2, \ldots a$),

(2) where within the array the observations lie between the limits $y_{tx} - \frac{1}{2}h$ and $y_{tx} + \frac{1}{2}h$ ($t = 1, 2, \ldots n_x$, $x = 1, 2, \ldots a$),

will be in the limit as $h \to 0$ asymptotic to

$$C = \frac{N!}{n_1! \ldots n_a!} (p_1)^{n_1} \ldots (p_a)^{n_a} \frac{1}{(\sqrt{2\pi}\sigma)^N} e^{-\frac{1}{2\sigma^2} \underset{x}{S} \underset{t}{S} (y_{tx} - Y_x)^2} h^N \quad \text{.........(11)}.$$

Taking logarithms we find

$$\log C = \text{constant} + \overset{a}{\underset{x=1}{S}} (n_x \log p_x) - N \log \sigma - \frac{1}{2} \left\{ \overset{a}{\underset{x=1}{S}} (n_x s_x^2 + n_x (\bar{y}_x - Y_x)^2) \right\} \Big/ \sigma^2,$$

where the constant term is a function of h and the sample frequencies only. To determine $\Pi(\Omega \max)$ we maximise $\log C$ with regard to the variables σ, Y_x and p_x ($x = 1, 2, \ldots a$). The result gives

$$\left. \begin{aligned} \sigma^2 &= \overset{a}{\underset{x=1}{S}} (n_x s_x^2)/N \\ Y_x &= \bar{y}_x, \ (x = 1, 2, \ldots a) \\ p_x &= n_x/N, \ (x = 1, 2, \ldots a) \end{aligned} \right\} \quad \text{........................(12)},$$

and hence as $h \to 0$,

$$C(\Omega \max) = \frac{1}{(\sqrt{2\pi})^N} e^{-\frac{1}{2}N} \left\{ \frac{S(n_x s_x^2)}{N} \right\}^{-\frac{N}{2}} \frac{N!}{N^N} \frac{n_1^{n_1}}{n_1!} \ldots \frac{n_a^{n_a}}{n_a!} h^N \quad \text{......(13)}.$$

To determine $\Pi(\omega \max)$ we maximise $\log C$ with regard to σ; $\alpha_1, \alpha_2, \ldots \alpha_c$; $p_1, p_2, \ldots p_a$; where the α's are the c undetermined parameters contained in the expression (9). The solution is now

$$\sigma^2 = \overset{a}{\underset{x=1}{S}} \{n_x s_x^2 + n_x (\bar{y}_x - Y_x)^2\}/N \quad \text{........................(14)},$$

$$p_x = n_x/N, \quad (x = 1, 2, \ldots a) \quad \text{............................(15)},$$

and for Y_x we have the values obtained by the solution of the equation

$$\overset{a}{\underset{x=1}{S}} \left\{ n_x (\bar{y}_x - Y_x) \frac{\partial Y_x}{\partial \alpha_t} \right\} = 0, \quad (t = 1, 2, \ldots c) \quad \text{..............(16)},$$

which are the same as those found by minimising $\overset{a}{\underset{x=1}{S}} \{n_x (\bar{y}_x - Y_x)^2\}$, or from fitting (9) by least squares to the observations. Inserting these values into (11) we obtain an expression for $C(\omega \max)$ identical with (13) except that the sample array means and standard deviations occur in a term of form

$$\overset{a}{\underset{x=1}{S}} \left\{ \frac{n_x s_x^2 + n_x (\bar{y}_x - Y_x)^2}{N} \right\}^{-\frac{N}{2}}.$$

It follows that the likelihood of the composite hypothesis becomes

$$\lambda = \frac{C(\omega \max)}{C(\Omega \max)} = \left\{ 1 + \frac{\overset{a}{\underset{x=1}{S}} n_x (\bar{y}_x - Y_x)^2}{\overset{a}{\underset{x=1}{S}} (n_x s_x^2)} \right\}^{-\frac{N}{2}} \quad \text{..............(17)},$$

the values of Y_x being obtained by fitting (9) to the weighted sample array means by the method of least squares. Following a common notation, if s_y be the standard deviation in the y-margin of the sample, we may write

$$\left.\begin{aligned}\overset{a}{\underset{x=1}{S}}(n_x s_x^2) &= N(1-\eta_{yx}^2)\,s_y^2\\ \overset{a}{\underset{x=1}{S}}\{n_x(\bar{y}_x-Y_x)^2\} &= N(\eta_{yx}^2-R^2)\,s_y^2\end{aligned}\right\} \quad\text{..................(18),}$$

so that

$$\lambda=\left\{1+\frac{\eta_{yx}^2-R^2}{1-\eta_{yx}^2}\right\}^{-\frac{N}{2}}=\left\{\frac{1-R^2}{1-\eta_{yx}^2}\right\}^{-\frac{N}{2}} \quad\text{..................(19).}$$

The hypothesis to be tested is most likely to be true when $R=\eta_{yx}$ and $\lambda=1$, and becomes more and more improbable as λ decreases. The completion of the solution depends upon finding the distribution of λ in sampling from a member of ω. This has been done by R. A. Fisher and is a special case of his general distribution (6) given above (p. 338). The quantity whose distribution he has obtained is not λ, but a function of λ which we may call θ, defined by the relation

$$\lambda=(1+\theta)^{-\frac{1}{2}N},$$

or

$$\theta=\frac{\eta_{yx}^2-R^2}{1-\eta_{yx}^2}=\frac{\overset{a}{\underset{x=1}{S}}\{n_x(\bar{y}_x-Y_x)^2\}}{\overset{a}{\underset{x=1}{S}}(n_x s_x^2)} \quad\text{.....................(20).}$$

As λ varies from 1 to 0, θ varies from 0 to ∞. If we divide the denominator of the expression for θ by $N-a$, it will be seen that it becomes the ratio of the weighted sum of the squares of the deviations of the sample array means from the fitted regression curve to a weighted estimate of the population array variance. Without therefore introducing the idea of likelihood, θ appears to be a natural criterion to use in judging the deviation of the observed regression from expected type.

(3) THE SAMPLING DISTRIBUTION OF θ.

The proof has been given by Fisher in somewhat condensed form*. It may be divided into the following steps:

(a) The set of all possible samples, Γ, from a population Π can be divided into a number of sub-sets within any one of which, say γ, the totals of the y-arrays have a fixed series of values $n_1, n_2, \dots n_a$. The chance of drawing a given sample Σ from Π can then be represented by the product of (1) the chance that Σ belongs to γ, or $C_\gamma=\frac{N!}{n_1!\dots n_a!}(p_1)^{n_1}\dots(p_a)^{n_a}$, and (2) the chance of obtaining the observed value of the variates on drawing a random sample of n_1 from the first population array, n_2 from the second, and so on. The solution is simplified immensely by first obtaining the distribution of θ among the samples of a single sub-set.

* *Journal of the Royal Statistical Society*, Vol. LXXXV. pp. 597—611.

(*b*) Take $$k = \mathop{S}_{x=1}^{a} \{n_x (\bar{y}_x - Y_x)^2\}/\sigma^2 \quad \text{.........................(21)}.$$

Then within the samples of the sub-set γ, if $\tilde{y}_x$ be a true population array mean, $\sqrt{n_x}(y_x - \tilde{y}_x)$ is a quantity normally distributed about zero with standard deviation σ. The sum contains the squares of a such quantities. The effect of using Y_x, found by fitting a regression curve to the observations, instead of $\tilde{y}_x$, can be shown, at any rate in certain important cases, to give for the distribution of k a curve of the form of (3), where $N = a$, the number of arrays, and c is the number of constants in the fitted regression curve (9)*. That is to say we have

$$f(k)\, dk = \text{constant} \times k^{\frac{a-c-2}{2}} e^{-\frac{1}{2}k} dk \quad \text{.................(22)}.$$

(*c*) Take $$q = \mathop{S}_{x=1}^{a} \left\{ \mathop{S}_{t=1}^{n_x} (y_{xt} - \bar{y}_x)^2 \right\} \Big/ \sigma^2 = \mathop{S}_{x=1}^{a} (n_x s_x^2)/\sigma^2 \text{.................(23)}.$$

This is the special case arising from equations (2) and (3) referred to on p. 337 above, where X_r is the mean of the group of n_r observations. The distribution of q is of form (3); there are N observations and the number of groups is a, hence

$$f(q)\, dq = \text{constant} \times q^{\frac{N-a-2}{2}} e^{-\frac{1}{2}q} dq \quad \text{.................(24)}.$$

(*d*) Finally within γ, as the population y-arrays are normal, k and q are independent, the first depending only on the variation in means, the second on that of standard deviations. It is therefore easy to obtain from (22) and (24) the distribution of $\theta = k/q$, namely,

$$f(\theta)\, d\theta = \text{constant} \times \theta^{\frac{a-c-2}{2}} (1+\theta)^{-\frac{N-c}{2}} d\theta \text{..............(25)}.$$

This distribution is not only independent of σ but also of the array totals $n_1, n_2, \ldots n_a$. It will therefore hold within all sub-sets γ, and hence for the aggregate of all possible samples Γ. The probability integral of (25) provides in fact the means of testing the hypothesis regarding the form of the regression curve. Various methods of obtaining this probability integral will be considered below.

(4) THE EFFECT OF NON-NORMALITY.

Dr Fisher's test can be used in examining the goodness of fit of linear and non-linear regression curves, but it has involved two large assumptions, first that the distributions in the y-arrays are normal, and next that they have the same standard deviations. In cases of non-linear regression it is often found that the array standard deviations change, while the form of the curve may pass from symmetry through increasing degrees of skewness. With linear regression the assumptions are more likely to be justified. As the population diverges from normal form the test will become less and less efficient, partly because the criterion

* This result appears to be exact provided that the constants $a_1, a_2, \ldots a_c$ appear in (9) in *linear* form; for example, if the curve be a parabola, or even a hyperbola of type $Y_x = a_1 + \frac{a_2}{x} + \ldots + \frac{a_c}{x^{c-1}}$. In the more general case it may perhaps be only true as an approximation.

θ is no longer the most appropriate one to use, and partly because its sampling distribution will cease to conform to (25), but it would be almost impossible to say at what point it becomes invalid. The practical situation seems, however, to be this; the statistician who is not dealing with very large samples has often no means of judging the exact form of his population distribution. It is therefore of first importance that he should feel some confidence that moderate deviations from normal homoscedasticity will not make worthless any conclusions which he may draw by referring z to Fisher's tables or $\theta = k/q$ to the distribution (25). The problem is a large one, seeing in how many directions non-normality may arise, but a simple illustration will throw some light upon it.

Suppose that the distributions in the y-arrays of the population are homoscedastic non-normal curves with the frequency constants σ, β_1 and β_2. If the means of the population arrays, $\tilde{y}_x$, were known, we could calculate

$$k' = \mathop{S}_{x=1}^{a} \{n_x(\bar{y}_x - \tilde{y}_x)^2\}/\sigma^2 = \mathop{S}_{x=1}^{a} (v_x^2)/\sigma^2 \quad \text{.....................(26).}$$

Within the sub-set of samples, γ, defined on p. 341 above, v_x will vary about zero with standard deviation σ and with a second "beta coefficient" defined by

$${}_xB_2 = 3 + (\beta_2 - 3)/n_x.$$

It follows that in repeated sampling within γ,

$$\text{Mean } k' = a,$$

$$\text{Mean } (k')^2 = \left\{ \mathop{S}_{x=1}^{a} (\text{Mean } v_x^4) + 2S' (\text{Mean } v_x^2 v_{x'}^2) \right\} \Big/ \sigma^{4}*$$

$$= \mathop{S}_{x=1}^{a} ({}_xB_2) + 2S' \{\text{Mean } v_x^2 \times \text{Mean } v_{x'}^2\}/\sigma^4$$

(since within γ, v_x and $v_{x'}$ are uncorrelated)

$$= \mathop{S}_{x=1}^{a} ({}_xB_2) + a(a-1) = a^2 + 2a + (\beta_2 - 3) \mathop{S}_{x=1}^{a} \left(\frac{1}{n_x}\right).$$

Hence $$\sigma_{k'}^2 = \text{Mean } (k')^2 - (\text{Mean } k')^2 = 2a + (\beta_2 - 3) \mathop{S}_{x=1}^{a} \left(\frac{1}{n_x}\right) \quad \text{.........(27).}$$

It is seen that the mean value of k' is the same whatever form be the population array, but (27) shows that the variability of k' depends upon β_2, and further is not the same within each of the sub-sets γ, varying according to the marginal totals n_x. The second term of (27) will usually, however, be very small compared with the first, and we may conclude that unless the population arrays are extremely leptokurtic or the sample very small, the distribution of k' will not differ seriously from that of "normal theory." The quantity with which we are really concerned is the k of (21), obtained by using the Y_x's of the fitted regression curve; its variability would appear harder to determine, but it seems likely that just as for k' the equation (22) will represent its sampling distribution with fair accuracy provided the sample is not very small or the array β_2 large.

* S' implies the summation for all possible pairs out of the a-arrays.

We may now consider the modifications connected with the q of (23). Within the sub-set γ, we know that

$$\begin{cases}\text{Mean}\,(s_x^2) = (n_x - 1)\,\sigma^2/n_x,\\ \text{Mean}\,(s_x^4) = (n_x - 1)\,\{(n_x - 1)\,\beta_2 + (n_x^2 - 2n_x + 3)\}\,\sigma^4/n_x^{3}*.\end{cases}$$

Hence
$$\text{Mean}\,q = \underset{x=1}{\overset{a}{S}}\,\{n_x\,\text{Mean}\,s_x^2\}/\sigma^2 = \underset{x=1}{\overset{a}{S}}\,(n_x - 1) = N - a,$$

$$\text{Mean}\,q^2 = \underset{x=1}{\overset{a}{S}}\,\{n_x^2\,\text{Mean}\,s_x^4\}/\sigma^4 + 2S'\,\{n_x n_{x'}\,\text{Mean}\,s_x^2 \times \text{Mean}\,s_{x'}^2\}/\sigma^4,$$

as within γ, s_x^2 and $s_{x'}^2$ are uncorrelated. Substituting the values for Mean s_x^2 and Mean s_x^4 it is found after reduction that

$$\text{Mean}\,q^2 = (N-a)^2 + 2\,(N-a) + (\beta_2 - 3)\left\{N - 2a + \underset{x=1}{\overset{a}{S}}\left(\frac{1}{n_x}\right)\right\},$$

or
$$\sigma_q^2 = 2\,(N-a)\left\{1 + (\beta_2 - 3)\,\frac{N-2a}{2\,(N-a)} + \frac{\beta_2 - 3}{2\,(N-a)}\underset{x=1}{\overset{a}{S}}\left(\frac{1}{n_x}\right)\right\} \quad \text{......(28).}$$

Again the mean value of q is independent of the population array form, but σ_q differs from the "normal theory" value of $\sqrt{2\,(N-a)}$. Although the third term within the brackets in (28) may be small, the second term will often not be negligible compared to unity. For example, if $\beta_2 = 4$, and we are dealing with very large samples, this second term will be of the order of 0·5. We must conclude therefore that if the population array distributions are distinctly platykurtic or leptokurtic, the variability of q will be affected and the "normal theory" law (24) begin to fail, although still giving the correct mean for q. The denominator of the ratio $\theta = k/q$ is in fact more sensitive to changes in population form than the numerator. If the array curves are skew, another feature is introduced owing to the correlation between deviations in mean and variance; that is to say $\bar{y}_x$ and s_x will be correlated. This will lead to a correlation between k and q which, provided that it is positive†, should have in the ratio θ somewhat the same compensating effect as in "Student's" ratio z when the population is not normal‡.

(5) Sampling Experiments.

To illustrate further the result of non-normality in this and certain other problems two series of sampling experiments have been carried out. The first, in which the arrays were both normal and homoscedastic, does no more than confirm the unquestioned accuracy of "normal theory" as set out in equations (22), (24) and (25), but it will be of more value in connection with the distribution of r. In the second experiment the standard deviation in the arrays was varied and the distributions were taken to be Type III curves.

* This value for the mean of the square of the variance is taken from Dr Church's paper in *Biometrika*, Vol. XVII. p. 81.

† If the array distributions are leptokurtic this correlation will presumably be positive. If, however, they were for instance "rectangular," large deviations in $\bar{y}_x$ would be associated with low values of s_x^2, leading to a negative correlation between k and q which would tend to increase the variability of θ.

‡ See *Biometrika*, Vol. XXI. p. 259 *et seq.*

Experiment I.

The population contained three arrays ($a=3$) with proportions $p_1 = \cdot 40$, $p_2 = \cdot 35$, $p_3 = \cdot 25$. The three array distributions were normal and homoscedastic, and the regression of y on x was linear, the coefficient of correlation being $\rho = \cdot 5346$. The sampling was carried out with the help of Tippett's Random Numbers*, the grouping unit for y being $\frac{1}{8}$ of the array standard deviation. 200 random samples of 20 were taken and k, q and θ, as defined above, calculated in each case. In fitting a sloping regression straight line to each sample we are using a law (9) of form

$$Y_x = a_1 + a_2 x \quad \text{.................................(29)},$$

that is to say $c=2$, while $N=20$, $a=3$.

Distribution of k.

Equation (22) becomes

$$f(k)\,dk = \text{constant} \times k^{-\frac{1}{2}} e^{-\frac{1}{2}k}\,dk \quad \text{....................(30)},$$

which is the distribution of χ^2 with $n'=2$. The following results were obtained:

Mean k; Theory 1·000, Observation 1·109, Standard Error† 0·100.
σ_k; „ 1·414, „ 1·538, „ 0·187.

The Goodness of Fit Test, using 11 groups, gave $P = \cdot 416$.

Distribution of q.

Equation (24) becomes

$$f(q)\,dq = \text{constant} \times q^{\frac{15}{2}} e^{-\frac{1}{2}q}\,dq \quad \text{....................(31)},$$

or the distribution of χ^2 with $n'=18$. The following results were obtained:

Mean q; Theory 17·000, Observation 17·035, Standard Error 0·412.
σ_q; „ 5·831, „ 5·586, „ 0·339.

The Goodness of Fit Test, using 16 groups, gave $P = \cdot 982$.

Distribution of $\theta = k/q$.

Equation (25) becomes

$$f(\theta)\,d\theta = \text{constant} \times \theta^{-\frac{1}{2}} (1+\theta)^{-9}\,d\theta \quad \text{................(32)}.$$

Using the transformation $\zeta = \theta/(1+\theta)$ we obtain the Type I distribution

$$f(\zeta)\,d\zeta = \text{constant} \times \zeta^{-\frac{1}{2}} (1-\zeta)^{\frac{15}{2}}\,d\zeta \quad \text{....................(33)},$$

whose probability integral depends on the Incomplete Beta Function. For the general distribution (25),

$$\text{Mean } \theta = \frac{a-c}{N-a-2}, \quad \sigma_\theta = \frac{1}{N-a-2}\sqrt{\frac{2(a-c)(N-c-2)}{N-a-4}} \quad \text{...(34)}.$$

* *Tracts for Computers*, No. xv.

† The standard errors are for Mean k and σ_k calculated from 200 samples. The first is $\sigma_k/\sqrt{N}$, and in the second case the approximation $\frac{1}{2}\sigma_k\sqrt{(\beta_2-1)/N}$ has been used, where here $N=200$ and β_2 refers to the theoretical distribution of k which for a χ^2 distribution has a value of $3+12/(n'-1)$. Similar expressions are used for q and θ.

Using these values with $N=20$, $a=3$, $c=2$, it was found that

Mean θ; Theory ·0667, Observation ·0727, Standard Error ·0074.
σ_θ; ·1046, „ ·1074.

The distribution of θ is a J-curve, and the expression used above for the standard error of a standard deviation will hardly be satisfactory. Using the transformation to a Type I curve, and comparing theory and observation for 11 groups, the Goodness of Fit Test gave $P=\cdot 409$.

Correlation between k and q.

As we should expect, there is no evidence for such a correlation. The observed values for the 200 samples are

$$r_{kq} = -\cdot 0342, \quad \eta_{kq}^2 = \cdot 1227.$$

The standard error for r_{kq} on "normal theory" is $1/\sqrt{200-1} = \cdot 0709$, while if we may consider the arrays of q in the k, q-correlation table sufficiently nearly normal to apply the test we are now discussing and as described in section (6) below, then equations (44) give Mean $\eta^2 = \cdot 1156$ and $\sigma_{\eta^2} = \cdot 0319$, so that the observed value of ·1227 is not significant.

Taken collectively these results show an admirable agreement between observation and theory.

Experiment II.

The population contained five arrays ($a=5$) with proportions and array standard deviations as follows:

Array	1	2	3	4	5
p_x	·1667	·2666	·2167	·1833	·1667
σ_x	5·7471	5·3191	5·0000	4·7619	4·5872

The standard deviations are in terms of the grouping unit employed for the sampling. The regression was linear and the coefficient of correlation was $\rho=+\cdot 4626$. The distribution in each array followed a Type III curve with $\beta_1 = 0\cdot 20$, $\beta_2 = 3\cdot 30$. If x is taken to be increasing as we pass from Array 1 to Array 5, and ρ is taken as positive, then these curves were negatively skew, the steeper tail pointing in the direction of increasing y. 300 random samples of 30 were now drawn, again using Tippett's Random Numbers. A sloping straight line

$$Y_x = \alpha_1 + \alpha_2 x \quad \text{.............................(29 \textit{bis})}$$

was fitted to each sample, so that $N=30$, $a=5$, $c=2$.

Distribution of k.

The population array standard deviations vary, but the weighted mean of the variances, or $\bar{\sigma}^2 = 26\cdot 1306$, has been substituted for σ^2 in the expression for k, (21), and also later in that for q, (23). Equation (22) becomes

$$f(k)\,dk = \text{constant} \times k^{\frac{1}{2}} e^{-\frac{1}{2}k}\,dk \quad \text{.......................(35).}$$

The following results were obtained:

Mean k; Theory 3·000, Observation 2·842, Standard Error* 0·141.
σ_k; „ 2·449, „ 2·457, „ 0·173.

Theoretical frequencies were obtained from the *Tables of the Incomplete Gamma Function*, taking $p=0{\cdot}5$ and $u=k/\sqrt{6}$; testing for goodness of fit with 14 groups it was found that $P=\cdot 329$. A comparison of cumulative frequencies is given in Table I.

Distribution of q.

Equation (24) becomes

$$f(q)\,dq = \text{constant} \times q^{\frac{23}{2}} e^{-\frac{1}{2}q}\,dq \quad \ldots\ldots\ldots\ldots\ldots\ldots(36).$$

The following results were obtained:

Mean q; Theory 25·000, Observation 25·127, Standard Error 0·408.
σ_q; „ 7·071, „ 8·052, „ 0·321.

Theoretical frequencies were again obtained from the *Incomplete Gamma Function Tables* taking $p=11{\cdot}5$, $u=q/\sqrt{50}$, and on testing for goodness of fit with 13 groups it was found that $P=\cdot 474$.

TABLE I.

Frequency Distributions from Experiment II.

Distribution of k			Distribution of q			Distribution of 25θ		
k greater than:	Observation	Normal Theory	q greater than:	Observation	Normal Theory	25θ greater than:	Observation	Normal Theory
0·0	300	300·0	8	300	299·8	0·0	300	300·0
0·5	272	275·7	10	297	299·0	0·4	281	281·4
1·0	237	240·4	12	293	296·0	0·8	245	254·2
1·5	192	204·7	14	283	288·5	1·2	219	225·8
2·0	158	171·7	16	269	274·4	1·6	194	198·6
2·5	128	142·6	18	244	252·7	2·0	171	173·6
3·0	111	117·5	20	216	224·0	2·4	141	151·2
3·5	87	96·2	22	183	190·7	2·8	124	131·3
4·0	66	78·4	24	154	155·8	3·2	97	113·8
4·5	53	63·7	26	122	122·3	3·6	79	98·6
5·0	42	51·5	28	96	92·4	4·0	73	85·4
5·5	37	41·6	30	68	67·3	4·4	63	73·9
6·0	32	33·5	32	54	47·4	4·8	57	64·0
7·0	26	21·6	34	39	32·4	5·2	47	55·4
8·0	13	13·8	36	29	21·5	5·6	42	48·0
9·0	6	8·8	38	22	13·9	6·0	37	41·6
10·0	5	5·6	40	17	8·8	6·4	31	36·1
11·0	4	3·5	42	12	5·4	7·2	21	27·2
			44	10	3·3	8·0	17	20·6
			46	7	2·0	8·8	15	15·6
			48	2	1·2	9·6	11	11·9
						10·4	8	9·1
						11·2	6	7·0
						12·0	4	5·4

* The standard errors were calculated as for Experiment I, using $N=300$ (see footnote to p. 345).

Distribution of $\theta = k/q$.

Equation (25) becomes

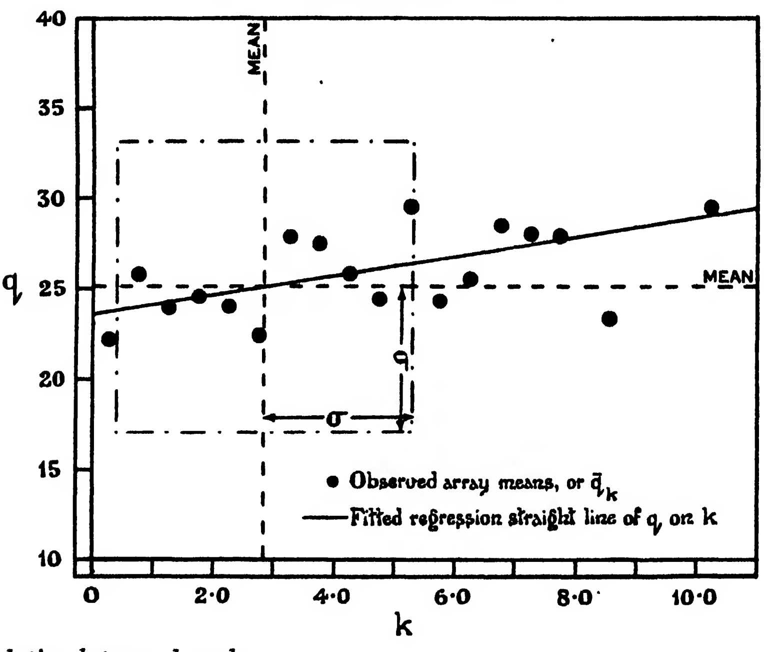
$$f(\theta)\,d\theta = \text{constant} \times \theta^{\frac{1}{2}}(1+\theta)^{-14}\,d\theta \quad \text{.....................(37).}$$

Using equations (34) we obtain the following comparison:

Mean θ;	Theory ·1304,	Observation ·1195,	Standard Error ·0068.
σ_θ;	„ ·1185,	„ ·1052.	
σ_θ^2;	„ ·01404,	„ ·01107,	„ ·00238.

The distribution of θ is so skew that the approximation to the standard error of a standard deviation used above is of doubtful value. The variances have therefore been compared, and differ by 1·25 times the standard error*. A comparison of goodness of fit was obtained by calculating the mid-ordinates of the Type VI curve for θ, (37), and correcting to obtain the group frequencies. Using 17 groups, a value of $P = {}$·270 was obtained. The cumulative frequencies are compared in Table I.

FIG. I. CORRELATION OF q AND k.



Correlation between k and q.

It was found that $r_{kq} = +$·1604; the standard error for zero theoretical correlation, were k and q normally distributed, is ·0578, so that r_{kq} differs from zero by about 2·8 times this standard error. The means of q for constant k are plotted in Fig. 1, where the observed regression straight line of q or k, or

$$\bar{q}_k = 23\text{·}6669 + \text{·}5278k \quad \text{.............................(38),}$$

has also been drawn.

* For 300 samples the standard error of the variance is very nearly $\sigma_\theta^2\sqrt{(\beta_2-1)/N}$, where the β_2 for distribution (37) is approximately 9·67.

If these results are taken as a whole it will be seen that there is nowhere any marked difference between the observed distributions and those of "normal theory." Owing to the changing array standard deviations the position is not as simple as that considered in section (4) above, but there seems to be evidence that the changes there contemplated are beginning to occur. We may note:

(*a*) The distribution of k is in good agreement with theory.

(*b*) The mean q differs from the expected value by only about one-third its standard error, but the observed σ_q is significantly greater than the "normal theory" value. This is as we should expect from (28), the array β_2 being 3·3; the slight excess of large values of q can be seen in Table I.

(*c*) The observed mean θ and σ_θ are a little low, but hardly significantly so, and a comparison of the cumulative frequencies of 25θ in Table I does not suggest that any serious error would be introduced by making use of the θ distribution (25).

(*d*) Finally a positive correlation between k and q has appeared which is probably significant.

There are, of course, so many directions in which the population form may be modified and so many changes to be rung in the values of N, a, c and ρ that it would be dangerous to draw too sweeping conclusions from a single experiment. Yet, as far as it goes, this appears to be a satisfactory result, and it suggests that in cases where we believe that the deviations from normal homoscedasticity in the y-arrays are of about the order existing in this experimental population, Fisher's test may be used with confidence.

(6) THE PRACTICAL DETERMINATION OF THE PROBABILITY INTEGRAL OF $f(\theta)$.

Let us first restate the problem; it is that of testing the hypothesis that a given sample comes from a population in which the regression of y on x follows a curve

$$Y_x = f(x; \alpha_1, \alpha_2, \dots \alpha_c) \qquad \text{(9 bis)}.$$

We either know that the population y-arrays are normal homoscedastic curves, or are prepared to take the risk of assuming that the deviation from this form is not sufficient to invalidate the test. We fit the regression curve to the sample by least squares and calculate

$$\theta = \frac{\eta_{yx}^2 - R^2}{1 - \eta_{yx}^2} = \frac{\overset{a}{\underset{x=1}{S}} \{n_x(\bar{y}_x - Y_x)^2\}}{\overset{a}{\underset{x=1}{S}} (n_x s_x^2)} \qquad \text{(20 bis)}.$$

Since it is when θ is large that the hypothesis is unlikely to be true, we refer this quantity to the distribution it would follow in repeated samples were the hypothesis true, namely,

$$f(\theta)\, d\theta = \text{constant} \times \theta^{\frac{a-c-2}{2}} (1+\theta)^{-\frac{N-c}{2}} d\theta \qquad \text{(25 bis)},$$

find $P_\theta = \int_\theta^\infty f(\theta)\, d\theta$, and on the basis of these odds judge whether or no so great a value of θ is likely to have arisen through chance fluctuations. The two simplest cases that arise are when:

(*a*) $c = 1$, and we wish to test the hypothesis that the population array means are constant. In this case the fitted regression line $Y_x = \alpha_1$ becomes $Y_x = \bar{y}$, the mean of the N individuals in the sample, while from the definition of (18) $R = 0$ and $\theta = \eta^2/1 - \eta^2$.

(*b*) $c = 2$, and we wish to test the hypothesis that the regression curve is linear but not necessarily parallel to the axis of x. Here $Y_x = \alpha_1 + \alpha_2 x$ is the ordinary regression straight line of y on x, and $R = r_{xy}$, the coefficient of correlation in the sample. Then $\theta = (\eta^2 - r^2)/(1 - \eta^2)$.

We shall now discuss several methods of calculating P_θ, the chance of obtaining in random sampling a more divergent result than that observed.

1. *R. A. Fisher's Method.*

(25) is a special case of Fisher's general distribution referred to in section (1), which he takes as

$$f(\theta)\, d\theta = \text{constant} \times \theta^{\frac{n_1 - 2}{2}} (1 + \theta)^{-\frac{n_1 + n_2}{2}}\, d\theta \quad \text{..............(39).}$$

Writing $z = \frac{1}{2} \log_e \left(\frac{n_2}{n_1} \theta\right)$, it follows that the distribution of z* is

$$f(z)\, dz = \text{constant} \times \frac{e^{n_1 z} dz}{(n_1 e^{2z} + n_2)^{\frac{1}{2}(n_1 + n_2)}} \quad \text{................(40).}$$

Tables VI of his *Statistical Methods for Research Workers* give for different values of n_1 and n_2 the values of z corresponding to the ·05 and ·01 proportionate tail areas of the z curve. In the present problem $n_1 = a - c$, $n_2 = N - a$, and

$$z = \frac{1}{2} \log_e \left\{\frac{\eta^2 - R^2}{1 - \eta^2} \cdot \frac{N - a}{a - c}\right\} \quad \text{.........................(41).}$$

The Tables can be entered with integral values of n_1 from 1 to 6, then for 8, 12, 24 and ∞; and of n_2 from 1 to 30, then for 60 and ∞. For many purposes this is adequate, but greater refinement is sometimes required.

2. *T. L. Woo's Tables.*

These have been published in the present volume of this Journal. They were primarily intended for testing the significance of a value of η^2, i.e. for the case $c = 1$. If we use the transformation $\zeta = \theta/(1 + \theta)$, equation (25) becomes of the form of (8) or

$$f(\zeta)\, d\zeta = \text{constant} \times \zeta^{\frac{a - c - 2}{2}} (1 - \zeta)^{\frac{N - a - 2}{2}}\, d\zeta \quad \text{..............(42),}$$

where

$$\zeta = \frac{\eta^2 - R^2}{1 - R^2} \quad \text{...................................(43).}$$

Then if $c = 1$, $\zeta = \eta^2$, and if $c = 2$, $\zeta = (\eta^2 - r^2)/(1 - r^2)$.

* This z must be distinguished from "Student's" z.

Mr Woo has taken $c=1$, and his tables are entered with N and n, which is a of the present paper. They may, however, be used for any value of c by equating his N to our $N-c+1$ and his n to our $a-c+1$. The tables give for a wide range of values of N and n*, (1) Mean ζ, (2) σ_ζ, and (3) the ratio $(\zeta - \text{Mean } \zeta)/\sigma_\zeta$ corresponding to tail areas of about ·02 and ·01.

3. *Other Methods of Approximation.*

The *Tables of the Incomplete Beta Function*, which are nearing completion in the Biometric Laboratory, will give the probability integral of (42) for a certain range of values of N, a and c, but it seems of interest to describe a form of approximation adequate for moderately large samples based on the Type III curve and the Incomplete Gamma Function. For the Type I curve written in the form

$$y = y_0 \zeta^{p-1}(1-\zeta)^{q-1}$$

we have the following moment constants:

$$\left.\begin{aligned} \text{Mean} &= \frac{p}{p+q} = \frac{a-c}{N-c} \text{ if } p = \tfrac{1}{2}(a-c),\ q = \tfrac{1}{2}(N-a) \\ \text{Variance} &= \frac{pq}{(p+q)^2(p+q+1)} = \frac{2(a-c)(N-a)}{(N-c)^2(N-c+2)} \\ \beta_1 &= \frac{4(p-q)^2(p+q+1)}{pq(p+q+2)^2} = \frac{8(N-2a+c)^2(N-c+2)}{(N-a)(a-c)(N-c+4)^2} \end{aligned}\right\} \quad \text{.(44).}$$

Further, we know that

$$6(\beta_2 - \beta_1 - 1)/(2\beta_2 - 3\beta_1 - 6) = -(p+q),$$

and consequently

$$2\beta_2 - 3\beta_1 - 6 = -12(\beta_2 - \beta_1 - 1)/(N-c) \quad \text{..............(45).}$$

The relation (45) suggests that if N be not too small the (β_1, β_2) point of the curve of ζ, (42), will lie close to the Type III line. The extent to which this is so is illustrated in Fig. 2, which shows for $c=2$ how for a constant number of arrays, a, the point converges on the Type III line as N increases. We shall therefore examine the adequacy of the following approximation · to represent the Type I curve (42) by a Type III curve with its mean, variance and β_1 having the values of (44), or approximations to these values.

The equation to the curve, whose integral $I(u, p)$ is given in the *Tables of the Incomplete Gamma Function*†, is

$$y = \text{constant} \times u^p e^{-\sqrt{p+1}\,u} \quad \text{..........................(46),}$$

$$u = (\text{deviation from start})/(\text{standard deviation}),$$

where $\left\{ \begin{array}{l} \sqrt{p+1} \times \text{standard deviation} = \text{distance from start to mean}, \\ p = 4/\beta_1 - 1. \end{array}\right.$

* $N=51$ to 1000 and $n=3$ to 20.

† His Majesty's Stationery Office, 1922.

Hence we must take

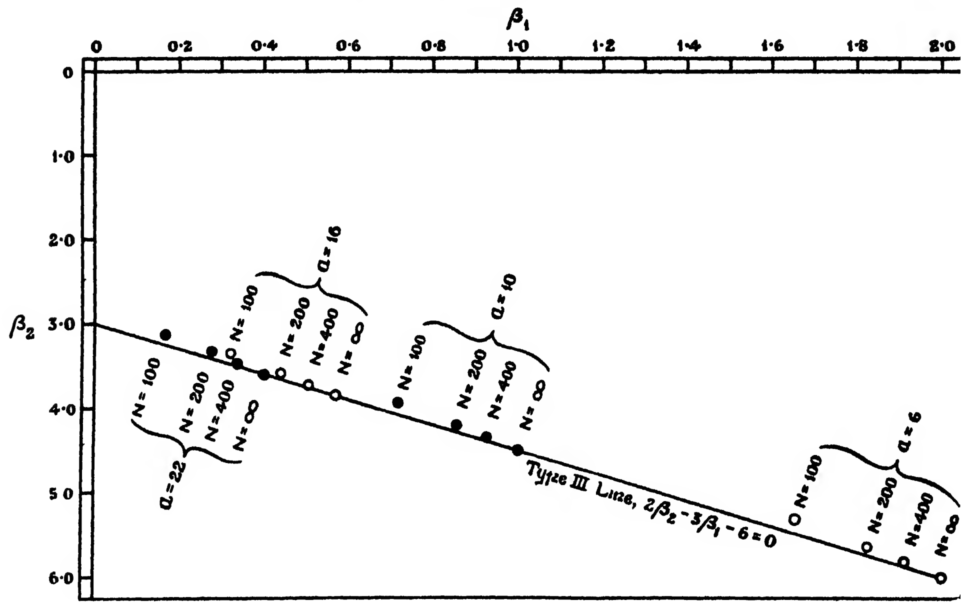
$$p = \frac{a-c}{2} \frac{(N-c-(a-c))(N-c+4)^2}{(N-c-2(a-c))^2(N-c+2)} - 1$$

$$= \frac{a-c}{2}\left(1-\frac{a-c}{N-c}\right)\left(1+\frac{8}{N-c}+\dots\right)\left(1+\frac{4(a-c)}{N-c}+\dots\right)\left(1-\frac{2}{N-c}+\dots\right)-1$$

$$= \frac{a-c}{2}\left\{1+\frac{3(a-c+2)}{N-c}+\dots\right\}-1 \quad\dots\dots(47),$$

where we have expanded in inverse powers of $N-c$.

FIG II. SHOWING THE β_1, β_2 POINTS FOR THE DISTRIBUTION OF ζ IN THE CASE $c=2$.



In the same way we may obtain an expansion for the expression for σ_ζ given in (44), namely,

$$\sigma_\zeta = \frac{\sqrt{2(a-c)}}{N-c}\left(1-\frac{a-c}{N-c}\right)^{\frac{1}{2}}\left(1+\frac{2}{N-c}\right)^{-\frac{1}{2}}$$

$$= \frac{\sqrt{2(a-c)}}{N-c}\left\{1-\frac{1}{2}\frac{a-c+2}{N-c}+\dots\right\} \quad\dots\dots(48).$$

Hence combining (47) and (48) we have

$$\text{Distance from start to mean} = \sqrt{p+1}\,\sigma_\zeta$$

$$= \frac{a-c}{N-c}\left\{1+\frac{a-c+2}{N-c}+\dots\right\} \quad\dots\dots(49).$$

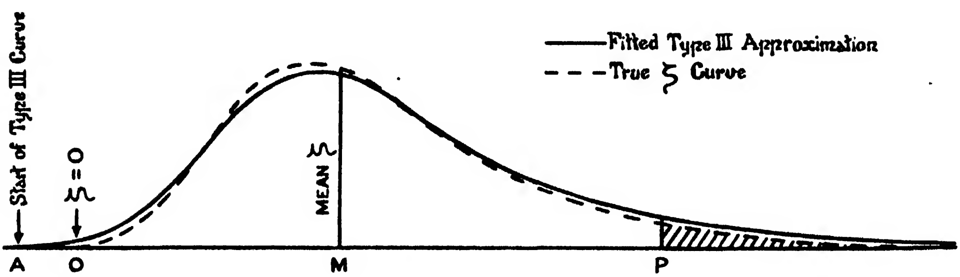
The fitted Type III curve does not start exactly at $\zeta=0$; the position is represented in Fig. 3. The Type III curve starts at A, the true curve at O, the means coincide at M, and it is desired to approximate to the tail area under the true curve beyond P by taking the corresponding area under the Type III curve,

$$\begin{aligned} AP &= OP + AO = OP + AM - OM \\ &= \zeta + \frac{a-c}{N-c}\left\{1+\frac{a-c+2}{N-c}\dots\right\} - \frac{a-c}{N-c} \\ &= \zeta + \frac{(a-c)(a-c+2)}{(N-c)^2} + \dots, \end{aligned}$$

using equations (44) and (49). The *Tables of the Incomplete Gamma Function* are now to be entered with the p of (47) and $u = AP/\sigma_\zeta$ or

$$u = \frac{(N-c)\left\{\zeta + \frac{(a-c)(a-c+2)}{(N-c)^2} + \dots\right\}}{\sqrt{2(a-c)}\left\{1 - \frac{1}{2}\frac{a-c+2}{N-c} + \dots\right\}} \quad \dots\dots\dots\dots\dots(50).$$

FIG. III



There are now possible two degrees of approximation.

Method I.

Take the p of (47) and the u of (50) as far as the terms given; this will involve interpolating for both p and u.

Method II.

Take $p=\frac{1}{2}(a-c)$, $u=N\zeta/\sqrt{2(a-c)}$, that is to say assume that a and c may be neglected compared with N. In this case it may only be necessary to interpolate for u*.

If the *Tables of the Incomplete Gamma Function* are not available, use can be made in certain cases of Elderton's χ^2 Tables in *Tables for Statisticians and Biometricians*. The χ^2 distribution is

$$y = \text{constant} \times (\chi^2)^{\frac{n'-3}{2}} e^{-\frac{1}{2}\chi^2} \quad \dots\dots\dots\dots\dots\dots\dots(51).$$

* In the *Incomplete Gamma Function Tables* (1922) the argument interval for p is 0·1 up to 5·0, but beyond this it is 0·2, e.g. there is a column for $p=4·5$, but for 5·5 we must interpolate between $p=5·4$ and 5·6.

This corresponds to (46) if we write

$$\chi^2 = 2u\sqrt{p+1}, \quad n' = 2p+3,$$

and consequently we have two approximations corresponding to Methods I and II.

Method III.

Enter Elderton's Tables with

$$\begin{cases} \chi^2 = (N-c)\left(1 + \dfrac{2(a-c+2)}{N-c}\right)\left(\zeta + \dfrac{(a-c)(a-c+2)}{(N-c)^2}\right), \\ n' = a-c+1+\dfrac{3(a-c)(a-c+2)}{N-c}. \end{cases}$$

It is here necessary to interpolate between the columns of n', which is not easy to do accurately.

Method IV.

To a rougher approximation use

$$\chi^2 = N\zeta, \quad n' = a-c+1.$$

Here n' will have an integral value and it is only necessary to interpolate for χ^2.

TABLE II.

Values of P_ζ.

Size of sample N	Number of arrays a	$\zeta = \frac{\eta^2 - r^2}{1-r^2}$	True P_ζ	P_ζ by I	P_ζ by II
100	6	·1141	·0193	·0190	·0284
200	6	·0618	·0136	·0136	·0145
	14	·1120	·0285	·0277	·0322
	14	·1191	·0176	·0173	·0208
	22	·1657	·0248	·0236	·0314
500	14	·0453	·0292	·0291	·0307
	14	·0482	·0184	·0183	·0196
	22	·0672	·0260	·0257	·0285
1000	22	·0338	·0264	·0263	·0277

By taking certain values from Mr Woo's tables, it has been possible to examine the closeness of approximation of Methods I and II; except for the difficulty in accurate interpolation III and IV would give the same results as I and II respectively. Suppose that we take the case $c = 2$, or are testing whether the regression of y or x is linear, and that we found in the nine samples with values for N and a shown in Table II, the values of $\zeta = (\eta^2 - r^2)/(1-r^2)$ given in the 3rd column. Then the true values of P_ζ* found by Mr Woo from the appropriate Type I distributions are set out in the 4th column, while those found by using the approximate

* That is to say the chance of ζ exceeding the observed value in random sampling were the hypothesis tested true.

Methods I and II are in the 5th and 6th columns. While not attempting to be mathematically exact, there can be little doubt that Method I gives values for P_ζ accurate enough for most practical statistical work. As we should expect for a given N the error increases as the number of arrays is increased. For N below 100 and a large number of arrays the approximation will no doubt become less satisfactory, but this field will be covered by the *Tables of the Incomplete Beta Function*. For large samples the gain in speed by using Method II may well be felt to compensate for the loss in accuracy.

These results only provide a comparison at the level of significance $P_\zeta = \cdot 03$ to $\cdot 01$. It seemed desirable to examine the degree of approximation throughout the whole range of the curve, and this has been done in three cases, namely, $N = 102$, $c = 2$, $a = 8$; $N = 202$, $c = 2$, $a = 14$; $N = 502$, $c = 2$, $a = 22$. The true probability integrals were found by quadrature of the curves

$$y = y_0 \zeta^2 (1-\zeta)^{46}; \quad y = y_0 \zeta^5 (1-\zeta)^{93}; \quad y = y_0 \zeta^9 (1-\zeta)^{239},$$

TABLE III.

Showing the Chance of Exceeding Certain Values of ζ.

$N=102$, $c=2$, $a=8$				$N=202$, $c=2$, $a=14$				$N=502$, $c=2$, $a=22$			
	P_ζ, or chance of exceeding ζ				P_ζ, or chance of exceeding ζ				P_ζ, or chance of exceeding ζ		
ζ	True value	Method I	Method II	ζ	True value	Method I	Method II	ζ	True value	Method I	Method II
·000	1·0000	·9996	1·0000	·000	1·0000	1·0000	1·0000	·000	1·0000	1·0000	1·0000
·012	·9789	·9731	·9940	·024	·9676	·9642	·9636	·016	·9926	·9919	·9918
·036	·7414	·7362	·8348	·050	·6250	·6170	·6116	·030	·7825	·7791	·7747
·060	·4301	·4233	·5295	·076	·2280	·2215	·2269	·038	·5256	·5212	·5193
·084	·2089	·2026	·2726	·102	·0540	·0522	·0582	·050	·1985	·1953	·1995
·108	·0893	·0859	·1219	·128	·0093	·0093	·0118	·064	·0399	·0391	·0427
·132	·0346	·0334	·0493	·154	·0012	·0013	·0020	·076	·0074	·0073	·0087
·156	·0123	·0122	·0186	·180	·0001	·0002	·0003	·090	·0008	·0008	·0011
·180	·0041	·0043	·0066					·102	·0001	·0001	·0002
·204	·0012	·0014	·0023								
·228	·0003	·0005	·0008								
·252	·0001	·0001	·0002								

ordinates being computed at intervals for ζ of ·003 in the first case and of ·002 in the other two cases. The results are shown in Table III. The adequacy of Method I for the common purposes of this test can hardly be questioned; Method II is less satisfactory, particularly for the sample of 102, but in all cases the agreement will be better as the number of arrays is decreased compared with the size of the sample.

(7) The Distribution of the Correlation Coefficient in the Experiments.

The sampling distribution of r first obtained by R. A. Fisher in 1915* is for two normally correlated and continuous variables. The population distributions of Experiments I and II are neither of them of this form. In the first case the y-arrays are normally distributed and contain five groups to the standard deviation, but there are only three alternative values of x, -1, 0 and $+1$; further, the proportions in these three x-marginal totals are $p_1 = \cdot 40$, $p_2 = \cdot 35$, $p_3 = \cdot 25$. That is to say, the x-distribution makes no approach either to normality or continuity. For Experiment II the y-arrays are skew curves with varying standard deviations, while there are five values for x, with proportional frequencies in the x-margin of $p_1 = \cdot 1667$, $p_2 = \cdot 2666$, $p_3 = \cdot 2167$, $p_4 = \cdot 1833$, $p_5 = \cdot 1667$. Here again there is no approach to a continuous normal distribution. Let us examine how closely the observed distributions of r conform to the sampling distributions of "normal theory."

TABLE IV.

Distribution of the Correlation Coefficient.

Experiment I			Experiment II		
r (Central Values)	Observed Frequency	Normal Theory Frequency	r (Central Values)	Observed Frequency	Normal Theory Frequency
−·05	1	0·8 (−·05 & below)	+·02	2	2·2 (+·02 & below)
·00	—	0·7	+·06	1	1·6
+·05	—	1·1	+·10	3	2·7
+·10	1	1·8	+·14	1	4·1
+·15	3	2·7	+·18	6	6·1
+·20	5	4·0	+·22	8	8·8
+·25	8	5·8	+·26	17	12·1
+·30	6	8·2	+·30	15	16·0
+·35	15	11·1	+·34	20	20·4
+·40	24	14·4	+·38	18	24·8
+·45	19	18·0	+·42	31	28·7
+·50	15	21·4	+·46	30	31·5
+·55	21	23·8	+·50	37	32·4
+·60	19	24·5	+·54	33	31·0
+·65	26	22·8	+·58	28	27·2
+·70	21	18·5	+·62	19	21·4
+·75	13	12·3	+·66	15	14·8
+·80	1	6·0	+·70	5	8·6
+·85	2	2·1 (+·85 & above)	+·74	5	4·0
			+·78	5	1·3
			+·82	1	0·3 (+·82 & above)
Total	200	200·0	Total	300	300·0

* *Biometrika*, Vol. x. pp. 507 *et seq.*

Experiment I.

Here $N = 20$, $\rho = \cdot 5346$ (population coefficient of correlation), and the theoretical distribution can be obtained by interpolating between the columns of ordinates for $\rho = \cdot 5$ and $\cdot 6$ given in Table A, p. 396, of the Cooperative Study on the distribution of r*. Second difference interpolation was used and a correction made to obtain group frequencies from mid-ordinates. The observed and theoretical results are compared in Table IV; the Goodness of Fit test with 11 groups gives $P = \cdot 223$. The following comparison was also made:

Mean r: Theory ·5244, Observation ·5160, Standard Error† ·0120.
σ_r: " ·1704, " ·1614, " ·0097.

These two quantities are somewhat less than the "normal theory" values, but the differences are less than the standard errors. The frequencies show some irregularity in the centre, but the numbers are not large enough to prove any significance in this.

Experiment II.

Here $N = 30$, $\rho = \cdot 4626$. We are now beyond the range of tables of ordinates contained in the "Cooperative Study." The theoretical frequencies given in Table IV were calculated with the help of Fisher's transformation by a method which will be described below. The agreement between "normal theory" and observation is excellent, the Goodness of Fit test with 14 groups giving $P = \cdot 916$. Further, we have the following comparison:

Mean r: Theory ·4563, Observation ·4631, Standard Error† ·0086;
σ_r: " ·1488, " ·1475, " ·0064;

the differences being again less than the standard errors.

These two series of results are of considerable interest and suggest that the normal bivariate surface can be mutilated and distorted to a remarkable degree without affecting the frequency distribution of r in samples as small as 20. The x-distribution in both cases has been made platykurtic, and it is possible that less satisfactory results would follow if the surface were pulled out into a more leptokurtic form.

(8) R. A. FISHER'S TRANSFORMATION OF THE r-DISTRIBUTION.

This method of transformation, which has been referred to in the preceding section, appears to be of such value in small sample work that it seems worth recording here the following examination of the degree of approximation involved. The equation for the distribution of r in samples of n may be written‡

$$f(r)\,dr = \text{constant} \times (1-r^2)^{\frac{n-4}{2}} \frac{\partial^{n-2}}{\partial(r\rho)^{n-2}} \left\{\frac{\cos^{-1}(-r\rho)}{\sqrt{1-r^2\rho^2}}\right\} dr \quad \text{.........(52).}$$

* *Biometrika*, Vol. XI. p. 396.

† The standard errors are calculated as described in the footnote to p. 345.

‡ *Biometrika*, Vol. X. p. 511.

Then the transformation

$$r = \tanh z, \quad \rho = \tanh \zeta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(53),$$

or

$$\left.\begin{aligned} z &= \tfrac{1}{2}\{\log_e(1+r) - \log_e(1-r)\} \\ \zeta &= \tfrac{1}{2}\{\log_e(1+\rho) - \log_e(1-\rho)\} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(54),$$

applied to (51) is such as to give for the distribution of z a close approximation to a normal curve with mean at ζ and standard deviation equal to $1/\sqrt{n-3}$. That is to say, the distribution of z is almost invariant in form with a standard deviation depending only on the size of the sample and not on ρ. The moment constants of the distribution of z have been given by Fisher in the form of series in inverse powers of $n-1$ *, and it is seen from these that the approximation is likely to be least satisfactory when ρ is large and n is small. The results shown in Table V have been computed for samples of 10 and of 20 from his series. Mean z differs from ζ by a quantity of the order of $\rho/2(n-1)$ and is the most variable of the expressions tabled. $1/\sqrt{n-3}$ is seen to be quite a good approximation to σ_z, at any rate in samples of 20, and if the distributions are slightly leptokurtic they are at any rate symmetrical.

TABLE V.

Moment Constants of Distribution of z.

ρ	$n=10$				$n=20$			
	Mean $z-\zeta$	σ_z	β_1	β_2	Mean $z-\zeta$	σ_z	β_1	β_2
·0	·0000	·375	·000 000	3·272	·0000	·2423	·000 000	3·116
·2	·0113	·375	·000 015	3·273	·0053	·2422	·000 002	3·117
·4	·0226	·374	·000 036	3·277	·0106	·2418	·000 004	3·118
·6	·0340	·372	·000 020	3·281	·0159	·2412	·000 002	3·118
·8	·0455	·369	·000 005	3·281	·0213	·2403	·000 001	3·116
·9	·0513	·367	·000 068	3·277	·0240	·2398	·000 007	3·114

$$\frac{1}{\sqrt{n-3}} = \cdot378 \qquad\qquad \frac{1}{\sqrt{n-3}} = \cdot2425$$

These results do not of course show whether sufficient terms are given in Fisher's series to insure convergence with n as low as 10, but it is possible to test the adequacy of the assumption that z is distributed normally in another way. Two tests were carried out.

Test (*a*). The moment coefficients of the true theoretical distribution of r are given as series in the Cooperative Study †. Taking a sample of 30 and $\rho = \cdot462\,579$ (as for Experiment II above), the following values were obtained:

$$\text{Mean } r = \cdot456\,265;\ \sigma_r = \cdot148\,818;\ \beta_1 = \cdot244;\ \beta_2 = 3\cdot252 \ldots\ldots\ldots(55).$$

* *Metron*, Vol. I. Part iv. pp. 13 and 14.

† *Biometrika*, Vol. XI. equations (xx), (xxi), (xxv), (xxvi).

Using Fisher's series from *Metron* we find

Mean $z = \cdot 508\,60$; $\sigma_z = \cdot 192\,04$ (N.B. $1/\sqrt{n-3} = \cdot 1925$); $\beta_1 = \cdot 000\,001$; $\beta_2 = 3\cdot 0742$ ....(56).

Next values of r at intervals of ·02 were taken between $-\cdot 24$ and $+\cdot 86$, and the corresponding values of z found from (54)*. The chance of a value of r lying in any of these subranges is the same as that of z lying in the corresponding subrange. We assume that z is normally distributed about ·508 60 with standard deviation ·192 04, obtain the proportional group frequencies from Sheppard's Tables of the Normal Curve, and hence have the grouped frequency distribution of r. The "normal theory" frequencies in the final column of Table IV above were obtained in this way. The moment constants of this distribution, were the process completely accurate, should be those of the series (55). Actually they were found to be

Mean $r = \cdot 4560$; $\sigma_r = \cdot 1489$; $\beta_1 = \cdot 229$; $\beta_2 = 3\cdot 175$ ....(57).

The agreement in the betas is not exact, but the z transformation seems to provide a quite adequate representation of the distribution of r.

TABLE VI.

Distributions of r.

r	Chance of r lying below values shown in 1st column: From quadrature with true ordinates	Chance of r lying below values shown in 1st column: From the z transformation
$-\cdot 6$	·000 008	·000 0003
$-\cdot 4$	·000 067	·000 011
$-\cdot 2$	·000 36	·000 13
·0	·0016	·0010
$+\cdot 2$	·0062	·0054
$+\cdot 4$	·0249	·0255
$+\cdot 6$	·1037	·1109
$+\cdot 8$	·4431	·4509
$+\cdot 85$	·6165†	·6193
$+\cdot 90$	·8133†	·8130
$+\cdot 95$	·9677†	·9688

Test (*b*). Suppose a sample of 10 taken from a normal population with $\rho = \cdot 8$. In this case the distribution of r is included in the Table A of the Cooperative Study (*loc. cit.* p. 386). It is seen to be a very skew curve with a modal ordinate at about $r = \cdot 85$, and $\beta_1 = 3\cdot 1377$, $\beta_2 = 8\cdot 0534$. Clearly it is not an easy distribution to handle, and but for these tables of ordinates we should be in difficulties when wanting to find the chance of r exceeding a certain value. The second column in

* A table of this function is given at the end of the *Metron* paper. Only about 1 sample in 10,000 lies outside the range $r = -\cdot 24$ to $+\cdot 86$.

† These values cannot be quite accurate as the r curve is too abrupt for a satisfactory quadrature from the tabled ordinates.

Table VI has been formed by applying quadrature to these ordinates. The z transformation leads to a distribution whose moment constants were calculated in forming Table V; they are

$$\text{Mean } z = \zeta + \cdot 0455 = 1\cdot 1441\,;\quad \sigma_z = \cdot 3691\,;\quad \beta_1 = \cdot 000\,005\,;\quad \beta_2 = 3\cdot 2808.$$

Now make the simplifying assumption that z is normally distributed about 1·1441 with a standard deviation of ·3691, and it is easy to find from Sheppard's Tables the chance of $z = \tanh^{-1} r$ exceeding any given value. Is the approximation adequate? The figures in Table VI suggest that for most purposes it is. It must also be remembered that in taking $n = 10$ and $\rho = \cdot 8$ we have chosen a most unfavourable case.

The author is very much indebted to Miss M. Page for the sampling and computing work for Experiment I; to Mr A. E. Stone for the sampling and Mrs L. J. Comrie for the computing for Experiment II; to Mr Ernest Martin for the computing required for the comparison of Table III; and to Miss Ida McLearn for the three diagrams.

INEQUALITIES FOR MOMENTS OF FREQUENCY FUNCTIONS AND FOR VARIOUS STATISTICAL CONSTANTS.

By J. SHOHAT (JACQUES CHOKHATE).

Introduction. The object of this paper is to derive certain inequalities for moments of frequency functions, and to show their applications, in particular, to the generalization of Bienaymé-Tchebycheff's criterion in the Theory of Probability. The following notations will be used: (a, b), finite or infinite, for the interval of distribution $(b > a)$; $F(x)$ for the law of distribution, so that $\int_a^b dF(x) = 1$; μ_s for the sth moment of the distribution about the origin, or

$$\mu_s = \int_a^b x^s dF(x) \qquad (s = 0, 1, 2, \ldots;\ \mu_0 = 1) \ldots (1);$$

$P_x^{c,d} \equiv P : [c \leqq x \leqq d]$ for the probability that the variable x satisfies the inequality $c \leqq x \leqq d$; $E(f) = \int_a^b f dF$ to denote the expected value of $f(x)$.

We shall use extensively Stieltjes's integrals, the advantage being that a single formula embraces the cases of a continuous, as well as of a discontinuous, distribution. The function $F(x)$ introduced above is non-decreasing in (a, b) and varies monotonically from $F(a) = 0$ to $F(b) = 1$. The case of a continuous distribution corresponds to the assumption $dF(x) = f(x)\,dx$, $F(x) = \int_a^x f(x)\,dx$, where $f(x)$ "the frequency function" is integrable on (a, b).

1. *Fundamental inequalities.* The basis of our discussion is formed by the following inequalities of Tchebycheff and Hölder which the writer has extended elsewhere* to Stieltjes's integrals:

$$\text{(A)} \qquad \int_a^b d\psi \cdot \int_a^b f_1 f_2 d\psi \gtrless \int_a^b f_1 d\psi \cdot \int_a^b f_2 d\psi,$$

$$\text{(B)} \qquad \int_a^b |f_1 f_2|\, d\psi \leqq \left[\int_a^b |f_1|^s d\psi\right]^{\frac{1}{s}} \cdot \left[\int_a^b |f_2|^{\frac{s}{s-1}} d\psi\right]^{\frac{s-1}{s}} \qquad (s > 1).$$

In (A) (Tchebycheff) and in (B) (Hölder) $\psi(x)$ denotes a monotonic non-increasing function, $f_{1,2}(x)$ are two continuous functions, which in (A) both vary, for $a \leqq x \leqq b$, in the same sense (sign >), or in the opposite sense (sign <). (B), with $f_2(x) \equiv 1$, $|f_1(x)| \equiv |f(x)|^{s_1}$, $s = s_2/s_1$ $(s_2 > s_1)$, gives

$$\text{(C)} \qquad \int_a^b |f|^{s_1} d\psi \leqq \left[\int_a^b |f|^{s_2} d\psi\right]^{\frac{s_1}{s_2}} \cdot \left[\int_a^b d\psi\right]^{\frac{s_2 - s_1}{s_2}} \qquad (s_2 > s_1 > 0).$$

We notice that Schwartz's inequality is a special case of (B), for $s = 2$.

* J. Chokhate, "Sur les intégrales de Stieltjes," *Comptes rendus*, T. CLXXXIX. (1929), pp. 618—620.

2. In order to illustrate at once the importance of the above inequalities, we proceed to show that they yield directly many important results the proof of which, otherwise, requires special considerations in each case.

(i) ξ denoting an arbitrary constant, take in (C): $f(x) = x - \xi$, $\psi(x) =$ law of distribution $F(x)$. This gives

$$\left[\int_a^b |x-\xi|^{s_1}\, dF(x)\right]^{\frac{1}{s_1}} \leq \left[\int_a^b |x-\xi|^{s_2}\, dF(x)\right]^{\frac{1}{s_2}} \quad (s_2 > s_1 > 0) \ldots (2).$$

Hence, *the quantity* $\nu_s = \left[\int_a^b |x-\xi|^s\, dF(x)\right]^{\frac{1}{s}}$ *increases with* s. This general property leads to many interesting results, by specifying s and ξ.

(α) *Take* $\xi = 0$: $\mu_{2s}^{\frac{1}{2s}}$ *increases with* s *for any distribution over any interval, and so does, more generally,* $\mu_s^{\frac{1}{s}}$, *in case* $a \geqq 0$.

(β) Take in (2) $\xi =$ arithmetical mean of the values of x, and denote by μ'_k the kth moment of the distribution about the mean

$$\frac{(\mu'_{2s})^r}{(\mu'_{2r})^s} > 1 \quad (s > r); \quad \beta_{2s-2} = \frac{\mu'_{2s}}{\sigma^{2s}} > 1 \quad (s = 2, 3, \ldots; \ \sigma^2 = \mu'_2) \ldots (3).$$

(ii) In (A) replace $\psi(x)$ by the law of distribution $F(x)$

$$E(f_1 f_2) \gtrless E(f_1)\, E(f_2) \quad \ldots\ldots\ldots\ldots (4),$$

$$E(f^n) > \{E(f)\}^n \quad \ldots\ldots\ldots\ldots (5)^*.$$

(4) holds for any two functions $f_{1,2}(x)$ continuous in (a, b), provided they both vary, for $a \leqq x \leqq b$, in the same sense (sign $>$) or in the opposite sense (sign $<$); (5) holds for any $f(x)$ continuous in (a, b).

The following remark is important. Suppose we are dealing with a *discrete* distribution, x attaining a finite number of values $x_1, x_2, \ldots, x_m$. Then Stieltjes's integrals reduce to finite sums, for example,

$$\int_a^b f(x)\, dF = \sum_{i=1}^m f(x_i)\, \sigma_i \quad [\sigma_i = F(x_i + 0) - F(x_i - 0)],$$

and the condition of continuity of $f(x)$ *evidently can be omitted.*

(iii) Integrating by parts the expression for μ_s, we get

$$\mu_s = \Big[F(x)\, x^s\Big]_a^b - s\int_a^b x^{s-1} F(x)\, dx = b^s - s\int_a^b x^{s-1} F(x)\, dx.$$

By (A)

$$\int_a^b dx \,.\int_a^b x^{s-1} F(x)\, dx > \int_a^b x^{s-1}\, dx \,.\int_a^b F(x)\, dx,$$

$$\mu_s < b^s - \frac{b^s - a^s}{b-a}\int_a^b F(x)\, dx \quad [(a, b) \text{ finite}, a \geqq 0;\ s = 1, 2, \ldots] \ldots (6).$$

* G. Bohlmann, "Formulierung und Begründung zweier Hülfssätze der mathematischen Statistik," *Mathematische Annalen*, Bd. LXXIV. (1913), pp. 341—412; pp. 374—5.

For a *symmetric* distribution over a finite interval $(-a, a)$ similarly

$$\mu_{2s} < 2a^{2s} - 2a^{2s-1}\int_0^a F(x)\,dx \qquad (s=1, 2, \ldots) \ldots\ldots(7).$$

(iv) Let $a \geqq 0$. In (A) introduce the non-decreasing function

$$\int_a^x x^k\,dF(x) \qquad (k \geqq 0),$$

where $F(x)$ represents the law of distribution over (a, b), and take $f_1(x) \equiv x^l$, $f_2(x) \equiv x^m$ (l, m positive integers or zero). This gives

$$\left.\begin{matrix}\mu_k\mu_{k+l+m} > \mu_{k+l}\mu_{k+m}\\ \mu_{l+m} > \mu_l\mu_m\end{matrix}\right\} \quad (k,\, l,\, m = 0, 1, 2, \ldots;\ l, m > 0) \ldots(8).$$

(8) *holds for any distribution over any interval* (a, b), *provided* $a \geqq 0$.

(v) Finally, we derive, by means of (A), two inequalities which we shall frequently use in our discussion:

(D) $$\int_\alpha^\beta x^{2s} f(x)\,dx \gtrless \frac{\beta^{2s+1}-\alpha^{2s+1}}{(2s+1)(\beta-\alpha)}\cdot\int_\alpha^\beta f(x)\,dx \qquad (0 \leqq \alpha < \beta),$$

where $f(x)$ is non-decreasing (sign $>$) or non-increasing (sign $<$) in (α, β);

(E) $$\int_\alpha^\beta x^{2s} f(x)\,dx \gtrless \frac{\beta^{2s+1}-\alpha^{2s+1}}{(2s+1)(\beta-\alpha)}\cdot P_x^{\alpha,\beta} \qquad (0 \leqq \alpha < \beta),$$

where $f(x)$ represents the frequency function over (a, b), with $a \leqq \alpha < \beta \leqq b$.

3. *Continuous* ∩*-shaped symmetric distribution over a finite interval.* Here the law of distribution is represented by $dF(x) = f(x)\,dx$, where $f(x)$ is an *even* function in the interval $(-a, a)$ with *a single maximum at* $x = 0$. Thus $\bar{x}$ (the mean value of x) $= 0$. Hence

$$\int_0^a f(x)\,dx = \tfrac{1}{2}, \quad \mu_{2s-1} = 0, \quad \mu_{2s} = 2\int_0^a x^{2s} f(x)\,dx \qquad (s = 1, 2, \ldots) \ldots(9).$$

Applying (D), we get

$$\mu_{2s} < \frac{a^{2s}}{2s+1} \quad (s = 1, 2, \ldots), \qquad \sigma = \sqrt{\mu_2} < \frac{a\sqrt{3}}{3} \ \ldots\ldots\ldots\ldots(10).$$

Now let $f(x)$ be subject to the following conditions:

(I) $x^{2k} f(x)$ *increases in* $(0, a)$ *for a certain positive integral* k [example: $x^2e^{-x^2}$ in $(0, 1)$]. We notice that (I) is satisfied *a fortiori* for any $k' > k$. Thus, we can apply (A) to

$$\tfrac{1}{2}\mu_{2s} = \int_0^a x^{2s-2k}\cdot x^{2k} f(x)\,dx \qquad (s > k),$$

$$\mu_{2s} > \frac{a^{2s-2k}}{2s-2k+1}\,\mu_{2k} \qquad (s > k) \ \ldots\ldots\ldots\ldots\ldots(11).$$

We can go further and find the asymptotic expression, for $s \to \infty$, of μ_{2s}, if

(II) *in a sufficiently small interval* $(a - \delta \leqq x \leqq a)$ $f(x) = (a-x)^\nu q(x)$ [$\nu > 0$, $q(a) \neq 0$, $q(x)$ continuous in $(a - \delta, a)$].

Then, as it has been shown by the writer*,

$$\mu_{2s} = \frac{2\Gamma(\nu+1)\,q(a)\,a^{2s+1+\nu}(1+e_s)}{(2s)^{\nu+1}} \quad (\lim_{s\to\infty} e_s = 0)\ldots(12).$$

The above inequalities (10, 11) enable us to find a lower bound for a in case the distribution is *known* to be of the type under consideration over a finite interval of *unknown length* ($2a$),

$$a > \sqrt{(2s+1)\,\mu_{2s}} \quad (s=1, 2, \ldots);\ a > \sigma\sqrt{3}\ldots(13),$$

$$a < \left[\frac{(2s-2k+1)\,\mu_{2s}}{\mu_{2k}}\right]^{\frac{1}{2s-2k}} \quad [\text{under condition (I); } s > k]\ldots(14).$$

4. *Continuous* ∩*-shaped asymmetric distribution over a finite interval.* Again we choose the origin at the maximum. Write

$$\mu_k = \int_a^0 x^k f(x)\,dx + \int_0^b x^k f(x)\,dx \qquad (a < 0 < b),$$

and apply to each integral the inequality (D),

$$\mu_{2s} < \frac{c^{2s}}{2s+1} \quad (s=1, 2, \ldots);\ \sigma < c\sqrt{3}\ldots(15),$$

$$\frac{a^{2s-1}}{2s} < \mu_{2s-1} < \frac{b^{2s-1}}{2s},\ |\mu_{2s-1}| < \frac{c^{2s-1}}{2s} \quad [c = \max.(|a|, b);\ s=1, 2, \ldots]\ldots(16).$$

5. ∪*-shaped continuous distribution over a finite interval.*

(i) *Symmetric distribution.* Taking the origin at the minimum and using the same notations as in § 3, we get (inequality (D))

$$\mu_{2s} > \frac{a^{2s}}{2s+1} \quad (s=1, 2, \ldots);\ \sigma > \frac{a\sqrt{3}}{3}\ldots(17).$$

Here certainly $x^k f(x)$ increases in $(0, a)$ for any $k \geqq 0$. Hence

$$\mu_{2s} > \frac{a^{2s-2k}}{2s-2k+1}\,\mu_{2k} \quad (s, k = 1, 2, \ldots;\ s > k)\ldots(18).$$

(ii) *Asymmetric distribution.* By (D)

$$\mu_{2s} > \frac{a^{2s}\int_a^0 f(x)\,dx + b^{2s}\int_0^b f(x)\,dx}{2s+1},$$

$$\mu_{2s} > \frac{d^{2s}}{2s+1} \quad [d = \min.(|a|, b);\ s=1, 2, \ldots]\ldots(19).$$

(For μ_{2s+1} we get, by Schwartz's inequality, the less satisfactory result

$$|\mu_{2s+1}| < \sqrt{\mu_{4s+2}} < c^{2s+1} \qquad [c = \max.(|a|, b)].)$$

* J. Shohat, "On the Asymptotic Expressions of Certain Definite Integrals," *Annals of Mathematics*, Vol. XXVII. (1925), pp. 8—11; p. 6.

6. *Generalization of Bienaymé-Tchebycheff's criterion.* The problem can be stated as follows. *Given two constants $h\,(>0)$ and ξ, find bounds for the probability* $P \equiv P:[|x-\xi| \leqq h]$, *in terms of the quantities* $\nu_s = \left[\int_a^b |x-\xi|^s\,dF\right]^{\frac{1}{s}}$. The simplest procedure generally adopted is the following*. We write

$$\nu_s^s = \int_{|x-\xi|\leqq h} |x-\xi|^s\,dF + \int_{|x-\xi|>h} |x-\xi|^s\,dF \equiv i_1 + i_2 \qquad (s>0) \ldots(20),$$

and then, *neglecting entirely* i_1,

$$\nu_s^s \geqq i_2 > h^s(1-P),$$

$$P \equiv P:[|x-\xi| \leqq h] > 1 - \left(\frac{\nu_s}{h}\right)^s \ldots\ldots\ldots\ldots\ldots\ldots(21),$$

$$P \equiv P:[|x-\xi| \leqq \lambda\nu_r] > 1 - \frac{1}{\lambda^s}\left(\frac{\nu_s}{\nu_r}\right)^s \qquad (s,\,r>0) \;\ldots(22).$$

The very procedure shows that the limitations (21, 22) are, in general, too gross. In order to get a better understanding of these inequalities, we give the following properties of ν_s:

1°. $1 = \nu_0 \leqq \nu_1 \leqq \nu_2 \leqq \ldots$, for any distribution over any interval.

2°. $\lim\limits_{s\to\infty} \nu_s = \lim\limits_{s\to\infty} \dfrac{\nu_{s+1}^{s+1}}{\nu_s^s} = M_\xi \equiv \max.\,(|a-\xi|,\ |b-\xi|)$ $[M_\xi = \infty$, if $(a,\,b)$ be infinite].

3°. $\lim \dfrac{\nu_s}{\nu_r} = \dfrac{M_\xi}{\nu_r} > 1;\ \lim\left(\dfrac{\nu_s}{\nu_r}\right)^s = \infty \qquad (s\to\infty;\ r>0$, fixed).

4°. $\lim \dfrac{\nu_s}{\nu_r} = \dfrac{\nu_s}{M_\xi} < 1 \qquad (r\to\infty;\ s>0$, fixed).

(1°) has been shown above (formula (2)); the first part of (2°) has been established by the writer†. To complete the proof of (2°), we notice that

$$\nu_s^{2s} \leqq \nu_{s-1}^{s-1}\,.\,\nu_{s+1}^{s+1};\quad \frac{\nu_s^s}{\nu_{s-1}^{s-1}} \leqq \frac{\nu_{s+1}^{s+1}}{\nu_s^s} \quad \text{(Schwartz's inequality)} \ldots(23),$$

$$\frac{\nu_{s+1}^{s+1}}{\nu_s^s} \leqq M_\xi \quad \text{(mean-value theorem)} \ldots(24);$$

hence, $\lim\limits_{s\to\infty} \dfrac{\nu_{s+1}^{s+1}}{\nu_s^s}$ exists and is equal (by a well-known proposition) to $\lim\limits_{s\to\infty} \nu_s$. Furthermore, if $(a,\,b)$ be infinite, then, for a sufficiently large s (since

$$\lim_{s\to\infty}\left[\int_{|x-\xi|\geqq 2G} dF(x)\right]^{\frac{1}{s}} = 1,$$

* Cf. Pearson, *Biometrika*, Vol. XII. (1918—19), pp. 284—296; Narumi, *ibid.* Vol. XV. pp. 245—254; Guldberg, *Comptes rendus*, T. CLXXV. (1922), pp. 418—420, 679—680, 1382—1384; Lurquin, *ibid.* pp. 681—683; Camp, *Bulletin of the American Mathematical Society*, Vol. XXVIII. (1922), pp. 427—432; Meidell, *Comptes rendus*, T. CLXXV. (1922), pp. 806—808; T. CLXXVI. (1923), pp. 280—282.

† J. Shohat, "On the polynomial and trigonometric approximation of measurable bounded functions on a finite interval," *Mathematische Annalen*, Bd. CII. (1929), pp. 157—175; pp. 168—4.

where $G > 0$ is arbitrarily large, but fixed)

$$\nu_s \geqq \left[\int_{|x-\xi| \geqq 2G} |x - \xi|^s \, dF\right]^{\frac{1}{s}} \geqq G \text{ ....(25).}$$

(3°, 4°) follow directly from (1°, 2°).

The aforesaid considerations show that *if we take in* (22) $r < s$, *the coefficient of* $1/\lambda^s$ *is* > 1. Moreover, *with* r *fixed, the usefulness of* (22) *generally decreases, as* s *increases,* for then the interval of admissible values for λ

$$\frac{\nu_s}{\nu_r} < \lambda < \frac{M_\xi}{\nu_r} \text{ ....(26)}$$

gets smaller and smaller; for s very large, $\lambda\nu_r$ must be taken very close to M_ξ, and (22) loses its meaning, whether (a, b) be finite or infinite. On the other hand, *if we take in* (22) $r > s$*, *the coefficient of* $1/\lambda^s$ *is* < 1, *and* (22) *becomes applicable, even with* $\lambda < 1$. Moreover, as r increases indefinitely, s remaining fixed, the interval of admissible values for λ approaches, in case of (a, b) finite, a limiting interval $(\nu_s/M_\xi, 1)$, whose length is different from zero. In case of (a, b) infinite, the upper bound for λ is ∞, and its lower bound approaches (under the said condition) 0.

7. *Criterion analogous to that of Bienaymé-Tchebycheff for certain distributions over a finite interval.*

(i) ∩*-shaped continuous symmetric distribution* (§ 3). We get from (10)

$$\frac{a^{2s}}{2s+1} > 2\int_h^a x^{2s} f(x)\,dx > h^{2s}(1 - P_x^{-h,h}),$$

$$P_x^{-h,h} \equiv P : [|x| \leqq h] > 1 - \frac{1}{2s+1}\left(\frac{a}{h}\right)^{2s} \text{ ....(27),}$$

$$P_x^{-h,h} \equiv P : [|x| \leqq \lambda\sigma] > 1 - \frac{1}{2s+1}\left(\frac{a}{\lambda\sigma}\right)^{2s} \text{ ....(28).}$$

(27, 28) do not require the computation of μ_{2s} (being useful, of course, so long as their right-hand members remain > 0).

Apply now the inequality (E) to the integrals i_1, i_2 in the expression

$$\mu_{2s} = 2\int_0^h x^{2s} f(x)\,dx + 2\int_h^a x^{2s} f(x)\,dx \equiv i_1 + i_2 \quad (h < a)\text{...(29),}$$

$$i_1 < \frac{h^{2s}}{2s+1} P_x^{-h,h}, \quad i_2 < \frac{a^{2s+1} - h^{2s+1}}{(2s+1)(a-h)}(1 - P_x^{-h,h}) \text{ ....(30),}$$

$$P_x^{-h,h} < \frac{a^{2s+1} - h^{2s+1} - (a-h)(2s+1)\mu_{2s}}{a(a^{2s} - h^{2s})} < \frac{a^{2s+1} - h^{2s+1} - (a-h)(2s+1)\sigma^{2s}}{a(a^{2s} - h^{2s})} \text{ ...(31).}$$

We thus obtain an upper bound for the probability $P_x^{-h,h}$ in addition to its lower bound given by (21)

$$P_x^{-h,h} > 1 - \frac{\mu_{2s}}{h^{2s}} \text{ ....(32).}$$

* For $r = s$, λ must be greater than 1.

(32) *becomes inapplicable for sufficiently large s and any $h < a$ or for any s and $h < \mu_{2s}^{\frac{1}{2s}}$, but in that case* (31) *becomes applicable, for its right-hand member is then necessarily* < 1. In fact, the contrary assumption leads to

$$\mu_{2s}^{\frac{1}{2s}} < \frac{h}{(2s+1)^{\frac{1}{2s}}} < h,$$

while $\lim_{s\to\infty} \mu_{2s}^{\frac{1}{2s}} = a > h$.

(ii) ∩*-shaped continuous asymmetric distribution.* The results of § 4 give (see formulae (32, 15))

$$P:[|x| \leqq h] > 1 - \frac{1}{2s+1}\left(\frac{c}{h}\right)^{2s} \qquad [c = \max.(|a|, b)] \ldots (33).$$

In order to get an upper bound for $P_x^{-h,h}$, write

$$\mu_{2s} = \int_a^{-h} x^{2s} f(x)\,dx + \int_{-h}^{0} + \int_0^h + \int_h^b \equiv i_1 + i_2 + i_3 + i_4 \qquad (a < 0 < h < b),$$

and apply (E) to each of these integrals, then

$$P_x^{-h,h} < \frac{H - (2s+1)\mu_{2s}}{H - h^{2s}} < \frac{H - (2s+1)\sigma^{2s}}{H - h^{2s}},$$

$$\left(H = \max.\left[\frac{a^{2s+1} + h^{2s+1}}{a+h}, \frac{b^{2s+1} - h^{2s+1}}{b-h}\right]\right) \ldots\ldots\ldots\ldots\ldots\ldots (34).$$

(iii) ∪*-shaped continuous symmetric distribution* (§ 5). Here

$$\mu_{2s} = 2\int_0^h x^{2s} f(x)\,dx + 2\int_h^a x^{2s} f(x)\,dx > \frac{h^{2s}}{2s+1} P_x^{-h,h} + \frac{a^{2s+1} - h^{2s+1}}{(2s+1)(a-h)}(1 - P_x^{-h,h}),$$

$$P_x^{-h,h} > \frac{a^{2s+1} - h^{2s+1} - (2s+1)\mu_{2s}(a-h)}{a(a^{2s} - h^{2s})} \qquad \ldots\ldots\ldots\ldots\ldots (35).$$

(iv) ∪*-shaped asymmetric continuous distribution.* Employing the now familiar reasoning, we get from

$$\mu_{2s} = \int_a^{-h} x^{2s} f(x)\,dx + \int_{-h}^0 + \int_0^h + \int_h^b :$$

$$\mu_{2s} > \frac{h^{2s}}{2s+1} P_x^{-h,h} + \frac{K}{2s+1}(1 - P_x^{-h,h}),$$

$$P_x^{-h,h} > \frac{K - (2s+1)\mu_{2s}}{K + h^{2s}} \left(K = \min.\left[\frac{a^{2s+1} + h^{2s+1}}{a+h}, \frac{b^{2s+1} - h^{2s+1}}{b-h}\right]\right) \ldots (36).$$

Here we can go further and discriminate between positive and negative values of x:

$$\mu_{2s} > \int_0^h x^{2s} f(x)\,dx + \int_h^b x^{2s} f(x)\,dx > \frac{h^{2s}}{2s+1} P_x^{0,h} + \frac{b^{2s+1} - h^{2s+1}}{(b-h)(2s+1)}(1 - P_x^{0,h}),$$

$$P_x^{0,h} > \frac{b^{2s+1} - h^{2s+1} - (b-h)(2s+1)\mu_{2s}}{b(b^{2s} - h^{2s})} \ldots\ldots\ldots\ldots\ldots\ldots (37),$$

$$P_x^{-h,0} > \frac{a^{2s+1} + h^{2s+1} - (a+h)(2s+1)\mu_{2s}}{a(a^{2s} + h^{2s})} \ldots\ldots\ldots\ldots (37\ bis).$$

8. *Distribution over an infinite interval.* Let the interval be $(-\infty, \infty)$ (other cases can be treated similarly). We assume the existence of all moments to be used.

(i) $\cap$-*shaped continuous symmetric distribution.* Here

$$\int_0^\infty f(x)\,dx = \tfrac{1}{2}; \quad \mu_{2s-1} = 0; \quad \mu_{2s} = 2\int_0^\infty x^{2s} f(x)\,dx \quad (s = 1, 2, \ldots).$$

The ratio $\dfrac{1}{\mu_{2s}} \cdot \displaystyle\int_0^t x^{2s} f(x)\,dx$ varies monotonically from 0 to $\tfrac{1}{2}$, as t varies from 0 to ∞. Hence, there always exists one and only one solution a_s of the equation

$$\int_0^{a_s} x^{2s} f(x)\,dx = \frac{\theta}{2}\mu_{2s} \qquad (0 < \theta < 1) \quad \ldots\ldots\ldots(38),$$

which leads to

$$\int_{a_s}^\infty x^{2s} f(x)\,dx = \frac{1-\theta}{\theta}\int_0^{a_s} x^{2s} f(x)\,dx; \quad \mu_{2s} = \frac{2}{\theta}\int_0^{a_s} x^{2s} f(x)\,dx \quad \ldots\ldots(39),$$

$$\mu_{2s} < \frac{1}{\theta}\cdot\frac{a_s^{2s}}{2s+1}\cdot P_x^{-a_s, a_s}\ (\text{by (E)}) < \frac{1}{\theta}\cdot\frac{a_s^{2s}}{2s+1} \quad (s = 1, 2, \ldots) \ldots(40).$$

Introduce now the probability $P_x^{-h,h}$. Then, if $h \geqq a_s$,

$$\mu_{2s} > 2\int_0^h x^{2s} f(x)\,dx + 2\int_h^\infty x^{2s} f(x)\,dx > h^{2s}(1 - P_x^{-h,h}) \ldots\ldots\ldots(41),$$

which leads again to the inequality

$$P_x^{-h,h} > 1 - \frac{\mu_{2s}}{h^{2s}} > 1 - \frac{1}{\theta}\cdot\frac{a_s^{2s}}{(2s+1)h^{2s}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(42),$$

and this, combined with (40), gives a very simple inequality for $P_x^{-h,h}$:

$$P_x^{-h,h} > 1 - \frac{\frac{1}{\theta}}{2s+1+\frac{1}{\theta}} \ (\text{for } P_x^{-h,h} \geqq P_x^{-a_s,a_s}, \text{ if } a_s \leqq h) \quad \ldots\ldots (43).$$

In order to use the above formulae for a given θ, it is sufficient to find an upper limit for a_s (and similarly for a_s, b_s below), for $t_1 > a_s$ implies

$$\int_0^{t_1} x^{2s} f(x)\,dx = \frac{\theta'}{2}\mu_{2s}\,(\theta' > \theta), \quad \mu_{2s} < \frac{1}{\theta}\cdot\frac{a_s^{2s}}{2s+1} < \frac{1}{\theta}\cdot\frac{t_1^{2s}}{2s+1}.$$

The simplest choice of θ is $\theta = \tfrac{1}{2}$, i.e. choose a_s so that

$$\int_{a_s}^\infty x^{2s} f(x)\,dx \leqq \int_0^{a_s} x^{2s} f(x)\,dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(44).$$

Then

$$\mu_{2s} < \frac{2a_s^{2s}}{2s+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(45),$$

$$P_x^{-h,h} > 1 - \frac{2}{2s+3} \qquad (h \geqq a_s) \quad \ldots\ldots\ldots\ldots\ldots(46).$$

(ii) *∩-shaped continuous asymmetric distribution.* We choose again the origin at the maximum. The same reasoning as in (i) holds, with proper modifications. Given any θ between 0 and 1, there exists a unique pair of numbers $a_s < 0$, $b_s > 0$ such that

$$\int_0^{b_s} x^{2s} f(x)\,dx = \theta \int_0^{\infty} x^{2s} f(x)\,dx\,; \quad \int_{a_s}^{0} x^{2s} f(x)\,dx = \theta \int_{-\infty}^{0} x^{2s} f(x)\,dx \quad \ldots(47).$$

Hence, by (15)

$$\mu_{2s} = \frac{1}{\theta}\int_{a_s}^{b_s} x^{2s} f(x)\,dx < \frac{1}{\theta}\cdot\frac{c_s^{2s}}{2s+1}\,P_x^{a_s, b_s} < \frac{1}{\theta}\cdot\frac{c_s^{2s+1}}{2s+1} \quad (c_s = \text{max. } [\,|a_s|, b_s]) \ldots(48).$$

We can follow now the discussion of § 7, (ii)

$$P_x^{-h,h} > 1 - \frac{1}{\theta}\cdot\frac{c_s^{2s}}{(2s+1)\,h^{2s}}\,; \quad P_x^{-h,h} > 1 - \frac{\frac{1}{\theta}}{2s+1+\frac{1}{\theta}} \quad (h \geqq c_s) \ldots(49),$$

$$P_x^{-h,h} < \frac{H_s - (2s+1)\,\theta\mu_{2s}}{H_s - h^{2s}} < \frac{H_s - (2s+1)\,\theta\sigma^{2s}}{H_s - h^{2s}}$$

$$\left(H_s = \text{max.}\left[\frac{a_s^{2s+1} + h^{2s+1}}{a_s + h},\ \frac{b_s^{2s+1} - h^{2s+1}}{b_s - h}\right]\right) \quad (a_s < 0 < h < b_s) \ldots(50).$$

In case $a_s = -b_s$ (which corresponds also to the symmetric case (i) above)

$$P_x^{-h,h} < \frac{b_s^{2s+1} - h^{2s+1} - (b_s - h)\,\theta\,(2s+1)\,\mu_{2s}}{b_s\,(b_s^{2s} - h^{2s})} < \frac{b_s^{2s+1} - h^{2s+1} - (b_s - h)\,\theta\sigma^{2s}}{b_s\,(b_s^{2s} - h^{2s})} \quad (h < b_s) \ldots\ldots(51).$$

9. Let a general distribution over $(-\infty, \infty)$ be determined by the law of distribution $F(x)$. Consider

$$\delta_s(h) = \int_{-\infty}^{-h} x^{2s} dF(x) + \int_h^{\infty} x^{2s} dF(x) > h^{2s}\,(1 - P_x^{-h,h}) \quad \ldots\ldots\ldots(52),$$

$$P_x^{-h,h} > 1 - \frac{\delta_s(h)}{h^{2s}}, \text{ with } \lim_{h\to\infty} \delta_s(h) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(53),$$

which, for h sufficiently large, is better than (32), provided we know, instead of μ_{2s}, the order, with respect to $1/h$, of $\delta_s(h)$. This can be done in some special cases which follow.

Continuous distribution, with the frequency function $f(x)$ such that

(III) $f(x)\,.\,|x|^k < M$ for $|x| \geqq X$ ($M, X > 0$, $k > 1$—certain constants).

Here we introduce

$$\delta(h) = \int_{-\infty}^{-h} f(x)\,dx + \int_h^{\infty} f(x)\,dx = 1 - P_x^{-h,h},$$

$$\delta(h) = \int_{-\infty}^{-h} \frac{1}{|x|^k}\,.\,|x|^k f(x)\,dx + \int_h^{\infty} \frac{1}{x^k}\,.\,x^k f(x)\,dx < \frac{2M}{(k-1)\,h^{k-1}} \ (h \geqq X),$$

$$P_x^{-h,h} > 1 - \frac{2M}{(k-1)\,h^{k-1}} \quad \text{(under condition (III); } h \geqq X) \ldots(54).$$

Notes. (i) (III) is *à fortiori* satisfied if

$$\lim_{|x|\to\infty} f(x)\,|x|^k = 0 \qquad (k>1) \quad \ldots\ldots\ldots\ldots\ldots(55).$$

Then in (54) $$\lim_{h\to\infty} M = 0.$$

(ii) If we replace (III) by a stronger condition

(IV) $f(x) < Me^{-r|x|^\nu}$ *for* $|x| \geq X$ (M, r, ν—positive constants),

then (54) is replaced by

$$P_x^{-h,h} > 1 - \frac{2M}{sh^s} \qquad \left(h > \max.\left(1,\, X,\, \frac{e^{2s+2}}{r\nu^2}\right);\ s>0\ \textit{arbitrary}\right) \ldots(56).$$

In fact, for such $h = |x|$ necessarily $e^{-r|x|^\nu} < \frac{1}{|x|^{s+1}}$*.

Editorial Note on the Limitation of Frequency Constants.

(1) My impression is that most inequalities hitherto found are not of great service in practical statistics; the limits are too wide. Some of the results reached by Dr Shohat are already familiar, others can be obtained by an analysis more customary with practical statisticians, and it is, perhaps, worth while considering them in connection with his memoir.

I use the following notation. Let there be n values of a variate x, namely $x_1, x_2, x_3, \ldots x_n$, and let

$$s_p = x_1^p + x_2^p + \ldots + x_n^p,$$

and $\mu'_p = s_p/n$ be a moment coefficient about an arbitrary origin from which the x's may be supposed measured.

Let $$\beta_{2r-2} = \frac{\mu_{2r}}{\mu_2^r} \quad\text{and}\quad \beta_{2r-1} = \frac{{}'}{\mu_2^{r+2}},$$

where μ_r = the rth moment coefficient when the arbitrary origin is taken about the mean. β'_p may be used, when we put μ'_q for μ_q in the above expression.

(2) *Lemma.* The determinant Δ written down below is always positive. I owe the following proof of this fact, which had been otherwise brought to my notice, to Professor G. N. Watson.

$$\begin{vmatrix} s_0, & s_1, & s_2 \\ s_1, & s_2, & s_3 \\ s_{2r-2}, & s_{2r-1}, & s_{2r} \end{vmatrix} = \Delta = \sum_{m=1}^{n} \begin{vmatrix} s_0, & s_1, & s_2 \\ s_1, & s_2, & s_3 \\ x_m^{2r-2}, & x_m^{2r-1}, & x_m^{2r} \end{vmatrix}, \text{ clearly.}$$

* For $z = \nu \log|x|$ must satisfy the inequality $e^z > \frac{k}{r\nu} z$; it holds for any $z > 0$ if $\frac{k}{r\nu} \leq 1$.

Expanding these determinants they may be written in the form:

$$\Delta = \mathop{S}_{m=1}^{n} \mathop{S}_{\binom{p>q}{p,\,q\neq m}} x_m^{2r-2}(x_p - x_q)^2(x_r - x_p)(x_r - x_q)$$

$$= \mathop{S}_{p\neq q\neq r} (x_p - x_q)(x_q - x_m)(x_m - x_p)$$

$$\times \{- x_p^{2r-2}(x_q - x_m) - x_q^{2r-2}(x_m - x_p) - x_m^{2r-2}(x_p - x_q)\},$$

where the summation extends over all *different sets* of values of p, q, r, a set such as 2, 7, 4 being regarded as the same as a set 2, 4, 7. It will be sufficient to prove that

$$k = (a-b)(b-c)(c-a)\{- a^{2r-2}(b-c) - b^{2r-2}(c-a) - c^{2r-2}(a-b)\}$$

is always positive, for then every term of the above expression will be positive. The expression is symmetrical in a, b, c and remains the same if the signs are all changed.

The possible cases are:

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
a	+	+	+	−	−	−
b	+	+	−	−	−	+
c	+	−	−	−	+	+

but by the second statement above (i) and (iv), (ii) and (v), (iii) and (vi) are really the same. It is sufficient therefore to consider (i), (ii) and (vi), i.e. a, b, c are all positive or two are positive and one negative. We have two cases then:

$$(\alpha)\ \ a > b > c, \ \text{ or }\ (\beta)\ \ a > b > 0 > c.$$

(α) We may write

$$k = (a-c)(a-b)^2(b-c)^2\left\{\frac{a^{2r-2} - b^{2r-2}}{a-b} - \frac{b^{2r-2} - c^{2r-2}}{b-c}\right\}$$

$$= (a-c)(a-b)^2(b-c)^2\{a^{2r-3} - c^{2r-3} + b(a^{2r-4} - c^{2r-4})$$

$$+ b^2(a^{2r-5} - c^{2r-5}) + \ldots\}.$$

Since $a > c$, this expression is always positive.

(β) Here $(a-b)(b-c)(a-c)$ is always > 0, while

$$a^{2r-2}(b-c) + b^{2r-2}(c-a) + c^{2r-2}(a-b)$$

will be positive when c is negative, if

$$ab(a^{2r-3} - b^{2r-3}) + (-c)(a^{2r-2} - b^{2r-2}) + c^{2r-2}(a-b)$$

is positive. But since $a > b$, and c is negative, all three parts are essentially positive.

Thus finally

$$\frac{\begin{vmatrix} s_0, & s_1, & s_2 \\ s_1, & s_2, & s_3 \\ s_{2r-2}, & s_{2r-1}, & s_{2r} \end{vmatrix}}{n^3 \begin{vmatrix} 1 & \mu'_1 & \mu'_2 \\ \mu'_1 & \mu'_2 & \mu'_3 \\ \mu'_{2r-2}, & \mu'_{2r-1}, & \mu'_{2r} \end{vmatrix}}$$

is essentially positive, or

$$\left(1 - \frac{\mu_1'^2}{\mu'_2}\right) \beta'_{2r-2} > \left(1 - \frac{\mu'_1 \mu'_2}{\mu'_3}\right) \beta'_{2r-3} + \left(1 - \frac{\mu'_1 \mu'_3}{\mu_2'^2}\right) \beta'_{2r-4} \quad \text{.........(i).}$$

For the particular case of moments about the mean this becomes

$$\beta_{2r-2} > \beta_{2r-3} + \beta_{2r-4} \quad \text{.................................(ii).}$$

Hence it follows that

$$\beta_{2r-2} > \beta_0 + \beta_1 + \beta_3 + \dots + \beta_{2r-3} \quad \text{.......................(iii);}$$

but $\beta_0 = 1$, hence, if all the odd moments are of the same sign, it follows that an even β will always be greater than unity. The relation for $r = 2$,

$$\beta_2 > 1 + \beta_1 \quad \text{...................................(iv),}$$

is very familiar as it gives the boundary to all frequency in the β_1, β_2 plane. The frequency system then corresponds to the limit of the U-curves, i.e., to two lumps of frequencies n_1 and n_2 at distance b, and lying on the line $\beta_2 - \beta_1 - 1 = 0$.

We may ask whether it is the two lump-frequency which bounds the possible frequency when we consider other β's than β_1, β_2. We have

$$\mu_q = \frac{b^q}{n_1 + n_2} \left(n_2 \left(\frac{n_1}{n_1 + n_2}\right)^q + (-1)^q n_1 \left(\frac{n_2}{n_1 + n_2}\right)^q\right)$$

$$= \frac{b^q n_1 n_2}{(n_1 + n_2)^{q+1}} (n_1^{q-1} + (-1)^q n_2^{q-1}).$$

Hence $$\mu_2 = \frac{b^2 n_1 n_2}{(n_1 + n_2)^2}, \qquad \mu_3 = \frac{b^3 n_1 n_2 (n_1 - n_2)}{(n_1 + n_2)^3},$$

$$\mu_{2r-1} = \frac{b^{2r-1} n_1 n_2}{(n_1 + n_2)^{2r}} (n_1^{2r-2} - n_2^{2r-2}).$$

It follows that

$$\beta_{2r-3} = \frac{\mu_3 \mu_{2r-1}}{\mu_2^{r+1}} = \frac{(n_1 - n_2)(n_1^{2r-2} - n_2^{2r-2})}{(n_1 n_2)^{r-1} (n_1 + n_2)},$$

$$\beta_{2r-2} = \frac{\mu_{2r}}{\mu_2^r} = \frac{n_1^{2r-1} + n_2^{2r-1}}{(n_1 n_2)^{r-1} (n_1 + n_2)},$$

$$\beta_{2r-4} = \frac{\mu_{2r-2}}{\mu_2^{r-1}} = \frac{n_1^{2r-3} + n_2^{2r-3}}{(n_1 n_2)^{r-2} (n_1 + n_2)}.$$

Hence $$\beta_{2r-2} = \beta_{2r-4} + \beta_{2r-3} \quad \text{.............................(v)},$$
or the two lumps form again the limiting condition, beyond which the possible frequencies, bounded by
$$\beta_{2r-2} > \beta_{2r-4} + \beta_{2r-3},$$
cannot pass.

At the point $\beta_1 = 0$, $\beta_2 = 1$, we have two equal lumps; all the odd β's vanish and
$$\beta_2 = \beta_4 = \ldots = \beta_{2r-2} = \text{etc.} = 1,$$
or all the even β's take their limiting value.

(3) For symmetrical curves with continuous convex curvature and range $2a$ the limit is the rectangle when we deal with the greatest value of any moment coefficient. Hence we must have
$$\mu_{2r} < a^{2r}/(2r+1) \quad \text{.................................(vi)},$$
or, for $r = 1$, $$\mu_2 < \tfrac{1}{3}a^2, \text{ i.e. } \sigma < \frac{a}{\sqrt{3}} \quad \text{...........................(vii)},$$
i.e. a rectangular distribution gives for such curves the maximum σ (or radius of gyration) about the mean.

(4) Now for U-shaped curves* the higher the moment coefficient the more closely its value will approach the two lumps moment coefficient and diverge from the moment coefficient of the rectangle, for the higher moment gives more weight to the outlying values than the lower moment. Hence, if s be $> k$,
$$\frac{\dfrac{\mu_{2s}}{a^{2s+1}}}{2s+1} > \frac{\dfrac{\mu_{2k}}{a^{2k+1}}}{2k+1},$$
or $$\mu_{2s} > \frac{a^{2s-2k}}{(2s+1)/(2k+1)}\,\mu_{2k} \quad \text{........................(viii)}.$$

This is a higher limit than that provided by Dr Shohat in his Equation (11).

Since $2k+1 < 2s+1$ because $k < s$ or $2k\,(2k+1) < 2k\,(2s+1)$,
$$2s+1 < (2s+1)(2k+1) - 2k\,(2k+1),$$
$$\frac{2s+1}{2k+1} < 2s+1-2k.$$
Thus $$\mu_{2s} > \frac{a^{2s-2k}}{2s-2k+1}\,\mu_{2k} \quad \text{..............................(ix)},$$
which is Dr Shohat's Equation (11).

Thus $$\beta_{2s-2} > \left(\frac{a}{\sigma}\right)^{2s-2k} \frac{2k+1}{2s+1}\,\beta_{2k-2},$$
or $$a < \sigma \left\{\frac{2s+1}{2k+1}\,\frac{\beta_{2s-2}}{\beta_{2k-2}}\right\}^{\frac{1}{2s-2k}} \quad \text{...........................(x)}.$$

* For ∩-curves the sign of the inequality must be reversed.

(5) Now consider the curve

$$y = y_0\left(1 - \frac{x^2}{a^2}\right)^{-p},$$

where p must lie between 0 and 1. In this case it is easy to show that

$$\mu_{s+1} = \frac{sa^2}{s+2-2p}\mu_{s-1},$$

and accordingly $$\mu_{2s} = \frac{1.3.5\ldots(2s-1)a^{2s}}{(3-2p)(5-2p)\ldots(2s+1-2p)},$$

and $$\frac{\dfrac{\mu_{2s}}{a^{2s}}}{2s+1} \text{ will be } > \frac{\dfrac{\mu_{2s-2}}{a^{2s-2}}}{2s-1},$$

since $$\frac{2s+1}{2s+1-2p} > 1.$$

Accordingly $$a^{2s-2k} < \frac{2s+1}{2k+1}\frac{\mu_{2s}}{\mu_{2k}} < (2s-2k+1)\frac{\mu_{2s}}{\mu_{2k}}.$$

Now, $\mu_2 = \dfrac{a^2}{3-2p}$, or we may put, as illustration, $a = 1$, $p = \frac{1}{2}$, which give

$$\sigma^2 = 1/2,$$

$$\mu_{2s} = \frac{1.3.5\ldots 2s-1}{2.4\ldots 2s}$$

$$\mu_{2k} = \frac{1.3.5\ldots 2k-1}{2.4\ldots 2k}.$$

Hence $$a^{2s-2k} < \frac{2.4\ldots 2k}{2.4\ldots 2s}\cdot\frac{1.3.5\ldots(2s+1)}{1.3.5\ldots(2k+1)}$$

$$< \frac{(2s+1)!\,2^{2k}(k!)^2}{2^{2s}(s!)^2(2k+1)!}.$$

For example, if we work from the fourth and second moments, i.e. $s = 2$, $k = 1$,

$$a^2 < 1{\cdot}25, \text{ or } a < 1{\cdot}12.$$

Using Dr Shohat's inequality we have

$$a < 1{\cdot}50.$$

In the former case we are 12 °/₀ and in the latter 50 °/₀ beyond the true value. If we take $s = 4$, $k = 2$, we have

$$a < 1{\cdot}07,$$

or we are 7 °/₀ above the true value. Dr Shohat's formula gives $a < 1{\cdot}38$, or 38 °/₀ above the true value.

In neither case should I personally feel justified in using an eighth moment coefficient, as the probable error of such a moment is too large in the case of the usual sized sample*.

* The safer $s = 3$, $k = 2$ gives $a < 1{\cdot}08$.

(6) If all the x's are >0, suppose them arranged in order of magnitude, then

$$x_p^k x_q^k (x_p^l - x_q^l)(x_p^m - x_q^m),$$

when $p > q$, will always be positive, and therefore

$$S(x_p^k x_q^k (x_p^l - x_q^l)(x_p^m - x_q^m)),$$

where S denotes that p and q are summed for every pair in 1, 2, 3, ... n, but $p > q$, will always be positive. Accordingly:

$$\overset{n}{\underset{p=1}{S}}\, x_p^k x_p^{k+l+m} + S x_p^k x_q^k (x_p^l - x_q^l)(x_p^m - x_q^m) > \overset{n}{\underset{p=1}{S}}\, x_p^{k+l} x_p^{k+m},$$

$$S x_p^k (x_p^{k+l+m} + \underset{q \neq p}{S'}\, x_q^{k+l+m}) > S x_p^{k+l} (x_p^{k+m} + \underset{q \neq p}{S'}\, x_q^{k+m}),$$

$$(x_1^k + x_2^k + \dots + x_n^k)(x_1^{k+l+m} + x_2^{k+l+m} + \dots + x_n^{k+l+m})$$
$$> (x_1^{k+l} + x_2^{k+l} + \dots + x_n^{k+l})(x_1^{k+m} + x_2^{k+m} + \dots + x_n^{k+m}),$$

$$n\mu'_k \times n\mu'_{k+l+m} > n\mu'_{k+l} \times n\mu'_{k+m},$$

or

$$\mu'_k \times \mu'_{k+l+m} > \mu'_{k+l} \times \mu'_{k+m} \quad \text{(xi)},$$

which agrees with Dr Shohat's Equation (8).

If we put $k = 0$, $\mu'_k = 1$,

$$\mu'_{l+m} > \mu'_l \times \mu'_m \quad \text{(xii)}.$$

Unfortunately this is not demonstrated for moment coefficients about the mean, but only for variates algebraically >0. It is true for moments about the mean, if k, l, m are even powers, or

$$\mu_{2k}\mu_{2(k+l+m)} > \mu_{2(k+l)}\mu_{2(k+m)} \quad \text{(xiii)}.$$

$$\mu_{2(l+m)} > \mu_{2l}\mu_{2m} \quad \text{(xiv)},$$

$$\beta_{2(l+m-1)} > \beta_{2(l-1)}\beta_{2(m-1)}.$$

Thus we see if $l = m = s$,

$$\beta_{4s-2} > \beta^2_{2s-2} \quad \text{(xv)},$$

$$\beta_{8s-2} > \beta^2_{4s-2} > \beta^4_{2s-2} \quad \text{(xvi)}.$$

Innumerable such relations may be deduced. But such inequalities teach us little. For example, in the case of the normal curve, (xv) and (xvi) tell us that, putting $s = 2$,

$$3 \,.\, 5 \,.\, 7 > 9, \text{ or, that } 35 > 3;$$

and

$$3 \,.\, 5 \,.\, 7 \,.\, 9 \,.\, 11 \,.\, 13 \,.\, 15 > 81, \text{ or, that } 25025 > 1.$$

Such limits are of small practical service.

K. P.

ON THE STANDARD ERROR OF THE MEAN SQUARE CONTINGENCY.

By TSUTOMU KONDO, Professor of Mathematics, Higher Commercial College, Yamaguchi, Japan.

I. *Introduction.*

(1) Let us consider a table of contingency and let n_{pq} be the frequency of the p, qth cell; $n_{p.}$, $n_{.q}$, as usual, the marginal totals of the pth row and qth column; N the total frequency and κ, λ the numbers of rows and columns respectively, then the mean square contingency is defined by

$$\frac{1}{N}\mathop{S}_{p=1}^{p=\kappa}\mathop{S}_{q=1}^{q=\lambda}\left\{\left(n_{pq}-\frac{n_{p.}n_{.q}}{N}\right)^2\Big/\left(\frac{n_{p.}n_{.q}}{N}\right)\right\},$$

which can be transformed into the simple form

$$\mathop{S}_{p=1}^{p=\kappa}\mathop{S}_{q=1}^{q=\lambda}\left(\frac{n_{pq}^2}{n_{p.}n_{.q}}\right)-1.$$

The mean square contingency is usually denoted by ϕ_1^2 and we have the following fundamental equation

$$\phi_1^2=\mathop{S}_{p=1}^{p=\kappa}\mathop{S}_{q=1}^{q=\lambda}\left(\frac{n_{pq}^2}{n_{p.}n_{.q}}\right)-1 \quad \text{..........................(1).}$$

In the right-hand side of this equation if, for $n_{p.}$ and $n_{.q}$, their expected means $\bar{n}_{p.}$ and $\bar{n}_{.q}$ are substituted, then we get an expression

$$\mathop{S}_{p=1}^{p=\kappa}\mathop{S}_{q=1}^{q=\lambda}\left(\frac{n_{pq}^2}{\bar{n}_{p.}\bar{n}_{.q}}\right)-1 \quad \text{............................(2),}$$

which is usually denoted by ϕ_2^2, and problems about the mean and standard error of ϕ_2^2 have been completely solved by Prof. Karl Pearson and A. W. Young*.

But problems of the same kind for ϕ_1^2 have not yet been fully solved by anyone, and I want, here, to consider these problems.

(2) Now, for simplicity, let us denote by m_s the contents of the sth division of the contingency table formed by the sampled population of size M; and let $\bar{n}_s$ be the mean or expected value of the frequency n_s of a sample of size N which corresponds to m_s; then, if p is the probability of an individual being in the s-class,

$$p=\frac{m_s}{M} \text{ or } p=\frac{\bar{n}_s}{N},$$

provided that the number of repeated samples is very large.

Again, let δn_s be the deviation of n_s from its mean, then

$$n_s=\bar{n}_s+\delta n_s,$$

* *Biometrika*, Vol. XI. p. 215.

and it has been proved* that deviations δn_s arrange themselves according to a hypergeometric series of which the moment coefficients are given by

$$\left.\begin{aligned} &\mu_2 = \chi_1 Npq, \quad \mu_3 = \chi_1\chi_2 Npq(p-q), \\ &\mu_4 = \chi_1 Npq(3\chi_3 Npq + \chi_4), \\ \text{where}\quad &q = 1-p, \quad \chi_1 = 1 - \frac{N-1}{M-1}, \\ &\chi_2 = 1 - \frac{2(N-1)}{M-2}, \\ &\chi_3 = \left(1-\frac{2}{N}\right)\left\{1 - \frac{N-1}{M-2}\left(\frac{N-10}{N-2} + \frac{9}{M-3}\right)\right\}, \\ \text{and}\quad &\chi_4 = 1 - 6\,\frac{N-1}{M-2}\left(1 - \frac{N-2}{M-3}\right) \end{aligned}\right\} \quad \text{.(3).}$$

But for sampling from infinite populations, to which I propose to confine my attention, we may put

$$\chi_1 = \chi_2 = \chi_4 = 1 \quad\text{and}\quad \chi_3 = 1 - \frac{2}{N} \quad\text{.......................(4).}$$

From these formulae, for infinite populations, we can deduce† the following expressions for means which are fundamental equations in the theory of this paper:

$$\text{Mean } (\delta n_s)^2 = \bar{n}_s\left(1 - \frac{\bar{n}_s}{N}\right),$$

$$\text{Mean } \delta n_s \delta n_{s'} = -\frac{\bar{n}_s \bar{n}_{s'}}{N},$$

$$\text{Mean } (\delta n_s)^3 = \bar{n}_s\left(1 - \frac{\bar{n}_s}{N}\right)\left(1 - \frac{2}{N}\bar{n}_s\right),$$

$$\text{Mean } (\delta n_s)^2 \delta n_{s'} = -\frac{\bar{n}_s \bar{n}_{s'}}{N}\left(1 - \frac{2}{N}\bar{n}_s\right),$$

$$\text{Mean } \delta n_s \delta n_{s'} \delta n_{s''} = \frac{2}{N^2}\bar{n}_s \bar{n}_{s'} \bar{n}_{s''},$$

$$\text{Mean } (\delta n_s)^4 = \bar{n}_s\left(1 - \frac{\bar{n}_s}{N}\right)\left\{1 + 3\bar{n}_s\left(1 - \frac{2}{N}\right)\left(1 - \frac{\bar{n}_s}{N}\right)\right\},$$

$$\text{Mean } (\delta n_s)^3 \delta n_{s'} = -\frac{\bar{n}_s \bar{n}_{s'}}{N}\left\{1 + 3\left(1 - \frac{2}{N}\right)\bar{n}_s\left(1 - \frac{\bar{n}_s}{N}\right)\right\},$$

$$\text{Mean } (\delta n_s)^2 (\delta n_{s'})^2 = \frac{\bar{n}_s \bar{n}_{s'}}{N}\left\{1 + \left(1 - \frac{2}{N}\right)\left(N - \bar{n}_s - \bar{n}_{s'} + \frac{3\bar{n}_s \bar{n}_{s'}}{N}\right)\right\},$$

$$\text{Mean } (\delta n_s)^2 \delta n_{s'} \delta n_{s''} = -\left(1 - \frac{2}{N}\right)\frac{\bar{n}_s \bar{n}_{s'} \bar{n}_{s''}}{N}\left(1 - \frac{3}{N}\bar{n}_s\right),$$

$$\text{and}\quad \text{Mean } \delta n_s \delta n_{s'} \delta n_{s''} \delta n_{s'''} = 3\left(1 - \frac{2}{N}\right)\frac{\bar{n}_s \bar{n}_{s'} \bar{n}_{s''} \bar{n}_{s'''}}{N^2} \quad\text{..........................(5).}$$

* K. Pearson, *Phil. Mag.* 1899, p. 239.

† These formulae are all given in *Biometrika*, Vol. XI. p. 217.

II. *The Deviation of* ϕ_1^2.

(3) The mean square contingency of the sampled population is given by the following equation

$$\tilde{\phi}^2 = \underset{p=1}{\overset{p=\kappa}{S}}\ \underset{q=1}{\overset{q=\lambda}{S}} \left(\frac{\bar{n}_{pq}^2}{\bar{n}_{p.}\bar{n}_{.q}}\right) - 1,$$

or simply by
$$\tilde{\phi}^2 = S\left(\frac{\bar{n}_{pq}^2}{\bar{n}_{p.}\bar{n}_{.q}}\right) - 1,$$

where, and hereafter, S stands for the double summation

$$\underset{p=1}{\overset{p=\kappa}{S}}\ \underset{q=1}{\overset{q=\lambda}{S}}.$$

Now let $\delta\phi_1^2$ be the deviation of ϕ_1^2 from the population value $\tilde{\phi}^2$, and if we write as follows,

$$\text{Mean } \delta\phi_1^2 = \mu_1', \quad \text{Mean } (\delta\phi_1^2)^2 = \mu_2',$$

then the mean and standard deviation of ϕ_1^2 are given by

$$\text{Mean } \phi_1^2 \ (= \bar{\phi}_1^2, \text{ say}) = \tilde{\phi}^2 + \mu_1' \quad \text{.......................(6)},$$

and
$$\sigma_{\phi_1^2} = \sqrt{\mu_2' - (\mu_1')^2} \quad \text{...............................(7)}.$$

Therefore, if we can find μ_1' and μ_2', we can at once find the mean and standard deviation of ϕ_1^2. But

$$\delta\phi_1^2 = S\left(\frac{n_{pq}^2}{n_{p.}n_{.q}}\right) - S\left(\frac{\bar{n}_{pq}^2}{\bar{n}_{p.}\bar{n}_{.q}}\right)$$
$$= S\left[\frac{\bar{n}_{pq}^2}{\bar{n}_{p.}\bar{n}_{.q}}\left\{\left(1 + \frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2 \Big/ \left(1 + \frac{\delta n_{p.}}{\bar{n}_{p.}}\right)\left(1 + \frac{\delta n_{.q}}{\bar{n}_{.q}}\right) - 1\right\}\right].$$

And, unless the expression

$$\left(1 + \frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2 \Big/ \left(1 + \frac{\delta n_{p.}}{\bar{n}_{p.}}\right)\left(1 + \frac{\delta n_{.q}}{\bar{n}_{.q}}\right)$$

be transformed into a form of simple summation of differential products, it is difficult to find the mean values of $\delta\phi_1^2$ or $(\delta\phi_1^2)^2$.

Now, in the usual cases, $\frac{\delta n_{p.}}{\bar{n}_{p.}}$ and $\frac{\delta n_{.q}}{\bar{n}_{.q}}$ may be considered as less than unity and we can expand the above expression into a convergent power series of the logarithmic differentials.

As the expression for $\delta\phi_1^2$ becomes very long and complicated, let us write

$$d_1 = \frac{\delta n_{p.}}{\bar{n}_{p.}} + \frac{\delta n_{.q}}{\bar{n}_{.q}},$$

$$d_2 = \left(\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)^2 + \frac{\delta n_{p.}}{\bar{n}_{p.}}\cdot\frac{\delta n_{.q}}{\bar{n}_{.q}} + \left(\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)^2,$$

$$d_3 = \left(\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)^3 + \left(\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)^2\frac{\delta n_{.q}}{\bar{n}_{.q}} + \frac{\delta n_{p.}}{\bar{n}_{p.}}\left(\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)^2 + \left(\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)^3,$$

and $u_{pq} = \dfrac{\bar{n}_{pq}^2}{\bar{n}_{p.}\bar{n}_{.q}}$,

then

$$\frac{n_{pq}^2}{n_{p.}n_{.q}}=\frac{\bar{n}_{pq}^2\left(1+\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2}{\bar{n}_{p.}\left(1+\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)\bar{n}_{.q}\left(1+\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)}$$

$$=\frac{\bar{n}_{pq}^2}{\bar{n}_{p.}\bar{n}_{.q}}\left(1+\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2\left(1-\frac{\delta n_{p.}}{\bar{n}_{p.}}+\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}-\frac{\delta n_{p.}^3}{\bar{n}_{p.}^3}+\dots\right)$$

$$\times\left(1-\frac{\delta n_{.q}}{\bar{n}_{.q}}+\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}-\frac{\delta n_{.q}^3}{\bar{n}_{.q}^3}+\dots\right)$$

$$=u_{pq}\left(1+\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2\left(1-\frac{\delta n_{p.}}{\bar{n}_{p.}}-\frac{\delta n_{.q}}{\bar{n}_{.q}}+\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}+\frac{\delta n_{p.}\,\delta n_{.q}}{\bar{n}_{p.}\,\bar{n}_{.q}}+\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}-\frac{\delta n_{p.}^3}{\bar{n}_{p.}^3}-\dots\right)$$

$$=u_{pq}\left(1+\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2(1-d_1+d_2-d_3+d_4-\dots)$$

$$=u_{pq}\left\{1-\left(d_1-2\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)+\left(d_2-2d_1\frac{\delta n_{pq}}{\bar{n}_{pq}}+\frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right)-\dots\right\}.$$

Or, if we write again as follows,

$$\delta_1=d_1-2\frac{\delta n_{pq}}{\bar{n}_{pq}}=\frac{\delta n_{p.}}{\bar{n}_{p.}}+\frac{\delta n_{.q}}{\bar{n}_{.q}}-2\frac{\delta n_{pq}}{\bar{n}_{pq}},$$

$$\delta_2=d_2-2d_1\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)+\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2,$$

$$\delta_3=d_3-2d_2\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)+d_1\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2 \quad\text{.............................(8)},$$

and so on, then

$$\frac{n_{pq}^2}{n_{p.}n_{.q}}=u_{pq}(1-\delta_1+\delta_2-\delta_3+\delta_4-\dots) \quad\text{.....................(9)},$$

and

$$1+\phi_1^2=S\left(\frac{n_{pq}^2}{n_{p.}n_{.q}}\right)$$

$$=S(u_{pq})+S\{u_{pq}(-\delta_1+\delta_2-\delta_3+\dots)\}$$

$$=1+\tilde{\phi}^2+S\{u_{pq}(-\delta_1+\delta_2-\delta_3+\dots)\},$$

therefore

$$\delta\phi_1^2=S\{u_{pq}(-\delta_1+\delta_2-\delta_3+\delta_4-\dots)\} \quad\text{.....................(10)}.$$

Thus $\delta\phi_1^2$ is expressed as a form of simple summation which is a convenient form for finding mean values, while δ_m is a homogeneous expression of order m, in the logarithmic differentials $\frac{\delta n_{pq}}{\bar{n}_{pq}}$, $\frac{\delta n_{p.}}{\bar{n}_{p.}}$ and $\frac{\delta n_{.q}}{\bar{n}_{.q}}$ of the form

$$\delta_m=d_m-2d_{m-1}\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)+d_{m-2}\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2$$

$$=\left(\frac{\delta n_{p.}^m}{\bar{n}_{p.}^m}+\frac{\delta n_{p.}^{m-1}}{\bar{n}_{p.}^{m-1}}\cdot\frac{\delta n_{.q}}{\bar{n}_{.q}}+\dots+\frac{\delta n_{.q}^m}{\bar{n}_{.q}^m}\right)$$

$$-2\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)\left\{\frac{\delta n_{p.}^{m-1}}{\bar{n}_{p.}^{m-1}}+\frac{\delta n_{p.}^{m-2}}{\bar{n}_{p.}^{m-2}}\cdot\frac{\delta n_{.q}}{\bar{n}_{.q}}+\dots+\frac{\delta n_{.q}^{m-1}}{\bar{n}_{.q}^{m-1}}\right\}$$

$$+\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2\left\{\frac{\delta n_{p.}^{m-2}}{\bar{n}_{p.}^{m-2}}+\frac{\delta n_{p.}^{m-3}}{\bar{n}_{p.}^{m-3}}\cdot\frac{\delta n_{.q}}{\bar{n}_{.q}}+\dots+\frac{\delta n_{.q}^{m-2}}{\bar{n}_{.q}^{m-2}}\right\}.$$

But as this expression for $\delta\phi_1^2$ is an infinite series, it is very difficult, sometimes impossible, to get any *finite* and *exact* expressions for μ_1' and μ_2'. What we can do is only to get certain approximate expressions for Mean $\delta\phi_1^2$ and Mean $(\delta\phi_1^2)^2$, and even then we have to consider the question of the degree of approximation to get any adequate estimates for practical purposes.

III. *Expressions for First Approximation.*

(4) Now, as the first approximation, let us retain terms of $\delta\phi_1^2$ and $(\delta\phi_1^2)^2$ only to the differential products of second order and let us use, for simplicity, a square bracket, [ ], as the symbol for "mean value in repeated samples"; then, from the Equation (10), we get

$$\delta\phi_1^2 = S\{u_{pq}(-\delta_1+\delta_2)\},$$

and

$$(\delta\phi_1^2)^2 = \{-S(u_{pq}\delta_1)\}^2 \quad \ldots\ldots(11),$$

therefore

$$\mu_1' = [\delta\phi_1^2] = S\{u_{pq}([\delta_2]-[\delta_1])\}.$$

Now

$$[\delta_1] = \left[\frac{\delta n_{p.}}{\bar{n}_{p.}} + \frac{\delta n_{.q}}{\bar{n}_{.q}} - 2\frac{\delta n_{pq}}{\bar{n}_{pq}}\right]$$

$$= \frac{[\delta n_{p.}]}{\bar{n}_{p.}} + \frac{[\delta n_{.q}]}{\bar{n}_{.q}} - \frac{2[\delta n_{pq}]}{\bar{n}_{pq}}.$$

But

$$[\delta n_{p.}] = [\delta n_{.q}] = [\delta n_{pq}] = 0,$$

therefore

$$[\delta_1] = 0.$$

Secondly, let us consider the mean $[\delta_2]$,

$$[\delta_2] = \frac{[\delta n_{p.}^2]}{\bar{n}_{p.}^2} + \frac{[\delta n_{p.}\,\delta n_{.q}]}{\bar{n}_{p.}\,\bar{n}_{.q}} + \frac{[\delta n_{.q}^2]}{\bar{n}_{.q}^2} - 2\left\{\frac{[\delta n_{p.}\,\delta n_{pq}]}{\bar{n}_{p.}\,\bar{n}_{pq}} + \frac{[\delta n_{.q}\,\delta n_{pq}]}{\bar{n}_{.q}\,\bar{n}_{pq}}\right\} + \frac{[\delta n_{pq}^2]}{\bar{n}_{pq}^2},$$

and the following equations can easily be deduced from the fundamental formulae (5),

$$[\delta n_{p.}^2] = \bar{n}_{p.}\left(1-\frac{\bar{n}_{p.}}{N}\right), \quad [\delta n_{pq}^2] = \bar{n}_{pq}\left(1-\frac{\bar{n}_{pq}}{N}\right),$$

$$[\delta n_{p.}\,\delta n_{pq}] = \bar{n}_{pq}\left(1-\frac{\bar{n}_{p.}}{N}\right),$$

and

$$[\delta n_{p.}\,\delta n_{.q}] = \bar{n}_{pq} - \frac{\bar{n}_{p.}\,\bar{n}_{.q}}{N} \quad \ldots\ldots(12).$$

And, from these equations, we get

$$[\delta_2] = \left(\frac{1}{\bar{n}_{p.}}-\frac{1}{N}\right) + \left(\frac{1}{\bar{n}_{.q}}-\frac{1}{N}\right) + \left(\frac{1}{\bar{n}_{pq}}-\frac{1}{N}\right)$$

$$-2\left(\frac{1}{\bar{n}_{p.}}-\frac{1}{N}\right) - 2\left(\frac{1}{\bar{n}_{.q}}-\frac{1}{N}\right) + \left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\,\bar{n}_{.q}}-\frac{1}{N}\right)$$

$$= \frac{1}{\bar{n}_{pq}}\left(1-\frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1-\frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right),$$

and

$$[\delta\phi_1^2] = S\left\{\frac{u_{pq}}{\bar{n}_{pq}}\left(1-\frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1-\frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right)\right\} \quad \ldots\ldots(13)$$

$$= \mu_1'{}_{(1)}, \text{ say}.$$

5) Now let us consider the mean of $(\delta\phi_1^2)^2$.

From the equation (11) we get

$$[(\delta\phi_1^2)^2] = \text{Mean}\,(S\{u_{pq}\delta_1\})^2$$
$$= \text{Mean}\left[S\left\{u_{pq}\left(\frac{\delta n_{p.}}{\bar{n}_{p.}} + \frac{\delta n_{.q}}{\bar{n}_{.q}} - 2\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)\right\}\right]^2,$$

and we have first to expand the right-hand side of this equation,

$$(S\{u_{pq}\delta_1\})^2 = \left\{S\left(u_{pq}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)\right\}^2 + \left\{S\left(u_{pq}\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)\right\}^2 + 4\left\{S\left(u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)\right\}^2$$
$$+ 2S\left(u_{pq}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)S\left(u_{pq}\frac{\delta n_{.q}}{\bar{n}_{.q}}\right) - 4S\left(u_{pq}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)S\left(u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)$$
$$- 4S\left(u_{pq}\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)S\left(u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right) \quad \text{.................................(14).}$$

Let us write, for simplicity, as follows:

$$\overset{p=\kappa}{\underset{p=1}{S}}(u_{pq}) = \overset{p=\kappa}{\underset{p=1}{S}}\left(\frac{\bar{n}_{pq}^2}{\bar{n}_{p.}\bar{n}_{.q}}\right) = u_{.q},$$

and

$$\overset{q=\lambda}{\underset{q=1}{S}}(u_{pq}) = u_{p.},$$

then

$$\left\{S\left(u_{pq}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)\right\}^2 = \left\{\overset{p=\kappa}{\underset{p=1}{S}}\overset{q=\lambda}{\underset{q=1}{S}} u_{pq}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right\}^2$$
$$= \left\{\overset{p=\kappa}{\underset{p=1}{S}}\frac{u_{p.}\,\delta n_{p.}}{\bar{n}_{p.}}\right\}^2$$
$$= \underset{p}{S}\left(\frac{u_{p.}^2\,\delta n_{p.}^2}{\bar{n}_{p.}^2}\right) + \underset{p}{S}\underset{p'}{S'}\left(\frac{u_{p.}\,u_{p'.}}{\bar{n}_{p.}\bar{n}_{p'.}}\delta n_{p.}\,\delta n_{p'.}\right) \quad \text{.........(15 a),}$$

where $\underset{p}{S}$ stands for $\overset{p=\kappa}{\underset{p=1}{S}}$ and $\underset{p'}{S'}$ is a symbol for the summation from $p'=1$ to $p'=\kappa$, excepting the case $p'=p$.

For instance,

$$\underset{p'}{S'}(u_{p.}) = u_{1.} + u_{2.} + \ldots + u_{(p-1).} + u_{(p+1).} + u_{(p+2).} + \ldots + u_{\kappa.}.$$

Similarly $$\underset{q'}{S'}(u_{.q}) = u_{.1} + u_{.2} + \ldots + u_{.(q-1)} + u_{.(q+1)} + \ldots + u_{.\lambda}.$$

We can also easily get the following expressions for the other terms of the expression (14):

$$\left\{S\left(u_{pq}\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)\right\}^2 = \underset{q}{S}\left(\frac{u_{.q}^2\,\delta n_{.q}^2}{\bar{n}_{.q}^2}\right) + \underset{q}{S}\underset{q'}{S'}\left(\frac{u_{.q}u_{.q'}}{\bar{n}_{.q}\bar{n}_{.q'}}\delta n_{.q}\delta n_{.q'}\right) \quad \text{......(15 b),}$$

$$S\left(u_{pq}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)S\left(u_{pq}\frac{\delta n_{.q}}{\bar{n}_{.q}}\right) = \underset{p}{S}\underset{q}{S}\left(\frac{u_{p.}\,u_{.q}}{\bar{n}_{p.}\bar{n}_{.q}}\delta n_{p.}\,\delta n_{.q}\right) \quad \text{.........(15 c),}$$

$$\left\{S\left(u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)\right\}^2 = S\left(\frac{u_{pq}^2}{\bar{n}_{pq}^2}\delta n_{pq}^2\right) + S\underset{p'}{S'}\left(\frac{u_{pq}u_{p'q}}{\bar{n}_{pq}\bar{n}_{p'q}}\delta n_{pq}\delta n_{p'q}\right)$$

$$+ S\underset{q'}{S'}\left(\frac{u_{pq}u_{pq'}}{\bar{n}_{pq}\bar{n}_{pq'}}\delta n_{pq}\delta n_{pq'}\right) + S\underset{p'}{S'}\underset{q'}{S'}\left(\frac{u_{pq}u_{p'q'}}{\bar{n}_{pq}\bar{n}_{p'q'}}\delta n_{pq}\delta n_{p'q'}\right) \quad \ldots\ldots(15\,d),$$

$$S\left(u_{pq}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)S\left(u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right) = S\left(\frac{u_{pq}u_{p.}}{\bar{n}_{pq}\bar{n}_{p.}}\delta n_{pq}\delta n_{p.}\right) + S\underset{p'}{S'}\left(\frac{u_{p.}u_{p'q}}{\bar{n}_{p.}\bar{n}_{p'q}}\delta n_{p.}\delta n_{p'q}\right)$$
$$\ldots\ldots\ldots(15\,e),$$

$$S\left(u_{pq}\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)S\left(u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right) = S\left(\frac{u_{pq}u_{.q}}{\bar{n}_{pq}\bar{n}_{.q}}\delta n_{pq}\delta n_{.q}\right) + S\underset{q'}{S'}\left(\frac{u_{.q}u_{pq'}}{\bar{n}_{.q}\bar{n}_{pq'}}\delta n_{.q}\delta n_{pq'}\right)$$
$$\ldots\ldots\ldots(15\,f).$$

It is necessary first to find the means $[\delta n_{p.}\delta n_{p'.}]$ and $[\delta n_{pq}\delta n_{p'.}]$ besides those already given.

But from the fundamental formulae (5), we can deduce at once

$$[\delta n_{p.}\delta n_{p'.}] = -\frac{\bar{n}_{p.}\bar{n}_{p'.}}{N}, \quad [\delta n_{pq}\delta n_{p'.}] = -\frac{\bar{n}_{pq}\bar{n}_{p'.}}{N} \quad \ldots\ldots\ldots\ldots\ldots(16).$$

Now let us substitute the expressions (15) in (14), and take mean values, then from the Equations (12), (14) and (16), after simplification and transformation, we get the following expression for μ_2' as its first approximation:

$$[(\delta\phi_1^2)^2] = [(S(u_{pq}\delta_1))^2]$$
$$= 4S\left(\frac{u_{pq}^2}{\bar{n}_{pq}}\right) - 3\underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}}\right) - 3\underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}}\right) + 2S\left(\frac{u_{p.}u_{.q}\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}\right)$$
$$= \mu'_{2\,(1)}, \text{ say} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17),$$

which, as we shall see below, is of order $\frac{1}{N}$ *.

Now
$$\mu_2 = \{\sigma_{\phi_1^2}\}^2 = \mu'_{2\,(1)} - (\mu'_{1\,(1)})^2.$$

But
$$\mu'_{1\,(1)} = S\left\{\frac{u_{pq}}{\bar{n}_{pq}}\left(1-\frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1-\frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right)\right\}$$

is of order $\frac{1}{N}$ as we shall see later*, and therefore $\{\mu'_{1\,(1)}\}^2$ is of order $\frac{1}{N^2}$, while $\mu'_{2\,(1)}$ is of order $\frac{1}{N}$.

Therefore as the expression of first approximation for μ_2, we may omit $\{\mu'_{1\,(1)}\}^2$ and we have

$$\{\sigma_{\phi_1^2}\}^2 = 4S\left(\frac{u_{pq}^2}{\bar{n}_{pq}}\right) - 3\underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}}\right) - 3\underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}}\right) + 2S\left(\frac{u_{p.}u_{.q}\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}\right) \quad \ldots\ldots(18),$$

and
$$\text{Mean } \phi_1^2 = S(u_{pq}) + S\left\{\frac{u_{pq}}{\bar{n}_{pq}}\left(1-\frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1-\frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right)\right\} \quad \ldots\ldots\ldots\ldots(19).$$

* See Article (18).

Note (i). If
$$\phi_{pq}^2 = \left(\bar{n}_{pq} - \frac{\bar{n}_{p.}\bar{n}_{.q}}{N}\right)^2 \Big/ \bar{n}_{p.}\bar{n}_{.q},$$
and ϕ_p^2 be the contribution to the contingency from a single row and ϕ_q^2 from a single column, then we have, approximately,
$$4\phi_1^2\{\sigma_{\phi_1}\}^2 = 4S\left(\phi_{pq}^2\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}\right) + 2S\left(\phi_p^2\phi_q^2\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}\right) - 3\underset{p}{S}\left(\frac{\phi_p^4}{\bar{n}_{p.}}\right) - 3S\left(\frac{\phi_q^4}{\bar{n}_{.q}}\right),$$
as has been shown by Prof. K. Pearson and Mr J. Blakeman*.

In my notation
$$\phi_{pq}^2 = u_{pq} - \frac{2}{N}\bar{n}_{pq} + \frac{\bar{n}_{p.}\bar{n}_{.q}}{N^2}, \quad \phi_p^2 = u_{p.} - \frac{\bar{n}_{p.}}{N} \quad \text{and} \quad \phi_q^2 = u_{.q} - \frac{\bar{n}_{.q}}{N}.$$

If we substitute these values in the right-hand side of those authors' equation and transform, then it becomes
$$4S\left(\frac{u_{pq}^2}{\bar{n}_{pq}}\right) + 2S\left(\frac{u_{p.}u_{.q}\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}\right) - 3\underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}}\right) - 3\underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}}\right) = \mu_2'{}_{(1)},$$
as we should expect, for
$$\{\sigma_{\phi_1^2}\}^2 = 4\phi_1^2\{\sigma_{\phi_1}\}^2 \text{ approximately.}$$

Note (ii). The expression (19) for the Mean ϕ_1^2 can be transformed into the form
$$\text{Mean } \phi_1^2 = \frac{(\kappa-1)(\lambda-1)}{N} + \left(1+\frac{3}{N}\right)\bar{\phi}^2 + S\left(\frac{\bar{\chi}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}\right)$$
$$+ S\left(\frac{\bar{\chi}_{pq}^3}{\bar{n}_{p.}^2\bar{n}_{.q}^2}\right) - S\left(\frac{\bar{\chi}_{pq}^2(\bar{n}_{p.}+\bar{n}_{.q})}{\bar{n}_{p.}^2\bar{n}_{.q}^2}\right),$$
where
$$\bar{\chi}_{pq} = \bar{n}_{pq} - \frac{\bar{n}_{p.}\bar{n}_{.q}}{N} = \text{contingency of the } p, q\text{th cell.}$$

The Mean ϕ_1^2, expressed in powers of $\bar{\chi}_{pq}$ in this form, was found by Prof. K. Pearson many years ago (1910).

Now $\bar{\chi}_{pq}$ is a relatively small quantity, and as it is partly positive and partly negative, the 3rd and 4th terms in this expression are likely to be small, and the first two terms depend on λ, κ, N and $\bar{\phi}^2$ only. Therefore, if we consider samples of the same size drawn from the same population, the term which contributes most to the variation of Mean ϕ_1^2 will be the 5th term, i.e.
$$S\left(\frac{\bar{\chi}_{pq}^2(\bar{n}_{p.}+\bar{n}_{.q})}{\bar{n}_{p.}^2\bar{n}_{.q}^2}\right).$$

Let C, X, Y and Z stand for
$$\frac{(\kappa-1)(\lambda-1)}{N}, \quad S\left(\frac{\bar{\chi}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}\right), \quad S\left(\frac{\bar{\chi}_{pq}^3}{\bar{n}_{p.}^2\bar{n}_{.q}^2}\right) \text{ and } S\left(\frac{\bar{\chi}_{pq}^2(\bar{n}_{p.}+\bar{n}_{.q})}{\bar{n}_{p.}^2\bar{n}_{.q}^2}\right)$$
respectively.

* See *Biometrika*, Vol. v. p. 195.

With this notation the sample value of ϕ_1^2 will be in the long run too large by an amount, which on the average is given by

$$\mu_1' = C + \frac{3}{N}\tilde{\phi}^2 + X + Y - Z.$$

We may now ask whether if we were to substitute the sample ϕ_1^2 for $\tilde{\phi}^2$ and the sample cell frequencies into the expressions X, Y, and Z, we should get a reasonable approximation to the true μ_1', and further what is the relative importance of the five different terms in this estimate. To examine these points I have taken two sets of ten samples drawn from the population of Tables V and VII of Article (15), which will be considered in another connection below (Article (19)). I obtained the following results, the line above each table giving the constants for the population.

TABLE I.

2×2-*table*, $\tilde{\phi}^2 = \cdot 494\,950$, $N = 100$, $C = \cdot 01$.

ϕ_1^2	$\frac{3}{N}\phi_1^2$	X	Y	Z	μ_1'
·274 350	·008 230	·000 000	− ·006 618	·011 135	·000 477
·401 296	·012 039	·000 497	− ·002 611	·016 708	·003 217
·440 000	·013 200	·000 000	− ·001 649	·014 930	·006 621
·460 000	·013 800	·000 956	− ·000 609	·020 234	·003 913
·481 668	·014 450	·000 000	− ·000 399	·020 069	·003 982
·519 592	·015 588	·000 472	·000 984	·021 783	·005 261
·540 000	·016 200	·000 000	·001 351	·022 500	·005 051
·560 022	·016 801	− ·000 120	·001 884	·022 663	·005 902
·600 126	·018 004	− ·001 352	·003 966	·023 671	·006 947
·725 556	·021 767	− ·000 206	·006 768	·029 420	·008 909

3×3-*table*, $\tilde{\phi}^2 = \cdot 188\,893$, $N = 200$, $C = \cdot 02$.

ϕ_1^2	$\frac{3}{N}\phi_1^2$	X	Y	Z	μ_1'
·080 840	·001 126	·000 769	− ·001 508	·002 992	·017 395
·150 002	·002 250	·000 502	− ·000 477	·005 038	·017 237
·172 689	·002 590	·001 445	·000 096	·006 440	·017 691
·184 136	·002 762	·000 713	·000 057	·005 290	·017 642
·195 403	·002 931	·002 342	·000 243	·006 682	·018 834
·206 624	·003 099	·000 612	·000 622	·007 456	·016 877
·230 464	·003 457	·001 819	·001 326	·008 784	·017 818
·249 042	·003 736	·002 469	·001 533	·009 371	·018 367
·281 469	·004 222	·001 462	·002 233	·010 075	·017 842
·370 502	·005 558	·001 053	·003 642	·012 692	·017 561

It will be seen that while C and the $\frac{3}{N}\phi_1^2$ and Z terms are the largest, X and Y being of a smaller order, it would yet scarcely be safe to neglect the latter. In the first

illustration we see that μ_1' is at most only about $\frac{1}{50}$th of $\tilde{\phi}^2$; in the second illustration μ_1' is about $\frac{1}{10}$th of $\tilde{\phi}^2$, but we see that it is remarkably steady. A larger amount of experimental work would be requisite, however, before we could interpret the exact meaning of these results *.

(6) *Numerical Illustrations.*

Now let us take a fourfold contingency table, where the frequencies of the sampled population, reduced to sample size of 100, are as given in Table II, and find the mean and standard deviation of ϕ_1^2, applying the formulae (18) and (19).

TABLE II.

15	25	40
40	20	60
55	45	$N=100$

* [It may not be without interest to indicate the method used by me many years ago to correct contingency by the result:

$$\text{Mean } \phi_1^2 = C + \left(1+\frac{3}{N}\right)\tilde{\phi}^2 + X + Y - Z.$$

I argued that the mean ϕ_1^2 was unlikely to be far removed from the modal ϕ_1^2 of samples. Hence although mean ϕ_1^2 was not the most likely value to be obtained in a sample, it was the best approach to such a value. We have then

$$\tilde{\phi}^2 = \frac{N}{N+3}\,(\text{Sample } \phi_1^2 + Z - C - X - Y),$$

where Z, X and Y are to be found from the sample. Our object is to find the correction on ϕ_1^2 which will give us a good value for $\tilde{\phi}^2$, the value ϕ_1^2 being too great. Applying this formula to our two tables we have for $\tilde{\phi}^2$ for the respective samples:

2×2-*table*	3×3-*table*
·273 789	·068 517
·398 173	·133 020
·433 572	·155 259
·456 201	·166 164
·477 802	·176 847
·514 484	·189 996
·535 096	·212 909
·554 292	·230 947
·593 382	·263 891
·717 323	·353 201

	ϕ_1^2	$\Delta\tilde{\phi}^2$		ϕ_1^2	$\Delta\tilde{\phi}^2$
Mean of 10 corrected	·495 411	·000 461	Mean of 10 corrected	·194 575	·005 682
Mean of 10 uncorrected ϕ_1^2	·500 261	·005 311	Mean of 10 uncorrected ϕ_1^2	·212 117	·023 224
Population value $\tilde{\phi}^2$	·494 950		Population value $\tilde{\phi}^2$	·188 893	

We see that while all ϕ_1^2 are *lowered* by correction so that the values of ϕ_1^2 below $\tilde{\phi}^2$ are slightly worsened, the average on ten samples is very close to the true value. I found, however, the corrections too troublesome to be made in the case of tables ranging from 4×4 to 7×7 cells, where they are chiefly needed.—ED.]

Here
$$\lambda = \kappa = 2;$$
$$\bar{n}_{11} = 15,\quad \bar{n}_{12} = 25,\quad \bar{n}_{21} = 40,\quad \bar{n}_{22} = 20;$$
$$\bar{n}_{1.} = 40,\quad \bar{n}_{2.} = 60,\quad \bar{n}_{.1} = 55,\quad \bar{n}_{.2} = 45.$$

From these numerical values we get
$$u_{11} = \cdot 102\,273,\quad u_{12} = \cdot 347\,222,$$
$$u_{21} = \cdot 484\,855,\quad u_{22} = \cdot 148\,148;$$
$$u_{1.} = u_{11} + u_{12} = \cdot 449\,495,\quad u_{2.} = \cdot 633\,003,$$
$$u_{.1} = \cdot 587\,128,\quad u_{.2} = \cdot 495\,370;$$

and, consequently,
$$S\left(\frac{u_{pq}^2}{\bar{n}_{pq}}\right) = \cdot 012\,494,\quad \underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}}\right) + \underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}}\right) = \cdot 023\,450,$$
$$S\left(\frac{u_{p.}u_{.q}\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}\right) = \cdot 011\,720,\qquad S(u_{pq}) = 1 + \tilde{\phi}^2 = 1\cdot 082\,498,$$

and
$$S\left\{\frac{u_{pq}}{\bar{n}_{pq}}\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right)\right\} = \cdot 009\,260;$$

therefore
$$\mu_1{'}_{(1)} = \cdot 009\,260,\quad \mu_2{'}_{(1)} = \cdot 003\,066,$$

and accordingly
$$\bar{\phi}_1^2 = \cdot 091\,758,$$

and
$$\sigma_{\phi_1^2} = \cdot 0554.$$

(7) *The Case of no Contingency.*

Now let us consider the special case of no contingency. In this case
$$\bar{n}_{pq} = \frac{\bar{n}_{p.}\bar{n}_{.q}}{N},\ \text{for any } p \text{ and } q.$$

and, consequently, for any such population, we have the following special relations:
$$u_{pq} = \frac{\bar{n}_{pq}}{N},\quad u_{p.} = \underset{q}{S}(u_{pq}) = \frac{1}{N}\bar{n}_{p.},$$
$$u_{.q} = \frac{1}{N}\bar{n}_{.q},\qquad S\left(\frac{u_{pq}^2}{\bar{n}_{pq}}\right) = \frac{1}{N},$$
$$S\left(\frac{u_{p.}u_{.q}\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}\right) = \frac{1}{N},\quad \underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}}\right) + \underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}}\right) = \frac{2}{N},$$

and
$$S\left\{\frac{u_{pq}}{\bar{n}_{pq}}\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right)\right\} = \frac{1}{N}S\left\{\left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(1 - \frac{\bar{n}_{.q}}{N}\right)\right\}$$
$$= \frac{1}{N}\left\{S(1) - \underset{p}{S}\underset{q}{S}\left(\frac{\bar{n}_{p.}}{N}\right) - \underset{p}{S}\underset{q}{S}\left(\frac{\bar{n}_{.q}}{N}\right) + \frac{1}{N^2}S(\bar{n}_{p.}\bar{n}_{.q})\right\}$$
$$= \frac{1}{N}\{\lambda\kappa - \kappa - \lambda + 1\}$$
$$= \frac{1}{N}(\kappa - 1)(\lambda - 1) \quad\ldots\ldots\ldots\ldots(20);$$

therefore $\mu_1'{}_{(1)} = \frac{1}{N}(\kappa-1)(\lambda-1)$ ....(21),

and
$$\mu_2'{}_{(1)} = 4S\left(\frac{u_{pq}^2}{\bar{n}_{pq}}\right) - 3\underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}}\right) - 3\underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}}\right) + 2S\left(\frac{u_{p.}u_{.q}\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}\right)$$
$$= \frac{4}{N} - \frac{6}{N} + \frac{2}{N}$$
$$= 0.$$

In this case, as μ_2' becomes identically zero, we cannot get any estimate for the standard deviation of ϕ_1^2 from the formulae (17) and (19) of the first approximation, and we have to find more exact expressions for μ_1' and μ_2' to get any adequate estimate of ϕ_1^2 and its standard error in such special cases.

Therefore, we need next to find expressions for μ_1' and μ_2' of one higher order of approximation.

IV. *Expression for the μ_1' of ϕ_1^2 to a Second Approximation.*

(8) Now "order" in statistical meaning is not the same as order in mathematics, and in the evaluation of $[\delta\phi_1^2]$ and $[(\delta\phi_1^2)^2]$ some terms of the 2nd order in statistical meaning come from the 4th order differential products, and we have now to retain terms of $\delta\phi_1^2$ to the 4th differential products.

Thus we get the following relations as equations for starting:

$$\delta\phi_1^2 = S\{u_{pq}(-\delta_1+\delta_2-\delta_3+\delta_4)\} \quad \text{.................................(22)},$$

and
$$(\delta\phi_1^2)^2 = \{S(u_{pq}\delta_1)\}^2 + \{S(u_{pq}\delta_2)\}^2$$
$$- 2S(u_{pq}\delta_1)S(u_{pq}\delta_2) + 2S(u_{pq}\delta_1)S(u_{pq}\delta_3) \quad \text{..............(23)}.$$

From the Equations (13) and (22),

$$\mu_1' = [\delta\phi_1^2] = \mu_1'{}_{(1)} - S\{u_{pq}([\delta_3]-[\delta_4])\} \quad \text{..................(24)},$$

and we have now to find the means of $[\delta_3]$ and $[\delta_4]$.

Now
$$[\delta_3] = \frac{[\delta n_{p.}^3]}{\bar{n}_{p.}^3} + \frac{[\delta n_{p.}^2\delta n_{.q}]}{\bar{n}_{p.}^2\bar{n}_{.q}} + \frac{[\delta n_{p.}\delta n_{.q}^2]}{\bar{n}_{p.}\bar{n}_{.q}^2} + \frac{[\delta n_{.q}^3]}{\bar{n}_{.q}^3}$$
$$- \frac{2}{\bar{n}_{pq}}\left\{\frac{[\delta n_{pq}\delta n_{p.}^2]}{\bar{n}_{p.}^2} + \frac{[\delta n_{pq}\delta n_{p.}\delta n_{.q}]}{\bar{n}_{p.}\bar{n}_{.q}} + \frac{[\delta n_{pq}\delta n_{.q}^2]}{\bar{n}_{.q}^2}\right\}$$
$$+ \frac{1}{\bar{n}_{pq}^2}\left\{\frac{[\delta n_{pq}^2\delta n_{p.}]}{\bar{n}_{p.}} + \frac{[\delta n_{pq}^2\delta n_{.q}]}{\bar{n}_{.q}}\right\} \text{.........(25)},$$

and, to find $[\delta_3]$, means of the following type must be found:

$$[\delta n_{p.}^3], \quad [\delta n_{p.}^2\delta n_{.q}], \quad [\delta n_{pq}^2\delta n_{p.}],$$
$$[\delta n_{pq}\delta n_{p.}^2] \quad \text{and} \quad [\delta n_{pq}\delta n_{p.}\delta n_{.q}].$$

But the following equations can easily be deduced from the fundamental formulae (5) and most of them are given in *Biometrika*, Vol. XII. p. 268:

$$[\delta n_{p.}^3] = \bar{n}_{p.}\left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(1 - \frac{2}{N}\bar{n}_{p.}\right),$$

$$[\delta n_{p.}^2 \delta n_{.q}] = \left(1 - \frac{2}{N}\bar{n}_{p.}\right)\left(\bar{n}_{pq} - \frac{\bar{n}_{p.}\bar{n}_{.q}}{N}\right),$$

$$[\delta n_{pq}^2 \delta n_{p.}] = \bar{n}_{pq}\left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(1 - \frac{2}{N}\bar{n}_{pq}\right),$$

$$[\delta n_{pq} \delta n_{p.}^2] = \bar{n}_{pq}\left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(1 - \frac{2}{N}\bar{n}_{p.}\right),$$

$$[\delta n_{pq} \delta n_{p.} \delta n_{.q}] = \bar{n}_{pq}\left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(1 - \frac{\bar{n}_{.q}}{N}\right) - \frac{\bar{n}_{pq}}{N}\left(\bar{n}_{pq} - \frac{\bar{n}_{p.}\bar{n}_{.q}}{N}\right) \quad \text{......(26).}$$

Therefore, from (25) and (26),

$$\begin{aligned}
[\delta_3] = &\left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)\left(\frac{1}{\bar{n}_{p.}} - \frac{2}{N}\right) + \left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right)\left(\frac{1}{\bar{n}_{.q}} - \frac{2}{N}\right) \\
&+ \left(\frac{1}{\bar{n}_{p.}} - \frac{2}{N}\right)\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}} - \frac{1}{N}\right) + \left(\frac{1}{\bar{n}_{.q}} - \frac{2}{N}\right)\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}} - \frac{1}{N}\right) \\
&- 2\left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)\left(\frac{1}{\bar{n}_{p.}} - \frac{2}{N}\right) - 2\left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right)\left(\frac{1}{\bar{n}_{.q}} - \frac{2}{N}\right) \\
&- 2\left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)\left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right) + \frac{2}{N}\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}} - \frac{1}{N}\right. \\
&+ \left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)\left(\frac{1}{\bar{n}_{pq}} - \frac{2}{N}\right) + \left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right)\left(\frac{1}{\bar{n}_{pq}} - \frac{2}{N}\right) \\
= &\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}} - \frac{2}{N}\right)\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}} - \frac{1}{\bar{n}_{p.}} - \frac{1}{\bar{n}_{.q}} + \frac{1}{\bar{n}_{pq}}\right) \quad \text{......(27);}
\end{aligned}$$

secondly,

$$\begin{aligned}
[\delta_4] = &[d_4] - 2\left[d_3 \frac{\delta n_{pq}}{\bar{n}_{pq}}\right] + \left[d_2 \frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right] \\
&\frac{[\delta n_{p.}^4]}{\bar{n}_{p.}^4} + \frac{[\delta n_{p.}^3 \delta n_{.q}]}{\bar{n}_{p.}^3 \bar{n}_{.q}} + \frac{[\delta n_{p.}^2 \delta n_{.q}^2]}{\bar{n}_{p.}^2 \bar{n}_{.q}^2} + \frac{[\delta n_{p.} \delta n_{.q}^3]}{\bar{n}_{p.}\bar{n}^3} \\
&+ \frac{[\delta n_{.q}^4]}{\bar{n}_{.q}^4} - \frac{2}{\bar{n}_{pq}}\left\{\frac{[\delta n_{pq}\delta n_{p.}^3]}{\bar{n}_{p.}^3} + \frac{[\delta n_{pq}\delta n_{p.}^2\delta n_{.q}]}{\bar{n}_{p.}^2\bar{n}_{.q}} + \frac{[\delta n_{pq}\delta n_{p.}\delta n_{.q}^2]}{\bar{n}_{p.}\bar{n}_{.q}^2} + \frac{[\delta n_{pq}\delta n_{.q}^3]}{\bar{n}_{.q}^3}\right\} \\
&+ \frac{1}{\bar{n}_{pq}^2}\left\{\frac{[\delta n_{pq}^2\delta n_{p.}^2]}{\bar{n}_{p.}^2} + \frac{[\delta n_{pq}^2\delta n_{p.}\delta n_{.q}]}{\bar{n}_{p.}\bar{n}_{.q}} + \frac{[\delta n_{pq}^2\delta n_{.q}^2]}{\bar{n}_{.q}^2}\right\} \quad \text{..................(28),}
\end{aligned}$$

and we have to find means of the following products of 4th order:

$[\delta n_{p.}^4]$, $[\delta n_{p.}^3 \delta n_{pq}]$, $[\delta n_{p.}^2 \delta n_{pq}^2]$, $[\delta n_{p.}^3 \delta n_{.q}]$, $[\delta n_{p.}^2 \delta n_{.q}^2]$,
$[\delta n_{pq} \delta n_{p.}^2 \delta n_{.q}]$, and $[\delta n_{pq}^2 \delta n_{p.} \delta n_{.q}]$.

But the expressions for the first three means are given in *Biometrika*, Vol. XII. p. 268, and we have here to find only the expressions for the last four means.

Let us, for example, find the expressions for the mean of $\delta n_{p.}{}^2 \delta n_{.q}{}^2$.

Now
$$\delta n_{p.}{}^2 \delta n_{.q}{}^2 = \delta n_{p.} \delta n_{.q} (\delta n_{pq} + \underset{q'}{S'} \delta n_{pq'}) (\delta n_{pq} + \underset{p'}{S'} \delta n_{p'q})$$
$$= \delta n_{pq}{}^2 \delta n_{p.} \delta n_{.q} + \underset{p'}{S'} (\delta n_{pq} \delta n_{p.} \delta n_{.q} \delta n_{p'q}) + \underset{q'}{S'} (\delta n_{pq} \delta n_{p.} \delta n_{.q} \delta n_{pq'})$$
$$+ \underset{p'}{S'} \underset{'}{S'} (\delta n_{p.} \delta n_{.q} \delta n_{pq'} \delta n_{p'q}) \qquad \text{.........................(29).}$$

But
$$\delta n_{pq}{}^2 \delta n_{p.} \delta n_{.q} = (\delta n_{pq} + \underset{q'}{S'} \delta n_{pq'}) (\delta n_{pq}{}^2 \delta n_{.q})$$
$$= \delta n_{pq}{}^3 \delta n_{.q} + \underset{q'}{S'} \delta n_{pq'} \delta n_{pq}{}^2 (\delta n_{pq} + \underset{p'}{S'} \delta n_{p'q})$$
$$= \delta n_{pq}{}^3 \delta n_{.q} + \underset{q'}{S'} (\delta n_{pq'} \delta n_{pq}{}^3) + \underset{q'}{S'} \underset{p'}{S'} (\delta n_{pq}{}^2 \delta n_{p'q} \delta n_{pq'}) \qquad \text{...(30),}$$

and from the fundamental formulae (5)

$$[\delta n_{pq}{}^3 \delta n_{.q}] = \bar{n}_{pq} \left(1 - \frac{\bar{n}_{.q}}{N}\right) \left\{1 + 3\left(1 - \frac{2}{N}\right) \bar{n}_{pq} \left(1 - \frac{\bar{n}_{pq}}{N}\right)\right\},$$
$$[\delta n_{pq}{}^3 \delta n_{pq'}] = -\frac{\bar{n}_{pq} \bar{n}_{pq'}}{N} \left\{1 + 3\left(1 - \frac{2}{N}\right) \bar{n}_{pq} \left(1 - \frac{\bar{n}_{pq}}{N}\right)\right\},$$
$$[\delta n_{pq}{}^2 \delta n_{pq'} \delta n_{p'q}] = -\left(1 - \frac{2}{N}\right) \frac{\bar{n}_{pq} \bar{n}_{pq'} \bar{n}_{p'q}}{N} \left(1 - \frac{3}{N} \bar{n}_{pq}\right) \qquad \text{.................(31).}$$

Therefore, from (30) and (31),

$$[\delta n_{pq}{}^2 \delta n_{p.} \delta n_{.q}] = [\delta n_{pq}{}^3 \delta n_{.q}] + \underset{q'}{S'} [\delta n_{pq}{}^3 \delta n_{pq'}] + \underset{p'}{S'} \underset{q'}{S'} [\delta n_{pq}{}^2 \delta n_{pq'} \delta n_{p'q}]$$
$$= \bar{n}_{pq} \left(1 - \frac{\bar{n}_{.q}}{N}\right) \left\{1 + 3\left(1 - \frac{2}{N}\right) \bar{n}_{pq} \left(1 - \frac{\bar{n}_{pq}}{N}\right)\right\}$$
$$- \frac{1}{N} \bar{n}_{pq} (\bar{n}_{p.} - \bar{n}_{pq}) \left\{1 + 3\left(1 - \frac{2}{N}\right) \bar{n}_{pq} \left(1 - \frac{\bar{n}_{pq}}{N}\right)\right\}$$
$$- \frac{1}{N} \left(1 - \frac{2}{N}\right) \bar{n}_{pq} (\bar{n}_{p.} - \bar{n}_{pq}) (\bar{n}_{.q} - \bar{n}_{pq}) \left(1 - \frac{3}{N} \bar{n}_{pq}\right)$$
$$= \bar{n}_{pq} \left(1 - \frac{\bar{n}_{p.}}{N} - \frac{\bar{n}_{.q}}{N} + \frac{\bar{n}_{pq}}{N}\right) + 3\bar{n}_{pq}{}^2 \left(1 - \frac{2}{N}\right) \left(1 - \frac{\bar{n}_{p.}}{N}\right) \left(1 - \frac{\bar{n}_{.q}}{N}\right)$$
$$- \frac{1}{N} \left(1 - \frac{2}{N}\right) \bar{n}_{pq} (\bar{n}_{p.} - \bar{n}_{pq}) (\bar{n}_{.q} - \bar{n}_{pq}) \qquad \text{...........(31 a).}$$

Similarly we can deduce

$$[\delta n_{p.} \delta n_{.q} \delta n_{pq} \delta n_{p'q}] = \bar{n}_{pq} \bar{n}_{p'q} \left(1 - \frac{2}{N}\right) \left(1 - \frac{2}{N} \bar{n}_{p.} - \frac{\bar{n}_{.q}}{N} - \frac{\bar{n}_{pq}}{N} + \frac{3}{N^2} \bar{n}_{p.} \bar{n}_{.q}\right) \text{...(31 b),}$$

$$[\delta n_{p.} \delta n_{.q} \delta n_{pq} \delta n_{pq'}] = \bar{n}_{pq} \bar{n}_{pq'} \left(1 - \frac{2}{N}\right) \left(1 - \frac{\bar{n}_{p}}{N} - \frac{2}{N} \bar{n}_{.q} - \frac{\bar{n}_{pq}}{N} + \frac{3}{N^2} \bar{n}_{p.} \bar{n}_{.q}\right) \text{...(31 c),}$$

and $[\delta n_{p.} \delta n_{.q} \delta n_{p'q} \delta n_{pq'}]$

$$= \bar{n}_{p'q} \bar{n}_{pq'} \left\{1 + \left(1 - \frac{2}{N}\right) \left(1 - \frac{\bar{n}_{p.}}{N} - \frac{\bar{n}_{.q}}{N} - \frac{\bar{n}_{pq}}{N} + \frac{3}{N^2} \bar{n}_{p.} \bar{n}_{.q}\right)\right\} \text{...(31 d).}$$

From these equations, after substitution, transformation and simplification, we get

$$[\delta n_{p.}^2 \delta n_{.q}^2] = \bar{n}_{pq}\left(1 - \frac{\bar{n}_{p.}}{N} - \frac{\bar{n}_{.q}}{N} + \frac{\bar{n}_{pq}}{N}\right) + \frac{1}{N}(\bar{n}_{p.} - \bar{n}_{pq})(\bar{n}_{.q} - \bar{n}_{pq})$$
$$+ \left(1 - \frac{2}{N}\right)\left\{2\left(\bar{n}_{pq} - \frac{\bar{n}_{p.}\bar{n}_{.q}}{N}\right)^2 + \bar{n}_{p.}\bar{n}_{.q}\left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(1 - \frac{\bar{n}_{.q}}{N}\right)\right\} \ldots (32).$$

In the same way we can deduce exact expressions for the other three means.

But all these differential products are of the 4th order and, as our present aim is to get approximate expressions for μ_1' and μ_2' to the 2nd order in statistical sense, some terms of them can be neglected in our calculation.

Let us, therefore, first examine the statistical order of these means.

Now let $\bar{f}_s$ be the proportion in the sth population group, then

$$\bar{f}_s = \frac{1}{N}\bar{n}_s \quad \text{and} \quad \delta f_s = \frac{1}{N}\delta n_s.$$

$$\therefore \quad [\delta f_s^4] = \frac{1}{N^4}[\delta n_s^4]$$
$$= \frac{1}{N^4}\bar{n}_s\left(1 - \frac{\bar{n}_s}{N}\right)\left\{1 + 3\bar{n}_s\left(1 - \frac{2}{N}\right)\left(1 - \frac{\bar{n}_s}{N}\right)\right\}$$
$$= \frac{\bar{n}_s}{N}\left(1 - \frac{\bar{n}_s}{N}\right)\left\{\frac{1}{N^3} + \frac{3}{N^3}N.\frac{\bar{n}_s}{N}\left(1 - \frac{2}{N}\right)\left(1 - \frac{\bar{n}_s}{N}\right)\right\}$$
$$= \bar{f}_s(1 - \bar{f}_s)\left\{\frac{1}{N^3} + \frac{3}{N^2}\bar{f}_s\left(1 - \frac{2}{N}\right)(1 - \bar{f}_s)\right\}$$
$$= \frac{3}{N^2}\bar{f}_s^2(1 - \bar{f}_s)^2 + \frac{1}{N^3}\bar{f}_s(1 - \bar{f}_s)(1 - 6\bar{f}_s + 6\bar{f}_s^2) \quad \ldots\ldots\ldots\ldots (33).$$

By the same method of deduction

$$[\delta f_s^3] = \frac{1}{N^2}\bar{f}_s(1 - \bar{f}_s)(1 - 2\bar{f}_s),$$

and we can see that the second term of $[\delta f_s^4]$, in the last expression of (33), is of one higher order in $\frac{1}{N}$ than the first term, or than $[\delta f_s^3]$, a mean of the 3rd order differential products, and we may therefore neglect this second term of $[\delta f_s^4]$.

Thus we have a very simple approximate expression for $[\delta f_s^4]$, which is exact enough for our present purposes, namely,

$$[\delta f_s^4] = \frac{3}{N^2}\bar{f}_s^2(1 - \bar{f}_s)^2 \quad \text{or} \quad [\delta n_s^4] = 3\bar{n}_s^2\left(1 - \frac{\bar{n}_s}{N}\right)^2 \ldots\ldots\ldots (34\,a).$$

The following equations are all deduced in the same way and are fundamental approximate expressions, in this work, for the mean differential products of the 4th order:

$$[\delta n_s^3 \delta n_{s'}] = -\frac{3}{N}\bar{n}_s^2\bar{n}_{s'}\left(1 - \frac{\bar{n}_s}{N}\right),$$

$$[\delta n_s^2 \delta n_{s'}^2] = \bar{n}_s\bar{n}_{s'}\left(1 - \frac{\bar{n}_s}{N} - \frac{\bar{n}_{s'}}{N} + \frac{3}{N^2}\bar{n}_s\bar{n}_{s'}\right),$$

$$[\delta n_s^2 \delta n_{s'} \delta n_{s''}] = -\frac{1}{N} \bar{n}_s \bar{n}_{s'} \bar{n}_{s''} \left(1 - \frac{3}{N} \bar{n}_s\right),$$

$$[\delta n_s \delta n_{s'} \delta n_{s''} \delta n_{s'''}] = \frac{3}{N^2} \bar{n}_s \bar{n}_{s'} \bar{n}_{s''} \bar{n}_{s'''} \qquad \text{(34 b)},$$

and also

$$[\delta n_{pq}^3 \delta n_{p.}] = 3\bar{n}_{pq}^2 \left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(1 - \frac{\bar{n}_{pq}}{N}\right),$$

$$[\delta n_{pq} \delta n_{p.}^3] = 3\bar{n}_{p.} \bar{n}_{pq} \left(1 - \frac{\bar{n}_{p.}}{N}\right)^2,$$

$$[\delta n_{pq}^2 \delta n_{p.}^2] = \bar{n}_{pq} \left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(2\bar{n}_{pq} + \bar{n}_{p.} - \frac{3}{N} \bar{n}_{p.} \bar{n}_{pq}\right),$$

$$[\delta n_{pq}^2 \delta n_{pq'} \delta n_{p.}] = \bar{n}_{pq} \bar{n}_{pq'} \left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(1 - \frac{3}{N} \bar{n}_{pq}\right),$$

$$[\delta n_{pq} \delta n_{pq'} \delta n_{pq''} \delta n_{p.}] = -\frac{3}{N} \bar{n}_{pq} \bar{n}_{pq'} \bar{n}_{pq''} \left(1 - \frac{\bar{n}_{p.}}{N}\right) \qquad \text{(34 c)}.$$

Now let us find the mean $[\delta n_{p.}^2 \delta n_{.q}^2]$ again, and also other necessary means, starting from these approximate fundamental equations; then we get very simple expressions for them as follows:

$$\left.\begin{aligned}
[\delta n_{p.}^2 \delta n_{.q}^2] &= \bar{n}_{p.} \bar{n}_{.q} \left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(1 - \frac{\bar{n}_{.q}}{N}\right) + 2\left(\bar{n}_{pq} - \frac{\bar{n}_{p.} \bar{n}_{.q}}{N}\right)^2 \\
[\delta n_{p.}^3 \delta n_{.q}] &= 3\bar{n}_{p.} \left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(\bar{n}_{pq} - \frac{\bar{n}_{p.} \bar{n}_{.q}}{N}\right) \\
[\delta n_{pq} \delta n_{p.}^2 \delta n_{.q}] &= \bar{n}_{pq} \left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(2\bar{n}_{pq} + \bar{n}_{p.} - \frac{3}{N} \bar{n}_{p.} \bar{n}_{.q}\right), \\
&\text{and} \\
[\delta n_{pq}^2 \delta n_{p.} \delta n_{.q}] &= 3\bar{n}_{pq}^2 \left(1 - \frac{\bar{n}_{p.}}{N}\right)\left(1 - \frac{\bar{n}_{.q}}{N}\right) - \frac{1}{N} \bar{n}_{pq} (\bar{n}_{p.} - \bar{n}_{pq})(\bar{n}_{.q} - \bar{n}_{pq})
\end{aligned}\right\} \quad \text{(35)}.$$

Now we can find the mean $[\delta_4]$ easily.

From the Equations (28), (34) and (35),

$$\begin{aligned}
[\delta_4] = {} & 3\left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)^2 + 3\left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right)^2 + 3\left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.} \bar{n}_{.q}} - \frac{1}{N}\right) \\
& + 3\left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right)\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.} \bar{n}_{.q}} - \frac{1}{N}\right) + \left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)\left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right) + 2\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.} \bar{n}_{.q}} - \frac{1}{N}\right)^2 \\
& + \left(\frac{1}{\bar{n}_{p}} - \frac{1}{N}\right)\left(\frac{2}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{pq}} - \frac{3}{N}\right) + \left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right)\left(\frac{2}{\bar{n}_{.q}} + \frac{1}{\bar{n}_{pq}} - \frac{3}{N}\right) \\
& + 3\left(\frac{1}{\bar{n}_{p}} - \frac{1}{N}\right)\left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right) - \frac{1}{N} \bar{n}_{pq} \left(\frac{1}{\bar{n}_{pq}} - \frac{1}{\bar{n}_{p.}}\right)\left(\frac{1}{\bar{n}_{pq}} - \frac{1}{\bar{n}_{.q}}\right) \\
& - 2\left\{3\left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)^2 + 3\left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right)^2 + \left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)\left(\frac{2\bar{n}_{pq}}{\bar{n}_{p.} \bar{n}_{.q}} + \frac{1}{\bar{n}_{.q}} - \frac{3}{N}\right)\right. \\
& \left. \qquad + \left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right)\left(\frac{2\bar{n}_{pq}}{\bar{n}_{p.} \bar{n}_{.q}} + \frac{1}{\bar{n}_{p.}} - \frac{3}{N}\right)\right\} \\
= {} & \frac{1}{\bar{n}_{pq}} \left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right)\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}} + 2\frac{\bar{n}_{pq}}{\bar{n}_{p.} \bar{n}_{.q}} - \frac{3}{N}\right) \qquad \text{(36)}.
\end{aligned}$$

Finally, from the Equations (24), (27), (36) and (13), and after transformation and simplification, we get the following equation for μ_1' as its second approximation:

$$\mu_1' = S\{u_{pq}([\delta_2]-[\delta_3]+[\delta_4])\}$$

$$= S\left\{\frac{u_{pq}}{\bar{n}_{pq}}\left(1-\frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1-\frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right)\left(1-\frac{1}{N}+2\frac{\bar{n}_{pq}}{\bar{n}_{p.}.\bar{n}_{.q}}\right)\right\} \quad \text{.........(37).}$$

V. *Expression for the μ_2' of ϕ_1^2 to a Second Approximation.*

(9) Now let us consider the second moment coefficient μ_2' of the mean square contingency ϕ_1^2.

From the Equation (23)

$$\mu_2' = \mu_2'_{(1)} - 2[S(u_{pq}\delta_1)S(u_{pq}\delta_2)] + [(S\{u_{pq}\delta_2\})^2] + 2[S(u_{pq}\delta_1)S(u_{pq}\delta_3)],$$

but the last three terms on the right-hand side are not in the form of simple summations and we have at first to transform them into forms of sums of differential products.

For instance, the mean values $[\delta n_{pq}^3]$, $[\delta n_{pq}^2\delta n_{pq'}]$ cannot be treated formally as special cases of their general form in mathematical meaning, $[\delta n_{pq}\delta n_{pq'}\delta n_{pq''}]$; we have not only to expand the above products and powers, but also to examine, classify and arrange all possible products so that we can apply at once the fundamental formulae (5) or (34), or those formulae which I have deduced already.

Now

$$S(u_{pq}\delta_1)S(u_{pq}\delta_2)$$

$$= S\left(u_{pq}\left\{d_1 - 2\frac{\delta n_{pq}}{\bar{n}_{pq}}\right\}\right)S\left(u_{pq}\left\{d_2 - 2d_1\frac{\delta n_{pq}}{\bar{n}_{pq}} + \frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right\}\right)$$

$$= S(u_{pq}d_1)S(u_{pq}d_2) - 2S(u_{pq}d_1)S\left(u_{pq}d_1\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)$$

$$S(u_{pq}d_1)S\left(u_{pq}\frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right) - 2S\left(u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)S(u_{pq}d_2)$$

$$+ 4S\left(u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)S\left(u_{pq}d_1\frac{\delta n_{pq}}{\bar{n}_{pq}}\right) - 2S\left(u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)S\left(u_{pq}\frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right)$$

$$T_1 - 2T_2 + T_3 - 2T_4 + 4T_5 - 2T_6, \text{ say} \quad \text{.................................(38),}$$

and

T_1 (the first term of (38))

$$= S(u_{pq}d_1)S(u_{pq}d_2)$$

$$\cdot S\left\{u_{pq}\left(\frac{\delta n_{p.}}{\bar{n}_{p.}} + \frac{\delta n_{.q}}{\bar{n}_{.q}}\right)\right\}S\left\{u_{pq}\left(\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2} + \frac{\delta n_{p.}}{\bar{n}_{p.}}.\frac{\delta n_{.q}}{\bar{n}_{.q}} + \frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right)\right\}$$

$$: \left\{\underset{p}{S}\left(u_{p.}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right) + \underset{q}{S}\left(u_{.q}\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)\right\}\left\{\underset{p}{S}\left(u_{p.}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right)\right.$$

$$\left. + \underset{q}{S}\left(u_{.q}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right) + \underset{p}{S}\underset{q}{S}\left(u_{pq}\frac{\delta n_{p.}.\delta n_{.q}}{\bar{n}_{p.}.\bar{n}_{.q}}\right)\right\}$$

$$= \underset{p}{S}\left(u_{p.}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)\underset{p}{S}\left(u_{p.}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right)+\underset{q}{S}\left(u_{.q}\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)\underset{q}{S}\left(u_{.q}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right)$$

$$+\underset{p}{S}\left(u_{p.}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)\underset{q}{S}\left(u_{.q}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right)+\underset{q}{S}\left(u_{.q}\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)\underset{p}{S}\left(u_{p.}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right)$$

$$+\underset{p}{S}\left(u_{p.}\frac{\delta n_{p.}}{\bar{n}_{p.}}\right)S\left(u_{pq}\frac{\delta n_{p.}.\delta n_{.q}}{\bar{n}_{p.}.\bar{n}_{.q}}\right)+\underset{q}{S}\left(u_{.q}\frac{\delta n_{.q}}{\bar{n}_{.q}}\right)S\left(u_{pq}\frac{\delta n_{p.}.\delta n_{.q}}{\bar{n}_{p.}.\bar{n}_{.q}}\right)$$

$$= \underset{p}{S}\left(u_{p.}^2\frac{\delta n_{p.}^3}{\bar{n}_{p.}^3}\right)+\underset{q}{S}\left(u_{.q}^2\frac{\delta n_{.q}^3}{\bar{n}_{.q}^3}\right)+\underset{p}{S}\underset{p'}{S'}\left(u_{p.}u_{p'.}\frac{\delta n_{p.}^2\delta n_{p'.}}{\bar{n}_{p.}^2\bar{n}_{p'.}}\right)$$

$$+\underset{q}{S}\underset{q'}{S'}\left(u_{.q}u_{.q'}\frac{\delta n_{.q}^2\delta n_{.q'}}{\bar{n}_{.q}^2\bar{n}_{.q'}}\right)+S\left(u_{p.}u_{.q}\left\{\frac{\delta n_{p.}^2\delta n_{.q}}{\bar{n}_{p.}^2\bar{n}_{.q}}+\frac{\delta n_{p.}.\delta n_{.q}^2}{\bar{n}_{p.}.\bar{n}_{.q}^2}\right\}\right.$$

$$\left.+u_{pq}u_{p.}\frac{\delta n_{p.}^2\delta n_{.q}}{\bar{n}_{p.}^2\bar{n}_{.q}}+u_{pq}u_{.q}\frac{\delta n_{p.}.\delta n_{.q}^2}{\bar{n}_{p.}.\bar{n}_{.q}^2}\right)+S\underset{p'}{S'}\left(u_{pq}u_{p'.}\frac{\delta n_{p.}.\delta n_{p'.}.\delta n_{.q}}{\bar{n}_{p.}.\bar{n}_{p'.}.\bar{n}_{.q}}\right)$$

$$+S\underset{q'}{S'}\left(u_{pq}u_{.q'}\frac{\delta n_{p.}.\delta n_{.q}\delta n_{.q'}}{\bar{n}_{p.}.\bar{n}_{.q}\bar{n}_{.q'}}\right)\quad\ldots\ldots\ldots\ldots(38\,a).$$

Similarly

$$T_2 = S\left(u_{pq}u_{p.}\left\{\frac{\delta n_{pq}\delta n_{p.}^2}{\bar{n}_{pq}\bar{n}_{p.}^2}+\frac{\delta n_{pq}\delta n_{p.}.\delta n_{.q}}{\bar{n}_{pq}\bar{n}_{p.}.\bar{n}_{.q}}\right\}+u_{pq}u_{.q}\left\{\frac{\delta n_{pq}\delta n_{.q}^2}{\bar{n}_{pq}\bar{n}_{.q}^2}+\frac{\delta n_{pq}\delta n_{p.}.\delta n_{.q}}{\bar{n}_{pq}\bar{n}_{p.}.\bar{n}_{.q}}\right\}\right)$$

$$+S\underset{p'}{S'}\left(u_{p.}u_{p'q}\left\{\frac{\delta n_{p'q}\delta n_{p.}.\delta n_{p'.}}{\bar{n}_{p'q}\bar{n}_{p.}.\bar{n}_{p'.}}+\frac{\delta n_{p'q}\delta n_{p.}.\delta n_{.q}}{\bar{n}_{p'q}\bar{n}_{p.}.\bar{n}_{.q}}\right\}\right)$$

$$+S\underset{q'}{S'}\left(u_{.q}u_{pq'}\left\{\frac{\delta n_{pq'}\delta n_{.q}\delta n_{.q'}}{\bar{n}_{pq'}\bar{n}_{.q}\bar{n}_{.q'}}+\frac{\delta n_{pq'}\delta n_{p.}.\delta n_{.q}}{\bar{n}_{pq'}\bar{n}_{p.}.\bar{n}_{.q}}\right\}\right)\quad\ldots\ldots\ldots\ldots(38\,b),$$

$$T_3 = S\left(u_{pq}u_{p.}\frac{\delta n_{pq}^2\delta n_{p.}}{\bar{n}_{pq}^2\bar{n}_{p.}}+u_{pq}u_{.q}\frac{\delta n_{pq}^2\delta n_{.q}}{\bar{n}_{pq}^2\bar{n}_{.q}}\right)$$

$$+S\underset{p'}{S'}\left(u_{p.}u_{p'q}\frac{\delta n_{p.}.\delta n_{p'q}^2}{\bar{n}_{p.}.\bar{n}_{p'q}^2}\right)+S\underset{q'}{S'}\left(u_{.q}u_{pq'}\frac{\delta n_{.q}\delta n_{pq'}^2}{\bar{n}_{.q}\bar{n}_{pq'}^2}\right)\ldots(38\,c),$$

$$T_4 = S\left(u_{pq}u_{p.}\frac{\delta n_{pq}\delta n_{p.}^2}{\bar{n}_{pq}\bar{n}_{p.}^2}+u_{pq}u_{.q}\frac{\delta n_{pq}\delta n_{.q}^2}{\bar{n}_{pq}\bar{n}_{.q}^2}+u_{pq}^2\frac{\delta n_{p.}.\delta n_{.q}\delta n_{pq}}{\bar{n}_{p.}.\bar{n}_{.q}\bar{n}_{pq}}\right)$$

$$+S\underset{p'}{S'}\left(u_{p.}u_{p'q}\frac{\delta n_{p.}^2\delta n_{p'q}}{\bar{n}_{p.}^2\bar{n}_{p'q}}+u_{pq}u_{p'q}\frac{\delta n_{p.}.\delta n_{.q}\delta n_{p'q}}{\bar{n}_{p.}.\bar{n}_{.q}\bar{n}_{p'q}}\right)$$

$$+S\underset{q'}{S'}\left(u_{.q}u_{pq'}\frac{\delta n_{.q}^2\delta n_{pq'}}{\bar{n}_{.q}^2\bar{n}_{pq'}}+u_{pq}u_{pq'}\frac{\delta n_{.q}\delta n_{p.}.\delta n_{pq'}}{\bar{n}_{.q}\bar{n}_{p.}.\bar{n}_{pq'}}\right)$$

$$+S\underset{p'}{S'}\underset{q'}{S'}\left(u_{pq}u_{p'q'}\frac{\delta n_{p.}.\delta n_{.q}\delta n_{p'q'}}{\bar{n}_{p.}.\bar{n}_{.q}\bar{n}_{p'q'}}\right)\quad\ldots\ldots\ldots\ldots(38\,d),$$

$$T_5 = S\left(u_{pq}^2\left\{\frac{\delta n_{pq}^2\delta n_{p.}}{\bar{n}_{pq}^2\bar{n}_{p.}}+\frac{\delta n_{pq}^2\delta n_{.q}}{\bar{n}_{pq}^2\bar{n}_{.q}}\right\}\right)$$

$$+S\underset{p'}{S'}\left(u_{pq}u_{p'q}\left\{\frac{\delta n_{pq}\delta n_{p'q}\delta n_{p'.}}{\bar{n}_{pq}\bar{n}_{p'q}\bar{n}_{p'.}}+\frac{\delta n_{pq}\delta n_{p'q}\delta n_{.q}}{\bar{n}_{pq}\bar{n}_{p'q}\bar{n}_{.q}}\right\}\right)$$

$$+S\underset{q'}{S'}\left(u_{pq}u_{pq'}\left\{\frac{\delta n_{pq}\delta n_{pq'}.\delta n_{.q'}}{\bar{n}_{pq}\bar{n}_{pq'}.\bar{n}_{.q'}}+\frac{\delta n_{pq}\delta n_{pq'}.\delta n_{p.}}{\bar{n}_{pq}\bar{n}_{pq'}.\bar{n}_{p.}}\right\}\right)$$

$$+S\underset{p'}{S'}\underset{q'}{S'}\left(u_{pq}u_{p'q'}\left\{\frac{\delta n_{pq}\delta n_{p'q'}.\delta n_{p'.}}{\bar{n}_{pq}\bar{n}_{p'q'}.\bar{n}_{p'.}}+\frac{\delta n_{pq}\delta n_{p'q'}.\delta n_{.q'}}{\bar{n}_{pq}\bar{n}_{p'q'}.\bar{n}_{.q'}}\right\}\right)\ldots(38\,e),$$

and $$T_6 = S\left(u_{pq}^2\frac{\delta n_{pq}^3}{\bar{n}_{pq}^3}\right) + \underset{p'}{S\,S'}\left(u_{pq}u_{p'q}\frac{\delta n_{pq}\delta n_{p'q}^2}{\bar{n}_{pq}\bar{n}_{p'q}^2}\right) + \underset{q'}{S\,S'}\left(u_{pq}u_{pq'}\frac{\delta n_{pq}\delta n_{pq'}^2}{\bar{n}_{pq}\bar{n}_{pq'}^2}\right)$$
$$+ \underset{p'}{S}\,\underset{q'}{S'}\,S'\left(u_{pq}u_{p'q'}\frac{\delta n_{pq}\delta n_{p'q'}^2}{\bar{n}_{pq}\bar{n}_{p'q'}^2}\right) \quad\text{.............................(38}\,f).$$

From these expressions, it is evident that we have to find the following kinds of means, besides those already given, which are all exact and can all be deduced by the same method of transformation as in Article (8):

$$\left.\begin{aligned}
[\delta n_{pq}\delta n_{p'q}\delta n_{p'.}] &= -\frac{1}{N}\bar{n}_{pq}\bar{n}_{p'q}\left(1-\frac{2}{N}\bar{n}_{p'.}\right)\\
[\delta n_{p.}\delta n_{p'.}\delta n_{.q}] &= -\frac{1}{N}\left(\bar{n}_{p.}\bar{n}_{p'q}+\bar{n}_{p'.}\bar{n}_{pq}-\frac{2}{N}\bar{n}_{p.}\bar{n}_{p'.}\bar{n}_{.q}\right)\\
[\delta n_{p'q}\delta n_{p.}\delta n_{p'.}] &= -\frac{1}{N}\bar{n}_{p.}\bar{n}_{p'q}\left(1-\frac{2}{N}\bar{n}_{p'.}\right)\\
[\delta n_{p'q'}\delta n_{p.}\delta n_{.q}] &= -\frac{1}{N}\bar{n}_{p'q'}\left(\bar{n}_{pq}-\frac{2}{N}\bar{n}_{p.}\bar{n}_{.q}\right)\\
[\delta n_{p.}\delta n_{p'q}^2] &= -\frac{1}{N}\bar{n}_{p.}\bar{n}_{p'q}\left(1-\frac{2}{N}\bar{n}_{p'q}\right)\\
[\delta n_{p.}^2\delta n_{p'q}] &= -\frac{1}{N}\bar{n}_{p.}\bar{n}_{p'q}\left(1-\frac{2}{N}\bar{n}_{p.}\right)\\
\text{and}\qquad [\delta n_{p.}\delta n_{.q}\delta n_{p'q}] &= -\frac{1}{N}\bar{n}_{p'q}\left(\bar{n}_{p.}+\bar{n}_{pq}-\frac{2}{N}\bar{n}_{p.}\bar{n}_{.q}\right)
\end{aligned}\right\}\quad\text{.........(39).}$$

Now we can easily find the means of any T-terms. For instance, let us find the mean of T_1.

From the Equations (26), (38 a) and (39), we get

$$\begin{aligned}
[T_1] = {} & \underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}^3}[\delta n_{p.}^3]\right) + \underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}^3}[\delta n_{.q}^3]\right) + \underset{p\ \ p'}{S\,S'}\left(\frac{u_{p.}u_{p'}}{\bar{n}_{p.}^2\bar{n}_{p'.}}[\delta n_{p.}^2\delta n_{p'.}]\right)\\
& + \underset{q\ \ q'}{S\,S'}\left(\frac{u_{.q}u_{.q'}}{\bar{n}_{.q}^2\bar{n}_{.q'}}[\delta n_{.q}^2\delta n_{.q'}]\right) + \underset{p'}{S\,S'}\left(\frac{u_{pq}u_{p'.}}{\bar{n}_{p.}\bar{n}_{p'.}\bar{n}_{.q}}[\delta n_{p.}\delta n_{p'.}\delta n_{.q}]\right)\\
& + \underset{q'}{S\,S'}\left(\frac{u_{pq}u_{.q'}}{\bar{n}_{p.}\bar{n}_{.q}\bar{n}_{.q'}}[\delta n_{p.}\delta n_{.q}\delta n_{.q'}]\right) + S\left\{\frac{u_{p.}u_{.q}}{\bar{n}_{p.}^2\bar{n}_{.q}}[\delta n_{p.}^2\delta n_{.q}]\right.\\
& \left. + \frac{u_{p.}u_{.q}}{\bar{n}_{p.}\bar{n}_{.q}^2}[\delta n_{p.}\delta n_{.q}^2] + \frac{u_{pq}u_{p.}}{\bar{n}_{p.}^2\bar{n}_{.q}}[\delta n_{p.}^2\delta n_{.q}] + \frac{u_{pq}u_{.q}}{\bar{n}_{p.}\bar{n}_{.q}^2}[\delta n_{p.}\delta n_{.q}^2]\right\}\\
= {} & \underset{p}{S}\left\{u_{p.}^2\left(\frac{1}{\bar{n}_{p.}}-\frac{1}{N}\right)\left(\frac{1}{\bar{n}_{p.}}-\frac{2}{N}\right)\right\} + \underset{q}{S}\left\{u_{.q}^2\left(\frac{1}{\bar{n}_{.q}}-\frac{1}{N}\right)\left(\frac{1}{\bar{n}_{.q}}-\frac{2}{N}\right)\right\}\\
& - \frac{1}{N}\underset{p\ \ p'}{S\,S'}\left\{u_{p.}u_{p'.}\left(\frac{1}{\bar{n}_{p.}}-\frac{2}{N}\right)\right\} - \frac{1}{N}\underset{q\ \ q'}{S\,S'}\left\{u_{.q}u_{.q'}\left(\frac{1}{\bar{n}_{.q}}-\frac{2}{N}\right)\right\}\\
& + S\left\{u_{p.}u_{.q}\left[\left(\frac{1}{\bar{n}_{p.}}-\frac{2}{N}\right)\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}-\frac{1}{N}\right)+\left(\frac{1}{\bar{n}_{.q}}-\frac{2}{N}\right)\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}-\frac{1}{N}\right)\right]\right.\\
& \left. + u_{pq}u_{p.}\left(\frac{1}{\bar{n}_{p.}}-\frac{2}{N}\right)\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}-\frac{1}{N}\right) + u_{pq}u_{.q}\left(\frac{1}{\bar{n}_{.q}}-\frac{2}{N}\right)\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}}-\frac{1}{N}\right)\right\}
\end{aligned}$$

$$-\frac{1}{N} S \underset{p'}{S'} \left\{u_{pq} u_{p'.} \left(\frac{\bar{n}_{p'q}}{\bar{n}_{p'.} \bar{n}_{.q}} + \frac{\bar{n}_{pq}}{\bar{n}_{p.} \bar{n}_{.q}} - \frac{2}{N}\right)\right\}$$

$$-\frac{1}{N} S \underset{q'}{S'} \left\{u_{pq} u_{.q'} \left(\frac{\bar{n}_{pq'}}{\bar{n}_{p.} \bar{n}_{.q'}} + \frac{\bar{n}_{pq}}{\bar{n}_{p.} \bar{n}_{.q}} - \frac{2}{N}\right)\right\}.$$

If we use for simplicity the expressions in (40) below, then after transformation and simplification, we get a fairly short expression (41 a) for the mean of T_1,

$$\left.\begin{aligned} S_2' &= \underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}}\right) + \underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}}\right) \\ S_2'' &= \underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}^2}\right) + \underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}^2}\right) \\ S_1' &= \underset{p}{S}\left(\frac{u_{p.}}{\bar{n}_{p.}}\right) + \underset{q}{S}\left(\frac{u_{.q}}{\bar{n}_{.q}}\right) \end{aligned}\right\} \quad .(40),$$

$$[T_1] = S_2'' - \frac{3}{N} S_2' - \frac{2}{N} S_1' (1 + \tilde{\phi}^2) + \frac{12}{N^2} (1 + \tilde{\phi}^2)^2 - \frac{6}{N} S\left(\frac{u_{p.} u_{.q} u_{pq}}{\bar{n}_{pq}}\right)$$
$$- \frac{2}{N} (1 + \tilde{\phi}^2) S\left(\frac{u_{pq}^2}{\bar{n}_{pq}}\right) + S\left\{\frac{u_{pq}^2}{\bar{n}_{pq}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\}$$
$$+ S\left\{\frac{u_{p.} u_{.q} u_{pq}}{\bar{n}_{pq}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}}\right)\right\} \quad \ldots\ldots(41\,a).$$

In the same way, we can obtain the following expressions for the other T-terms:

$$[T_2] = S_2'' - \frac{3}{N} S_2' - \frac{2}{N} S_1' (1 + \tilde{\phi}^2) + \frac{8}{N^2} (1 + \tilde{\phi}^2)^2$$
$$- \frac{2}{N} S\left(\frac{u_{p.} u_{.q} u_{pq}}{\bar{n}_{pq}}\right) + S\left\{\frac{u_{pq}}{\bar{n}_{p.} \bar{n}_{.q}} (u_{p.} + u_{.q})\right\} \quad \ldots\ldots(41\,b),$$

$$[T_3] = -\frac{2}{N} S_2' + \frac{4}{N^2} (1 + \tilde{\phi}^2)^2 + S\left\{\frac{u_{pq}}{\bar{n}_{pq}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\}$$
$$- \frac{2}{N} S\left(\frac{u_{pq}}{\bar{n}_{pq}}\right) (1 + \tilde{\phi}^2) \quad \ldots\ldots(41\,c),$$

$$[T_4] = S_2'' - \frac{3}{N} S_2' + \frac{6}{N^2} (1 + \tilde{\phi}^2)^2 - \frac{1}{N} S_1' (1 + \tilde{\phi}^2)$$
$$+ S\left(\frac{u_{pq}^2}{\bar{n}_{p.} \bar{n}_{.q}}\right) - \frac{1}{N} (1 + \tilde{\phi}^2) S\left(\frac{u_{pq}^2}{\bar{n}_{pq}}\right) \quad \ldots\ldots(41\,d),$$

$$[T_5] = -\frac{1}{N} S_2' - \frac{1}{N} S_1' (1 + \tilde{\phi}^2) + \frac{4}{N^2} (1 + \tilde{\phi}^2)^2$$
$$+ S\left\{\frac{u_{pq}^2}{\bar{n}_{pq}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}} - \frac{2}{N}\right)\right\} \quad \ldots\ldots(41\,e),$$

and $$[T_6] = \frac{2}{N^2} (1 + \tilde{\phi}^2)^2 - \frac{1}{N} (1 + \tilde{\phi}^2) S\left(\frac{u_{pq}}{\bar{n}_{pq}}\right) + S\left\{\frac{u_{pq}^2}{\bar{n}_{pq}}\left(\frac{1}{\bar{n}_{pq}} - \frac{2}{N}\right)\right\} \quad \ldots\ldots(41\,f).$$

Substituting these results in the expression (38), and after transformation and simplification, I have obtained the following equation:

Mean $S(u_{pq}\delta_1)S(u_{pq}\delta_2)$

$$= -3S_2'' + \frac{3}{N}S_2' - 2S\left\{\frac{u_{pq}^2}{\bar{n}_{pq}^2}(1+u_{pq})\right\} - \frac{4}{N}S\left(\frac{u_{pq}^2}{\bar{n}_{pq}}\right)$$

$$+ S\left\{\frac{u_{p.}u_{.q}u_{pq}}{\bar{n}_{pq}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}} - \frac{2}{N}\right)\right\} + 4S\left\{\frac{u_{pq}^2}{\bar{n}_{pq}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}}\right)\right\}$$

$$- 2S\left\{\frac{u_{pq}^2}{\bar{n}_{pq}^2}(u_{p.} + u_{.q})\right\} + S\left\{\frac{u_{pq}}{\bar{n}_{pq}}(1+u_{pq})\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\} \quad \text{.........(42).}$$

(10) *Transformation of the last two Terms of μ_2' for ϕ_1^2.*

Finally, let us consider the terms $(S\{u_{pq}\delta_2\})^2$ and $S\{u_{pq}\delta_1\}S\{u_{pq}\delta_3\}$ of the 4th order.

Now $$\delta_2 = d_2 - 2d_1\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right) + \left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2,$$

and $$\delta_1 = d_1 - 2\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right),$$

$$\delta_3 = d_3 - 2d_2\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right) + d_1\left(\frac{\delta n_{pq}}{\bar{n}_{pq}}\right)^2.$$

$$\therefore (S\{u_{pq}\delta_2\})^2 = \left(S\left\{u_{pq}\left(d_2 - 2d_1\frac{\delta n_{pq}}{\bar{n}_{pq}} + \frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right)\right\}\right)^2$$

$$= (S\{u_{pq}d_2\})^2 + 4\left(S\left\{u_{pq}d_1\frac{\delta n_{pq}}{\bar{n}_{pq}}\right\}\right)^2 + \left(S\left\{u_{pq}\frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right\}\right)^2$$

$$- 4S\{u_{pq}d_2\}S\left\{u_{pq}d_1\frac{\delta n_{pq}}{\bar{n}_{pq}}\right\} + 2S\{u_{pq}d_2\}S\left\{u_{pq}\frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right\}$$

$$- 4S\left\{u_{pq}d_1\frac{\delta n_{pq}}{\bar{n}_{pq}}\right\}S\left\{u_{pq}\frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right\}$$

$$= T_1 + 4T_2 + T_3 - 4T_4 + 2T_5 - 4T_6, \text{ say } \quad \text{........................(43),}$$

and $S\{u_{pq}\delta_1\}S\{u_{pq}\delta_3\}$

$$= S\{u_{pq}d_1\}S\{u_{pq}d_3\} - 2S\left\{u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right\}S\{u_{pq}d_3\}$$

$$- 2S\{u_{pq}d_1\}S\left\{u_{pq}d_2\frac{\delta n_{pq}}{\bar{n}_{pq}}\right\} + 4S\left\{u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right\}S\left\{u_{pq}d_2\frac{\delta n_{pq}}{\bar{n}_{pq}}\right\}$$

$$+ S\{u_{pq}d_1\}S\left\{u_{pq}d_1\frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right\} - 2S\left\{u_{pq}\frac{\delta n_{pq}}{\bar{n}_{pq}}\right\}S\left\{u_{pq}d_1\frac{\delta n_{pq}^2}{\bar{n}_{pq}^2}\right\}$$

$$= T_1' - 2T_2' - 2T_3' + 4T_4' + T_5' - 2T_6', \text{ say} \quad \text{......................(43 \textit{bis}).}$$

Among these twelve terms, let us consider the T_1-term only as an example.

Now $$T_1 = (S\{u_{pq}d_2\})^2 = \left(S\left\{u_{pq}\left(\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2} + \frac{\delta n_{p.}}{\bar{n}_{p.}}\cdot\frac{\delta n_{.q}}{\bar{n}_{.q}} + \frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right)\right\}\right)^2$$

$$= \left(S\left\{u_{pq}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right\}\right)^2 + \left(S\left\{u_{pq}\frac{\delta n_{p.}\,\delta n_{.q}}{\bar{n}_{p.}\,\bar{n}_{.q}}\right\}\right)^2 + \left(S\left\{u_{pq}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right\}\right)^2$$

$$+2S\left\{u_{pq}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right\}S\left\{u_{pq}\frac{\delta n_{p.}\delta n_{.q}}{\bar{n}_{p.}\bar{n}_{.q}}\right\}+2S\left\{u_{pq}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right\}S\left\{u_{pq}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right\}$$

$$+2S\left\{u_{pq}\frac{\delta n_{p.}\delta n_{.q}}{\bar{n}_{p.}\bar{n}_{.q}}\right\}S\left\{u_{pq}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right\}$$

$$\left(\underset{p}{S}\left\{u_{p.}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right\}\right)^2+\left(\underset{q}{S}\left\{u_{.q}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right\}\right)^2+\left(S\left\{u_{pq}\frac{\delta n_{p.}\delta n_{.q}}{\bar{n}_{p.}\bar{n}_{.q}}\right\}\right)^2$$

$$+2\underset{p}{S}\left\{u_{p.}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right\}\underset{p}{S}\left(\underset{q}{S}\left\{u_{pq}\frac{\delta n_{p.}\delta n_{.q}}{\bar{n}_{p.}\bar{n}_{.q}}\right\}\right)$$

$$+2\underset{q}{S}\left\{u_{.q}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right\}\underset{q}{S}\left(\underset{p}{S}\left\{u_{pq}\frac{\delta n_{p.}\delta n_{.q}}{\bar{n}_{p.}\bar{n}_{.q}}\right\}\right)$$

$$+2\underset{p}{S}\left\{u_{p.}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right\}\underset{q}{S}\left\{u_{.q}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(44),$$

and

$$\left(\underset{p}{S}\left\{u_{p.}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right\}\right)^2+\left(\underset{q}{S}\left\{u_{.q}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right\}\right)^2+2\underset{p}{S}\left\{u_{p.}\frac{\delta n_{p.}^2}{\bar{n}_{p.}^2}\right\}\underset{q}{S}\left\{u_{.q}\frac{\delta n_{.q}^2}{\bar{n}_{.q}^2}\right\}$$

$$=\underset{p}{S}\left(u_{p.}^2\frac{\delta n_{p.}^4}{\bar{n}_{p.}^4}\right)+\underset{q}{S}\left(u_{.q}^2\frac{\delta n_{.q}^4}{\bar{n}_{.q}^4}\right)+2S\left(u_{p.}u_{.q}\frac{\delta n_{p.}^2\delta n_{.q}^2}{\bar{n}_{p.}^2\bar{n}_{.q}^2}\right)$$

$$+\underset{p}{S}\underset{p'}{S'}\left(u_{p.}u_{p'.}\frac{\delta n_{p.}^2\delta n_{p'.}^2}{\bar{n}_{p.}^2\bar{n}_{p'.}^2}\right)+\underset{q}{S}\underset{q'}{S'}\left(u_{.q}u_{.q'}\frac{\delta n_{.q}^2\delta n_{.q'}^2}{\bar{n}_{.q}^2\bar{n}_{.q'}^2}\right)\quad\ldots\ldots(44\,a),$$

$$\left(S\left\{u_{pq}\frac{\delta n_{p.}\delta n_{.q}}{\bar{n}_{p.}\bar{n}_{.q}}\right\}\right)^2=S\left\{u_{pq}^2\frac{\delta n_{p.}^2\delta n_{.q}^2}{\bar{n}_{p.}^2\bar{n}_{.q}^2}\right\}+S\underset{p'}{S'}\left\{u_{pq}u_{p'q}\frac{\delta n_{p.}\delta n_{.q}^2\delta n_{p'.}}{\bar{n}_{p.}\bar{n}_{.q}^2\bar{n}_{p'.}}\right\}$$

$$+S\underset{q'}{S'}\left\{u_{pq}u_{pq'}\frac{\delta n_{p.}^2\delta n_{.q}\delta n_{.q'}}{\bar{n}_{p.}^2\bar{n}_{.q}\bar{n}_{.q'}}\right\}+S\underset{p'}{S'}\underset{q'}{S'}\left\{u_{pq}u_{p'q'}\frac{\delta n_{p.}\delta n_{.q}\delta n_{p'.}\delta n_{.q'}}{\bar{n}_{p.}\bar{n}_{.q}\bar{n}_{p'.}\bar{n}_{.q'}}\right\}\ldots(44\,b),$$

and the other two terms of (44)

$$=2S\left\{u_{pq}u_{p.}\frac{\delta n_{p.}^3\delta n_{.q}}{\bar{n}_{p.}^3\bar{n}_{.q}}+u_{pq}u_{.q}\frac{\delta n_{p.}\delta n_{.q}^3}{\bar{n}_{p.}\bar{n}_{.q}^3}\right\}+2S\underset{p'}{S'}\left\{u_{p.}u_{p'q}\frac{\delta n_{p.}^2\delta n_{p'.}\delta n_{.q}}{\bar{n}_{p.}^2\bar{n}_{p'.}\bar{n}_{.q}}\right\}$$

$$+2S\underset{q'}{S'}\left\{u_{.q}u_{pq'}\frac{\delta n_{p.}\delta n_{.q}^2\delta n_{.q'}}{\bar{n}_{p.}\bar{n}_{.q}^2\bar{n}_{.q'}}\right\}\ \ldots\ldots\ldots\ldots\ldots(44\,c).$$

From these Equations (44), (44 *a*), (44 *b*) and (44 *c*), we get

$$T_1=\underset{p}{S}\left(u_{p.}^2\frac{\delta n_{p.}^4}{\bar{n}_{p.}^4}\right)+\underset{q}{S}\left(u_{.q}^2\frac{\delta n_{.q}^4}{\bar{n}_{.q}^4}\right)+\underset{p}{S}\underset{p'}{S'}\left(u_{p.}u_{p'.}\frac{\delta n_{p.}^2\delta n_{p'.}^2}{\bar{n}_{p.}^2\bar{n}_{p'.}^2}\right)$$

$$+\underset{q}{S}\underset{q'}{S'}\left(u_{.q}u_{.q'}\frac{\delta n_{.q}^2\delta n_{.q'}^2}{\bar{n}_{.q}^2\bar{n}_{.q'}^2}\right)$$

$$+2S\left\{u_{pq}u_{p.}\frac{\delta n_{p.}^3\delta n_{.q}}{\bar{n}_{p.}^3\bar{n}_{.q}}+u_{pq}u_{.q}\frac{\delta n_{p.}\delta n_{.q}^3}{\bar{n}_{p.}\bar{n}_{.q}^3}\right\}+S\left\{(u_{pq}^2+2u_{p.}u_{.q})\frac{\delta n_{p.}^2\delta n_{.q}^2}{\bar{n}_{p.}^2\bar{n}_{.q}^2}\right\}$$

$$+S\underset{p'}{S'}\left\{u_{pq}u_{p'q}\frac{\delta n_{p.}\delta n_{p'.}\delta n_{.q}^2}{\bar{n}_{p.}\bar{n}_{p'.}\bar{n}_{.q}^2}+2u_{p.}u_{p'q}\frac{\delta n_{p.}^2\delta n_{p'.}\delta n_{.q}}{\bar{n}_{p.}^2\bar{n}_{p'.}\bar{n}_{.q}}\right\}$$

$$+S\underset{q'}{S'}\left\{u_{pq}u_{pq'}\frac{\delta n_{p.}^2\delta n_{.q}\delta n_{.q'}}{\bar{n}_{p.}^2\bar{n}_{.q}\bar{n}_{.q'}}+2u_{.q}u_{pq'}\frac{\delta n_{p.}\delta n_{.q}^2\delta n_{.q'}}{\bar{n}_{p.}\bar{n}_{.q}^2\bar{n}_{.q'}}\right\}$$

$$+S\underset{p'}{S'}\underset{q'}{S'}\left\{u_{pq}u_{p'q'}\frac{\delta n_{p.}\delta n_{p'.}\delta n_{.q}\delta n_{.q'}}{\bar{n}_{p.}\bar{n}_{p'.}\bar{n}_{.q}\bar{n}_{.q'}}\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(45\,a).$$

Similarly

$$\begin{aligned}
T_2 = S\Bigg\{&u_{pq}^2\left(\frac{\delta n_{pq}^2\,\delta n_{p.}^2}{\bar{n}_{pq}^2\,\bar{n}_{p.}^2} + 2\,\frac{\delta n_{pq}^2\,\delta n_{p.}\,.\,\delta n_{.q}}{\bar{n}_{pq}^2\,\bar{n}_{p.}\,.\,\bar{n}_{.q}} + \frac{\delta n_{pq}^2\,\delta n_{.q}^2}{\bar{n}_{pq}^2\,\bar{n}_{.q}^2}\right)\Bigg\} \\
&+ S\,\underset{p'}{S'}\left\{u_{pq}u_{p'q}\left(\frac{\delta n_{pq}\,\delta n_{p'q}\,\delta n_{p.}\,.\,\delta n_{p'.}}{\bar{n}_{pq}\,\bar{n}_{p'q}\,\bar{n}_{p.}\,.\,\bar{n}_{p'.}} + 2\,\frac{\delta n_{pq}\,\delta n_{p'q}\,\delta n_{p.}\,.\,\delta n_{.q}}{\bar{n}_{pq}\,\bar{n}_{p'q}\,\bar{n}_{p.}\,.\,\bar{n}_{.q}} + \frac{\delta n_{pq}\,\delta n_{p'q}\,\delta n_{.q}^2}{\bar{n}_{pq}\,\bar{n}_{p'q}\,\bar{n}_{.q}^2}\right)\right\} \\
&+ S\,\underset{q'}{S'}\left\{u_{pq}u_{pq'}\left(\frac{\delta n_{pq}\,\delta n_{pq'}\,\delta n_{.q}\,\delta n_{.q'}}{\bar{n}_{pq}\,\bar{n}_{pq'}\,\bar{n}_{.q}\,\bar{n}_{.q'}} + 2\,\frac{\delta n_{pq}\,\delta n_{pq'}\,\delta n_{p.}\,.\,\delta n_{.q}}{\bar{n}_{pq}\,\bar{n}_{pq'}\,\bar{n}_{p.}\,.\,\bar{n}_{.q}} + \frac{\delta n_{pq}\,\delta n_{pq'}\,\delta n_{p.}^2}{\bar{n}_{pq}\,\bar{n}_{pq'}\,\bar{n}_{p.}^2}\right)\right\} \\
&+ S\,\underset{p'}{S'}\,\underset{q'}{S'}\left\{u_{pq}u_{p'q'}\left(\frac{\delta n_{pq}\,\delta n_{p'q'}\,\delta n_{p.}\,.\,\delta n_{p'.}}{\bar{n}_{pq}\,\bar{n}_{p'q'}\,\bar{n}_{p.}\,.\,\bar{n}_{p'.}} + \frac{\delta n_{pq}\,\delta n_{p'q'}\,\delta n_{.q}\,\delta n_{.q'}}{\bar{n}_{pq}\,\bar{n}_{p'q'}\,\bar{n}_{.q}\,\bar{n}_{.q'}}\right)\right. \\
&\qquad\qquad\left. + 2u_{p'q}u_{pq'}\left(\frac{\delta n_{p'q}\,\delta n_{pq'}\,\delta n_{p.}\,.\,\delta n_{.q}}{\bar{n}_{p'q}\,\bar{n}_{pq'}\,\bar{n}_{p.}\,.\,\bar{n}_{.q}}\right)\right\} \quad \ldots\ldots\ldots\ldots(45\,b),
\end{aligned}$$

$$\begin{aligned}
T_3 = S\left\{u_{pq}^2\,\frac{\delta n_{pq}^4}{\bar{n}_{pq}^4}\right\} &+ S\,\underset{p'}{S'}\left\{u_{pq}u_{p'q}\,\frac{\delta n_{pq}^2\,\delta n_{p'q}^2}{\bar{n}_{pq}^2\,\bar{n}_{p'q}^2}\right\} \\
&+ S\,\underset{q'}{S'}\left\{u_{pq}u_{pq'}\,\frac{\delta n_{pq}^2\,\delta n_{pq'}^2}{\bar{n}_{pq}^2\,\bar{n}_{pq'}^2}\right\} + S\,\underset{p'}{S'}\,\underset{q'}{S'}\left\{u_{pq}u_{p'q'}\,\frac{\delta n_{pq}^2\,\delta n_{p'q'}^2}{\bar{n}_{pq}^2\,\bar{n}_{p'q'}^2}\right\} \quad \ldots\ldots\ldots\ldots(45\,c),
\end{aligned}$$

$$\begin{aligned}
T_4 = S\Bigg\{&u_{pq}u_{p.}\left(\frac{\delta n_{p.}^3\,\delta n_{pq}}{\bar{n}_{p.}^3\,\bar{n}_{pq}} + \frac{\delta n_{p.}^2\,\delta n_{.q}\,\delta n_{pq}}{\bar{n}_{p.}^2\,\bar{n}_{.q}\,\bar{n}_{pq}}\right) \\
&+ u_{pq}^2\left(\frac{\delta n_{pq}\,\delta n_{p.}^2\,\delta n_{.q}}{\bar{n}_{pq}\,\bar{n}_{p.}^2\,\bar{n}_{.q}} + \frac{\delta n_{pq}\,\delta n_{p.}\,.\,\delta n_{.q}^2}{\bar{n}_{pq}\,\bar{n}_{p.}\,.\,\bar{n}_{.q}^2}\right) + u_{pq}u_{.q}\left(\frac{\delta n_{.q}^3\,\delta n_{pq}}{\bar{n}_{.q}^3\,\bar{n}_{pq}} + \frac{\delta n_{.q}^2\,\delta n_{p.}\,.\,\delta n_{pq}}{\bar{n}_{.q}^2\,\bar{n}_{p.}\,.\,\bar{n}_{pq}}\right)\Bigg\} \\
&+ S\,\underset{p'}{S'}\left\{u_{p.}\,u_{p'q}\left(\frac{\delta n_{p'q}\,\delta n_{p.}^2\,\delta n_{p'.}}{\bar{n}_{p'q}\,\bar{n}_{p.}^2\,\bar{n}_{p'.}} + \frac{\delta n_{p'q}\,\delta n_{p.}^2\,\delta n_{.q}}{\bar{n}_{p'q}\,\bar{n}_{p.}^2\,\bar{n}_{.q}}\right)\right. \\
&\qquad\qquad\left. + u_{pq}u_{p'q}\left(\frac{\delta n_{pq}\,\delta n_{p.}\,.\,\delta n_{.q}\,\delta n_{p'.}}{\bar{n}_{pq}\,\bar{n}_{p.}\,.\,\bar{n}_{.q}\,\bar{n}_{p'.}} + \frac{\delta n_{pq}\,\delta n_{.q}^2\,\delta n_{p'.}}{\bar{n}_{pq}\,\bar{n}_{.q}^2\,\bar{n}_{p'.}}\right)\right\} \\
&+ S\,\underset{q'}{S'}\left\{u_{.q}u_{pq'}\left(\frac{\delta n_{pq'}\,\delta n_{.q}^2\,\delta n_{.q'}}{\bar{n}_{pq'}\,\bar{n}_{.q}^2\,\bar{n}_{.q'}} + \frac{\delta n_{pq'}\,\delta n_{.q}^2\,\delta n_{p.}}{\bar{n}_{pq'}\,\bar{n}_{.q}^2\,\bar{n}_{p.}}\right)\right. \\
&\qquad\qquad\left. + u_{pq}u_{pq'}\left(\frac{\delta n_{pq}\,\delta n_{p.}\,.\,\delta n_{.q}\,\delta n_{.q'}}{\bar{n}_{pq}\,\bar{n}_{p.}\,.\,\bar{n}_{.q}\,\bar{n}_{.q'}} + \frac{\delta n_{pq}\,\delta n_{.q'}\,\delta n_{p.}^2}{\bar{n}_{pq}\,\bar{n}_{.q'}\,\bar{n}_{p.}^2}\right)\right\} \\
&+ S\,\underset{p'}{S'}\,\underset{q'}{S'}\left\{u_{pq}u_{p'q'}\left(\frac{\delta n_{pq}\,\delta n_{p.}\,.\,\delta n_{p'.}\,.\,\delta n_{.q'}}{\bar{n}_{pq}\,\bar{n}_{p.}\,.\,\bar{n}_{p'.}\,.\,\bar{n}_{.q'}} + \frac{\delta n_{pq}\,\delta n_{.q}\,\delta n_{p'.}\,.\,\delta n_{.q'}}{\bar{n}_{pq}\,\bar{n}_{.q}\,\bar{n}_{p'.}\,.\,\bar{n}_{.q'}}\right)\right\} \quad \ldots\ldots\ldots(45\,d),
\end{aligned}$$

$$\begin{aligned}
T_5 = S\Bigg\{&u_{pq}u_{p.}\,\frac{\delta n_{pq}^2\,\delta n_{p.}^2}{\bar{n}_{pq}^2\,\bar{n}_{p.}^2} + u_{pq}u_{.q}\,\frac{\delta n_{pq}^2\,\delta n_{.q}^2}{\bar{n}_{pq}^2\,\bar{n}_{.q}^2} + u_{pq}^2\,\frac{\delta n_{pq}^2\,\delta n_{p.}\,.\,\delta n_{.q}}{\bar{n}_{pq}^2\,\bar{n}_{p.}\,.\,\bar{n}_{.q}}\Bigg\} \\
&+ S\,\underset{p'}{S'}\left\{u_{p.}\,u_{p'q}\,\frac{\delta n_{p'q}^2\,\delta n_{p.}^2}{\bar{n}_{p'q}^2\,\bar{n}_{p.}^2} + u_{pq}u_{p'q}\,\frac{\delta n_{p'q}^2\,\delta n_{p.}\,.\,\delta n_{.q}}{\bar{n}_{p'q}^2\,\bar{n}_{p.}\,.\,\bar{n}_{.q}}\right\} \\
&+ S\,\underset{q'}{S'}\left\{u_{.q}u_{pq'}\,\frac{\delta n_{pq'}^2\,\delta n_{.q}^2}{\bar{n}_{pq'}^2\,\bar{n}_{.q}^2} + u_{pq}u_{pq'}\,\frac{\delta n_{pq'}^2\,\delta n_{p.}\,.\,\delta n_{.q}}{\bar{n}_{pq'}^2\,\bar{n}_{p.}\,.\,\bar{n}_{.q}}\right\} \\
&+ S\,\underset{p'}{S'}\,\underset{q'}{S'}\left\{u_{pq}u_{p'q'}\,\frac{\delta n_{p.}\,.\,\delta n_{.q}\,\delta n_{p'q'}^2}{\bar{n}_{p.}\,.\,\bar{n}_{.q}\,\bar{n}_{p'q'}^2}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(45\,e),
\end{aligned}$$

$$\begin{aligned}
T_6 = S\Bigg\{&u_{pq}^2\,\frac{\delta n_{p.}\,.\,\delta n_{pq}^3}{\bar{n}_{p.}\,.\,\bar{n}_{pq}^3} + u_{pq}^2\,\frac{\delta n_{.q}\,\delta n_{pq}^3}{\bar{n}_{.q}\,\bar{n}_{pq}^3}\Bigg\} \\
&+ S\,\underset{p'}{S'}\left\{u_{pq}u_{p'q}\left(\frac{\delta n_{pq}^2\,\delta n_{p'q}\,\delta n_{p'.}}{\bar{n}_{pq}^2\,\bar{n}_{p'q}\,\bar{n}_{p'.}} + \frac{\delta n_{pq}^2\,\delta n_{p'q}\,\delta n_{.q}}{\bar{n}_{pq}^2\,\bar{n}_{p'q}\,\bar{n}_{.q}}\right)\right\}
\end{aligned}$$

$$+\underset{q'}{S}S'\left\{u_{pq}u_{pq'}\left(\frac{\delta n_{pq}^2\delta n_{pq'}\delta n_{.q'}}{\bar n_{pq}^2\bar n_{pq'}\bar n_{.q'}}+\frac{\delta n_{pq}^2\delta n_{pq'}\delta n_{p.}}{\bar n_{pq}^2\bar n_{pq'}\bar n_{p.}}\right)\right\}$$

$$+\underset{p'}{S}\,\underset{q'}{S'}S'\left\{u_{pq}u_{p'q'}\left(\frac{\delta n_{pq}^2\delta n_{p'q'}\delta n_{p'.}}{\bar n_{pq}^2\bar n_{p'q'}\bar n_{p'.}}+\frac{\delta n_{pq}^2\delta n_{p'q'}\delta n_{.q'}}{\bar n_{pq}^2\bar n_{p'q'}\bar n_{.q'}}\right)\right\}\quad\ldots\ldots\ldots\ldots\ldots(45f),$$

and

$$T_1'=\underset{p}{S}\left\{u_{p.}^2\frac{\delta n_{p.}^4}{\bar n_{p.}^4}\right\}+\underset{q}{S}\left\{u_{.q}^2\frac{\delta n_{.q}^4}{\bar n_{.q}^4}\right\}+\underset{p\;p'}{S\,S'}\left\{u_{p.}u_{p'.}\frac{\delta n_{p.}^3\delta n_{p'.}}{\bar n_{p.}^3\bar n_{p'.}}\right\}$$

$$+\underset{q\;q'}{S\,S'}\left\{u_{.q}u_{.q'}\frac{\delta n_{.q}^3\delta n_{.q'}}{\bar n_{.q}^3\bar n_{.q'}}\right\}+S\left\{u_{pq}u_{p.}\frac{\delta n_{p.}^3\delta n_{.q}}{\bar n_{p.}^3\bar n_{.q}}+u_{pq}u_{.q}\frac{\delta n_{.q}^3\delta n_{p.}}{\bar n_{.q}^3\bar n_{p.}}\right.$$

$$\left.+u_{pq}(u_{p.}+u_{.q})\frac{\delta n_{p.}^2\delta n_{.q}^2}{\bar n_{p.}^2\bar n_{.q}^2}+u_{p.}u_{.q}\left(\frac{\delta n_{p.}^3\delta n_{.q}}{\bar n_{p.}^3\bar n_{.q}}+\frac{\delta n_{p.}\delta n_{.q}^3}{\bar n_{p.}\bar n_{.q}^3}\right)\right\}$$

$$+\underset{p'}{S}S'\left\{u_{pq}u_{p'.}\left(\frac{\delta n_{p.}^2\delta n_{.q}\delta n_{p'.}}{\bar n_{p.}^2\bar n_{.q}\bar n_{p'.}}+\frac{\delta n_{p.}\delta n_{.q}^2\delta n_{p'.}}{\bar n_{p.}\bar n_{.q}^2\bar n_{p'.}}\right)\right\}$$

$$+\underset{q'}{S}S'\left\{u_{pq}u_{.q'}\left(\frac{\delta n_{p.}\delta n_{.q}^2\delta n_{.q'}}{\bar n_{p.}\bar n_{.q}^2\bar n_{.q'}}+\frac{\delta n_{p.}^2\delta n_{.q}\delta n_{.q'}}{\bar n_{p.}^2\bar n_{.q}\bar n_{.q'}}\right)\right\}\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(46a),$$

$$T_2'=S\left\{u_{pq}u_{p.}\frac{\delta n_{p.}^3\delta n_{pq}}{\bar n_{p.}^3\bar n_{pq}}+u_{pq}u_{.q}\frac{\delta n_{.q}^3\delta n_{pq}}{\bar n_{.q}^3\bar n_{pq}}+u_{pq}^2\left(\frac{\delta n_{p.}^2\delta n_{.q}\delta n_{pq}}{\bar n_{p.}^2\bar n_{.q}\bar n_{pq}}+\frac{\delta n_{p.}\delta n_{.q}^2\delta n_{pq}}{\bar n_{p.}\bar n_{.q}^2\bar n_{pq}}\right)\right\}$$

$$+\underset{p'}{S}S'\left\{u_{p'q}u_{p.}\frac{\delta n_{p.}^3\delta n_{p'q}}{\bar n_{p.}^3\bar n_{p'q}}+u_{pq}u_{p'q}\left(\frac{\delta n_{p.}^2\delta n_{.q}\delta n_{p'q}}{\bar n_{p.}^2\bar n_{.q}\bar n_{p'q}}+\frac{\delta n_{.q}^2\delta n_{p.}\delta n_{p'q}}{\bar n_{.q}^2\bar n_{p.}\bar n_{p'q}}\right)\right\}$$

$$+\underset{q'}{S}S'\left\{u_{pq'}u_{.q}\frac{\delta n_{.q}^3\delta n_{pq'}}{\bar n_{.q}^3\bar n_{pq'}}+u_{pq}u_{pq'}\left(\frac{\delta n_{p.}\delta n_{.q}^2\delta n_{pq'}}{\bar n_{p.}\bar n_{.q}^2\bar n_{pq'}}+\frac{\delta n_{p.}^2\delta n_{.q}\delta n_{pq'}}{\bar n_{p.}^2\bar n_{.q}\bar n_{pq'}}\right)\right\}$$

$$+\underset{p'}{S}\,\underset{q'}{S'}S'\left\{u_{pq}u_{p'q'}\left(\frac{\delta n_{p.}^2\delta n_{.q}\delta n_{p'q'}}{\bar n_{p.}^2\bar n_{.q}\bar n_{p'q'}}+\frac{\delta n_{p.}\delta n_{.q}^2\delta n_{p'q'}}{\bar n_{p.}\bar n_{.q}^2\bar n_{p'q'}}\right)\right\}\quad\ldots\ldots\ldots\ldots\ldots\ldots(46b),$$

$$T_3'=S\left\{u_{pq}u_{p.}\left(\frac{\delta n_{pq}\delta n_{p.}^3}{\bar n_{pq}\bar n_{p.}^3}+\frac{\delta n_{pq}\delta n_{p.}\delta n_{.q}^2}{\bar n_{pq}\bar n_{p.}\bar n_{.q}^2}+\frac{\delta n_{pq}\delta n_{p.}^2\delta n_{.q}}{\bar n_{pq}\bar n_{p.}^2\bar n_{.q}}\right)\right.$$

$$\left.+u_{pq}u_{.q}\left(\frac{\delta n_{pq}\delta n_{.q}^3}{\bar n_{pq}\bar n_{.q}^3}+\frac{\delta n_{pq}\delta n_{p.}^2\delta n_{.q}}{\bar n_{pq}\bar n_{p.}^2\bar n_{.q}}+\frac{\delta n_{pq}\delta n_{p.}\delta n_{.q}^2}{\bar n_{pq}\bar n_{p.}\bar n_{.q}^2}\right)\right\}$$

$$+\underset{p'}{S}S'\left\{u_{pq}u_{p'.}\left(\frac{\delta n_{pq}\delta n_{p.}^2\delta n_{p'.}}{\bar n_{pq}\bar n_{p.}^2\bar n_{p'.}}+\frac{\delta n_{pq}\delta n_{.q}^2\delta n_{p'.}}{\bar n_{pq}\bar n_{.q}^2\bar n_{p'.}}+\frac{\delta n_{pq}\delta n_{p.}\delta n_{.q}\delta n_{p'.}}{\bar n_{pq}\bar n_{p.}\bar n_{.q}\bar n_{p'.}}\right)\right\}$$

$$+\underset{q'}{S}S'\left\{u_{pq}u_{.q'}\left(\frac{\delta n_{pq}\delta n_{.q}^2\delta n_{.q'}}{\bar n_{pq}\bar n_{.q}^2\bar n_{.q'}}+\frac{\delta n_{pq}\delta n_{p.}^2\delta n_{.q'}}{\bar n_{pq}\bar n_{p.}^2\bar n_{.q'}}+\frac{\delta n_{pq}\delta n_{p.}\delta n_{.q}\delta n_{.q'}}{\bar n_{pq}\bar n_{p.}\bar n_{.q}\bar n_{.q'}}\right)\right\}\ (46c),$$

$$T_4'=S\left\{u_{pq}^2\left(\frac{\delta n_{pq}^2\delta n_{p.}^2}{\bar n_{pq}^2\bar n_{p.}^2}+\frac{\delta n_{pq}^2\delta n_{p.}\delta n_{.q}}{\bar n_{pq}^2\bar n_{p.}\bar n_{.q}}+\frac{\delta n_{pq}^2\delta n_{.q}^2}{\bar n_{pq}^2\bar n_{.q}^2}\right)\right\}$$

$$+\underset{p'}{S}S'\left\{u_{pq}u_{p'q}\left(\frac{\delta n_{pq}\delta n_{p'q}\delta n_{p'.}^2}{\bar n_{pq}\bar n_{p'q}\bar n_{p'.}^2}+\frac{\delta n_{pq}\delta n_{p'q}\delta n_{p'.}\delta n_{.q}}{\bar n_{pq}\bar n_{p'q}\bar n_{p'.}\bar n_{.q}}+\frac{\delta n_{pq}\delta n_{p'q}\delta n_{.q}^2}{\bar n_{pq}\bar n_{p'q}\bar n_{.q}^2}\right)\right\}$$

$$+\underset{q'}{S}S'\left\{u_{pq}u_{pq'}\left(\frac{\delta n_{pq}\delta n_{pq'}\delta n_{.q'}^2}{\bar n_{pq}\bar n_{pq'}\bar n_{.q'}^2}+\frac{\delta n_{pq}\delta n_{pq'}\delta n_{p.}\delta n_{.q'}}{\bar n_{pq}\bar n_{pq'}\bar n_{p.}\bar n_{.q'}}+\frac{\delta n_{pq}\delta n_{pq'}\delta n_{p.}^2}{\bar n_{pq}\bar n_{pq'}\bar n_{p.}^2}\right)\right\}$$

$$+\underset{p'}{S}\,\underset{q'}{S'}S'\left\{u_{pq}u_{p'q'}\left(\frac{\delta n_{pq}\delta n_{p'q'}\delta n_{p'.}^2}{\bar n_{pq}\bar n_{p'q'}\bar n_{p'.}^2}+\frac{\delta n_{pq}\delta n_{p'q'}\delta n_{p'.}\delta n_{.q'}}{\bar n_{pq}\bar n_{p'q'}\bar n_{p'.}\bar n_{.q'}}+\frac{\delta n_{pq}\delta n_{p'q'}\delta n_{.q'}^2}{\bar n_{pq}\bar n_{p'q'}\bar n_{.q'}^2}\right)\right\}$$

$$\ldots\ldots\ldots(46d),$$

$$T_5' = S\left\{u_{pq}u_{p.}\left(\frac{\delta n_{pq}^2\delta n_{p.}^2}{\bar{n}_{pq}^2\bar{n}_{p.}^2}+\frac{\delta n_{pq}^2\delta n_{p.}\delta n_{.q}}{\bar{n}_{pq}^2\bar{n}_{p.}\bar{n}_{.q}}\right)+u_{pq}u_{.q}\left(\frac{\delta n_{pq}^2\delta n_{.q}^2}{\bar{n}_{pq}^2\bar{n}_{.q}^2}+\frac{\delta n_{pq}^2\delta n_{p.}\delta n_{.q}}{\bar{n}_{pq}^2\bar{n}_{p.}\bar{n}_{.q}}\right)\right\}$$
$$+\underset{p'}{S}S'\left\{u_{pq}u_{p'.}\left(\frac{\delta n_{pq}^2\delta n_{p.}\delta n_{p'.}}{\bar{n}_{pq}^2\bar{n}_{p.}\bar{n}_{p'.}}+\frac{\delta n_{pq}^2\delta n_{.q}\delta n_{p'.}}{\bar{n}_{pq}^2\bar{n}_{.q}\bar{n}_{p'.}}\right)\right\}$$
$$+\underset{q'}{S}S'\left\{u_{pq}u_{.q'}\left(\frac{\delta n_{pq}^2\delta n_{.q}\delta n_{.q'}}{\bar{n}_{pq}^2\bar{n}_{.q}\bar{n}_{.q'}}+\frac{\delta n_{pq}^2\delta n_{p.}\delta n_{.q'}}{\bar{n}_{pq}^2\bar{n}_{p.}\bar{n}_{.q'}}\right)\right\} \quad\ldots\ldots(46\,e),$$

$$T_6' = S\left\{u_{pq}^2\left(\frac{\delta n_{pq}^3\delta n_{p.}}{\bar{n}_{pq}^3\bar{n}_{p.}}+\frac{\delta n_{pq}^3\delta n_{.q}}{\bar{n}_{pq}^3\bar{n}_{.q}}\right)\right\}$$
$$+\underset{p'}{S}S'\left\{u_{pq}u_{p'q}\left(\frac{\delta n_{pq}^2\delta n_{p'q}\delta n_{p.}}{\bar{n}_{pq}^2\bar{n}_{p'q}\bar{n}_{p.}}+\frac{\delta n_{pq}^2\delta n_{p'q}\delta n_{.q}}{\bar{n}_{pq}^2\bar{n}_{p'q}\bar{n}_{.q}}\right)\right\}$$
$$+\underset{q'}{S}S'\left\{u_{pq}u_{pq'}\left(\frac{\delta n_{pq}^2\delta n_{pq'}\delta n_{p.}}{\bar{n}_{pq}^2\bar{n}_{pq'}\bar{n}_{p.}}+\frac{\delta n_{pq}^2\delta n_{pq'}\delta n_{.q}}{\bar{n}_{pq}^2\bar{n}_{pq'}\bar{n}_{.q}}\right)\right\}$$
$$+\underset{p'}{S}\underset{q'}{S'}S'\left\{u_{pq}u_{p'q'}\left(\frac{\delta n_{pq}^2\delta n_{p'q'}\delta n_{p.}}{\bar{n}_{pq}^2\bar{n}_{p'q'}\bar{n}_{p.}}+\frac{\delta n_{pq}^2\delta n_{p'q'}\delta n_{.q}}{\bar{n}_{pq}^2\bar{n}_{p'q'}\bar{n}_{.q}}\right)\right\} \quad\ldots\ldots(46\,f).$$

(11) *Approximate Expressions for Means to the fourth Order.*

From these expressions (45) and (46), it is evident that we have still to find further means, besides those already given.

I have deduced all of them from the fundamental formulae (34) by the same method of transformation as in Article (8), but I will write here only the results which I obtained. The mean values of the quantities placed in round brackets below each equation are of the same type as those immediately above them.

(a) $[\delta n_{p.}^3\delta n_{p'q}] = -\frac{3}{N}\bar{n}_{p.}^2\bar{n}_{p'q}\left(1-\frac{\bar{n}_{p.}}{N}\right)$,

($[\delta n_{p.}^3\delta n_{p'.}]$, $[\delta n_{p.}\delta n_{p'q}^3]$);

(b) $[\delta n_{p.}^2\delta n_{p'q}^2] = \bar{n}_{p.}\bar{n}_{p'q}\left(1-\frac{\bar{n}_{p.}}{N}-\frac{\bar{n}_{p'q}}{N}+\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{p'q}\right)$,

($[\delta n_{p.}^2\delta n_{p'.}^2]$);

$[\delta n_{p.}^2\delta n_{pq}\delta n_{pq'}] = \bar{n}_{pq}\bar{n}_{pq'}\left(1-\frac{\bar{n}_{p.}}{N}\right)\left(2-\frac{3}{N}\bar{n}_{p.}\right)$,

$[\delta n_{p.}^2\delta n_{pq}\delta n_{p'q}] = -\frac{3}{N}\bar{n}_{pq}\bar{n}_{p'q}\bar{n}_{p.}\left(1-\frac{\bar{n}_{p.}}{N}\right)$,

($[\delta n_{p.}^2\delta n_{pq}\delta n_{p'q'}]$, $[\delta n_{p.}^2\delta n_{p'q}\delta n_{pq'}]$);

$[\delta n_{p.}^2\delta n_{pq}\delta n_{p'.}] = -\frac{3}{N}\bar{n}_{p.}\bar{n}_{p'.}\bar{n}_{pq}\left(1-\frac{\bar{n}_{p.}}{N}\right)$,

$[\delta n_{p.}^2\delta n_{pq}\delta n_{.q}] = \bar{n}_{pq}\left(1-\frac{\bar{n}_{p.}}{N}\right)\left(2\bar{n}_{pq}+\bar{n}_{p.}-\frac{3}{N}\bar{n}_{p.}\bar{n}_{.q}\right)$,

$[\delta n_{p.}^2\delta n_{pq}\delta n_{.q'}] = \bar{n}_{pq}\left(1-\frac{\bar{n}_{p.}}{N}\right)\left(2\bar{n}_{pq'}-\frac{3}{N}\bar{n}_{p.}\bar{n}_{.q'}\right)$,

$[\delta n_{p.}^2\delta n_{p'q}\delta n_{.q}] = \bar{n}_{p.}\bar{n}_{p'q}\left(1-\frac{\bar{n}_{p.}}{N}-\frac{\bar{n}_{.q}}{N}-\frac{2}{N}\bar{n}_{pq}+\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{.q}\right)$,

$$[\delta n_{p.}{}^2 \delta n_{p'q'} \delta n_{.q}] = -\bar{n}_{p.}\bar{n}_{p'q}\left(\frac{\bar{n}_{.q}}{N}+\frac{2}{N}\bar{n}_{pq}-\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{.q}\right),$$

$$[\delta n_{p.}{}^2 \delta n_{p'q} \delta n_{p'.}] = \bar{n}_{p.}\bar{n}_{p'q}\left(1-\frac{\bar{n}_{p.}}{N}-\frac{\bar{n}_{p'.}}{N}+\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{p'.}\right),$$

(c) $$[\delta n_{p.}\delta n_{p'.}\delta n_{pq}\delta n_{p'q}] = \bar{n}_{pq}\bar{n}_{p'q}\left(1-\frac{\bar{n}_{p.}}{N}-\frac{\bar{n}_{p'.}}{N}+\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{p'.}\right),$$

$([\delta n_{p.}\delta n_{p'.}\delta n_{pq}\delta n_{p'q'}])$;

$$[\delta n_{p.}\delta n_{.q}\delta n_{pq}\delta n_{p'q}] = \bar{n}_{pq}\bar{n}_{p'q}\left(1-\frac{2}{N}\bar{n}_{p.}-\frac{\bar{n}_{.q}}{N}-\frac{\bar{n}_{pq}}{N}-\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{.q}\right),$$

$$[\delta n_{p.}\delta n_{.q}\delta n_{pq}\delta n_{p'q'}] = -\frac{1}{N}\bar{n}_{pq}\bar{n}_{p'q'}\left(\bar{n}_{pq}+\bar{n}_{p.}+\bar{n}_{.q}-\frac{3}{N}\bar{n}_{p.}\bar{n}_{.q}\right),$$

$$[\delta n_{p.}\delta n_{.q}\delta n_{pq'}\delta n_{p'q}] = \bar{n}_{p'q}\bar{n}_{pq'}\left(1-\frac{\bar{n}_{pq}}{N}-\frac{\bar{n}_{p.}}{N}-\frac{\bar{n}_{.q}}{N}+\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{.q}\right),$$

$$[\delta n_{p.}\delta n_{pq}{}^2\delta n_{p'q'}] = -\frac{1}{N}\bar{n}_{pq}\bar{n}_{p'q'}\left(\bar{n}_{p.}+2\bar{n}_{pq}-\frac{3}{N}\bar{n}_{p.}\bar{n}_{pq}\right),$$

$([\delta n_{p.}\delta n_{pq}{}^2\delta n_{p'.}],\ [\delta n_{p.}\delta n_{pq}{}^2\delta n_{p'q}])$;

$$[\delta n_{p'.}\delta n_{pq}{}^2\delta n_{p'q}] = \bar{n}_{pq}\bar{n}_{p'q}\left(1-\frac{\bar{n}_{p'.}}{N}-\frac{\bar{n}_{pq}}{N}+\frac{3}{N^2}\bar{n}_{pq}\bar{n}_{p'.}\right),$$

$([\delta n_{p'.}\delta n_{pq}{}^2\delta n_{p'q'}])$;

(d) $$[\delta n_{p.}\delta n_{.q}\delta n_{p'q}{}^2] = \bar{n}_{p'q}\left(\bar{n}_{pq}-\frac{\bar{n}_{pq}\bar{n}_{p'q}}{N}-\frac{\bar{n}_{p.}\bar{n}_{.q}}{N}-\frac{2}{N}\bar{n}_{p.}\bar{n}_{p'q}+\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{.q}\bar{n}_{p'q}\right),$$

$$[\delta n_{p.}\delta n_{.q}\delta n_{p'q'}{}^2] = \bar{n}_{p'q'}\left(\bar{n}_{pq}-\frac{1}{N}\bar{n}_{pq}\bar{n}_{p'q'}-\frac{1}{N}\bar{n}_{p.}\bar{n}_{.q}+\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{.q}\bar{n}_{p'q'}\right),$$

$$[\delta n_{p.}\delta n_{p'.}\delta n_{pq}{}^2] = -\frac{1}{N}\bar{n}_{pq}\bar{n}_{p'.}\left(2\bar{n}_{pq}+\bar{n}_{p.}-\frac{3}{N}\bar{n}_{pq}\bar{n}_{p.}\right),$$

(e) $$[\delta n_{p.}\delta n_{p'.}\delta n_{.q}\delta n_{pq}] = \bar{n}_{pq}\left(\bar{n}_{p'q}-\frac{1}{N}\bar{n}_{p'.}\bar{n}_{pq}-\frac{1}{N}\bar{n}_{p.}\bar{n}_{p'q}-\frac{1}{N}\bar{n}_{p'.}\bar{n}_{.q}\right.$$
$$\left.-\frac{1}{N}\bar{n}_{p.}\bar{n}_{p'.}+\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{p'.}\bar{n}_{.q}\right),$$

$$[\delta n_{p.}\delta n_{p'.}\delta n_{.q}\delta n_{pq'}] = \bar{n}_{pq'}\left(\bar{n}_{p'q}-\frac{1}{N}\bar{n}_{p.}\bar{n}_{p'q}-\frac{1}{N}\bar{n}_{p'.}\bar{n}_{pq}\right.$$
$$\left.-\frac{1}{N}\bar{n}_{p'.}\bar{n}_{.q}+\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{p'.}\bar{n}_{.q}\right),$$

(f) $$[\delta n_{p.}\delta n_{.q}\delta n_{p'.}\delta n_{.q'}] = \bar{n}_{pq}\bar{n}_{p'q'}+\bar{n}_{p'q}\bar{n}_{pq'}+\frac{3}{N^2}\bar{n}_{p.}\bar{n}_{p'.}\bar{n}_{.q}\bar{n}_{.q'}$$
$$-\frac{1}{N}(\bar{n}_{p'q'}\bar{n}_{p.}\bar{n}_{.q}+\bar{n}_{pq'}\bar{n}_{p'.}\bar{n}_{.q}+\bar{n}_{p'q}\bar{n}_{p.}\bar{n}_{.q'}+\bar{n}_{pq}\bar{n}_{p'.}\bar{n}_{.q'}),$$

$$[\delta n_{p.}\delta n_{p'.}\delta n_{.q}{}^2] = 2\bar{n}_{p'q}\left(\bar{n}_{pq}-\frac{1}{N}\bar{n}_{p.}\bar{n}_{.q}\right)-\frac{1}{N}\bar{n}_{p'.}\bar{n}_{.q}\left(2\bar{n}_{pq}+\bar{n}_{p.}-\frac{3}{N}\bar{n}_{p.}\bar{n}_{.q}\right),$$

$$[\delta n_{p.}{}^2\delta n_{p'.}\delta n_{.q}] = \bar{n}_{p'q}\bar{n}_{p.}\left(1-\frac{\bar{n}_{p.}}{N}\right)-\frac{1}{N}\bar{n}_{p.}\bar{n}_{p'.}\left(2\bar{n}_{pq}+\bar{n}_{.q}-\frac{3}{N}\bar{n}_{p.}\bar{n}_{.q}\right)$$

......(47).

(12) *The mean values of the last two terms of μ_2'.*

It is now possible to find the means of $\{S(u_{pq}\delta_2)\}^2$ and $S(u_{pq}\delta_1)\,S(u_{pq}\delta_2)$. However, as this involves also long and rather complicated calculations and transformations, I will give here only the results I have obtained and the outline of one of the deductions.

Let us take, for instance, the first term (T_1) of $(S\{u_{pq}\delta_2\})^2$ and find its mean.

From the Equation (45 *a*) and Formulae (34), (35), (47), we get

$$\begin{aligned}
[T_1] = {} & \underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}^4}[\delta n_{p.}^4]\right) + \underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}^4}\right)[\delta n_{.q}^4] + \underset{p}{S}\underset{p'}{S'}\left(\frac{u_{p.}u_{p'.}}{\bar{n}_{p.}^2\bar{n}_{p'.}^2}[\delta n_{p.}^2\delta n_{p'.}^2]\right) \\
& + \underset{q}{S}\underset{q'}{S'}\left(\frac{u_{.q}u_{.q'}}{\bar{n}_{.q}^2\bar{n}_{.q'}^2}[\delta n_{.q}^2\delta n_{.q'}^2]\right) + 2S\left\{u_{pq}u_{p.}\frac{[\delta n_{p.}^3\delta n_{.q}]}{\bar{n}_{p.}^3\bar{n}_{.q}}\right. \\
& \qquad\qquad \left. + u_{pq}.u_{.q}\frac{[\delta n_{p.}\delta n_{.q}^3]}{\bar{n}_{p.}\bar{n}_{.q}^3}\right\} + S\left\{(u_{pq}^2 + 2u_{p.}u_{.q})\frac{[\delta n_{p.}^2\delta n_{.q}^2]}{\bar{n}_{p.}^2\bar{n}_{.q}^2}\right\} \\
& + S\underset{p'}{S'}\left\{\frac{u_{pq}u_{p'q}}{\bar{n}_{p.}\bar{n}_{p'.}\bar{n}_{.q}^2}[\delta n_{p.}\delta n_{p'.}\delta n_{.q}^2] + 2\frac{u_{p.}u_{p'q}}{\bar{n}_{p.}^2\bar{n}_{p'.}\bar{n}_{.q}}[\delta n_{p.}^2\delta n_{p'.}\delta n_{.q}]\right\} \\
& + S\underset{q'}{S'}\left\{\frac{u_{pq}u_{pq'}}{\bar{n}_{.q}\bar{n}_{.q'}\bar{n}_{p.}^2}[\delta n_{p.}^2\delta n_{.q}\delta n_{.q'}] + 2\frac{u_{.q}u_{pq'}}{\bar{n}_{.q}^2\bar{n}_{.q'}\bar{n}_{p.}}[\delta n_{.q}^2\delta n_{p.}\delta n_{.q'}]\right\} \\
& + S\underset{p'}{S'}\underset{q'}{S'}\left\{\frac{u_{pq}u_{p'q'}}{\bar{n}_{p.}\bar{n}_{p'.}\bar{n}_{.q}\bar{n}_{.q'}}[\delta n_{p.}\delta n_{.q}\delta n_{p'.}\delta n_{.q'}]\right\} \\
= {} & 3\underset{p}{S}\left\{u_{p.}^2\left(\frac{1}{\bar{n}_{p.}^2} - \frac{2}{N\bar{n}_{p.}} + \frac{1}{N^2}\right)\right\} + 3\underset{q}{S}\left\{u_{.q}^2\left(\frac{1}{\bar{n}_{.q}^2} - \frac{2}{N\bar{n}_{.q}} + \frac{1}{N^2}\right)\right\} \\
& + \underset{p}{S}\underset{p'}{S'}\left\{u_{p.}u_{p'.}\left(\left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)\frac{1}{\bar{n}_{p'.}} - \frac{1}{N}\left(\frac{1}{\bar{n}_{p.}} - \frac{3}{N}\right)\right)\right\} \\
& + \underset{q}{S}\underset{q'}{S'}\left\{u_{.q}u_{.q'}\left(\left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right)\frac{1}{\bar{n}_{.q'}} - \frac{1}{N}\left(\frac{1}{\bar{n}_{.q}} - \frac{3}{N}\right)\right)\right\} \\
& + 6S\left\{u_{pq}u_{p.}\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}^2\bar{n}_{.q}} - \frac{1}{N\bar{n}_{p.}} - \frac{\bar{n}_{pq}}{N\bar{n}_{p.}\bar{n}_{.q}} + \frac{1}{N^2}\right)\right. \\
& \qquad\qquad \left. + u_{pq}u_{.q}\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}^2} - \frac{1}{N\bar{n}_{.q}} - \frac{\bar{n}_{pq}}{N\bar{n}_{p.}\bar{n}_{.q}} + \frac{1}{N^2}\right)\right\} \\
& + S\left\{(u_{pq}^2 + 2u_{p.}u_{.q})\left(\left(\frac{1}{\bar{n}_{p.}} - \frac{1}{N}\right)\left(\frac{1}{\bar{n}_{.q}} - \frac{1}{N}\right) + 2\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}} - \frac{1}{N}\right)^2\right)\right\} \\
& + 2S\underset{p'}{S'}\left\{u_{p.}u_{p'q}\left(\frac{\bar{n}_{p'q}}{\bar{n}_{p'.}}\left(\frac{1}{\bar{n}_{p.}\bar{n}_{.q}} - \frac{1}{N\bar{n}_{.q}}\right) - \frac{1}{N}\left(\frac{2\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}} + \frac{1}{\bar{n}_{p.}} - \frac{3}{N}\right)\right)\right\} \\
& + S\underset{p'}{S'}\left\{u_{pq}u_{p'q}\left(2\frac{\bar{n}_{p'q}}{\bar{n}_{p'.}}\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}^2} - \frac{1}{N\bar{n}_{.q}}\right) - \frac{1}{N}\left(\frac{2\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}} + \frac{1}{\bar{n}_{.q}} - \frac{3}{N}\right)\right)\right\} \\
& + 2S\underset{q'}{S'}\left\{u_{.q}u_{pq'}\left(\frac{\bar{n}_{pq'}}{\bar{n}_{.q'}}\left(\frac{1}{\bar{n}_{p.}\bar{n}_{.q}} - \frac{1}{N\bar{n}_{p.}}\right) - \frac{1}{N}\left(\frac{2\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}} + \frac{1}{\bar{n}_{.q}} - \frac{3}{N}\right)\right)\right\} \\
& + S\underset{q'}{S'}\left\{u_{pq}u_{pq'}\left(2\frac{\bar{n}_{pq'}}{\bar{n}_{.q'}}\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}^2\bar{n}_{.q}} - \frac{1}{N\bar{n}_{p.}}\right) - \frac{1}{N}\left(\frac{2\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}} + \frac{1}{\bar{n}_{p.}} - \frac{3}{N}\right)\right)\right\} \\
& + S\underset{p}{S'}\underset{q'}{S'}\left\{u_{pq}u_{p'q'}\left(\left(\frac{3}{N^2} - \frac{\bar{n}_{pq}}{N\bar{n}_{p.}\bar{n}_{.q}}\right) + \frac{\bar{n}_{p'q'}}{\bar{n}_{p'.}\bar{n}_{.q'}}\left(\frac{\bar{n}_{pq}}{\bar{n}_{p.}\bar{n}_{.q}} - \frac{1}{N}\right)\right.\right. \\
& \qquad\qquad \left.\left. + \frac{1}{\bar{n}_{p.}\bar{n}_{.q}}\cdot\frac{\bar{n}_{p'q}\bar{n}_{pq'}}{\bar{n}_{p'.}\bar{n}_{.q'}} - \frac{1}{N\bar{n}_{.q}}\cdot\frac{\bar{n}_{p'q}}{\bar{n}_{p'.}} - \frac{1}{N\bar{n}_{p.}}\cdot\frac{\bar{n}_{pq'}}{\bar{n}_{.q'}}\right)\right\}.
\end{aligned}$$

As there are double and triple summations, the simplification of the above expression is not very easy. It is long and rather complicated, but the result I have reached is not so unmanageable and is as follows:

If we write $\bar{n}_{pq}$ and u_{pq} without the suffix, for simplicity, as follows,

$$\bar{n}_{pq} = \bar{n} \text{ and } u_{pq} = u,$$

when there is no ambiguity in the meaning of these symbols, then, after simplification, the above expression for $[T_1]$ becomes, when we put

$$A_{pq} = \underset{p'=1}{\overset{p'=\kappa}{S}} \underset{q'=1}{\overset{q'=\lambda}{S}} \left(\frac{u_{p'q'}\bar{n}_{p'q}\bar{n}_{pq'}}{\bar{n}_{p.}\bar{n}_{.q'}}\right) \text{ ....(48)},$$

$$[T_1] = 2S_2'' - \frac{9}{N}S_2' + \frac{27}{N^2}(1+\tilde{\phi}^2)^2 + (S_1')^2 - \frac{6}{N}S_1'(1+\tilde{\phi}^2)$$
$$-\frac{6}{N}(1+\tilde{\phi}^2)S\left(\frac{u^2}{\bar{n}}\right) + 2S_1'S\left(\frac{u^2}{\bar{n}}\right) + \left(S\left\{\frac{u^2}{\bar{n}}\right\}\right)^2$$
$$+4S\left\{\frac{u^2}{\bar{n}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\} + S\left\{\frac{u^2}{\bar{n}^2}(u + A_{pq})\right\}$$
$$+2S\left\{\frac{u\,u_{p.}u_{.q}}{\bar{n}}\left(2\frac{u}{\bar{n}} - \frac{9}{N}\right)\right\} \text{.......................................(49 a).}$$

Similarly

$$[T_2] = S_2'' - \frac{5}{N}S_2' - \frac{4}{N}S_1'(1+\tilde{\phi}^2) + \frac{12}{N^2}(1+\tilde{\phi}^2)^2 + 4S\left(\frac{u^3}{\bar{n}^2}\right)$$
$$+2S\left(\frac{u_{p.}u_{.q}}{\bar{n}_{p.}\bar{n}_{.q}}\right) - \frac{2}{N}S\left(\frac{u\,u_{p.}u_{.q}}{\bar{n}}\right) + S\left\{\frac{u^2}{\bar{n}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}} - \frac{4}{N}\right)\right\}$$
$$+\left(\underset{p}{S}\left\{\frac{u_{p.}}{\bar{n}_{p.}}\right\}\right)^2 + \left(\underset{q}{S}\left\{\frac{u_{.q}}{\bar{n}_{.q}}\right\}\right)^2 \text{ ....(49 b),}$$

$$[T_3] = \frac{3}{N^2}(1+\tilde{\phi}^2)^2 - \frac{2}{N}(1+\tilde{\phi}^2)S\left(\frac{u}{\bar{n}}\right) + \left(S\left\{\frac{u}{\bar{n}}\right\}\right)^2 + 2S\left\{\frac{u^2}{\bar{n}}\left(\frac{1}{\bar{n}} - \frac{2}{N}\right)\right\} \text{...(49 c),}$$

$$[T_4] = 2S_2'' - \frac{9}{N}S_2' - \frac{5}{N}S_1'(1+\tilde{\phi}^2) + \frac{18}{N^2}(1+\tilde{\phi}^2)^2 + S_1'S\left(\frac{u^2}{\bar{n}}\right)$$
$$+2S\left(\frac{u^3}{\bar{n}^2}\right) - \frac{2}{N}(1+\tilde{\phi}^2)S\left(\frac{u^2}{\bar{n}}\right) + 2S\left\{\frac{u\,u_{p.}u_{.q}}{\bar{n}}\left(\frac{1}{\bar{n}} - \frac{3}{N}\right)\right\}$$
$$+\left(\underset{p}{S}\left\{\frac{u_{p.}}{\bar{n}_{p.}}\right\}\right)^2 + \left(\underset{q}{S}\left\{\frac{u_{.q}}{\bar{n}_{.q}}\right\}\right)^2 + 3S\left\{\frac{u^2}{\bar{n}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\} \text{ ....(49 d),}$$

$$[T_5] = 2S_2'' - \frac{6}{N}S_2' + \frac{9}{N^2}(1+\tilde{\phi}^2)^2 - \frac{1}{N}S_1'(1+\tilde{\phi}^2) + S_1'S\left(\frac{u}{\bar{n}}\right) + 2S\left(\frac{u^3}{\bar{n}^2}\right)$$
$$-\frac{1}{N}(1+\tilde{\phi}^2)S\left(\frac{u^2}{\bar{n}}\right) - \frac{3}{N}(1+\tilde{\phi}^2)S\left(\frac{u}{\bar{n}}\right) + S\left(\frac{u}{\bar{n}}\right)S\left(\frac{u^2}{\bar{n}}\right) \text{......(49 e),}$$

and $$[T_6] = -\frac{2}{N}S_2' + \frac{6}{N^2}(1+\tilde{\phi}^2)^2 - \frac{1}{N}S_1'(1+\tilde{\phi}^2) - \frac{4}{N}S\left(\frac{u^2}{\bar{n}}\right)$$
$$-\frac{2}{N}(1+\tilde{\phi}^2)S\left(\frac{u}{\bar{n}}\right) + S_1'S\left(\frac{u}{\bar{n}}\right) + 2S\left\{\frac{u^2}{\bar{n}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}}\right)\right\} \text{ ....(49 f).}$$

And also when

$$V_{p.} = \overset{q'=\lambda}{\underset{q'=1}{S}} \left(\frac{u_{.q'} \bar{n}_{pq'}}{\bar{n}_{.q'}}\right), \qquad V_{.q} = \overset{p'=\kappa}{\underset{p'=1}{S}} \left(\frac{u_{p'.} \bar{n}_{p'q}}{\bar{n}_{p'.}}\right) \quad \text{..............(50)},$$

$$[T_1'] = 3S_2'' - \frac{6}{N} S_2' - \frac{8}{N}(1+\tilde{\phi}^2) S_1' + \frac{24}{N^2}(1+\tilde{\phi}^2)^2 - \frac{8}{N}(1+\tilde{\phi}^2) S\left(\frac{u^2}{\bar{n}}\right)$$
$$+ 2S\left\{\frac{u^2}{\bar{n}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\} + S\left\{\frac{u^2}{\bar{n}^2}(u_{p.} + u_{.q})\right\} + S\left\{\frac{u^2}{\bar{n}^2}(V_{p.} + V_{.q})\right\}$$
$$+ 2S\left\{\frac{u^2}{\bar{n}}\left(\frac{V_{p.}}{\bar{n}_{p.}} + \frac{V_{.q}}{\bar{n}_{.q}}\right)\right\} + 3S\left\{\frac{u\, u_{p.} u_{.q}}{\bar{n}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}} - \frac{4}{N}\right)\right\} \quad \text{......(51 } a\text{)},$$

$$[T_2'] = 3S_2'' - \frac{6}{N} S_2' - \frac{4}{N}(1+\tilde{\phi}^2) S_1' + \frac{12}{N^2}(1+\tilde{\phi}^2)^2 - \frac{4}{N}(1+\tilde{\phi}^2) S\left(\frac{u^2}{\bar{n}}\right)$$
$$+ S\left\{\frac{u^2}{\bar{n}^2}(u_{p.} + u_{.q})\right\} + 2S\left\{\frac{u^2}{\bar{n}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\} \quad \text{......................(51 } b\text{)},$$

$$[T_3'] = 3S_2'' - \frac{6}{N} S_2' - \frac{8}{N}(1+\tilde{\phi}^2) S_1' + \frac{18}{N^2}(1+\tilde{\phi}^2)^2 - \frac{2}{N}(1+\tilde{\phi}^2) S\left(\frac{u^2}{\bar{n}}\right)$$
$$+ 2S\left\{\frac{u^2}{\bar{n}^2}(u_{p.} + u_{.q})\right\} + S\left\{\frac{u^2}{\bar{n}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\} - \frac{6}{N} S\left(\frac{u\, u_{p.} u_{.q}}{\bar{n}}\right)$$
$$+ S\left\{\frac{u^2}{\bar{n}^2}(V_{p.} + V_{.q})\right\} + 2S\left\{\frac{u\, u_{p.} u_{.q}}{\bar{n}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}}\right)\right\} \quad \text{..............(51 } c\text{)},$$

$$[T_4'] = 2S_2'' - \frac{3}{N} S_2' - \frac{4}{N}(1+\tilde{\phi}^2) S_1' + \frac{9}{N^2}(1+\tilde{\phi}^2)^2 - \frac{1}{N}(1+\tilde{\phi}^2) S\left(\frac{u^2}{\bar{n}}\right)$$
$$+ S\left(\frac{u^3}{\bar{n}^2}\right) + S\left\{\frac{u^2}{\bar{n}^2}(u_{p.} + u_{.q})\right\} + S\left\{\frac{u^2}{\bar{n}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}} - \frac{3}{N}\right)\right\} \quad \text{......(51 } d\text{)},$$

$$[T_5'] = 2S_2'' - \frac{5}{N} S_2' + \frac{12}{N^2}(1+\tilde{\phi}^2)^2 - \frac{4}{N}(1+\tilde{\phi}^2) S_1' - \frac{4}{N}(1+\tilde{\phi}^2) S\left(\frac{u}{\bar{n}}\right)$$
$$- \frac{2}{N} S\left(\frac{u\, u_{p.} u_{.q}}{\bar{n}}\right) + 2S\left\{\frac{u^2}{\bar{n}^2}(u_{p.} + u_{.q})\right\} + S\left\{\frac{u}{\bar{n}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\}$$
$$+ S\left\{\frac{u}{\bar{n}}\left(\frac{V_{p.}}{\bar{n}_{p.}} + \frac{V_{.q}}{\bar{n}_{.q}}\right)\right\} \quad \text{..(51 } e\text{)},$$

and
$$[T_6'] = -\frac{1}{N} S_2' + \frac{6}{N^2}(1+\tilde{\phi}^2)^2 - \frac{2}{N}(1+\tilde{\phi}^2) S_1' - \frac{2}{N}(1+\tilde{\phi}^2) S\left(\frac{u}{\bar{n}}\right)$$
$$- \frac{4}{N} S\left(\frac{u^2}{\bar{n}}\right) + 2S\left\{\frac{u^2}{\bar{n}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}}\right)\right\} + S\left\{\frac{u^2}{\bar{n}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\} \quad \text{......(51} f\text{)}.$$

From these equations (49) and (51), it is easy to prove that

Mean $(S\{u_{pq}\delta_z\})^2$

$$= 2S_2'' + \frac{3}{N} S_2' - \frac{4}{N} S\left(\frac{u^2}{\bar{n}}\right) + 2S\left(\frac{u^2}{\bar{n}^2}\right) + 13S\left(\frac{u^3}{\bar{n}^3}\right) - \frac{2}{N} S\left(\frac{u\, u_{p.} u_{.q}}{\bar{n}}\right)$$
$$+ 4S\left(\frac{u\, u_{p.} u_{.q}}{\bar{n}_{p.} \bar{n}_{.q}}\right) - 4S\left(\frac{u^2}{\bar{n}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}}\right)\right) - 8S\left(\frac{u^2}{\bar{n}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right)$$
$$+ S\left\{\frac{A_{pq} u^2}{\bar{n}^2}\right\} + \left(S\left\{\frac{u}{\bar{n}}\left(1 - \frac{\bar{n}}{\bar{n}_{p.}}\right)\left(1 - \frac{\bar{n}}{\bar{n}_{.q}}\right)\right\}\right)^2 \quad \text{......................(52)},$$

Mean $S(u_{pq}\delta_1)\, S(u_{pq}\delta_3)$

$$= S_2'' + \frac{3}{N} S_2' - \frac{4}{N} S\left(\frac{u^2}{\bar{n}}\right) + 4S\left(\frac{u^3}{\bar{n}^2}\right) - \frac{2}{N} S\left(\frac{u\, u_{p.} u_{.q}}{\bar{n}}\right) + S\left(\frac{u^2}{\bar{n}^2}(u_{p.} + u_{.q})\right)$$
$$- 4S\left(\frac{u^2}{\bar{n}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right) - S\left(\frac{u^2}{\bar{n}^2}(V_{p.} + V_{.q})\right) + 2S\left(\frac{u^2}{\bar{n}}\left(\frac{V_{p.}}{\bar{n}_{p.}} + \frac{V_{.q}}{\bar{n}_{.q}}\right)\right)$$
$$+ S\left(\frac{u}{\bar{n}}\left(\frac{V_{p.}}{\bar{n}_{p.}} + \frac{V_{.q}}{\bar{n}_{.q}}\right)\right) - S\left(\frac{u\, u_{p.} u_{.q}}{\bar{n}}\left(\frac{1}{\bar{n}_{p.}} + \frac{1}{\bar{n}_{.q}}\right)\right) - S\left(\frac{u}{\bar{n}}\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right) \quad \text{......(53)},$$

and, consequently, from the Equations (17), (23), (42), (52) and (53),

$$\mu_2' = \mu_2'{}_{(1)} + 10S_2'' + \frac{3}{N} S_2' - \frac{4}{N} S\left(\frac{u^2}{\bar{n}}\right) + 13S\left(\frac{u^3}{\bar{n}^2}\right) - 6S\left(\frac{u^2}{\bar{n}^2}\right)$$
$$\cdot 6S\left(\frac{u^2}{\bar{n}^2}(u_{p.} + u_{.q})\right) - 4S\left(\frac{u_{p.} u_{.q}}{\bar{n}_{p.} \bar{n}_{.q}}\right) - \frac{2}{N} S\left(\frac{u\, u_{p.} u_{.q}}{\bar{n}}\right)$$
$$\left\{S\left(\frac{u^2}{\bar{n}}\right) + S\left(\frac{u}{\bar{n}}\right) - S_1'\right\}^2 + 12S\left\{\frac{u^2}{\bar{n}^2}\left(1 + u - \frac{\bar{n}}{\bar{n}_{p.}} - \frac{\bar{n}}{\bar{n}_{.q}}\right)\right\}$$
$$+ 4S\left\{\frac{u\, u_{p.} u_{.q}}{\bar{n}^2}\left(1 + u - \frac{\bar{n}}{\bar{n}_{p.}} - \frac{\bar{n}}{\bar{n}_{.q}}\right)\right\} + S\left\{\frac{u^2}{\bar{n}^2}(A_{pq} - 2V_{p.} - 2V_{.q})\right\}$$
$$- 2S\left\{\frac{u}{\bar{n}}(2 + 9u)\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\} + 2S\left\{\frac{u}{\bar{n}}(1 + 2u)\left(\frac{V_{p.}}{\bar{n}_{p.}} + \frac{V_{.q}}{\bar{n}_{.q}}\right)\right\}$$
$$= \mu_2'{}_{(1)} + 10S_2'' + \frac{3}{N} S_2' - \frac{4}{N} S\left(\frac{u^2}{\bar{n}}\right) + 13S\left(\frac{u^3}{\bar{n}^2}\right) - 6S\left(\frac{u^2}{\bar{n}^2}\right)$$
$$+ 6S\left(\frac{u^2}{\bar{n}^2}(u_{p.} + u_{.q})\right) - 4S\left(\frac{u_{p.} u_{.q}}{\bar{n}_{p.} \bar{n}_{.q}}\left(1 + \frac{\bar{n}}{2N}\right)\right) + (S\{W_{pq}\})^2$$
$$+ 12S\left(\frac{u}{\bar{n}} W_{pq}\right) + 4S\left(\frac{u_{p.} u_{.q}}{\bar{n}} W_{pq}\right) + S\left\{\frac{u^2}{\bar{n}^2}(A_{pq} - 2V_{p.} - 2V_{.q})\right\}$$
$$- 2S\left\{\frac{u}{\bar{n}}(2 + 9u)\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\} + 2S\left\{\frac{u}{\bar{n}}(1 + 2u)\left(\frac{V_{p.}}{\bar{n}_{p.}} + \frac{V_{.q}}{\bar{n}_{.q}}\right)\right\} \quad \text{......(54)},$$

where
$$W_{pq} = \frac{u_{pq}}{\bar{n}_{pq}}\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right) \quad \text{.....................(55)*},$$

and
$$\mu_2'{}_{(1)} = 4S\left(\frac{u^2}{\bar{n}}\right) - 3S_2' + 2S\left(\frac{u\, u_{p.} u_{.q}}{\bar{n}_{p.} \bar{n}_{.q}}\right).$$

VI. *Mean and Standard Deviation of* ϕ_1^2.

(13) Now we can find the expressions of the second approximation for the mean and standard deviation of ϕ_1^2.

From the Equations (17) and (55),

$$\mu_1' = S\left\{W_{pq}\left(1 - \frac{1}{N} + 2\frac{u}{\bar{n}}\right)\right\} \quad \text{........................(56)}.$$

* $S\left(\frac{u^2}{\bar{n}}\right) + S\left(\frac{u}{\bar{n}}\right) - S_1' = S\left(\frac{u}{\bar{n}}\left(1 + u - \frac{\bar{n}}{\bar{n}_{p.}} - \frac{\bar{n}}{\bar{n}_{.q}}\right)\right) = S\left(\frac{u}{\bar{n}}\left(1 - \frac{\bar{n}}{\bar{n}_{p.}}\right)\left(1 - \frac{\bar{n}}{\bar{n}_{.q}}\right)\right)$
$= S\left\{\frac{u_{pq}}{\bar{n}_{pq}}\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right)\right\} = S(W_{pq}).$

Therefore

$$\text{Mean } \phi_1^2 = \bar{\phi}_1^2 = \bar{\phi}^2 + \mu_1' = S(u) + S\left\{W_{pq}\left(1 - \frac{1}{N} + 2\frac{u}{\bar{n}}\right)\right\} \quad \text{......(57)},$$

and also from the Equations (7) and (54),

$$\begin{aligned}(\sigma_{\phi_1^2})^2 = \mu_2' - \mu_1'^2 &= 4S\left(\frac{u^2}{\bar{n}}\right) - 3\underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}}\right) - 3\underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}}\right) + 2S\left(\frac{u u_{p.} u_{.q}}{\bar{n}}\right)\\
&+ 10\underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}^2}\right) + 10\underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}^2}\right) + \frac{3}{N}\left\{\underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}}\right) + \underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}}\right)\right\} - \frac{4}{N}S\left(\frac{u^2}{\bar{n}}\right)\\
&+ 13S\left(\frac{u^3}{\bar{n}^2}\right) - 6S\left(\frac{u^2}{\bar{n}^2}\right) - 2S\left(\frac{u_{p.}u_{.q}}{\bar{n}_{p.}\bar{n}_{.q}}\left(2 + \frac{\bar{n}}{N}\right)\right) + (S\{W_{pq}\})^2\\
&+ 12S\left(\frac{u}{\bar{n}}W_{pq}\right) + 4S\left(\frac{u_{p.}u_{.q}}{\bar{n}}W_{pq}\right) + 6S\left(\frac{u^2}{\bar{n}^2}(u_{p.} + u_{.q})\right)\\
&+ S\left(\frac{u^2}{\bar{n}^2}(A_{pq} - 2V_{p.} - 2V_{.q})\right) - 2S\left(\frac{u}{\bar{n}}(2 + 9u)\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right)\\
&+ 2S\left(\frac{u}{\bar{n}}(1 + 2u)\left(\frac{V_{p.}}{\bar{n}_{p.}} + \frac{V_{.q}}{\bar{n}_{.q}}\right)\right) - (\mu_1')^2 \quad \text{........................(58)},\end{aligned}$$

where $\bar{n} = \bar{n}_{pq}$, $\quad u = u_{pq} = \bar{n}_{pq}^2/(\bar{n}_{p.}\bar{n}_{.q})$, $\quad u_{p.} = \overset{q=\lambda}{\underset{q=1}{S}} u_{pq}$,

$u_{.q} = \overset{p=\kappa}{\underset{p=1}{S}} u_{pq}$, $\quad V_{p.} = \overset{q=\lambda}{\underset{q=1}{S}}\left(\frac{u_{.q}\bar{n}_{pq}}{\bar{n}_{.q}}\right)$, $\quad V_{.q} = \overset{p=\kappa}{\underset{p=1}{S}}\left(\frac{u_{p.}\bar{n}_{pq}}{\bar{n}_{p.}}\right)$,

$W_{pq} = \frac{u_{pq}}{\bar{n}_{pq}}\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{p.}}\right)\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{.q}}\right)$, $\quad A_{pq} = \underset{p'}{S}\underset{q'}{S}\left(\frac{u_{p'q'}\bar{n}_{p'q}\bar{n}_{pq'}}{\bar{n}_{p'.}\bar{n}_{.q'}}\right)$,

and S stands for the double summation $\overset{p=\kappa}{\underset{p=1}{S}}\ \overset{q=\lambda}{\underset{q=1}{S}}$.

Now if r_{pq}, $r_{p.}$, $r_{.q}$ are the proportional values of $\bar{n}_{pq}$, $\bar{n}_{p.}$, $\bar{n}_{.q}$ respectively, taking the total frequency as unity, then

$$\begin{aligned}u_{pq} = \frac{\bar{n}_{pq}^2}{\bar{n}_{p.}\bar{n}_{.q}} &= \left(\frac{\bar{n}_{pq}}{N}\right)^2 \Big/ \left(\frac{\bar{n}_{p.}}{N}\cdot\frac{\bar{n}_{.q}}{N}\right) = r_{pq}^2/r_{p.}r_{.q}\\
&= \text{a function of proportions and of the same form,}\end{aligned}$$

and, consequently, $u_{p.}u_{.q}$; $V_{p.}$, $V_{.q}$; and A_{pq} may also be considered as functions of proportions only and of similar forms respectively.

Let us consider $\bar{\phi}_1^2$ and $\sigma_{\phi_1^2}$ again from this point of view, then

$$\begin{aligned}\mu_{2\,(1)}' &= 4S\left(\frac{u_{pq}^2}{\bar{n}_{pq}}\right) - 3\underset{p}{S}\left(\frac{u_{p.}^2}{\bar{n}_{p.}}\right) - 3\underset{q}{S}\left(\frac{u_{.q}^2}{\bar{n}_{.q}}\right) + 2S\left(\frac{u_{p.}u_{.q}u_{pq}}{\bar{n}_{pq}}\right)\\
&= 4S\left(\frac{u_{pq}^2}{Nr_{pq}}\right) - 3\underset{p}{S}\left(\frac{u_{p.}^2}{Nr_{p.}}\right) - 3\underset{q}{S}\left(\frac{u_{.q}^2}{Nr_{.q}}\right) + 2S\left(\frac{u_{pq}u_{p.}u_{.q}}{Nr_{pq}}\right)\\
&= \frac{1}{N}\left\{4S\left(\frac{u_{pq}^2}{r_{pq}}\right) - 3\underset{p}{S}\left(\frac{u_{p.}^2}{r_{p.}}\right) - 3\underset{q}{S}\left(\frac{u_{.q}^2}{r_{.q}}\right) + 2S\left(\frac{u_{pq}u_{p.}u_{.q}}{r_{pq}}\right)\right\}\\
&\ \frac{1}{N}f_1(r_{pq}), \text{ say,}\end{aligned}$$

or simply

$$\mu_{2\,(1)}' = \frac{1}{N}f_1 \quad \text{..................................(59)},$$

where f_1 is a function of proportions only and of the form

$$f_1 = 4S\left(\frac{u^2}{r_{pq}}\right) - 3\underset{p}{S}\left(\frac{u_{p\cdot}^2}{r_{p\cdot}}\right) - 3\underset{q}{S}\left(\frac{u_{\cdot q}^2}{r_{\cdot q}}\right) + 2S\left(\frac{u\,u_{p\cdot}u_{\cdot q}}{r_{pq}}\right) \quad \text{.........(60).}$$

For the same reason and by the same method of transformation, the expressions for μ_1', μ_2', $\bar{\phi}_1^2$ and $\sigma_{\phi_1^2}$ can be transformed into very simple forms as follows:

$$\mu_1' = \frac{1}{N}\psi_1 + \frac{1}{N^2}\psi_2 \quad \text{.................................(61),}$$

$$\mu_2' = \frac{1}{N}f_1 + \frac{1}{N^2}f_2 \quad \text{.................................(62),}$$

and, consequently,

$$\bar{\phi}_1^2 = \tilde{\phi}^2 + \frac{1}{N}\psi_1 + \frac{1}{N^2}\psi_2 \quad \text{.............................(63),}$$

$$\sigma_{\phi_1^2} = \frac{1}{\sqrt{N}}\left\{f_1 + \frac{1}{N}(f_2 - \psi_1^2)\right\}^{\frac{1}{2}} \quad \text{.......................(64),}$$

where

$$\psi_1 = S(W'_{pq}); \qquad W'_{pq} = \frac{u_{pq}}{r_{pq}}\left(1 - \frac{r_{pq}}{r_{p\cdot}}\right)\left(1 - \frac{r_{pq}}{r_{\cdot q}}\right) \text{...........(65),}$$

$$\psi_2 = S\left\{W'_{pq}\left(2\frac{u_{pq}}{r_{pq}} - 1\right)\right\} \quad \text{..................................(66).}$$

f_1 is as in (60) and

$$\begin{aligned} f_2 = {} & 10\left\{\underset{p}{S}\left(\frac{u_{p\cdot}^2}{r_{p\cdot}^2}\right) + \underset{q}{S}\left(\frac{u_{\cdot q}^2}{r_{\cdot q}^2}\right)\right\} + 3\left\{\underset{p}{S}\left(\frac{u_{p\cdot}^2}{r_{p\cdot}}\right) + \underset{q}{S}\left(\frac{u_{\cdot q}^2}{r_{\cdot q}}\right)\right\} - 4S\left(\frac{u_{pq}^2}{r_{pq}}\right) - 6S\left(\frac{u_{pq}^2}{r_{pq}^2}\right) \\ & + 13S\left(\frac{u_{pq}^3}{r_{pq}^2}\right) + 6S\left\{\frac{u_{pq}^2}{r_{pq}^2}(u_{p\cdot} + u_{\cdot q})\right\} - 4S\left\{\frac{u_{p\cdot}u_{\cdot q}}{r_{p\cdot}r_{\cdot q}}\left(1 + \frac{r_{pq}}{2}\right)\right\} + (S\{W'_{pq}\})^2 \\ & + 12S\left(\frac{u_{pq}W'_{pq}}{r_{pq}}\right) + 4S\left(\frac{u_{p\cdot}u_{\cdot q}W'_{pq}}{r_{pq}}\right) + S\left\{\frac{u_{pq}^2}{r_{pq}^2}(A_{pq} - 2V_{p\cdot} - 2V_{\cdot q})\right\} \\ & - 2S\left\{\frac{u_{pq}}{r_{pq}}(2 + 9u_{pq})\left(\frac{u_{p\cdot}}{r_{p\cdot}} + \frac{u_{\cdot q}}{r_{\cdot q}}\right)\right\} + 2S\left\{\frac{u_{pq}}{r_{pq}}(1 + 2u_{pq})\left(\frac{V_{p\cdot}}{r_{p\cdot}} + \frac{V_{\cdot q}}{r_{\cdot q}}\right)\right\} \end{aligned}$$

......(67).

(14) *The Case of no Contingency (continued from Article* (7)).

Now let us consider the special case of no contingency again.

In this case we have the following special relations, besides those of Equations (20) obtained on pp. 386—7:

$$\frac{u^3}{\bar{n}^2} = \frac{\bar{n}}{N^3}, \qquad V_{p\cdot} = \underset{q}{S}\left(\frac{u_{\cdot q}\bar{n}_{pq}}{\bar{n}_{\cdot q}}\right) = \frac{\bar{n}_{p\cdot}}{N}, \qquad V_{\cdot q} = \frac{\bar{n}_{\cdot q}}{N},$$

$$S_2' = \underset{p}{S}\left(\frac{u_{p\cdot}^2}{\bar{n}_{p\cdot}}\right) + \underset{q}{S}\left(\frac{u_{\cdot q}^2}{\bar{n}_{\cdot q}}\right) = \frac{2}{N},$$

$$S_2'' = \underset{p}{S}\left(\frac{u_{p\cdot}^2}{\bar{n}_{p\cdot}^2}\right) + \underset{q}{S}\left(\frac{u_{\cdot q}^2}{\bar{n}_{\cdot q}^2}\right) = \frac{1}{N^2}(\kappa + \lambda),$$

$$A_{pq} = \underset{p'}{S}\,\underset{q'}{S}\left(\frac{u_{p'q'}\bar{n}_{p'q}\bar{n}_{pq'}}{\bar{n}_{p'\cdot}\bar{n}_{\cdot q'}}\right) = \frac{1}{N^2}\bar{n}_{p\cdot}\bar{n}_{\cdot q},$$

and

$$S(W_{pq}) = S\left\{\frac{u_{pq}}{\bar{n}_{pq}}\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{p\cdot}}\right)\left(1 - \frac{\bar{n}_{pq}}{\bar{n}_{\cdot q}}\right)\right\} = \frac{1}{N}(\kappa - 1)(\lambda - 1) \quad \text{......(68).}$$

By these special relations and relations (20), the general expressions for μ_1' and μ_2' become

$$\mu_1' = \frac{1}{N}\left(1+\frac{1}{N}\right)(\kappa-1)(\lambda-1) = \frac{1}{N}\left(1+\frac{1}{N}\right)\omega \quad \text{.........(69)},$$

where $\omega = (\kappa-1)(\lambda-1)$ ....(70),

and $$\mu_2' = \frac{1}{N^2}\{2(\kappa-1)(\lambda-1)+(\kappa-1)^2(\lambda-1)^2\} = \frac{1}{N^2}(2\omega+\omega^2) \quad \text{...(71)};$$

and, consequently, we have

$$\bar{\phi}_1^2 = \frac{1}{N}\left(1+\frac{1}{N}\right)\omega \quad \text{.........(72)},$$

$$\sigma_{\phi_1^2} = \frac{\sqrt{2\omega}}{N}\left(1-\frac{\omega}{N}-\frac{\omega}{2N^2}\right)^{\frac{1}{2}} \quad \text{.........(73)},$$

for the case of no contingency.

In the case of no contingency, the formula for μ_2', obtained as the first approximation, became identically zero and we could not estimate the standard error of the mean square contingency ϕ_1^2. But now we have got out of this difficulty and the formula obtained for this special case is very simple.

In this case, if the size N of samples becomes very large compared to ω,

$$\text{Mean } \phi_1^2 \to 0 \quad \text{as } N\to\infty,$$

i.e. $$\bar{\phi}_1^2 \to \tilde{\phi}^2 \quad \text{as } N\to\infty,$$

and $$\sigma_{\phi_1^2} \to \frac{\sqrt{2\omega}}{N} \quad \text{as } N\to\infty \quad \text{.........(74)}.$$

This last limiting value of $\sigma_{\phi_1^2}$ is not of course new*.

(15) *Numerical Illustrations.*

Ex. (1). Let us take first the numerical example treated in Article (6), where the population frequencies (reduced to the sample size of 100) are as follows:

TABLE III.

15	25	40
40	20	60
55	45	$N=100$

* *Journal of Royal Statistical Society*, Vol. LXXXV. pp. 87—94, R. A. Fisher; pp. 95—104, G. U. Yule; *Biometrika*, Vol. XX[A]. p. 284, J. Neyman and E. S. Pearson. It may also be compared with the value for $\sigma_{\phi_2^2} = \frac{\sqrt{2c}}{N} = \frac{\sqrt{2\kappa\lambda}}{N}$ when there is no contingency and ϕ_2^2 and not ϕ_1^2 is used. See *Biometrika*, Vol. XI. p. 230 (1915).

In this case, after calculation, we have

$$S_2' = \cdot 023\,450, \qquad S_2'' = \cdot 000\,473, \qquad S\left(\frac{u^2}{\bar{n}}\right) = \cdot 012\,494,$$

$$S\left(\frac{u_{p.}u_{.q}u}{\bar{n}}\right) = \cdot 011\,720, \qquad \tilde{\phi}^2 = \cdot 082\,498,$$

$$S\left(\frac{u^2}{\bar{n}^2}\right) = \cdot 000\,441, \qquad S\left(\frac{u^3}{\bar{n}^2}\right) = \cdot 000\,151,$$

$$S\left\{\frac{u^2}{\bar{n}^2}(u_{p.} + u_{.q})\right\} = \cdot 000\,472, \qquad S\left\{\frac{u_{p.}u_{.q}}{\bar{n}_{p.}\bar{n}_{.q}}\left(1 + \frac{u}{2N}\right)\right\} = \cdot 000\,531,$$

$$S\left\{\frac{u}{\bar{n}}(2 + 9u)\left(\frac{u_{p.}}{\bar{n}_{p.}} + \frac{u_{.q}}{\bar{n}_{.q}}\right)\right\} = \cdot 004\,189, \qquad S\{W_{pq}\} = \cdot 009\,260,$$

$$S\left\{\frac{u}{\bar{n}}(1 + 2u)\left(\frac{V_{p.}}{\bar{n}_{p.}} + \frac{V_{.q}}{\bar{n}_{.q}}\right)\right\} = \cdot 001\,413, \qquad [S(W_{pq})]^2 = \cdot 000\,086,$$

$$S\left\{\frac{u}{\bar{n}}W_{pq}\right\} = \cdot 000\,087, \qquad S\left\{\frac{u_{p.}u_{.q}}{\bar{n}}W_{pq}\right\} = \cdot 000\,128,$$

$$S\left\{W_{pq}\left(1 - \frac{1}{N} + \frac{2u}{\bar{n}}\right)\right\} = \cdot 009\,300,$$

and
$$S\left\{\frac{u}{\bar{n}^2}(A_{pq} - 2V_{p.} - 2V_{.q})\right\} = -\cdot 000\,820.$$

Substituting these numerical values in the expressions for μ_1' and μ_2' directly, we get
$$\mu_1' = \cdot 009\,300 \quad \text{and} \quad \mu_2' = \cdot 003\,291.$$

Or finding functional values at first,

$$\psi_1 = \cdot 9260, \qquad \psi_2 = \cdot 4017,$$

and
$$f_1 = \cdot 3066, \qquad f_2 = 2\cdot 2486,$$

therefore
$$\mu_1' = \frac{\psi_1}{N} + \frac{\psi_2}{N^2} = \cdot 009\,300,$$

$$\mu_2' = \frac{f_1}{N} + \frac{f_2}{N^2} = \cdot 003\,291;$$

and finally
$$\bar{\phi}_1^2 = \tilde{\phi}^2 + \mu_1' = \cdot 091\,798,$$

and
$$\sigma_{\phi_1^2} = \sqrt{\mu_2' - \mu_1'^2} = \cdot 056\,60.$$

Ex. (2). Let us take a 3×3-table of contingency as the second example, wherein population frequencies (reduced to the sample size of 1000) are as given in Table IV.

TABLE IV.

82	73	23	178
101	319	99	519
33	129	141	303
216	521	263	$N = 1000$

In this case, after calculation, we get

$$\mu_1' = \cdot 004\,297, \qquad \mu_2' = \cdot 000\,751,$$

and $$\tilde{\phi}^2 = \cdot 154\,321,$$

therefore $$\bar{\phi}_1^2 = \cdot 158\,618, \quad \text{and} \quad \sigma_{\phi_1^2} = \cdot 027\,07.$$

Ex. (3). Let us consider a 2 × 2-table where the reduced population frequencies are as given in Table V.

In this case $$\psi_1 = \cdot 485\,100, \qquad \psi_2 = \cdot 0049,$$
$$f_1 = \cdot 970\,158, \qquad f_2 = -\cdot 4488,$$

and $$\tilde{\phi}^2 = \cdot 494\,950,$$

therefore $$\mu_1' = \cdot 004\,851, \qquad \mu_2' = \cdot 009\,657,$$

and $$\bar{\phi}_1^2 = \cdot 499\,801, \qquad \sigma_{\phi_1^2} = \cdot 098\,15.$$

TABLE V.

45	5	50
10	40	50
55	45	$N = 100$

Now, with regard to this population, the following experimental sampling results were given to me by Prof. K. Pearson. 804 samples of size 100 were drawn at random from this population and the distribution of ϕ_1^2 was observed. The following is the result of this sampling:

TABLE VI.

	Mean ϕ_1^2	$\sigma_{\phi_1^2}$	Number of samples
1st series of samples	·496 278	·097 926	352
2nd series of samples	·499 336	·095 697	452
Whole series	·497 998	·096 717	804

The standard errors of this $\bar{\phi}_1^2$ and $\sigma_{\phi_1^2}$ are as follows:

the standard error of $\bar{\phi}_1^2 = \cdot 003\,4614$ *,

the standard error of $\sigma_{\phi_1^2} = \cdot 002\,3125$ *.

* The standard errors of $\bar{\phi}_1^2$ and $\sigma_{\phi_1^2}$ are given, the latter as its first approximation, by the equations

$$\text{S.E. of } \bar{\phi}_1^2 = \frac{\tilde{\sigma}}{\sqrt{n}}, \quad \text{S.E. of } \sigma_{\phi_1^2} = \frac{\tilde{\sigma}}{2}\sqrt{\frac{\tilde{\beta}_2 - 1}{n}},$$

where n is the number of samples and $\tilde{\sigma}$, $\tilde{\beta}_2$ are the values of the theoretical standard deviation and of β_2. Here $n = 804$, $\tilde{\sigma} = \cdot 098\,15$, and the value of $\tilde{\beta}_2$ is given by the distribution law (81), Article (17).

These observed values of $\bar{\phi}_1^2$ and $\sigma_{\phi_1^2}$ and the theoretical values obtained by my formulae for this example are in good agreement having regard to their standard errors; this is interesting from a practical point of view.

Ex. (4). Let us take one more population, in the proportions given in Table VII, and find the mean and standard deviation of ϕ_1^2 for samples of 200.

TABLE VII.

·0831	·0786	·0270	·1887
·1032	·2864	·0862	·4758
·0335	·1235	·1785	·3355
·2198	·4885	·2917	1·0000

In this case

$$r_{11} = \cdot 0831, \quad r_{12} = \cdot 0786, \quad r_{13} = \cdot 0270, \quad r_{21} = \cdot 1032,$$

$$r_{22} = \cdot 2864, \quad r_{23} = \cdot 0862, \quad r_{1\cdot} = \frac{\bar{n}_{1\cdot}}{N} = \cdot 1887, \quad r_{2\cdot} = \cdot 4758,$$

and so on.

Further:
$$\tilde{\phi}^2 = S(u_{pq}) - 1 = \cdot 188\,893,$$

$$\psi_1 = S(W'_{pq}) = 3\cdot 500\,959, \qquad \psi_2 = S\left\{W'_{pq}\left(2\frac{u_{pq}}{r_{pq}} - 1\right)\right\} = 4\cdot 192\,990,$$

$$f_1 = 4S\left(\frac{u_{pq}^2}{r_{pq}}\right) - 3\underset{p}{S}\left(\frac{u_{p\cdot}^2}{r_{p\cdot}}\right) - 3\underset{q}{S}\left(\frac{u_{\cdot q}^2}{r_{\cdot q}}\right) + 2S\left(\frac{u_{p\cdot}\,u_{\cdot q}\,u_{pq}}{r_{pq}}\right) = \cdot 801\,842,$$

and finally
$$f_2 = 25\cdot 558\,896.$$

Therefore Mean $\phi_1^2 = \cdot 206\,503$, $\sigma_{\phi_1^2} = \sqrt{\cdot 004\,338} = \cdot 065\,86$

With regard to the last population, I have made a sampling experiment. In my case, the sampling was carried out with the help of Tippett's Random Numbers*.

250 samples of size 200 were drawn and the 250 observed values of ϕ_1^2, thus obtained, are recorded in Table VIII.

The mean value and the standard deviation of ϕ_1^2 obtained in this experiment were

$$\bar{\phi}_1^2 = \cdot 200\,72 \quad (\text{S.E.} = \cdot 004\,1655)\dagger,$$

and
$$\sigma_{\phi_1^2} = \cdot 060\,86 \quad (\text{S.E.} = \cdot 003\,2328)\dagger,$$

which are also in good agreement with the theoretical values obtained by my formulae for this example.

* *Tracts for Computers*, No. xv.

† These two values of the standard error are obtained in the same way as in Example (3).

TABLE VIII.

250 *observed values of* ϕ_1^2.

·370 502	·260 342	·226 946	·196 717	·175 246	·149 476	·108 222
·367 532	·259 954	·226 664	·196 035	·175 094	·148 303	·107 749
·362 669	·259 520	·226 046	·195 403	·173 507	·146 151	·104 882
·343 805	·259 413	·224 281	·193 993	·172 689	·139 878	·101 528
·342 499	·257 989	·223 655	·193 733	·172 668	·139 776	·099 940
·333 963	·257 461	·222 414	·193 366	·171 994	·138 489	·098 898
·323 583	·256 603	·222 398	·192 621	·170 971	·138 444	·098 408
·317 275	·256 069	·221 677	·192 578	·170 358	·137 102	·080 840
·313 279	·255 371	·221 340	·192 519	·169 973	·135 931	·063 055
·310 627	·253 547	·221 209	·192 471	·169 758	·132 744	·054 201
·309 207	·252 659	·221 149	·190 377	·169 707	·131 848	
·308 969	·252 478	·220 887	·189 337	·169 369	·130 481	
·306 961	·251 270	·220 296	·189 269	·167 995	·130 332	
·305 968	·249 042	·219 244	·188 763	·167 982	·129 939	
·300 418	·248 347	·218 992	·188 640	·167 031	·127 415	
·298 320	·245 817	·218 488	·187 250	·166 349	·127 205	
·297 917	·244 628	·218 175	·186 388	·165 052	·126 732	
·297 674	·243 629	·218 015	·185 480	·164 243	·124 849	
·293 353	·243 613	·217 070	·184 789	·164 239	·124 810	
·292 560	·243 238	·216 009	·184 667	·162 711	·124 489	
·291 491	·243 118	·215 974	·184 136	·162 266	·123 836	
·290 145	·243 001	·215 844	·183 903	·161 765	·123 812	
·289 985	·242 921	·213 582	·183 355	·161 686	·122 143	
·289 051	·242 213	·212 336	·182 900	·161 603	·120 799	
·288 586	·241 357	·210 517	·182 841	·161 602	·120 244	
·288 480	·240 918	·209 856	·182 334	·161 006	·120 097	
·287 323	·239 461	·209 827	·182 247	·159 840	·118 360	
·284 562	·239 216	·209 801	·181 588	·159 544	·117 135	
·281 469	·237 815	·208 716	·180 964	·159 220	·116 869	
·276 664	·237 075	·206 624	·180 957	·158 757	·116 355	
·276 582	·234 214	·205 565	·180 618	·158 391	·116 274	
·275 174	·234 103	·204 131	·179 411	·156 700	·115 913	
·275 082	·233 776	·202 800	·177 843	·156 316	·115 724	
·272 184	·232 690	·202 653	·177 842	·154 973	·115 238	
·268 507	·232 592	·202 077	·177 014	·154 359	·114 232	
·268 218	·230 464	·202 019	·176 487	·154 290	·114 054	
·267 703	·229 020	·201 639	·176 190	·153 763	·113 263	
·266 022	·228 391	·201 141	·176 085	·153 347	·112 327	
·266 009	·228 355	·199 410	·175 774	·151 797	·112 003	
·261 559	·227 032	·198 342	·175 522	·150 002	·110 667	

(16) We have now reached three approximate formulae for $\mu_2'(\phi_1^2)$, e.g.

$$\mu_2' = \mu_{2(1)} = \frac{f_1}{N} \quad \text{.................................(A)},$$

$$\mu_2' = \frac{1}{N^2}(2\omega + \omega^2) \quad \text{...........................(B)},$$

for the case of no contingency,

and

$$\mu_2' = \frac{f_1}{N} + \frac{f_2}{N^2} \quad \text{.................................(C)}.$$

Of these formulae, the first two are very simple and very convenient for practical application. But the second one is applicable only in the case of no contingency, or in cases where at least the mean square contingency of the population is very small; in such cases, the formula (A) loses its accuracy. Therefore in some cases the third formula becomes very necessary, but this formula is not simple; it is too complicated for practical purposes and, from the practical point of view, it is desirable to use the simple formulae as much as possible.

What are the limits to the application of the formulae (A) and (B)? This is an important question and, although it is difficult to give any general criterion as a guide, the following considerations provide some suggestions on this point.

Let us consider the four examples in Article (15) again, finding the values of $\bar{\phi}_1^2$ and $\sigma_{\phi_1^2}$ by applying the formulae for the first approximations only, and then let us compare the results with those already obtained. We get the results given in Table IX.

TABLE IX.

	$\tilde{\phi}^2$	Range	Mean and S.D.	1st Approximations	2nd Approximations
Ex. (1) 2 × 2-table	·082 498	from 0 to 1	$\bar{\phi}_1^2$ $\sigma_{\phi_1^2}$	·091 758 ·054 58	·091 798 ·056 60
Ex. (2) 3 × 3-table	·154 321	from 0 to 2	$\bar{\phi}_1^2$ $\sigma_{\phi_1^2}$	·157 925 ·026 84	·158 618 ·027 07
Ex. (3) 2 × 2-table	·494 950	from 0 to 1	$\bar{\phi}_1^2$ $\sigma_{\phi_1^2}$	·499 801 ·098 50	·499 801 ·098 15
Ex. (4) 3 × 3-table	·188 893	from 0 to 2	$\bar{\phi}_1^2$ $\sigma_{\phi_1^2}$	·206 398 ·060 85	·206 503 ·065 86

In these examples, although, except in one case, the values $\tilde{\phi}^2$ are rather small as compared with the ranges, the values $\bar{\phi}_1^2$ and $\sigma_{\phi_1^2}$ of the first and second approximations are in good agreement.

Now $$\mu_1' = \frac{1}{N}\psi_1 + \frac{1}{N^2}\psi_2, \quad \text{and} \quad \mu_2' = \frac{1}{N}f_1 + \frac{1}{N^2}f_2;$$

where ψ_1, ψ_2, f_1 and f_2 are all functions of proportional frequencies only, and accordingly may be considered as constants when N, the size of repeated samples, alone changes its value.

In this case it is evident that

$$\mu_1' \to \frac{1}{N}\psi_1 \quad \text{and} \quad \mu_2' \to \frac{1}{N}f_1,$$

as $N \to \infty$.

Therefore, unless the mean square contingency of the sampled population be zero, i.e. if f_1 is not identically zero, we can get good estimates of ϕ_1^2 always from the simple formula (A) by making the size of the samples large enough.

For instance, for a given population, if N satisfies the following condition,

$$f_1 \times 0{\cdot}1 \geqq \frac{1}{N}\,|f_2|,$$

or

$$N \geqq \frac{10\,.\,|f_2|}{f_1}\ (=N_c,\text{ say})\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(75),$$

then the second term f_2/N is numerically less than 10 °/$_\circ$ of the first term f_1, and we can get for many purposes adequate estimates from the first approximation formula.

Let us take once more the four numerical examples above and find the value of N_c, and see also how the second term changes its value as compared with the first term when N increases.

The results are given in Table X.

TABLE X.

	Ex. (1). 2×2-table $\bar{\phi}^2=\cdot082\,498$ Range: 0→1 $f_1=\cdot3066$ $f_2=2\cdot2486$ $N_c=73\cdot340$		Ex. (2). 3×3-table $\bar{\phi}^2=\cdot154\,321$ Range: 0→2 $f_1=\cdot7332$ $f_2=18\cdot19$ $N_c=248\cdot1$		Ex. (3). 2×2-table $\bar{\phi}^2=\cdot494\,950$ Range: 0→1 $f_1=\cdot9702$ $f_2=\cdot4488$ $N_c=4\cdot626$		Ex. (4). 3×3-table $\bar{\phi}^2=\cdot188\,893$ Range: 0→2 $f_1=\cdot8018$ $f_2=25\cdot559$ $N_c=318\cdot8$	
N (size of sample)	$\frac{f_2}{N}$	f_1	$\frac{f_2}{N}$	f_1	$\frac{f_2}{N}$	f_1	$\frac{f_2}{N}$	f_1
$N=$ 50	·0450	·3066	·3638	·7332	·0090	·9702	·5110	·8018
$N=$ 100	·0225	·3066	·1819	·7332	·0045	·9702	·2556	·8018
$N=$ 200	·0112	·3066	·0910	·7332	·0022	·9702	·1278	·8018
$N=$ 500	·0045	·3066	·0364	·7332	·0009	·9702	·0511	·8018
$N=$1000	·0022	·3066	·0182	·7332	·0004	·9702	·0256	·8018
$N=$2000	·0011	·3066	·0091	·7332	·0002	·9702	·0128	·8018
Notes	$\psi_1=\cdot9260$ $\psi_2=\cdot4017$ when $N=100$ $\bar{\phi}_1^2=\cdot091\,798$ $\sigma_{\phi_1^2}=\cdot056\,60$		$\psi_1=3\cdot6044$ $\psi_2=692\cdot60$ when $N=1000$ $\bar{\phi}_1^2=\cdot158\,618$ $\sigma_{\phi_1^2}=\cdot027\,07$		$\psi_1=\cdot4851$ $\psi_2=\cdot0049$ when $N=100$ $\bar{\phi}_1^2=\cdot499\,801$ $\sigma_{\phi_1^2}=\cdot098\,15$		$\psi_1=3\cdot5010$ $\psi_2=4\cdot1930$ when $N=200$ $\bar{\phi}_1^2=\cdot206\,503$ $\sigma_{\phi_1^2}=\cdot065\,86$	

We see from these numerical values that the application of the simple formula (A) is not so very limited; if the size N be large and $\bar{\phi}^2$ be not very small we can get a good estimation of $\sigma_{\phi_1^2}$ from the simple formula (A).

VII. *Distribution of* ϕ_1^2.

(17) From the definition of the mean square contingency ϕ_1^2, it is evident ϕ_1^2 is always positive. And the closest relation between two characters A and B that can be shown in a contingency table consisting of a given number of classes will occur when all the individuals having a given A character fall into a single class for the B character and *vice versa.*

This form of relationship can only occur in a square table, i.e. when

$$\lambda = \kappa \ (= m, \text{ say}),$$

and this closest relationship corresponds to a value of $\phi_1^2 = m - 1$, which is the maximum value of ϕ_1^2 that can occur in a sample from any $m \times m$-table*.

Therefore, we must assume that the mean square contingency ϕ_1^2 can vary from 0 to $m - 1$ according to a uni-modal curve of limited range; thus a Pearson's Type I curve

$$y = y_0 x^{p_1} (b - x)^{p_2}$$

may be expected to give a reasonable approximation to the distribution law of ϕ_1^2.

Now let us assume that the distribution law of ϕ_1^2 is of the form

$$y = y_0 (\phi_1^2)^{p_1} (b - \phi_1^2)^{p_2},$$

then

$$\text{Range } b = \phi_1^2 \,(\text{max.}) = m - 1 \quad \text{......(76)},$$

and by well-known theorems†

$$\bar{\phi}_1^2 = \frac{bp}{r} \quad \text{......(77 a)},$$

$$(\bar{\phi}_1^2)^2 + (\sigma_{\phi_1^2})^2 = \frac{b^2 p (p+1)}{r(r+1)} \quad \text{......(77 b)},$$

and

$$M \text{ (total frequency)} = y_0 b^{(r-1)} \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(r)} \quad \text{......(77 c)},$$

where $p = p_1 + 1$, $q = p_2 + 1$, and $r = p + q$.

From the Equations (77 *a*) and (77 *b*), we can easily deduce

$$r = \frac{\bar{\phi}_1^2 (b - \bar{\phi}_1^2)}{\mu_2} - 1, \qquad p = \frac{r}{b} \cdot \bar{\phi}_1^2, \qquad q = r - p,$$

where

$$\mu_2 = \{\sigma_{\phi_1^2}\}^2 \quad \text{......(78)}.$$

* This case corresponds to the case where the coefficient $C_2 = \sqrt{1 - \frac{1}{m}}$, which is the maximum of C_2 for $m \times m$ classes.

† If $y = y_0 x^{p_1} (b - x)^{p_2}$ be the distribution law of the variate x, then

$$\text{Mean } x = \bar{x} = \frac{1}{M} \int_0^b yx\,dx = \frac{bp}{r} = \frac{b(p_1+1)}{p_1+p_2+2},$$

$$M \text{ (total area)} = \int_0^b y\,dx = y_0 b^{(r-1)} \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(r)},$$

and

$$\frac{1}{M} \int_0^b yx^2\,dx = \frac{b^2 p(p+1)}{r(r+1)} = \mu_2(x) + \bar{x}^2.$$

Thus if we know M, m, $\bar{\phi}_1^2$ and μ_2, i.e. $(\sigma_{\phi_1^2})^2$, we can at once find the values of b, p_1, p_2 and y_0, and we thus obtain the following equations giving the distribution law of ϕ_1^2:

$$y = y_0 (\phi_1^2)^{p_1} (b - \phi_1^2)^{p_2} \quad \text{.........................(79)},$$

where

$$b = m - 1,$$

$$p_1 = \frac{\bar{\phi}_1^2}{b} \left\{ \frac{\bar{\phi}_1^2 (b - \bar{\phi}_1^2)}{\mu_2} - 1 \right\} - 1, \qquad p_2 = \frac{\bar{\phi}_1^2 (b - \bar{\phi}_1^2)}{\mu_2} - p_1 - 3;$$

and

$$y_0 = \frac{M \Gamma (p_1 + p_2 + 2)}{b^{(p_1+p_2+1)} \Gamma (p_1 + 1) \Gamma (p_2 + 1)} \quad \text{.....................(80)}.$$

Let us, for instance, take the population in Table V. In this case evidently

$$m = 2,$$

and, as we have already calculated,

$$\bar{\phi}_1^2 = \cdot 499\,8005, \qquad \mu_2 = \{\sigma_{\phi_1^2}\}^2 = \cdot 009\,6332.$$

Therefore $b = 1$, and from the Equations (80)

$$p_1 = 11 \cdot 471\,0186, \qquad p_2 = 11 \cdot 480\,9744.$$

Accordingly the required distribution law of ϕ_1^2 in this case becomes

$$y = y_0 (\phi_1^2)^{11 \cdot 4710186} (1 - \phi_1^2)^{11 \cdot 4809744} \quad \text{.....................(81)*},$$

very nearly a symmetrical curve.

(18) *Verification of the Distribution Law of* ϕ_1^2.

It is necessary and also interesting to examine the accuracy of the distribution law (79) with the help of our experimental results.

(i) The First Experiment.

From the population in Table V, 804 samples of size 100 were drawn at random, as I mentioned before, and the observed distribution of these 804 ϕ_1^2's, which were also given me by Prof. K. Pearson, is n_s', given in Table XI.

The theoretical frequencies in the third and sixth columns of this table are frequencies in groups, given by Equation (81).

In this case

$$M = 804,$$

and from the Equations (77 *c*) and (81), we get

$$\log y_0 = 10 \cdot 410\,6351,$$

or

$$y_0 = 2 \cdot 574\,157 \times 10^{10}.$$

Now we can find the mid-ordinate y_s of any group from the Equation (81).

* If the frequency function $= y_0 x^{p-1} (b - x)^{q-1}$, then β_2 for this distribution is given by

$$\beta_2 = \frac{3 (r+1) \{2r^2 + pq (r - 6)\}}{pq (r+2) (r+3)},$$

and for the distribution law (81), we have $\beta_2 = 2 \cdot 78535$, which was needed to find the standard error of the standard deviation $\sigma_{\phi_1^2}$ in Article (15).

The theoretical group frequency n_s in the table was calculated from these mid-ordinates, applying the formula

$$n_s = \frac{h}{5760}\{5178y_s + 208(y_{s-1}+y_{s+1}) - 17(y_{s-2}+y_{s+2})\},$$

where h is the class interval.

We can now compare the two sets of frequencies by the test for goodness of fit.

TABLE XI.

ϕ_1^2 (central values)	n_s' (obs. freqs.)	n_s (theor. freqs.)	ϕ_1^2 (central values)	n_s' (obs. freqs.)	n_s (theor. freqs.)
·25	2	2·377	·55	84	56·408
·27	5	4·202	·57	31	50·435
·29	5	6·916			
·31	10	10·686	·59	32	43·376
·33	19	15·594	·61	46	35·827
·35	23·5	21·600	·63	21	28·365
·37	31·5	28·513	·65	19	21·468
·39	27	35·985	·67	23	15·482
·41	50	43·533	·69	5	10·601
·43	51	50·576	·71	6	6·856
·45	54	56·519	·73	5	4·159
·47	76	60·822	·75	2	2·353
·49	63	63·069	·77	2	1·225
·51	66	63·045	·79		·582
·53	44	60·749	·81	1	·247

In each group the frequencies are in good agreement, except in a few groups where the irregularity in the observation arises from the fact that, in sampling, we must have whole numbers only in the cells*.

Grouping together as indicated by brackets to avoid this irregularity, we get

$$\chi^2 = 7{\cdot}329\,931,$$

and consequently $$P = {\cdot}770\,775.$$

From this value of P we can say that the distribution of ϕ_1^2, obtained by the above method, is well satisfied by this experimental result.

* It must be remembered that ϕ_1^2 is *not* really a continuous variate for tables with a limited number of cells; whole numbers only in the cells can be provided by the samples, and these lead to a discontinuity in ϕ_1^2, the more noticeable the smaller the number of cells.

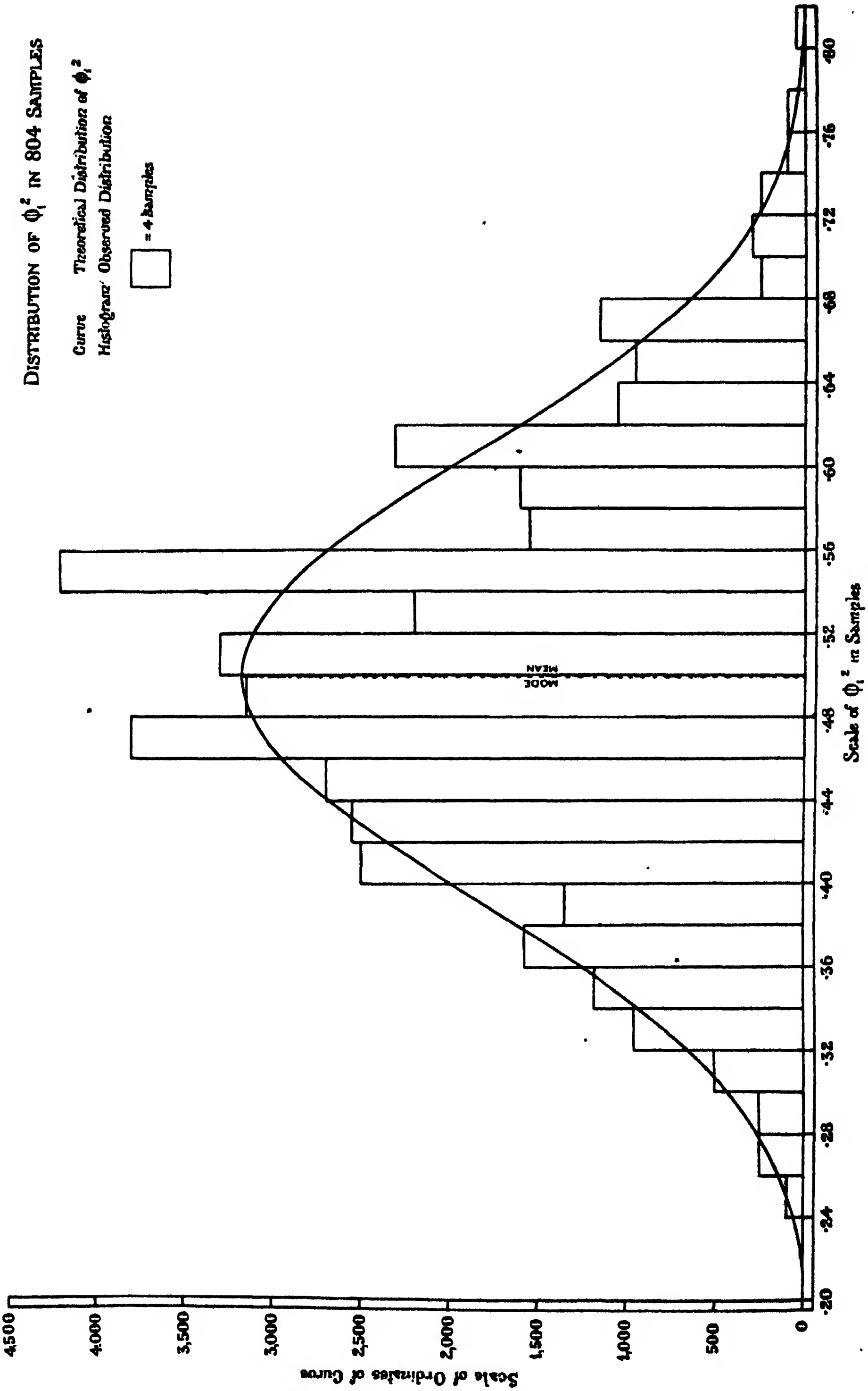

DISTRIBUTION OF ϕ_1^2 IN 804 SAMPLES
Curve Theoretical Distribution of ϕ_1^2
Histogram Observed Distribution
= 4 Samples
MODE
MEAN
Scale of ϕ_1^2 in Samples
·20 ·24 ·28 ·32 ·36 ·40 ·44 ·48 ·52 ·56 ·60 ·64 ·68 ·72 ·76 ·80
Scale of Ordinates of Curve
0 500 1,000 1,500 2,000 2,500 3,000 3,500 4,000 4,500

Fig. 1.

(ii) The Second Experiment.

In this experiment, mentioned in Article (15), for the population in Table VII, I reached the distribution of ϕ_1^2 given in the first column in Table XII.

Now the following theoretical values have been already obtained:

$$\bar{\phi}_1^2 = \cdot 206\,503, \qquad \sigma_{\phi_1^2} = \cdot 065\,863,$$

and further $$m = 3, \qquad M = 250.$$

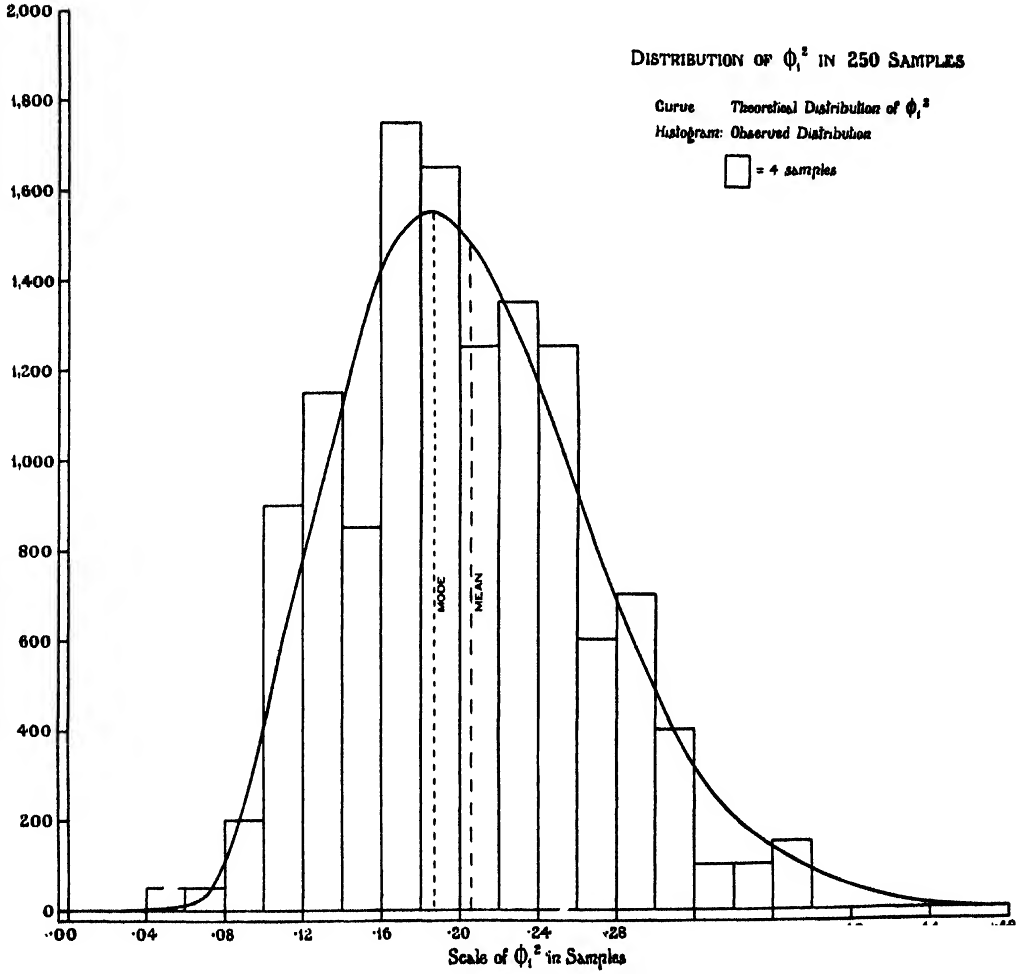


Fig. 2.

From these numerical values and the distribution (79), we get the following equation for the theoretical frequency of ϕ_1^2 for this case,

$$\left.\begin{array}{ll} & y = y_0 (\phi_1^2)^{p_1} (2 - \phi_1^2)^{p_2} \\ \text{where} & p_1 = 7\cdot 712\,036, \quad p_2 = 74\cdot 664\,405 \\ \text{and} & y_0 = 7\cdot 648\,80 \times 10^{10} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(82).$$

The frequencies n_s in the third column of Table XII are the theoretical values obtained from equation (82), and n_s' in the same table gives the corresponding observed frequencies.

We can now compare the theoretical frequencies with the observed ones.

For the same reason as before we group together as indicated by brackets, and testing for goodness of fit, we have

$$\chi^2 = 11{\cdot}9181,$$

and consequently $$P = {\cdot}369\,245.$$

With this value of P, we can say that the distribution (79) of $\phi_1{}^2$ is satisfied again by another experimental result.

TABLE XII.

$\phi_1{}^2$ (central values)	n_s' (obs. freqs.)	n_s (theor. freqs.)
0·45		·228
·43		·418
·41		·743
·39		1·285
·37	3	2·148
·35	2	3·473
·33	2	5·415
·31	8	8·123
·29	14	11·670
·27	12	16·003
·25	25	20·820
·23	27	25·530
·21	25	29·250
·19	33	30·958
·17	35	29·810
·15	17	25·575
·13	23	18·975
·11	18	11·655
·09	4	5·460
·07	1	·735
·05	1	·263
·03		·005

(19) Now, in practice, what we really have are samples, and the population, from which these samples are drawn, is usually unknown. Therefore, it is important to examine what we can find out about a population by the above theory from the individual samples which may be drawn from the population.

We have two sets of samples. One set of 804 samples is drawn at random from the population given in Table V, and the other set of 250 samples is drawn at random from the population given in Table VII.

Suppose we take from each set ten representative samples; this I have done by taking the two theoretical distribution curves of ϕ_1^2 (81) and (82), breaking each of them up into ten strips of almost equal area and choosing from each set of samples, ten samples with a ϕ_1^2 as near the central value of each of these strips as possible*.

The ϕ_1^2's of these representative samples are given in the first and the fifth columns of Table XIII. Substituting the cell and marginal frequencies of these 20 samples in my formulae for μ_1' and μ_2' instead of the population frequencies, I obtained the values of E, the estimate of $\tilde{\phi}_1^2$ from sample, and of $\sigma_{\phi_1^2}$ given in the same table.

TABLE XIII.

The 2 × 2-table				The 3 × 3-table			
True $\phi_1^2=\tilde{\phi}^2=\cdot 494\,950$, Theoretical $\sigma_{\phi_1^2}=\cdot 098\,15$				True $\phi_1^2=\tilde{\phi}^2=\cdot 188\,893$, Theoretical $\sigma_{\phi_1^2}=\cdot 065\,86$			
ϕ_1^2 (sample)	E (estimate from sample)	$\sigma_{\phi_1^2}$ (from sample)	$\lambda=\left(\frac{E-\tilde{\phi}^2}{\sigma_{\phi_1^2}}\right)$	ϕ_1^2 (sample)	E (estimate from sample)	$\sigma_{\phi_1^2}$ (from sample)	$\lambda=\left(\frac{E-\tilde{\phi}^2}{\sigma_{\phi_1^2}}\right)$
·725 556	·723 565	·079 29	2·883	·370 502	·355 666	·064 70	2·578
·600 126	·596 334	·097 63	1·038	·281 469	·265 016	·079 31	·960
·560 022	·556 072	·095 15	·642	·249 042	·231 579	·072 79	·586
·540 000	·536 300	·091 96	·450	·230 464	·213 270	·072 93	·334
·519 592	·515 070	·098 19	·205	·206 624	·190 012	·060 86	·018
·481 668	·477 287	·093 81	− ·188	·195 403	·176 666	·056 34	− ·217
·460 000	·455 038	·097 72	− ·408	·184 136	·165 823	·054 11	− ·426
·440 000	·431 731	·097 70	− ·647	·172 689	·154 755	·058 76	− ·581
·401 296	·395 269	·098 72	−1·010	·150 002	·132 181	·046 98	−1·207
·274 350	·267 254	·088 19	−2·582	·080 840	·061 738	·038 42	−3·310

From these values of E and $\sigma_{\phi_1^2}$, we see that, if the sample deviation in ϕ_1^2 is not large, we can obtain good estimates of the true populations, but the estimates become worse as the sample ϕ_1^2 diverges from the true value $\tilde{\phi}^2$ as we should expect.

Now if we find μ_1' and μ_2' from my formulae by introducing sample values and estimate the true ϕ_1^2, i.e. $\tilde{\phi}^2$ and $\sigma_{\phi_1^2}$ from them, then we might practically use the rough rule that the population ϕ_1^2 "almost certainly" lies within the range

$$E\left(\begin{matrix}\text{sample estimate}\\ \text{of true } \phi_1^2\end{matrix}\right) \pm 3\sigma_{\phi_1^2}\left(\begin{matrix}\text{calculated}\\ \text{from samples}\end{matrix}\right),$$

for we have only one exception in twenty random samples, or 95 °/₀ lie within the range, and indeed "most probably" lie within

$$E \pm 2\sigma_{\phi_1^2},$$

since 80 °/₀ lie within this range.

* In the tail strips, the medians are used instead of their centres, as there are no samples very near the centres.

For this reason I have found the ratio

$$\lambda = \frac{E - \tilde{\phi}^2}{\sigma_{\phi_1^2}},$$

and the results I obtained are given in the fourth and eighth columns of Table XIII, which throw some light on how far this rule would be justified by showing the result of application of this rule to a typical series of samples.

Now if the distribution law of a variate x be known its modal value will be the single value which will most probably occur in a single sample, and if the distribution law is of the form

$$y = y_0 x^{p-1} (b - x)^{q-1},$$

and $\bar{x}$, μ_2 are the mean of x and the second moment coefficient about the mean, then the probable value of x—the modal value of x—is given by

$$b \left\{ \frac{(p+q)\bar{x}}{b} - 1 \right\} \bigg/ \left\{ \frac{(b-\bar{x})\bar{x}}{\mu_2} - 3 \right.$$

The distribution law of ϕ_1^2 is of the above form in my theory and as the probable value of ϕ_1^2 (Φ_1^2, say) is one of the most important estimates of ϕ_1^2, I have found Φ_1^2 from the representative samples and from the population.

The results obtained are given in Table XIV, where $\Phi_1'^2$ is the estimate of Φ_1^2 from samples and the values of the ratio $(\Phi_1'^2 - \Phi_1^2)/\Phi_1^2$ show that we can get adequate estimates of Φ_1^2 from samples, except in the case of a few samples from extreme strips.

TABLE XIV.

The 2×2-table			The 3×3-table		
Theoretical probable value $\Phi_1^2 = \cdot 499\,784$			Theoretical probable value $\Phi_1^2 = \cdot 187\,238$		
ϕ_1^2 (sample)	$\Phi_1'^2$ (from sample)	$\frac{\Phi_1'^2 - \Phi_1^2}{\Phi_1^2}$	ϕ_1^2 (sample)	$\Phi_1'^2$ (from sample)	$\frac{\Phi_1'^2 - \Phi_1^2}{\Phi_1^2}$
·725 556	·714 293	·429	·370 502	·346 240	·849
·600 126	·604 991	·211	·281 469	·244 044	·303
·560 022	·560 694	·122	·249 042	·210 893	·126
·540 000	·539 049	·079	·230 464	·190 350	·017
·519 592	·516 386	·033	·206 624	·171 983	− ·081
·481 668	·475 495	− ·049	·195 403	·159 945	− ·146
·460 000	·480 486	− ·039	·184 136	·149 283	− ·203
·440 000	·425 716	− ·148	·172 689	·133 545	− ·287
·401 296	·385 539	− ·229	·150 002	·116 239	− ·379
·274 350	·246 266	− ·507	·080 840	·022 321	− ·881

Moreover, if we know the distribution law of ϕ_1^2, we should expect more samples in random sampling about the probable value Φ_1^2 than about the mean $\bar{\phi}_1^2$ or the population value $\tilde{\phi}^2$.

To examine this point I counted the numbers of samples found in the two sets of observed samples which lie within the ranges

$$\Phi_1^2 \pm \tfrac{1}{4}\sigma_{\phi_1^2}, \quad \tilde{\phi}^2 \pm \tfrac{1}{4}\sigma_{\phi_1^2} \quad \text{and} \quad \bar{\phi}_1^2 \pm \tfrac{1}{4}\sigma_{\phi_1^2},$$

where $\sigma_{\phi_1^2}$ is the theoretical standard deviation already found, i.e.

$$\sigma_{\phi_1^2} = {\cdot}065\,86 \text{ for the set of 250 samples,}$$

and

$$\sigma_{\phi_1^2} = {\cdot}098\,15 \text{ for the set of 804 samples;}$$

and obtained the following results as might be expected:

	Number of samples within $\Phi_1^2 \pm \frac{1}{4}\sigma_{\phi_1^2}$	Number of samples within $\tilde{\phi}^2 \pm \frac{1}{4}\sigma_{\phi_1^2}$	Number of samples within $\bar{\phi}_1^2 \pm \frac{1}{4}\sigma_{\phi_1^2}$
The set of 250 samples	55	54	46
The set of 804 samples	68	60	68

In the second case

$$\Phi_1^2 = {\cdot}499\,784 \quad \text{and} \quad \bar{\phi}_1^2 = {\cdot}499\,801,$$

and the difference $= {\cdot}000\,017$.

Therefore Φ_1^2 and $\bar{\phi}_1^2$ are almost the same and this accounts for my obtaining the same number of samples about Φ_1^2 and $\bar{\phi}_1^2$ in this case.

If they had not been almost the same we should probably have found more samples about Φ_1^2 than about $\bar{\phi}_1^2$ as in the first case.

[*Remarks.*] (i) Mean ϕ_1^2 in samples = pop. $\phi_1^2 + \mu_1'$.

Therefore in the long run the sample ϕ_1^2 will be larger than the population ϕ_1^2 by a quantity μ_1' and the estimate of population ϕ_1^2, obtained from the sample, which is denoted by E in Table XIII, would be given by

$$E = \text{sample } \phi_1^2 - \mu_1'.$$

(ii) The last sample in Table XIII, drawn from the 3×3-table, illustrates a difficulty which may always arise in a problem of this kind. The observed ϕ_1^2 in this sample is ·080 840; for a 3×3-table with no contingency we have, neglecting terms in $1/N^2$,

$$\text{Mean } \phi_1^2 \text{ in sample} = \frac{(\kappa - 1)(\lambda - 1)}{N} = {\cdot}02,$$

and

$$\sigma_{\phi_1^2} = \frac{\sqrt{2(\kappa - 1)(\lambda - 1)}}{N} = {\cdot}014\,14.$$

Hence we should conclude that the sample could not have come from such a population. If we were, however, to form the possible upper limit of ϕ_1^2 in the population and to say that it could not lie above

$$E + 3\sigma_{\phi_1^2} \text{ (obtained from sample)} = {\cdot}176\,998,$$

we should in fact be in error as the population value of ϕ_1^2 actually $= {\cdot}188\,893$.

The erroneous conclusion arises in this case, and might well arise in other samples from this population, because the standard deviation of ϕ_1^2 calculated from the sample changes very considerably with changes in the sample frequencies. This is partly due to the fact that the ϕ_1^2 in the population lies relatively close to zero.

When we come to the other samples where the population ϕ_1^2 is larger, although the size of sample be less, the $\sigma_{\phi_1^2}$'s calculated from our ten representative samples are much better estimates of the true $\sigma_{\phi_1^2}$'s.

Our estimates will improve (*a*) as the size of the samples increases and (*b*) as ϕ_1^2 increases.

(iii) In the calculations of Tables XIII and XIV, I have used for practical reasons the formulae of the first approximation only. If I had used the formulae of the second approximation, I should have obtained better values for all my estimates.

Lastly it must be noted that the calculation of $\sigma_{\phi_1^2}$ from my formulae (even those of the first approximation), using either the population or the sample frequencies, is a somewhat lengthy business. For a sample from a 2×2-table and by the formulae of the first approximation, I have taken about 45 minutes, and for one from a 3×3-table, a little more than $1\frac{1}{2}$ hours.

But if problems occur when it is of real importance to know the reliability of ϕ_1^2, the result of the labour may be well worth its expenditure, especially if the samples be large.

VIII. *Mean and Standard Deviation of the Coefficient of Mean Square Contingency.*

(20) The coefficient of mean square contingency C_2 is defined by

$$C_2 = \sqrt{\phi_1^2/(1 + \phi_1^2)} \quad \text{.............................(83)}.$$

If we write, for simplicity, as follows:

$$\tilde{f} = \frac{1}{1 + \tilde{\phi}^2} \quad \text{and} \quad \tilde{C}_2 = \sqrt{\tilde{\phi}^2/(1 + \tilde{\phi}^2)},$$

and let δC_2, $\delta \phi_1^2$ be the deviations of C_2, ϕ_1^2 respectively from their population values $\tilde{C}_2$, $\tilde{\phi}^2$, then

$$C_2 = \sqrt{1 - \frac{1}{1 + \tilde{\phi}^2 + \delta\phi_1^2}} = \sqrt{1 - \frac{\tilde{f}}{1 + \tilde{f}\delta\phi_1^2}}$$

$$= 1 + \sum_{m=1}^{\infty} (-1)^m \frac{\rho_m \tilde{f}^m}{(1 + \tilde{f}\delta\phi_1^2)^m} \quad \text{.............................(84)},$$

where ρ_m is the coefficient of the $(m+1)$th term of $(1+x)^{\frac{1}{2}}$,

and $$(1 + \tilde{f}\delta\phi_1^2)^{-m} = 1 - m\tilde{f}\delta\phi_1^2 + \frac{m(m+1)}{2}\tilde{f}^2(\delta\phi_1^2)^2 \text{...........(85)}.$$

The infinite series (84) is convergent, if $|1+\tilde{\phi}^2+\delta\phi_1^2|>1$. For $\tilde{\phi}^2 \neq 0$, this is true if (a) $\delta\phi_1^2 \geqq 0$; or (b) $\delta\phi_1^2<0$, provided $|\delta\phi_1^2|<\tilde{\phi}^2$. For $\tilde{\phi}^2=0$, $\delta\phi_1^2$ is always $\geqq 0$. The infinite series (85) is convergent, provided

$$|\tilde{f}\,\delta\phi_1^2|<1, \quad \text{i.e. } |\delta\phi_1^2|<1+\tilde{\phi}^2.$$

Therefore if $|\delta\phi_1^2|<\tilde{\phi}^2$ or $0<\delta\phi_1^2<1$ when $\tilde{\phi}^2=0$, then both infinite series (84) and (85) are convergent. But these conditions, in the more usual cases, may be assumed quite reasonably.

Now from the equations (84) and (85),

$$C_2=1+\sum_{m=1}^{\infty}(-1)^m\rho_m\tilde{f}^m\left\{1-m\tilde{f}(\delta\phi_1^2)+\frac{m(m+1)}{2}\tilde{f}^2(\delta\phi_1^2)^2-\dots\right\},$$

and

$$\tilde{C}_2=\sqrt{1-\frac{1}{1+\tilde{\phi}^2}}=1+\sum_{m=1}^{\infty}(-1)^m\rho_m\tilde{f}^m.$$

Therefore

$$\begin{aligned}\delta C_2&=C_2-\tilde{C}_2\\&=\sum_{m=1}^{\infty}(-1)^{m+1}m\rho_m\tilde{f}^{(m+1)}(\delta\phi_1^2)\\&\qquad+\sum_{m=1}^{\infty}(-1)^m\frac{m(m+1)}{2}\rho_m\tilde{f}^{(m+2)}(\delta\phi_1^2)^2+\dots\\&=k_1(\delta\phi_1^2)+k_2(\delta\phi_1^2)^2+\dots \qquad (86),\end{aligned}$$

where

$$k_1=\sum_{m=1}^{\infty}(-1)^{m+1}m\rho_m\tilde{f}^{(m+1)},$$

and

$$k_2=\sum_{m=1}^{\infty}(-1)^m\frac{m(m+1)}{2}\rho_m\tilde{f}^{(m+2)}.$$

Therefore, we have

$$\delta C_2=k_1(\delta\phi_1^2)+k_2(\delta\phi_1^2)^2 \text{ (approximately)} \qquad (87),$$

and thus

$$\begin{aligned}\mu_1'(C_2)&=\text{Mean }\delta C_2\\&=k_1\mu_1'(\phi_1^2)+k_2\mu_2'(\phi_1^2)\text{ (approximately)} \qquad (88\,a),\\ \mu_2'(C_2)&=\text{Mean }(\delta C_2)^2\\&=k_1^2\mu_2'(\phi_1^2)\text{ (approximately)} \qquad (88\,b).\end{aligned}$$

Now

$$(1-x)^{\frac{1}{2}}=1+\sum_{m=1}^{\infty}(-1)^m\rho_m x^m \qquad (89).$$

Differentiating (89) in respect to x, and after that multiplying by $-x^2$, we have

$$\frac{x^2}{2\sqrt{1-x}}=\sum_{m=1}^{\infty}(-1)^{m+1}m\rho_m x^{(m+1)}.$$

Therefore

$$\begin{aligned}k_1&=\sum_{m=1}^{\infty}(-1)^{m+1}m\rho_m\tilde{f}^{(m+1)}=\frac{\tilde{f}^2}{2\sqrt{1-\tilde{f}}}\\&=\frac{1}{2\tilde{C}_2(1+\tilde{\phi}^2)^2} \qquad (90).\end{aligned}$$

Similarly $$k_2 = -\frac{1+3\tilde{C}_2^2}{8\tilde{C}_2^3(1+\tilde{\phi}^2)^3} \quad\text{.............................(91).}$$

From (88), (90) and (91),

$$\text{Mean } C_2 = \tilde{C}_2 + \frac{\mu_1'(\phi_1^2)}{2\tilde{C}_2(1+\tilde{\phi}^2)^2} - \frac{(1+3\tilde{C}_2^2)\,\mu_2'(\phi_1^2)}{8\tilde{C}_2^3(1+\tilde{\phi}^2)^3} \text{ (approximately) ...(92),}$$

and $$\sigma_{C_2} = \frac{\sigma_{\phi_1^2}}{2\tilde{C}_2(1+\tilde{\phi}^2)^3} \text{ in rough approximation ....(93).}$$

But we have already found that

$$\mu_1'(\phi_1^2) = \frac{\psi_1}{N} + \frac{\psi_2}{N^2}, \qquad \mu_2'(\phi_1^2) = \frac{f_1}{N} + \frac{f_2}{N^2},$$

therefore we can find the theoretical values of $\bar{C}_2$ and σ_{C_2} approximately, and the following results are obtained:

$$\bar{C}_2 = \tilde{C}_2 + \frac{1}{N}(k_1\psi_1 + k_2 f_1) + \frac{1}{N^2}(k_1\psi_2 + k_2 f_2),$$

and $$\sigma_{C_2} = k_1\sqrt{\mu_2' - (\mu_1')^2} = k_1\sigma_{\phi_1^2}{}^* \quad\text{....................(94).}$$

For instance, let us consider again my experiment of sampling from the population in Table VII, Article (15).

In this case $\tilde{\phi}^2 = \cdot188\,893$, and

$$\mu_1'(\phi_1^2) = \cdot017\,610, \qquad \mu_2'(\phi_1^2) = \cdot004\,648,$$

as already found.

Therefore from the equations (90) and (91),

$$k_1 = \cdot887\,459, \qquad k_2 = -1\cdot734\,399,$$

and finally from (92) and (93) we get

$$\bar{C}_2 = \cdot406\,166, \text{ and } \sigma_{C_2} = \cdot058\,451.$$

Now for this population we can get 250 observed values of C_2 from the 250 values of ϕ_1^2, obtained in my experiment.

I calculated these values of C_2 and found the observed values of $\bar{C}_2$ and σ_{C_2}.

The following results were obtained:

$$\text{Mean } C_2 = \cdot4028 \text{ (its S.E.} = \cdot003\,697)\dagger,$$

and $$\sigma_{C_2} = \cdot0528 \text{ (its S.E.} = \cdot002\,536)\dagger,$$

while their theoretical values, given by my formulae, are

$$C_2 = \cdot406\,166, \qquad \sigma_{C_2} = \cdot058\,451,$$

and therefore we can say that they are in fair agreement.

* Thus $\sigma_{C_2} = \frac{\sigma_{\phi_1^2}}{2\tilde{C}_2(1+\tilde{\phi}^2)^2} = \frac{\sigma_{\phi_1}}{(1+\tilde{\phi}^2)^{\frac{3}{2}}}$ to a first approximation, the value given by Blakeman and Pearson, *Biometrika*, Vol. v. p. 191, eqn. (xxxv).

† These standard errors are obtained in the same manner as indicated in the footnote to Article (15), p. 410.

(21) **Finally let us consider the distribution of C_2.**

It is well known that C_2 is always positive and ranges from 0 to $\sqrt{1-\frac{1}{m}}$, where m is the number of categories, the latter value resulting when there is as close a relation as is possible between the variates.

Therefore let us assume in the same way as in Article (17) that a Pearson's Type I curve will give us approximately the theoretical distribution of C_2; also by the same method of deduction as on p. 415 we get the following equation as the frequency distribution of C_2:

$$y = y_0 C_2^{p_1} (b - C_2)^{p_2},$$

where
$$b = \sqrt{1-\frac{1}{m}}, \qquad p_1 = \frac{\bar{C}_2}{b}\left\{\frac{\bar{C}_2(b-\bar{C}_2)}{\sigma_{C_2}^2} - 1\right\} - 1,$$

$$p_2 = \frac{\bar{C}_2(b-\bar{C}_2)}{\sigma_{C_2}^2} - p_1 - 3,$$

and
$$y_0 = \frac{M\Gamma(p_1+p_2+2)}{b^{(p_1+p_2+1)}\Gamma(p_1+1)\Gamma(p_2+1)} \quad \text{.....................(95)}.$$

For instance, if we consider the distribution of the 250 values of C_2, given in Table VIII on p. 412,

$$\bar{C}_2 = \cdot 4062, \qquad \sigma_{C_2} = \cdot 0585, \qquad m = 3, \quad \text{and} \quad M = 250,$$

and from the equation (95) we get the following equation as the frequency distribution of C_2 for this case:

$$y = y_0 C_2^{22\cdot 7653} (\cdot 8165 - C_2)^{23\cdot 0090},$$

where
$$y_0 = 4\cdot 33150 \times 10^{20} \text{.....................................(96)}.$$

TABLE XV.

C_2 (central values)	Observed frequencies of C_2	Theoretical frequencies of C_2	C_2 (central values)	Observed frequencies of C_2	Theoretical frequencies of C_2
·17		·003	·41	28	33·370
·19		·019	·43	36	30·930
·21		·087			
·23	1	·313			
·25	1	·925	·45	31	25·682
·27	1	2·304	·47	20	19·054
·29	1	4·928			
·31	10	9·166			
			·49	9	12·578
			·51	6	7·342
·33	25	14·984	·53		3·758
·35	10	21·697	·55		1·668
			·57		·633
			·59		·202
·37	29	27·987	·61		·053
·39	42	32·282			·012

The theoretical frequencies in the third column of Table XV are provided by the above equation.

I examined the distribution of the 250 observed values of C_2 and the results obtained are given in the second and fifth columns of the same table and in the diagram below, Fig. 3.

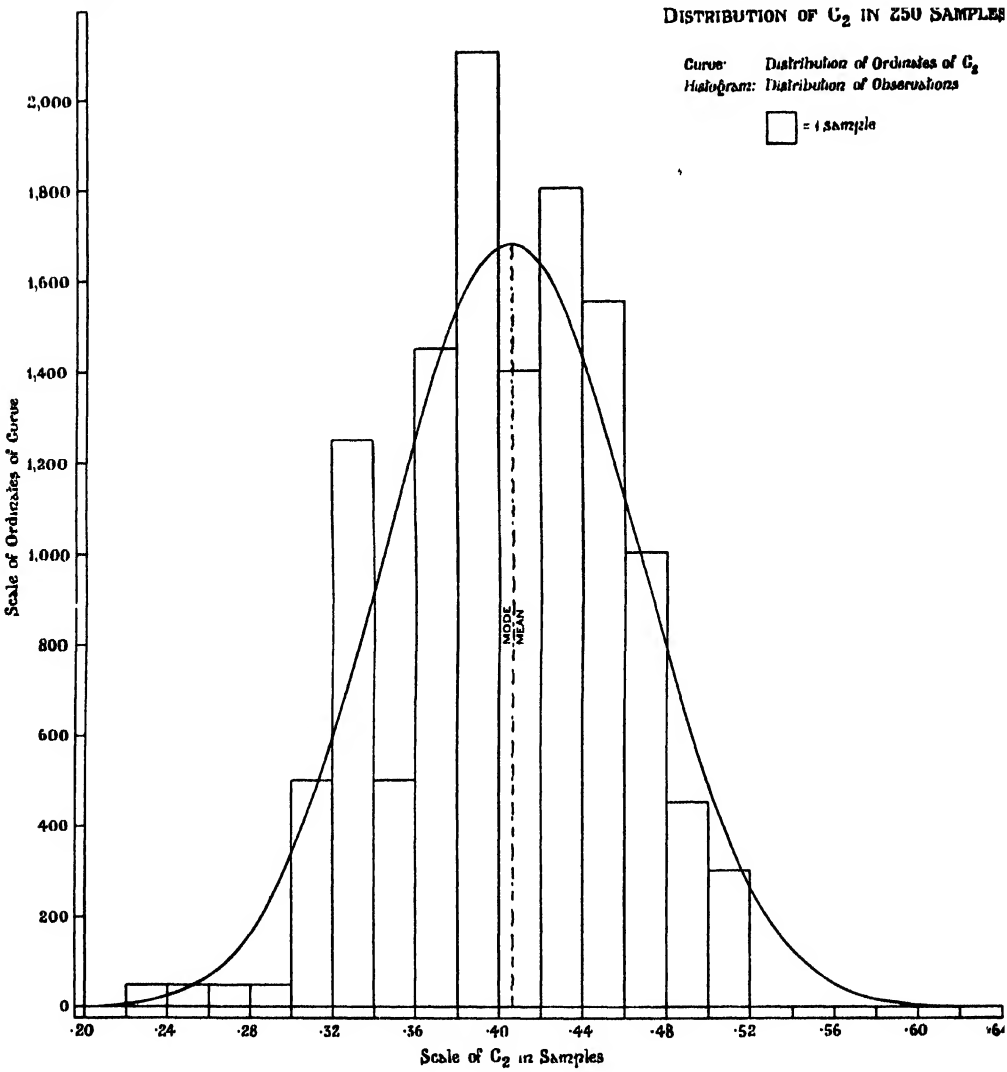


Fig. 3.

Grouping the frequencies together as indicated by brackets in Table XV for the same reason as in Article (18) (i), we obtain by applying the χ^2-test,

$$\chi^2 = 8{\cdot}4764,$$

and consequently $$P = {\cdot}133\,763.$$

This value of P is not so good as in the case of ϕ_1^2 discussed above, but it is reasonable, and we can say that the distribution law (95) for the coefficient of mean square contingency C_2 is not contradicted by this experimental result.

MISCELLANEA.

Note on Variability in Girls and Boys (Glasgow) for Height and Weight.

BY ETHEL M. ELDERTON.

IN a memoir published in Vol. x. (pp. 288—339) of this Journal, I dealt with the regression curves of weight and height on age of a very large number of Glasgow children. These children were grouped in four classes of schools, A, B, C, D, according to the economic conditions of the district. In that paper although they were necessarily computed I did not publish the standard deviations for each age-group, as I was seeking only the change in height and weight with age. My numbers were so large that it was possible to obtain standard deviations with a relatively small probable error. I have since found several investigators inquiring for reliable measures of the variability in stature and weight of boys and girls. Accordingly I give here two Tables. The first provides the Means, Standard Deviations, and Coefficients of Variation of the entire population of children not divided into the groups A, B, C and D. The second gives the Means, Standard Deviations, and Regression Coefficients of Weight on Height for school ages of the four grades. The ages are central ages.

TABLE I.

Means, Standard Deviations, and Coefficients of Variation of Glasgow Girls and Boys at various ages for all Schools combined.

GIRLS

No. of cases and age	Means		Standard Deviations		Coefficients of Variation	
	Height	Weight	Height	Weight	Height	Weight
5 (894)	40·188	38·129	2·4151	4·2959	6·01 ± ·10	11·27 ± ·18
6 (3104)	41·727	40·570	2·5472	4·6896	6·10 ± ·05	11·56 ± ·10
7 (3828)	43·538	43·921	2·6000	5·1844	5·97 ± ·05	11·80 ± ·09
8 (3928)	45·375	47·519	2·6694	5·7708	5·88 ± ·04	12·15 ± ·09
9 (3819)	47·315	51·792	2·7978	6·5072	5·91 ± ·05	12·56 ± ·10
10 (3762)	49·163	55·968	2·8630	7·3037	5·82 ± ·05	13·05 ± ·10
11 (3518)	50·973	61·151	2·9822	8·2768	5·85 ± ·05	13·54 ± ·11
12 (3658)	53·019	67·114	3·1152	10·1768	5·88 ± ·05	15·16 ± ·12
13 (3225)	55·211	74·778	3·3046	12·0868	5·99 ± ·05	16·16 ± ·14
14 (1229)	57·261	82·757	3·1098	13·6973	5·43 ± ·07	16·55 ± ·23

BOYS

No. of cases and age	Means Height	Means Weight	S.D. Height	S.D. Weight	C.V. Height	C.V. Weight
5 (990)	40·126	38·829	2·4349	4·3547	6·07 ± ·09	11·22 ± ·17
6 (3322)	41·937	41·775	2·4364	4·7906	5·81 ± ·05	11·47 ± ·10
7 (3903)	43·726	45·274	2·6294	5·3286	6·01 ± ·05	11·77 ± ·09
8 (4200)	45·784	49·275	2·8832	5·8846	6·30 ± ·05	11·94 ± ·09
9 (4017)	47·680	53·696	2·8154	6·4161	5·90 ± ·04	11·95 ± ·09
10 (3881)	49·526	58·371	2·8226	7·0398	5·70 ± ·04	12·06 ± ·09
11 (3761)	51·208	63·020	2·8504	7·7292	5·57 ± ·04	12·26 ± ·10
12 (3632)	52·922	68·199	2·9412	8·7298	5·56 ± ·04	12·80 ± ·10
13 (3638)	54·552	73·569	3·2333	10·2444	5·93 ± ·05	13·92 ± ·11
14 (1467)	56·297	79·180	3·3144	12·3372	5·89 ± ·07	15·58 ± ·20

TABLE II. *Means, Standard Deviations, and Regression Coefficients of Weight on Height of Glasgow Girls and Boys at various ages in four grades of Schools.*

GIRLS

Age	Schools A					Schools B				
	Mean		Standard Deviations		Regression	Mean		Standard Deviations		Regression
	Height	Weight	Height	Weight		Height	Weight	Height	Weight	
6	41·016	41·949	2·6159	4·7425	1·329	42·038	40·556	2·5039	4·5309	1·345
7	42·872	43·045	2·5987	5·0780	1·465	43·743	43·934	2·6459	5·2103	1·556
8	44·643	46·377	2·8054	5·6879	1·503	45·601	47·721	2·6033	5·7693	1·718
9	46·606	50·495	2·7964	6·2936	1·730	47·352	51·844	2·8415	6·5804	1·709
10	48·548	54·695	2·9703	7·1460	1·878	49·162	55·771	2·8622	7·1783	1·925
11	50·252	59·486	2·8855	8·0625	2·153	51·054	60·839	3·0620	8·2168	2·034
12	52·368	65·311	3·1165	9·7898	2·360	53·017	66·779	3·1763	10·0822	2·465
13	54·412	72·430	3·4658	11·8328	2·694	55·243	74·316	3·1901	11·8143	2·939
14	55·846	76·765	3·0899	12·4446	3·084	57·062	81·326	3·0074	12·6811	2·906

BOYS

Age	Schools A					Schools B				
	Mean		Standard Deviations		Regression	Mean		Standard Deviations		Regression
	Height	Weight	Height	Weight		Height	Weight	Height	Weight	
6	41·283	40·884	2·4988	4·8692	1·503	42·139	41·981	2·4539	4·7634	1·532
7	42·951	44·178	2·6881	5·2635	1·519	43·976	45·593	2·6300	5·3651	1·635
8	45·066	47·993	2·9152	5·7188	1·495	45·885	49·648	2·7796	5·9046	1·695
9	47·037	52·253	2·9094	6·3297	1·635	47·680	53·856	2·6538	6·4067	1·818
10	48·815	56·716	2·9170	6·7429	1·768	49·542	58·384	2·7774	7·0686	1·899
11	50·598	61·567	2·8865	7·5099	2·302	51·072	62·674	2·8096	7·4417	2·005
12	52·284	66·401	2·8980	8·3221	2·217	52·786	67·808	2·8698	8·7074	2·476
13	53·849	71·686	2·9911	9·6112	2·547	54·341	72·934	3·1165	9·8383	2·511
14	55·186	75·593	3·1839	11·1960	2·888	55·498	77·258	3·0905	10·7314	2·775

GIRLS

Age	Schools C					Schools D				
	Mean		Standard Deviations		Regression	Mean		Standard Deviations		Regression
	Height	Weight	Height	Weight		Height	Weight	Height	Weight	
6	41·943	41·326	2·2479	4·4488	1·551	42·712	41·801	2·1091	4·4710	1·624
7	43·740	44·740	2·3389	5·2686	1·693	44·777	45·626	2·0998	4·8120	1·694
8	45·641	48·066	2·4040	5·6187	1·691	46·428	49·311	2·3041	5·7138	1·927
9	47·636	52·694	2·5368	6·4095	1·917	48·583	54·286	2·4677	6·5745	2·045
10	49·411	56·907	2·6594	7·1487	2·088	50·398	58·789	2·3581	7·2193	2·397
11	51·190	61·933	2·6826	7·8814	2·209	52·214	64·375	2·6005	8·3187	2·339
12	53·261	68·353	2·8113	9·9720	2·735	54·146	70·505	2·7977	10·3370	2·859
13	55·362	76·147	3·2003	12·0201	2·892	56·475	78·764	2·8643	11·7571	3·229
14	57·032	83·042	2·8995	12·7213	3·317	58·679	89·018	2·8011	13·5880	3·804

BOYS

Age	Schools C					Schools D				
	Mean		Standard Deviations		Regression	Mean		Standard Deviations		Regression
	Height	Weight	Height	Weight		Height	Weight	Height	Weight	
6	42·125	42·532	2·1714	4·5626	1·591	43·031	43·301	2·0419	4·5533	1·603
7	43·968	45·900	2·2944	5·0110	1·667	44·809	46·642	2·1620	4·9927	1·818
8	46·223	50·076	3·0231	5·7679	1·227	46·893	51·161	2·3488	5·4587	1·790
9	48·050	54·383	2·7165	6·2327	1·562	49·009	56·254	2·4549	6·3646	1·883
10	49·872	59·545	2·4867	6·6832	2·055	50·856	61·163	2·3787	6·8466	2·218
11	51·507	63·919	2·7007	7·4161	2·088	52·605	66·268	2·4318	7·7596	2·546
12	53·256	69·073	2·8880	8·9508	2·337	54·244	70·848	2·7103	8·6799	2·450
13	54·996	75·617	3·0704	10·9614	2·854	55·974	76·885	2·7154	10·5429	3·160
14	57·183	82·241	3·1467	12·6626	3·251	57·710	83·214	3·0595	11·3919	3·633

Note on a Paper published in *Biometrika*, Vol. XIX.

BY J. O. IRWIN, M.A., M.Sc.

IN my paper, *Biometrika*, Vol. XIX. p. 225, "On the Frequency Distribution of the Means of Samples from a Population having any Law of Frequency with Finite Moments, with special reference to Pearson's Type II," I had occasion to evaluate the integral

$$w(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin^n\theta}{\theta^n}\cos 2x\theta\,d\theta.$$

The result obtained was correct, but there is an error in the demonstration which requires pointing out. I obtained the identities

$$n \text{ even}=2r. \quad w(x)=\frac{(-1)^r}{2^{2r}\pi}\int_{-\infty}^{\infty}\left[\sum_{s=0}^{r-1}(-1)^s\,{}^{2r}C_s\{\cos 2(r-s+x)\theta+\cos 2(r-s-x)\theta\}\right.$$
$$\left.+(-1)^r\,{}^{2r}C_r\cos 2x\theta\right]\frac{d\theta}{\theta^{2r}} \quad \text{.....................................(1)},$$

$$n \text{ odd}=2r-1. \quad w(x)=\frac{(-1)^r}{2^{2r-1}\pi}\int_{-\infty}^{\infty}\left[\sum_{s=0}^{r-1}(-1)^s\,{}^{2r-1}C_s\{\sin[2(r-s+x)-1]\theta\right.$$
$$\left.+\sin[2(r-s-x)-1]\theta\}\right]\frac{d\theta}{\theta^{2r-1}} \quad \text{.........................(1) } bis.$$

I then asserted that we had to evaluate a number of integrals of the types

$$(1)\quad \int_{-\infty}^{\infty}\frac{\cos 2k\theta}{\theta^{2r}}\,d\theta. \qquad (2)\quad \int_{-\infty}^{\infty}\frac{\sin(2k-1)\theta}{\theta^{2r-1}}\,d\theta \quad \text{....................(2)},$$

and by integrating each of these by parts I obtained the results

$$\int_{-\infty}^{\infty}\frac{\cos 2k\theta}{\theta^{2r}}\,d\theta=\pm\frac{(-1)^r(2k)^{2r-1}}{(2r-1)!} \text{ according as } 2k \gtrless 0,$$

and $$\int_{-\infty}^{\infty}\frac{\sin(2k-1)\theta}{\theta^{2r-1}}\,d\theta=\pm\frac{(-1)^{r-1}(2k-1)^{2r-2}}{(2r-2)!} \text{ according as } 2k-1 \gtrless 0 \quad \text{............(3)}.$$

Now owing to a pole at $\theta=0$, each of the integrals (2) is divergent and the results (3) are incorrect, the true results being in fact infinite.

The proof may be made rigorous as follows:

The integral $$w(x)=2\lim_{\epsilon\to 0}\int_{\epsilon}^{\infty}\left(\frac{\sin\theta}{\theta}\right)^n\cos 2x\theta\,d\theta.$$

Now the integrand of $w(x)$ contains no pole at $\theta=0$, therefore its Laurent expansion in positive and negative powers of ϵ (if it has one) will be such that the coefficients of the negative powers of ϵ all vanish.

By making use of the relations

$$\int_{\epsilon}^{\infty}\frac{\cos 2k\theta}{\theta^{2r}}\,d\theta=\sum_{p=0}^{2r-2}\frac{(2k)^p\cos\left(2k\epsilon+\frac{p\pi}{2}\right)}{(2r-1)\ldots(2r-p-1)\,\epsilon^{2r-p-1}}+\frac{(-1)^r(2k)^{2r-1}}{(2r-1)!}\int_{\epsilon}^{\infty}\frac{\sin 2k\theta}{\theta}\,d\theta,$$

$$\int_{\epsilon}^{\infty}\frac{\sin(2k-1)\theta}{\theta^{2r-1}}\,d\theta=\sum_{p=0}^{2r-3}\frac{(2k-1)^p\sin\left((2k-1)\epsilon+\frac{p\pi}{2}\right)}{(2r-2)\ldots(2r-p-2)\,\epsilon^{2r-p-2}}+\frac{(-1)^{r-1}(2k-1)^{2r-2}}{(2r-2)!}\int_{\epsilon}^{\infty}\frac{\sin(2k-1)\theta}{\theta}\,d\theta \quad \text{......(4)},$$

we can obtain $$\int_{\epsilon}^{\infty}f(\theta)\,d\theta \quad\text{and}\quad \int_{\epsilon}^{\infty}\phi(\theta)\,d\theta,$$

where f and ϕ are the integrands of (1) and (1) *bis*.

By making use of the expressions

$$w_\epsilon(x) = \int_\epsilon^\infty f(\theta)\,d\theta, \quad n \text{ even},$$

and

$$= \int_\epsilon^\infty \phi(\theta)\,d\theta, \quad n \text{ odd},$$

and expanding all the sines and cosines and the integrals in powers of ϵ, we obtain the Laurent expansion for $w_\epsilon(x)$. As we have seen, the coefficients of all the negative powers of ϵ must vanish; further no contribution is made to the term independent of ϵ by the series in (4) (because each series consists of cosines divided by odd powers of ϵ and sines divided by even powers of ϵ). Thus these series only contribute to the positive powers of ϵ in the Laurent expansion, and these vanish in the limit. Thus only the integrals on the right-hand side of (4) need be taken into account in evaluating

$$w(x) = 2 \lim_{\epsilon \to 0} w_\epsilon(x),$$

and these terms were the ones taken into account in my paper. Hence the final result given there is correct.

The following misprints in the paper may be pointed out:

p. 234, line 12 for $\frac{1}{2}(r)^{r\,2r}C_r$ read $\frac{1}{2}(-1)^{r\,2r}C_r$; line 15 should read

$$\sin^{2r}\theta \cos 2x\theta = \frac{(-1)^r}{2^{2r-1}} \sum_{s=0}^{r-1} \{\tfrac{1}{2}(-1)^{s\,2r}C_s[\cos 2(r-s+x)\theta + \cos 2(r-s+x)\theta]\} + \frac{1}{2^{2r}}\,{}^{2r}C_r \cos 2x\theta.$$

p. 235, line 6, for $\frac{d\theta}{\theta^r}$ read $\frac{d\theta}{\theta^{2r}}$.

p. 236, line 5, for $\sin[2(r-s+x-1)-1]\theta$ read $\sin[2(r-s+x)-1]\theta$.

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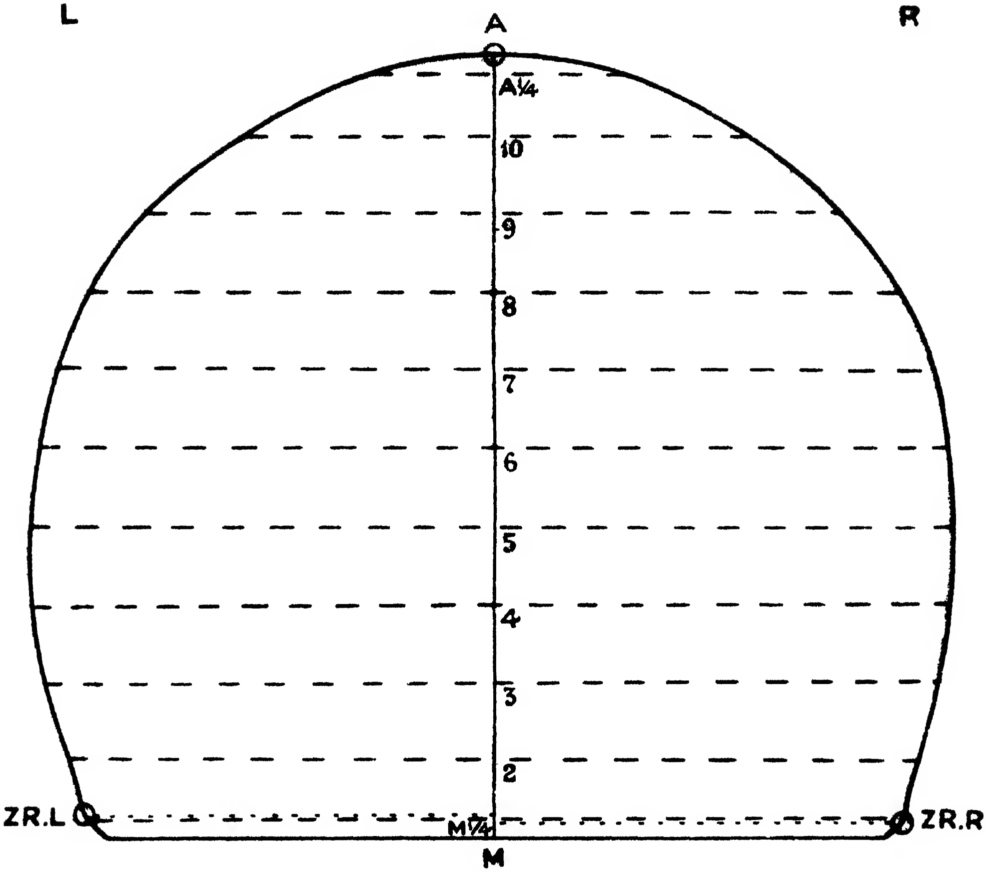


FIG. I. Transverse Type Contour, based on 37 ♂ Basque Skulls.

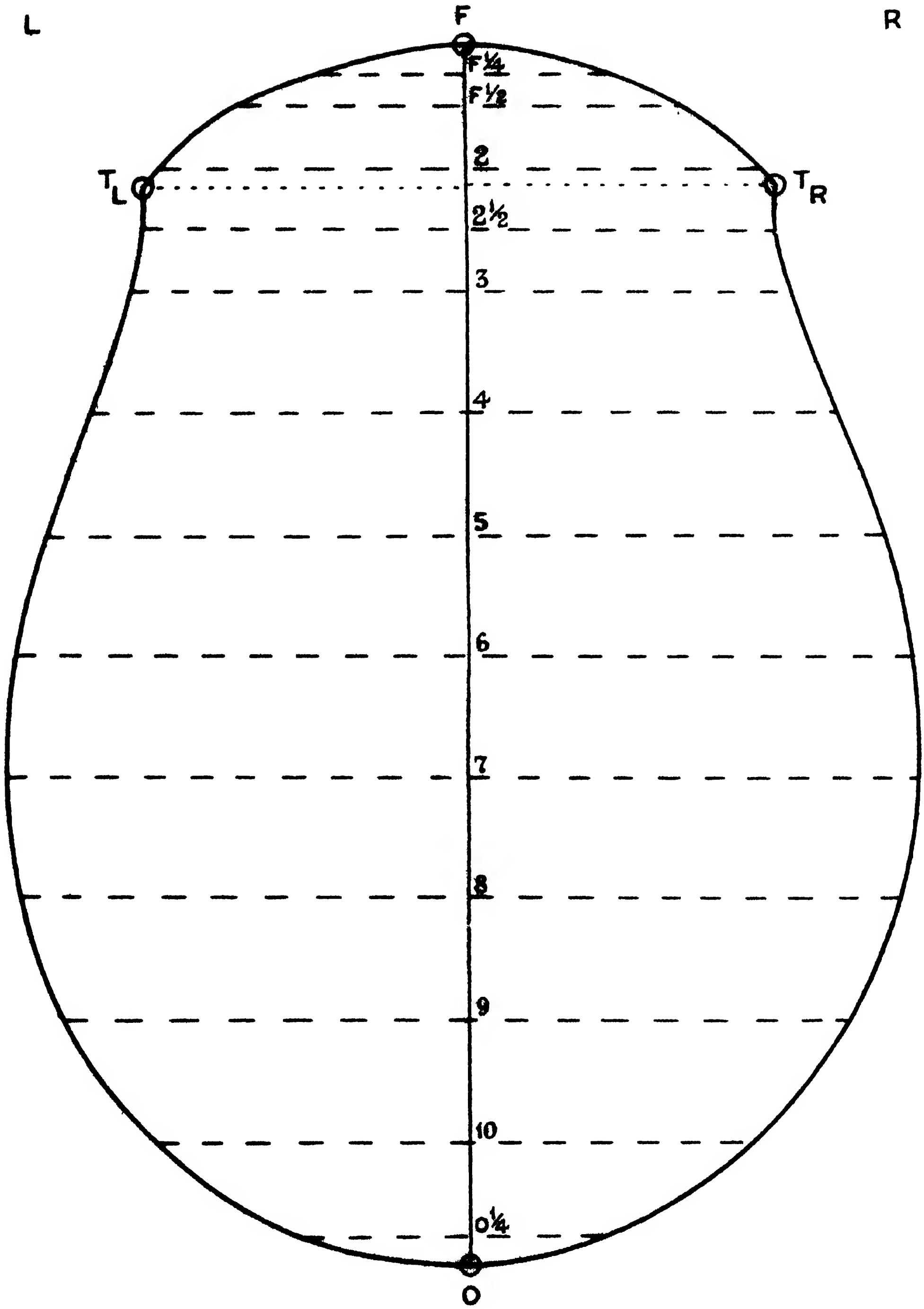
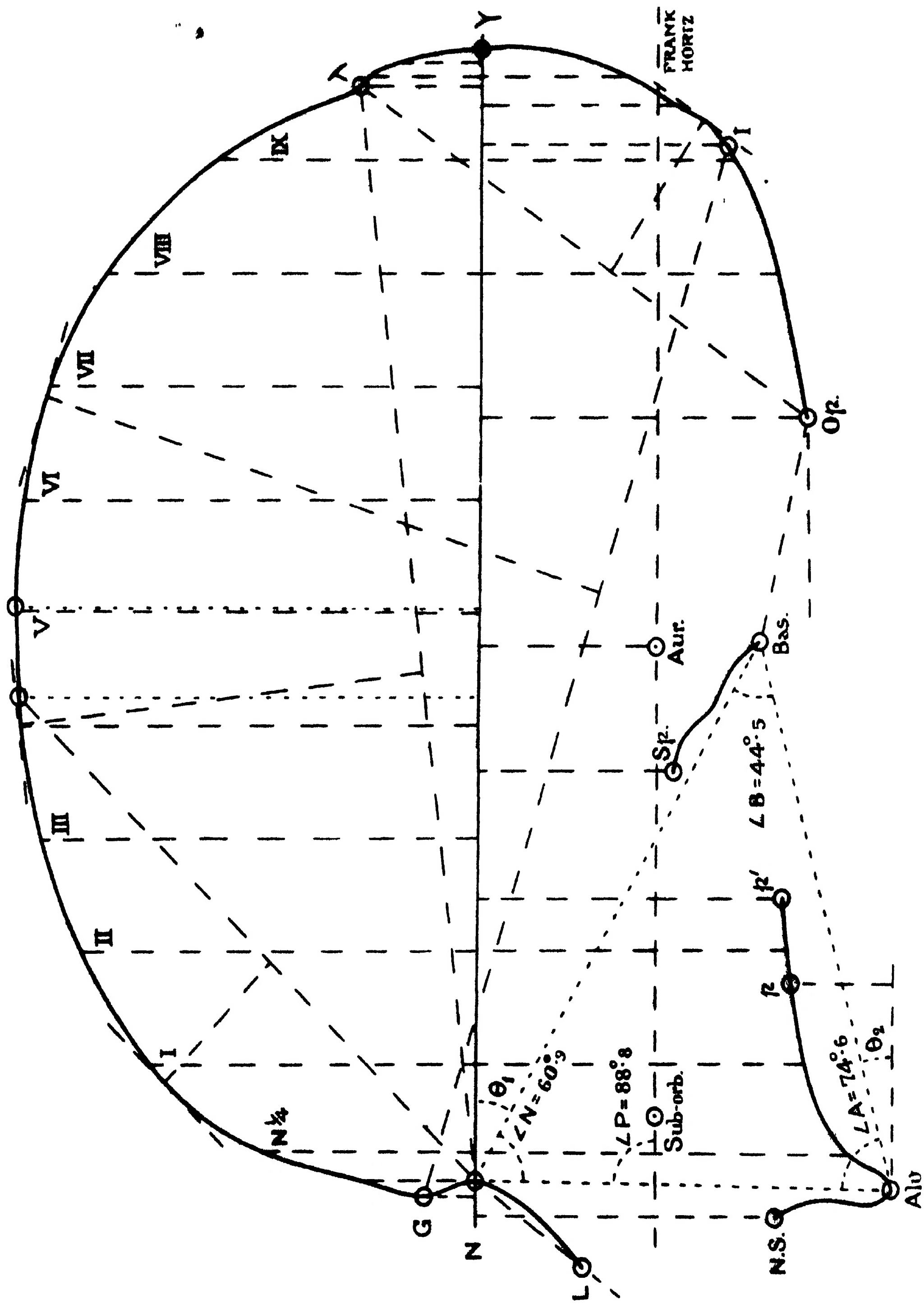


FIG. II. Horizontal Type Contour, based on 37 ♂ Basque Skulls.



FRANK HORIZ
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∠P = 88°·8
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